Théorie des ensembles et robotique navale autonome

Journée Sciences navales - 6/02/2014

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1 Calcul par intervalles

Problème. Soit $f : \mathbb{R}^n \to \mathbb{R}$. Montrer que

$$\forall \mathbf{x} \in \mathbb{R}^n, f(\mathbf{x}) \geq 0.$$

Exemple. La fonction

$$f(\mathbf{x}) = x_1 x_2 - (x_1 + x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2$$

est-elle toujours positive pour $x_1, x_2 \in [-1, 1]$?

Arithmétique des intervalles

$$
\begin{aligned}
[-1,3]+[2,5] &= [1,8], \\
[-1,3]\cdot[2,5] &= [-5,15], \\
\operatorname{abs}([-7,1]) &= [0,7]
\end{aligned}
$$

L'extension intervalle de

$$\begin{aligned} f(x_1, x_2) &= x_1 \cdot x_2 - (x_1 + x_2) \cdot \cos x_2 \\ &\quad + \sin x_1 \cdot \sin x_2 + 2 \end{aligned}$$

est

$$\begin{aligned} [f]([x_1], [x_2]) &= [x_1] \cdot [x_2] - ([x_1] + [x_2]) \cdot \cos [x_2] \\ &\quad + \sin [x_1] \cdot \sin [x_2] + 2. \end{aligned}$$

Théorème (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

2 Calcul ensembliste

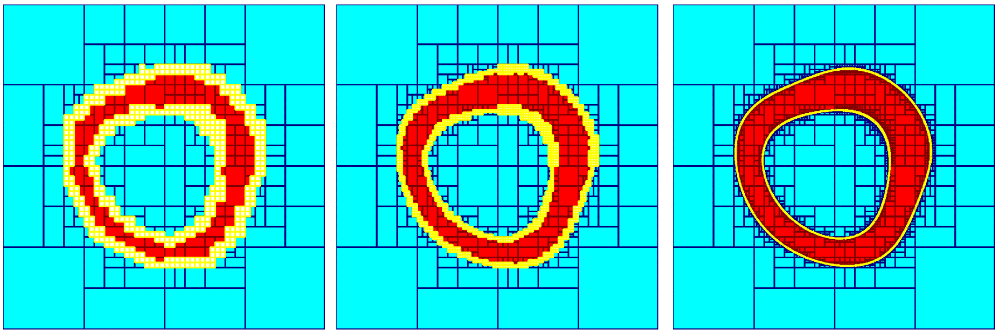
Les sous-ensembles $\mathbb{X} \subset \mathbb{R}^n$ peuvent être encadrés par des sous pavages :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

grâce au calcul par intervalles.

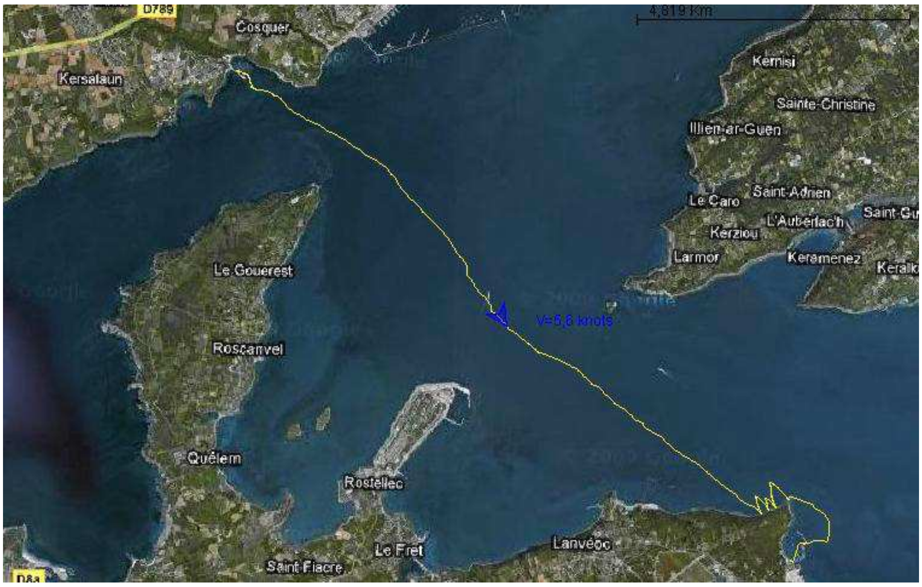
Exemple.

