Secure the Bay of Biscay with Robots

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Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1, \ldots, \mathscr{R}_n$ at positions $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are moving in a 2D or 3D world.
- If the intruder is in the visibility zone of one robot, it is detected.
- The robots collaborate to guarantee that they is no moving intruder inside a subzone of $\mathbb{G}$.

Complementary approach

- We assume that there exists a virtual intruder with a limited speed inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies a state inclusion

$$\frac{d\mathbf{x}}{dt}(t) \in \mathbb{F}(\mathbf{x}(t)).$$

- Each robot $\mathscr{R}_i$ has a visibility zone of the form $g_{\mathbf{a}_i}^{-1}([0, d_i])$ where d_i is the scope.

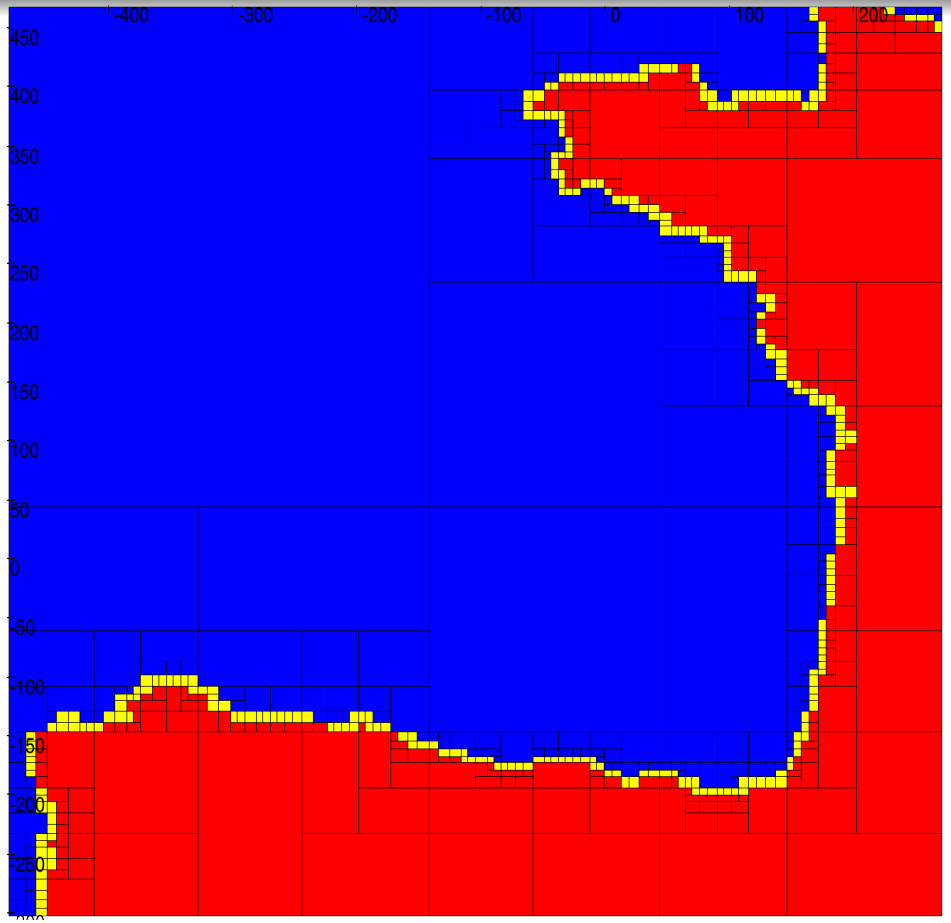
Theorem. The intruder has a state vector $\mathbf{x}(t)$ inside the set