$$\mathbb{X} = \{(x_1, x_2) \mid x_1^2 + x_2^2 + \sin(x_1 + x_2) \in [4, 9]\}.$$



3 Sur la mer


BREIZH
ENSIETA
SPIRIT
ENSIETA


D789
4,819 Km
Cosquer
Kersalaun
Kernisi
Sainte-Christine
Illien-ar-Guen
Le Caro
Saint-Adrien
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
V=5,8 knots
Roscanvel
Quélern
Rostellec
Lanvéoc
Le Fret
Saint-Fiacre

ERWAN 1 (Ecole Navale)

3.1 Vaimos

Collaboration ENSTA/IFREMER

Vaimos à la WRSC (ENSTA-IFREMER-Ecole Navale).

La dynamique d'un robot est décrite par une équation d'état

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}).$$

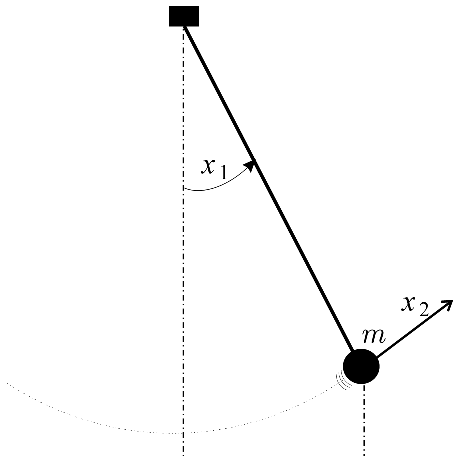
$$
\left\{
\begin{array}{rcl}
\dot{x} & = & v\cos\theta + p_1 a\cos\psi \\
\dot{y} & = & v\sin\theta + p_1 a\sin\psi \\
\dot{\theta} & = & \omega \\
\dot{v} & = & \frac{f_s\sin\delta_s - f_r\sin u_1 - p_2 v^2}{p_9} \\
\dot{\omega} & = & \frac{f_s(p_6 - p_7\cos\delta_s) - p_8 f_r\cos u_1 - p_3\omega}{p_{10}} \\
f_s & = & p_4 a\sin\left(\theta - \psi + \delta_s\right) \\
f_r & = & p_5 v\sin u_1 \\
\sigma & = & \cos\left(\theta - \psi\right) + \cos\left(u_2\right) \\
\delta_s & = & \left\{
\begin{array}{cc}
\pi - \theta + \psi & \text{si } \sigma \leq 0 \\
sign\left(\sin\left(\theta - \psi\right)\right).u_2 & \text{sinon.}
\end{array}
\right.
\end{array}
\right.
$$

Après insertion d'un régulateur $\mathbf{u} = \mathbf{g}(\mathbf{x})$, le système

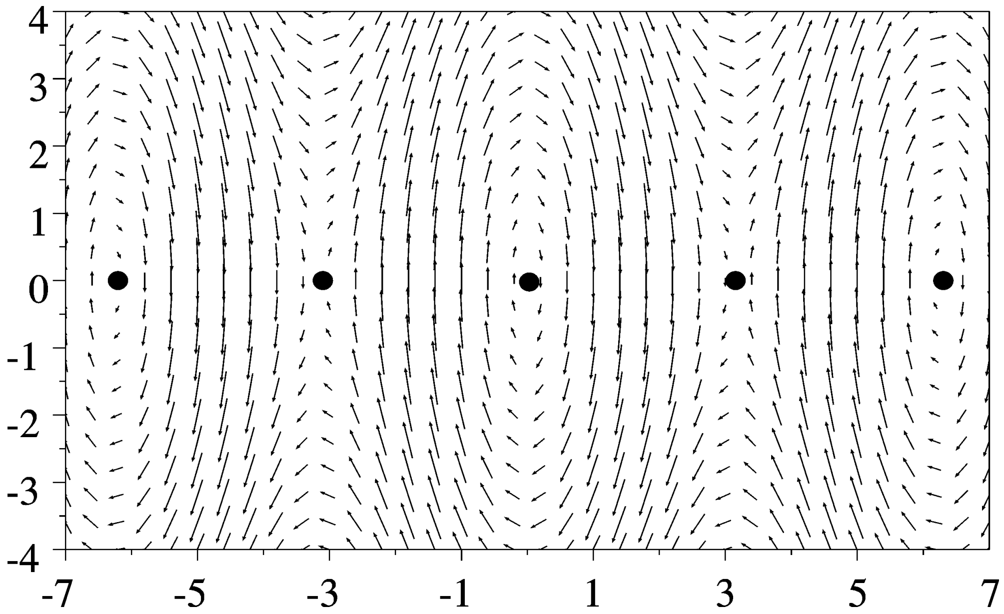
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

devient un robot

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$



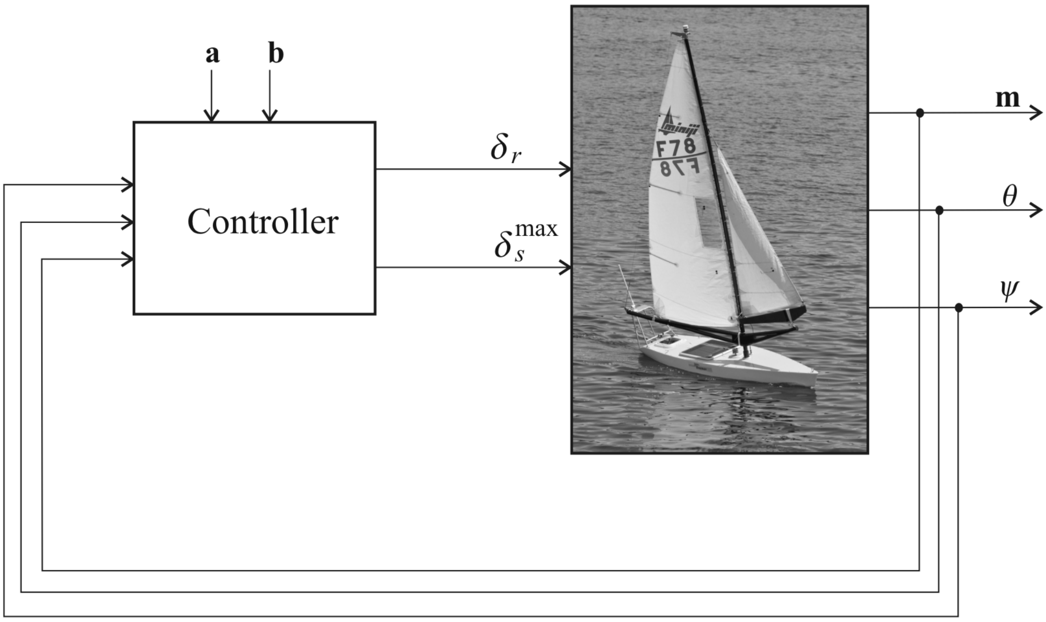
$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -\sin x_1. \end{cases} \qquad \text{c'est-à-dire .} \ \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