$$\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

$$\mathbb{S}(t) = \overline{proj_{world}(\mathbb{X}(t))}.$$

Set $\mathbb{G}$ in white


-400
-300
-200
-100
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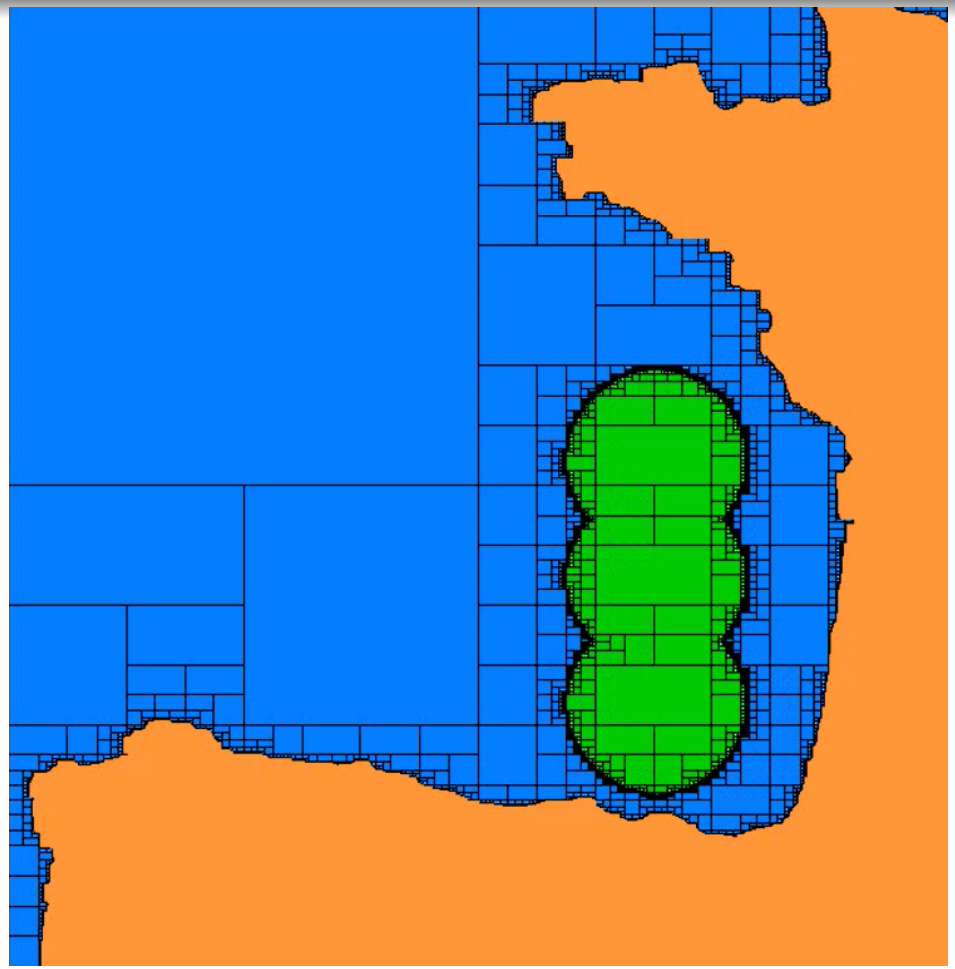
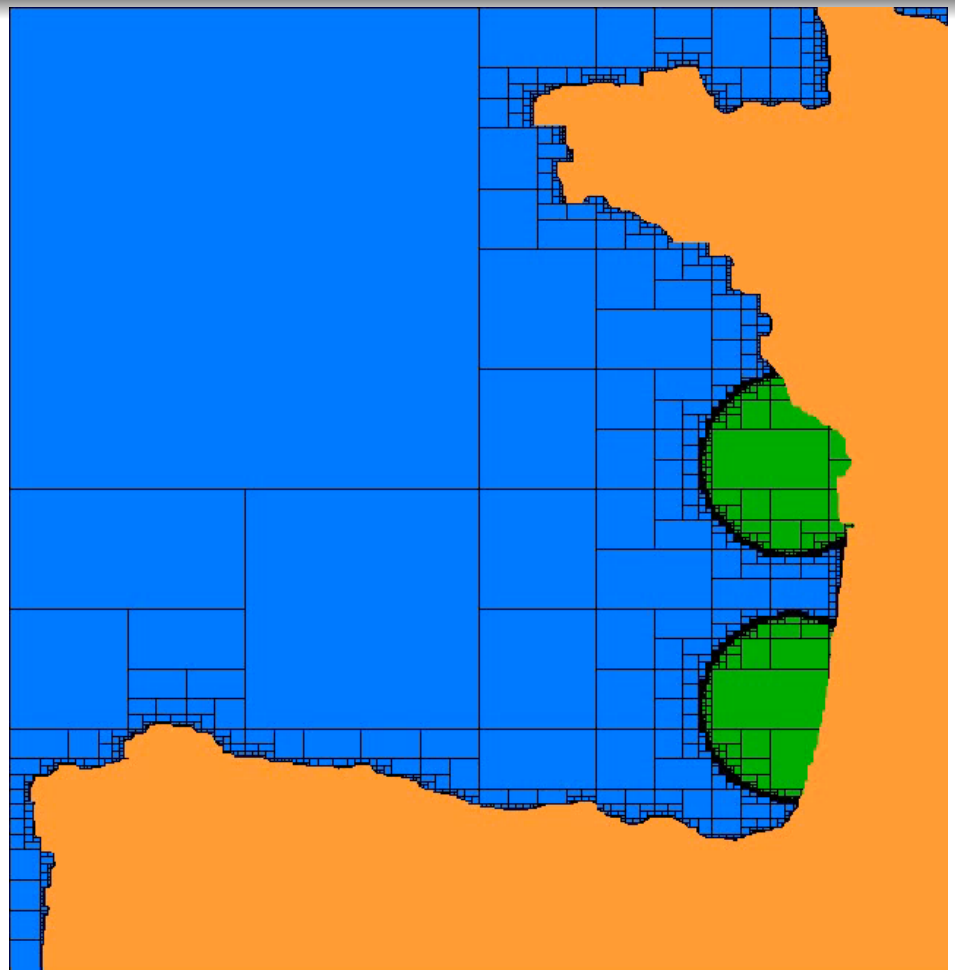
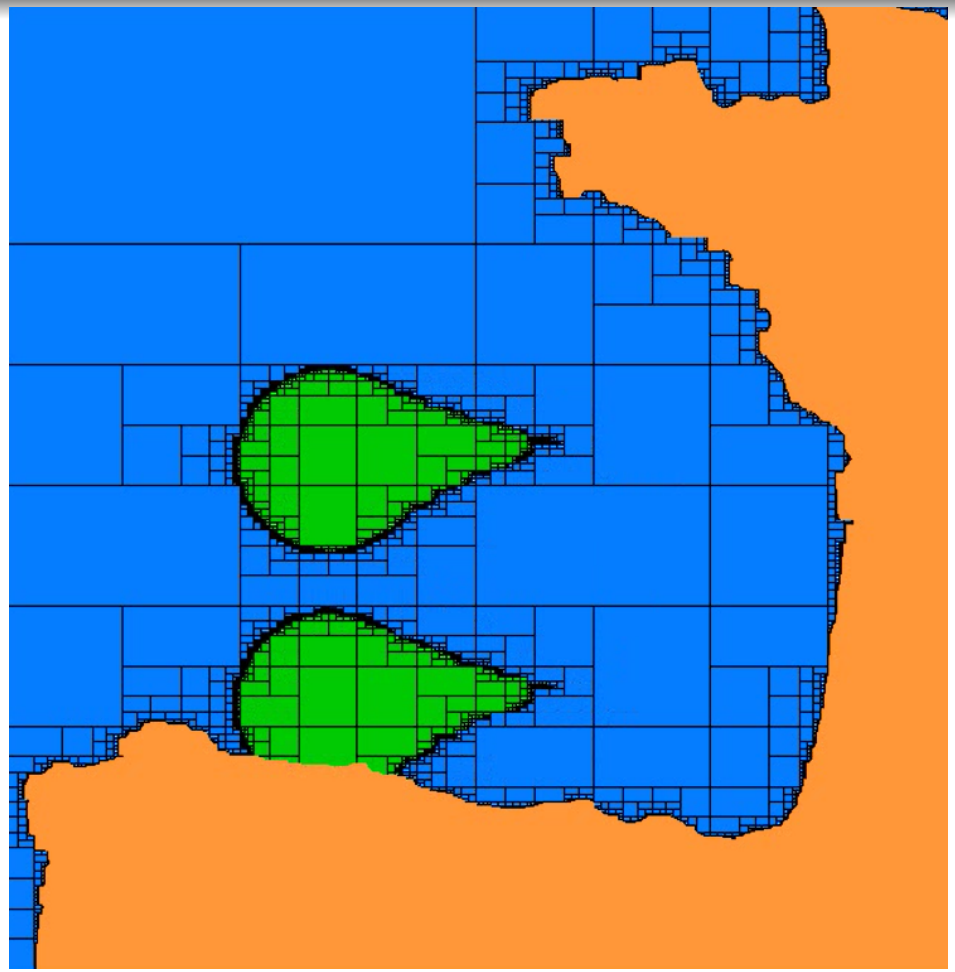