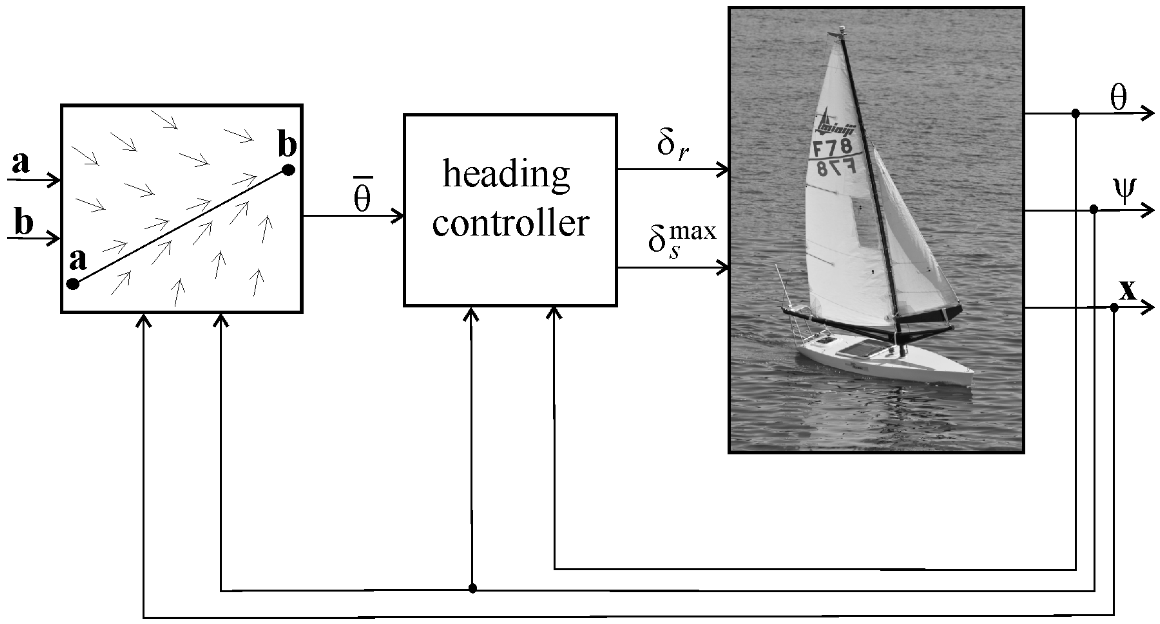

4
3
2
1
0
-1
-2
-3
-4
-7
-5
-3
-1
1
3
5
7

Avec toutes les incertitudes, le robot satisfait une *inclusion différentielle*

$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$$

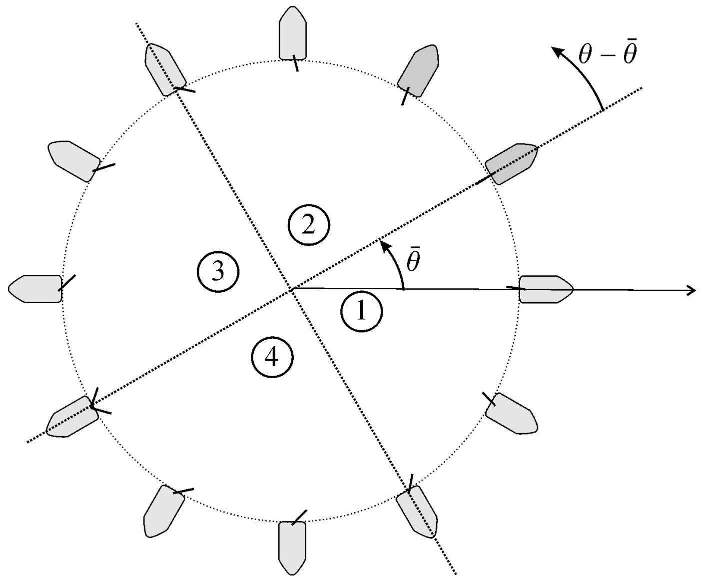
3.2 Suivi de ligne


a
b
Controller
δ_r
$\delta_s^{\max}$
F78
m
θ
ψ


a
b
a
b
$\bar{\theta}$
heading controller
δ_r
δ_s^{max}
θ
ψ
x

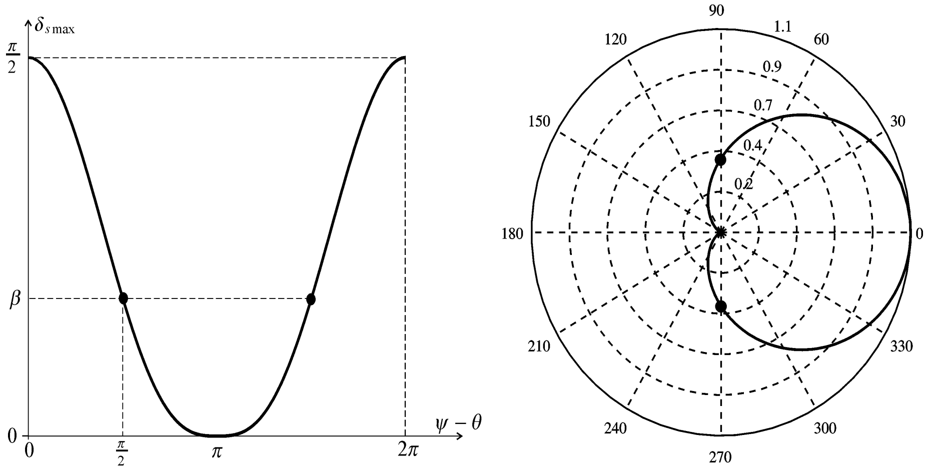
Régulation en cap

$$\left\{\begin{array}{lll}\delta_r & = & \frac{\delta_r^{\mathsf{max}}}{\pi}.\mathsf{atan}(\mathsf{tan}\,\frac{\theta-\bar{\theta}}{2}) \\ \delta_s^{\mathsf{max}} & = & \frac{\pi}{2}.\left(\frac{\cos(\psi-\bar{\theta})+1}{2}\right).\end{array}\right.$$

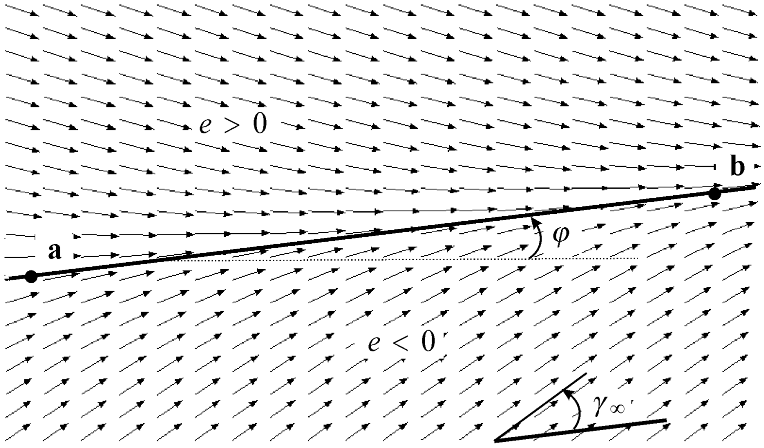


$\theta - \bar{\theta}$
2
3
$\bar{\theta}$
1
4

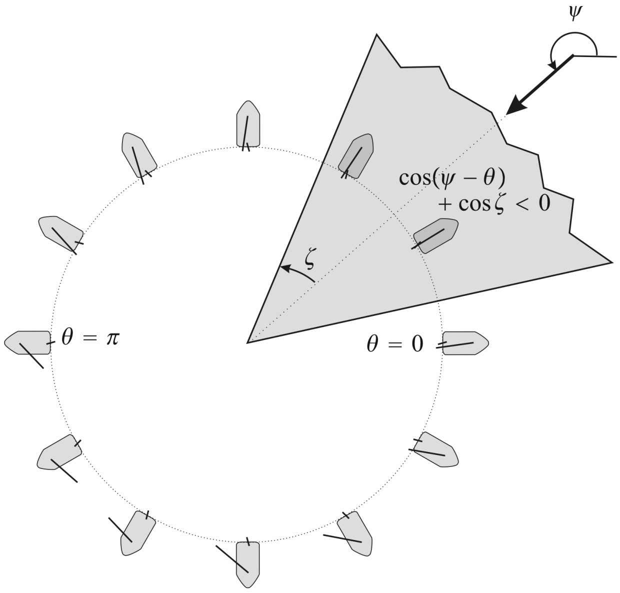
$$\delta_s^{\mathsf{max}} = \frac{\pi}{2} \cdot \left(\frac{\cos\left(\psi - \bar{\theta}\right) + 1}{2} \right)$$