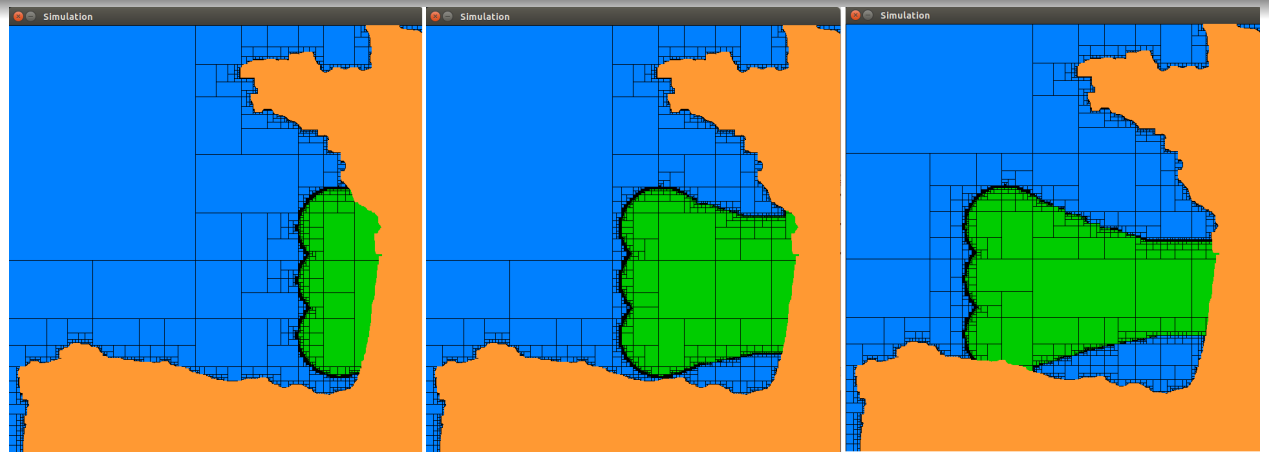
green: $\bigcup_i g^{-1}_{\mathbf{a}_i(t)}([0, d_i(t)])$