3.3 Champ de vecteur nomimal



$$\theta^* = \varphi - \frac{1}{2}.\mathsf{atan}\left(\frac{e}{r}\right).$$



ψ
$\cos(\psi - \theta)$
$+ \cos\zeta < 0$
ζ
$\theta = \pi$
$\theta = 0$

3.4 Régulateur

Régulateur : in: $\mathbf{m}, \theta, \psi, \mathbf{a}, \mathbf{b}$; out: $\delta_r, \delta_s^{\mathsf{max}}$; inout: q

$e = \frac{\mathsf{det}(\mathbf{b}-\mathbf{a},\mathbf{m}-\mathbf{a})}{\|\mathbf{b}-\mathbf{a}\|}$

if $|e| > \frac{r}{2}$ then $q = \mathsf{sign}(e)$

$\bar{\theta} = \mathsf{atan2}(\mathbf{b} - \mathbf{a}) - \frac{1}{2}.\mathsf{atan}\left(\frac{e}{r}\right)$

if $\cos\left(\psi - \bar{\theta}\right) + \cos\zeta < 0$ then $\bar{\theta} = \pi + \psi - q.\zeta.$

$\delta_r = \frac{\delta_r^{\mathsf{max}}}{\pi}.\mathsf{atan}(\tan\frac{\theta-\bar{\theta}}{2})$

$\delta_s^{\mathsf{max}} = \frac{\pi}{2}.\left(\frac{\cos(\psi-\bar{\theta})+1}{2}\right).$

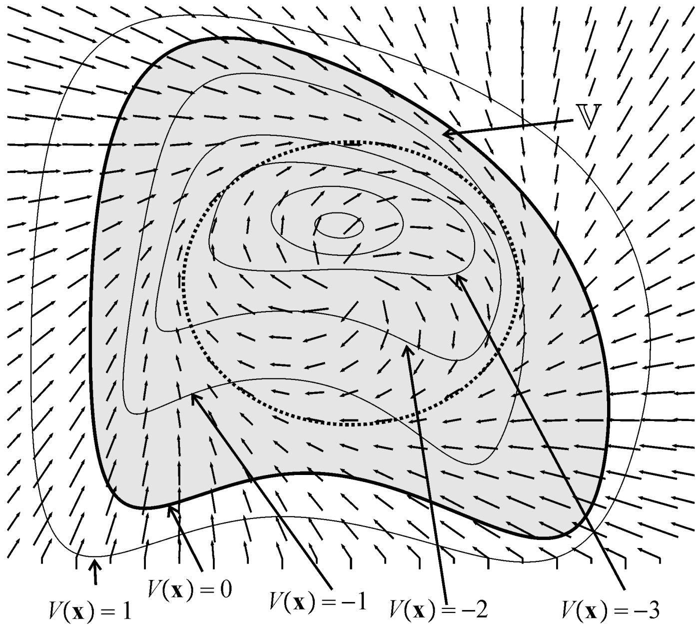
3.5 Validation théorique

Quand le vent est connu, le robot satisfait

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

Définition. Soit $V(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$. Le système est V-stable si

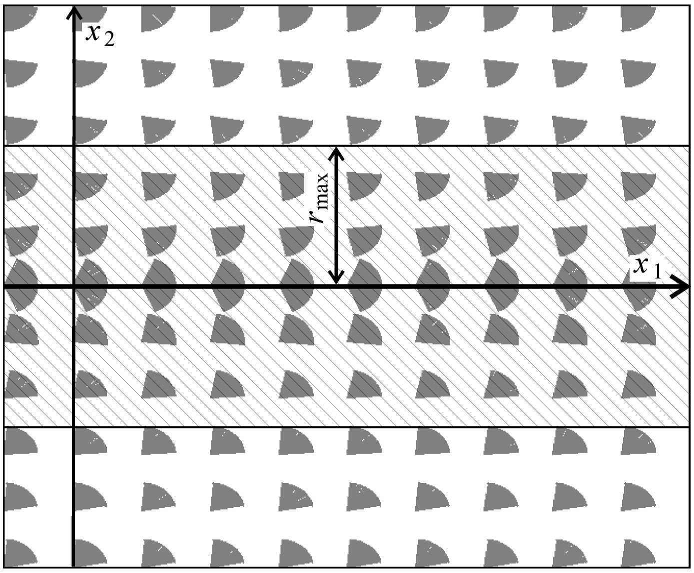
$$\left(V(\mathbf{x}) \geq 0 \;\Rightarrow\; \dot{V}(\mathbf{x}) < 0 \right).$$



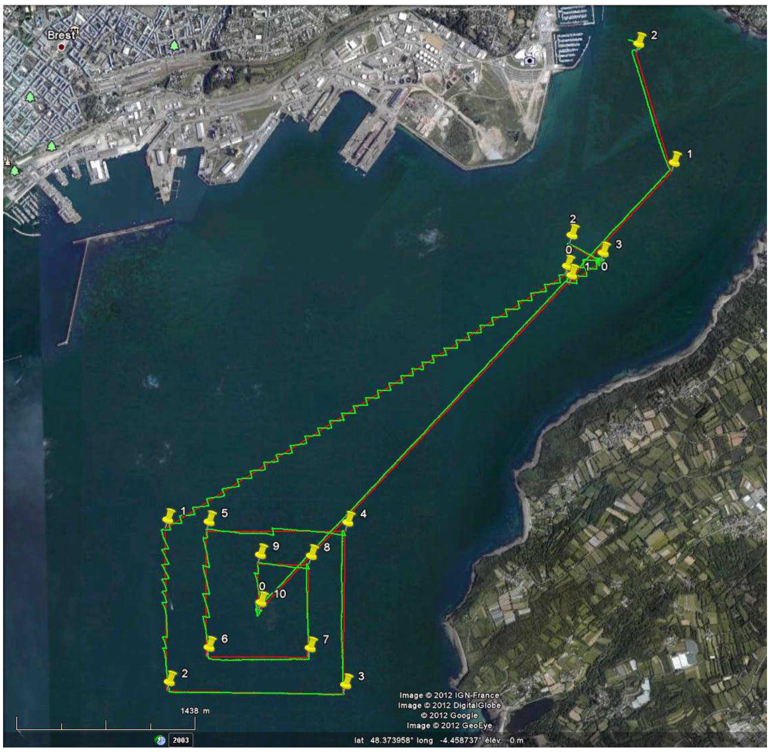
$\mathbb{V}$
$V(\mathbf{x}) = 1$
$V(\mathbf{x}) = 0$
$V(\mathbf{x}) = -1$
$V(\mathbf{x}) = -2$
$V(\mathbf{x}) = -3$

Théorème.

$$\left\{\begin{array}{c} \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) \geq 0 \\ V(\mathbf{x}) \geq 0 \end{array}\right. \text{ impossible } \Leftrightarrow \dot{\mathbf{x}}=\mathbf{f}(\mathbf{x}) \text{ is } V\text{-stable.}$$