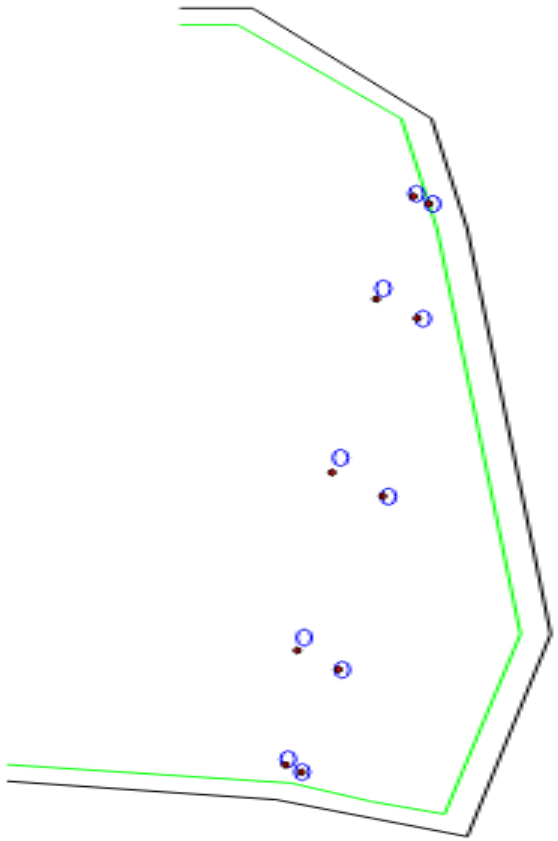


Blue: $\mathbb{G} \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty])$

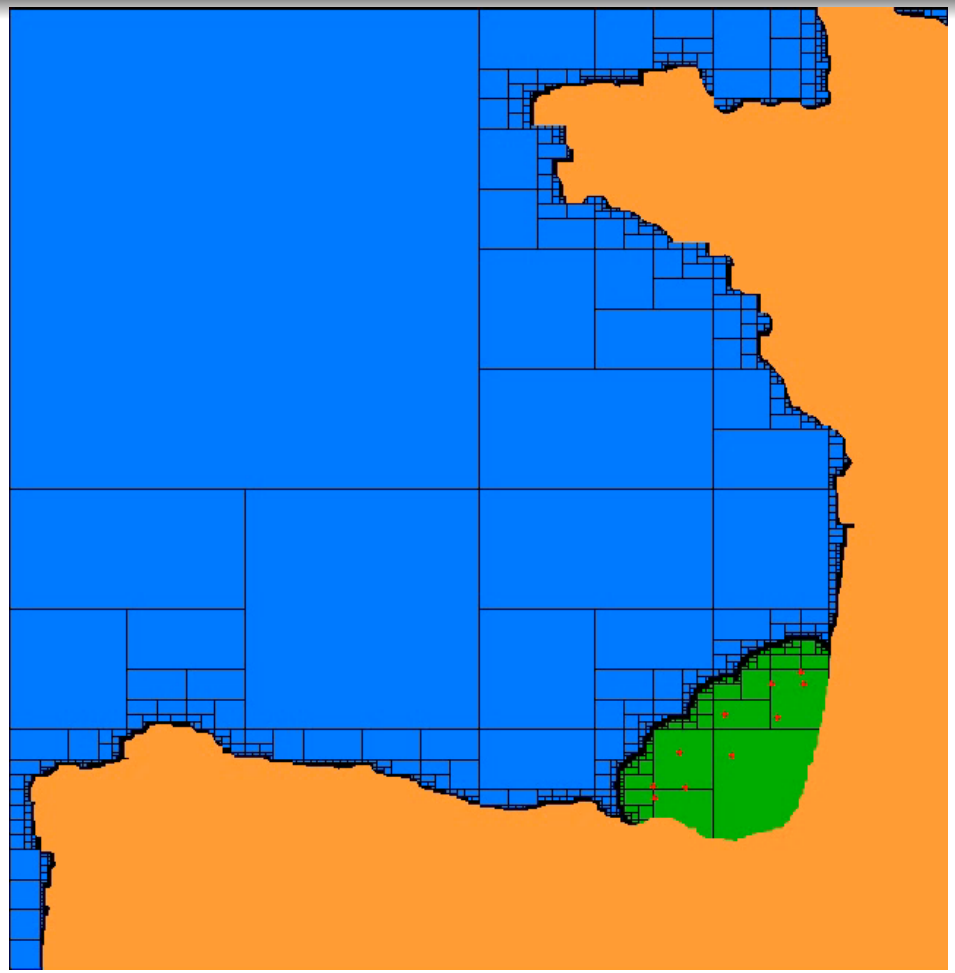


Blue: $\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty])$.


Simulation
Simulation
Simulation

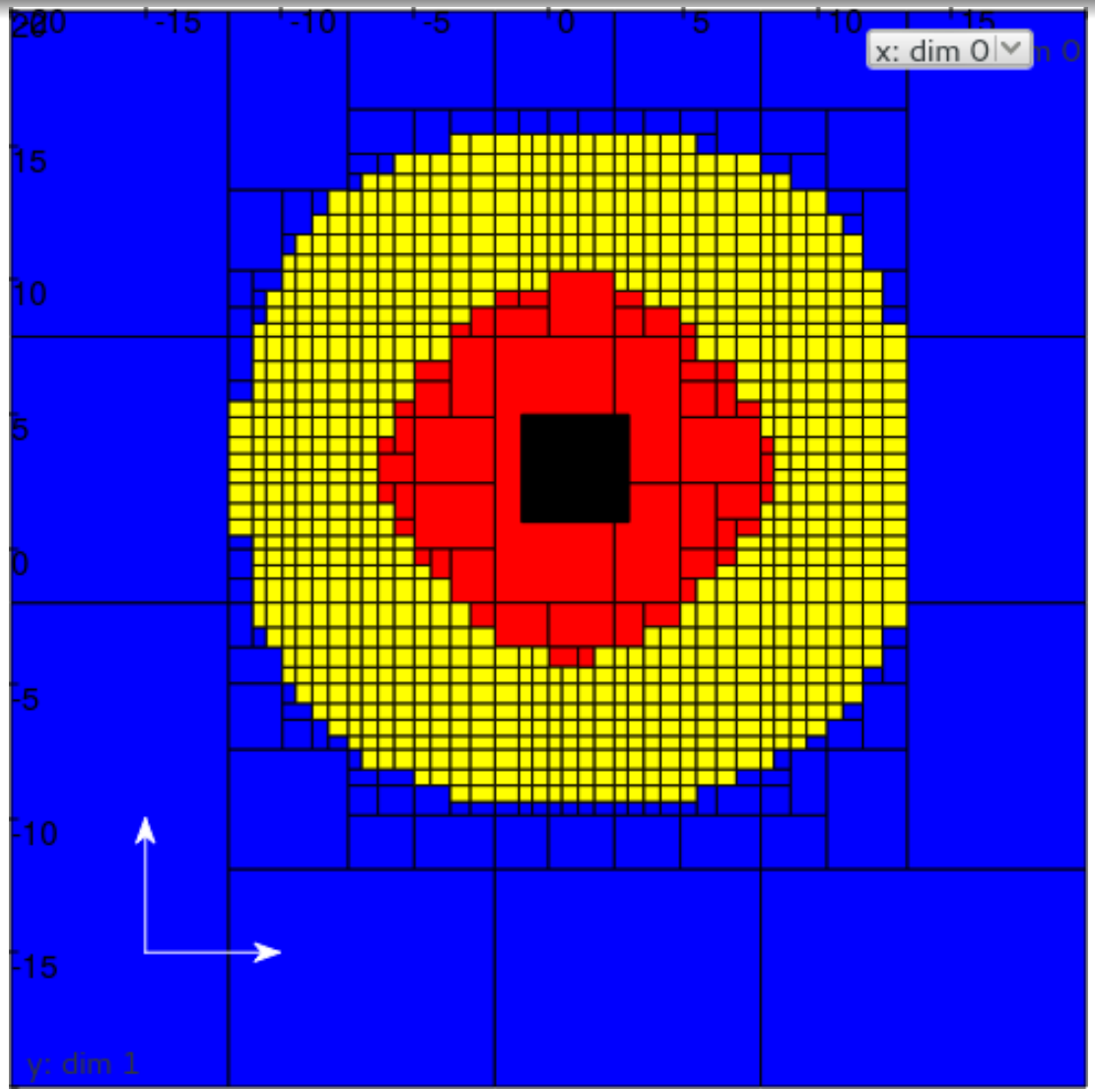


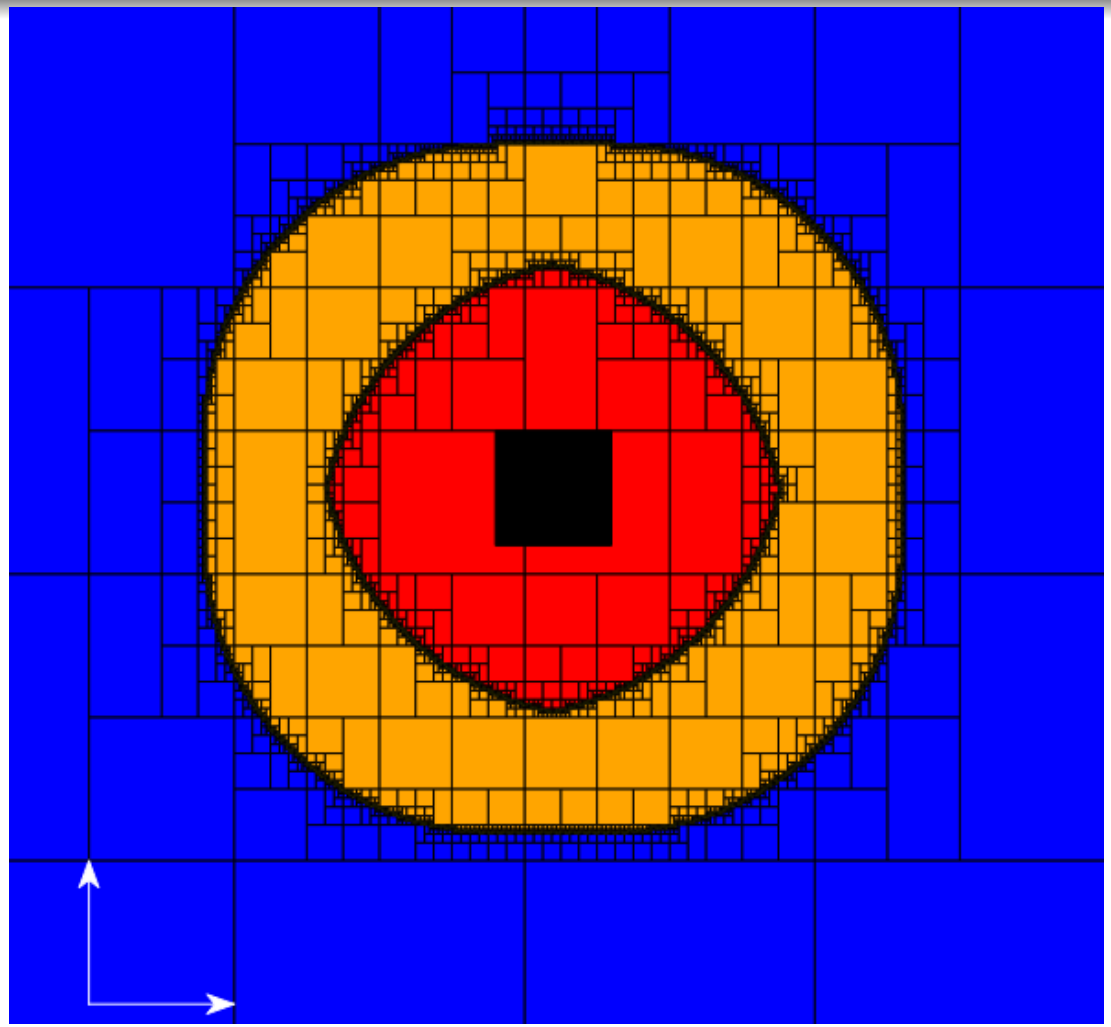
Strategy of the ellipse

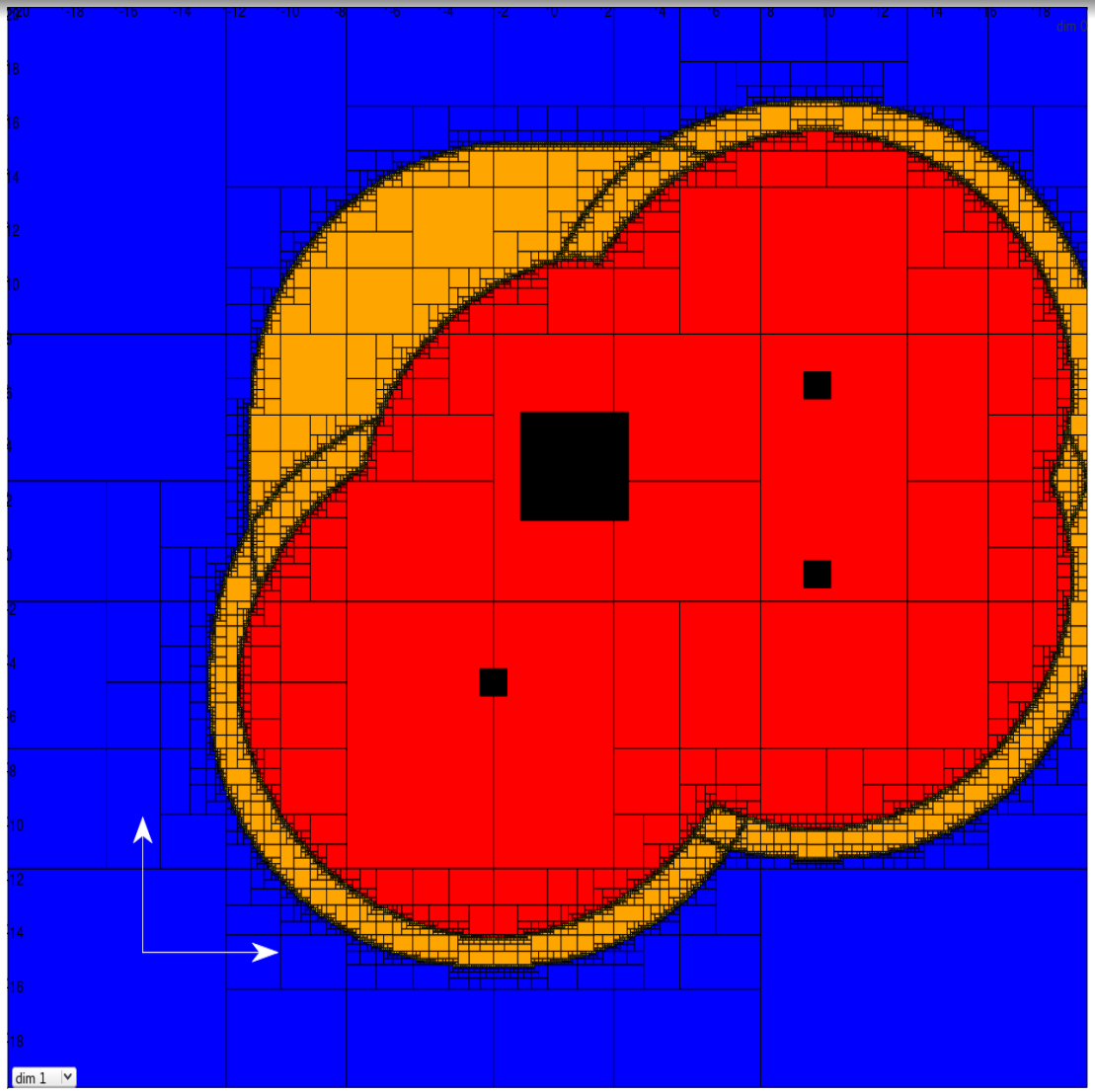


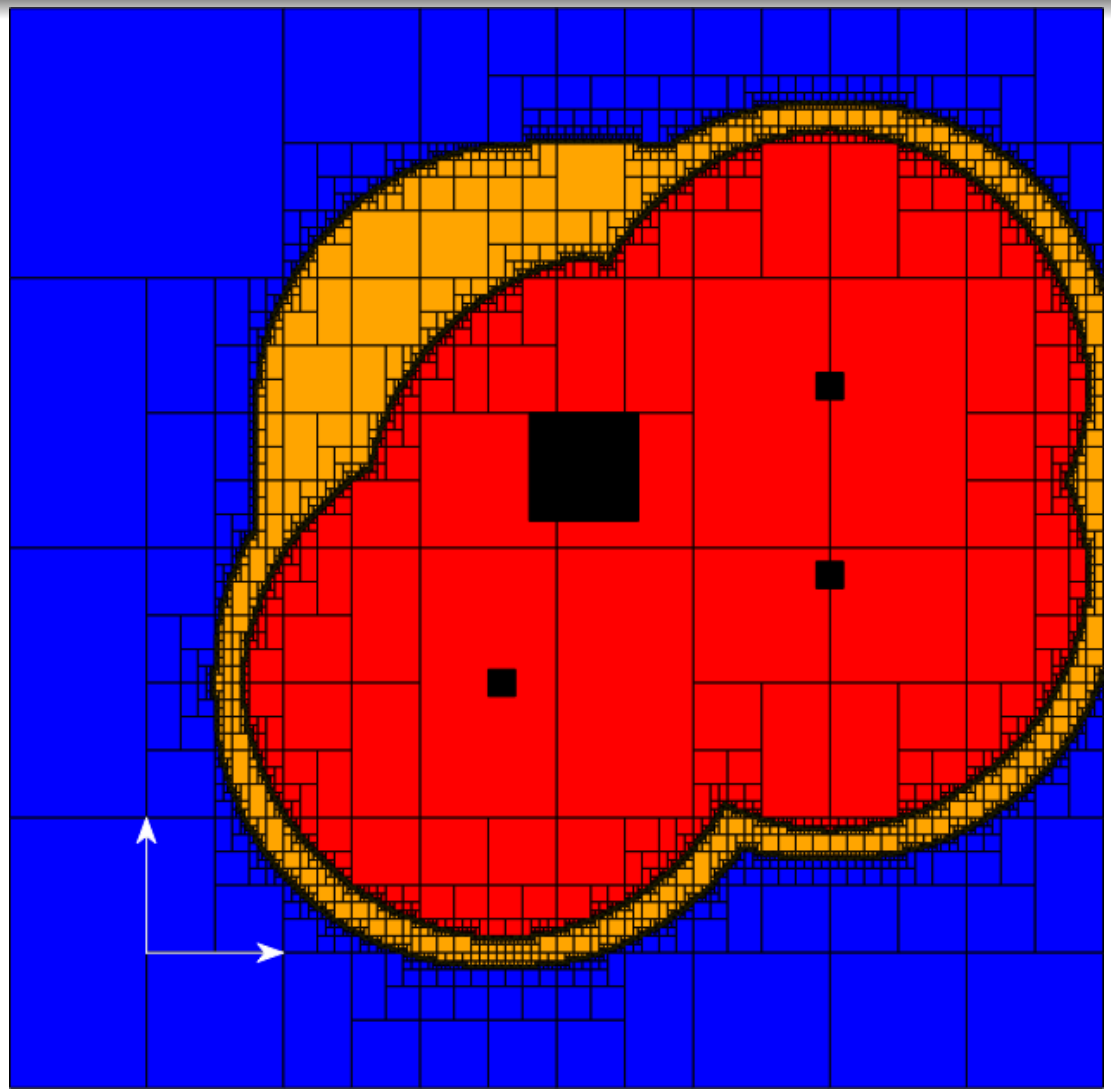
Video : https://youtu.be/rNcDW6npLfE

Thick sets


x: dim 0
y: dim 1
-15
-10
-5
0
5
10
15




dim 1



We have $d_i(t) \in [d_i(t)]$ and $\mathbf{a}_i(t) \in [\mathbf{a}_i](t)$. Thus the thick observer is

$$[\![\mathbb{X}]\!](t) = \mathbb{G} \cap dt \cdot \mathbb{F}([\![\mathbb{X}]\!](t - dt)) \cap \bigcap_i g^{-1}_{[\mathbf{a}_i](t)}([\![[d_i(t)], \infty]\!]).$$

Non causal secure zone

Idea: Take into account the future to reduce the number of robots.

If $\mathbb{H}$ is the set-membership flow, we have

$$\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_{t_1 \geq t} \mathbb{H}_{t-t_1}(\bigcap_i g^{-1}_{\mathbf{a}_i(t_1)}([d_i(t_1), \infty]).$$