x_2
$r_{\max}$
x_1

3.6 Validation expérimentale

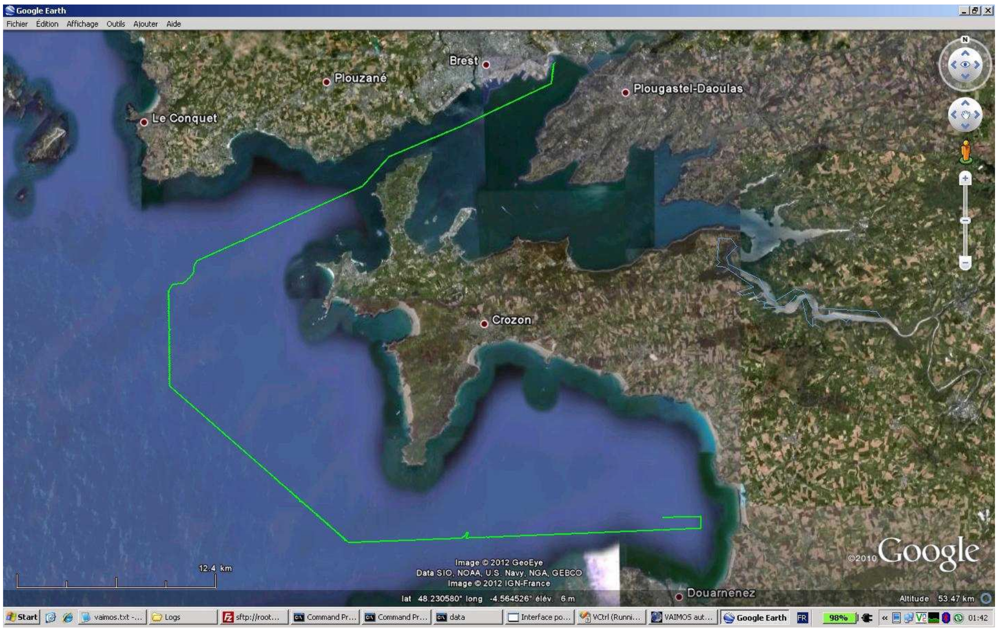


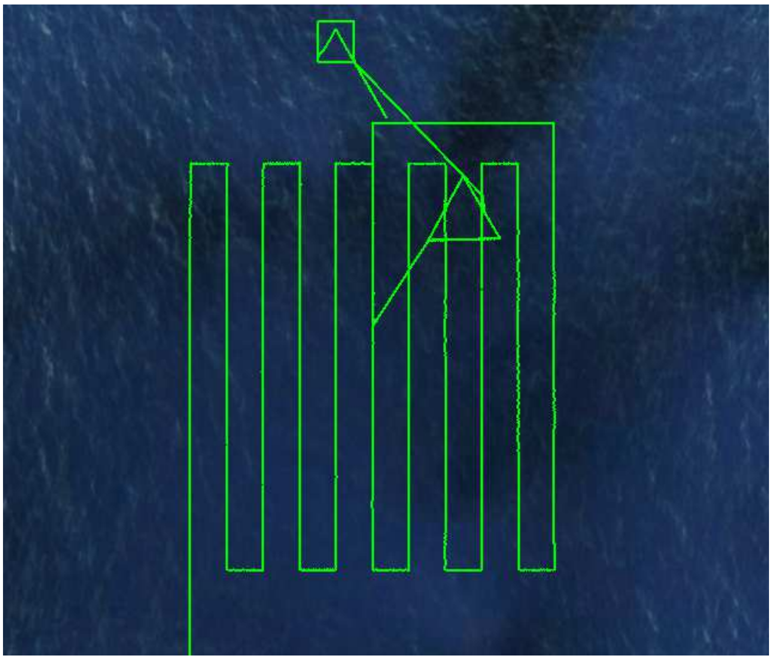
Rade de Brest

Brest-Douarnenez. January 17, 2012, 8am


Brest
YAMAHA


minij
F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Plougastel-Daoulas
Le Conquet
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
©2010 Google
lat 48.230580° long -4.564526° élév 6 m
Altitude 53.47 km
Start
Logs
data
Google Earth
98%
01:42



Au milieu de l'Atlantique, 350 km fait par Vaimos en 53h, sept. 6-9, 2012.

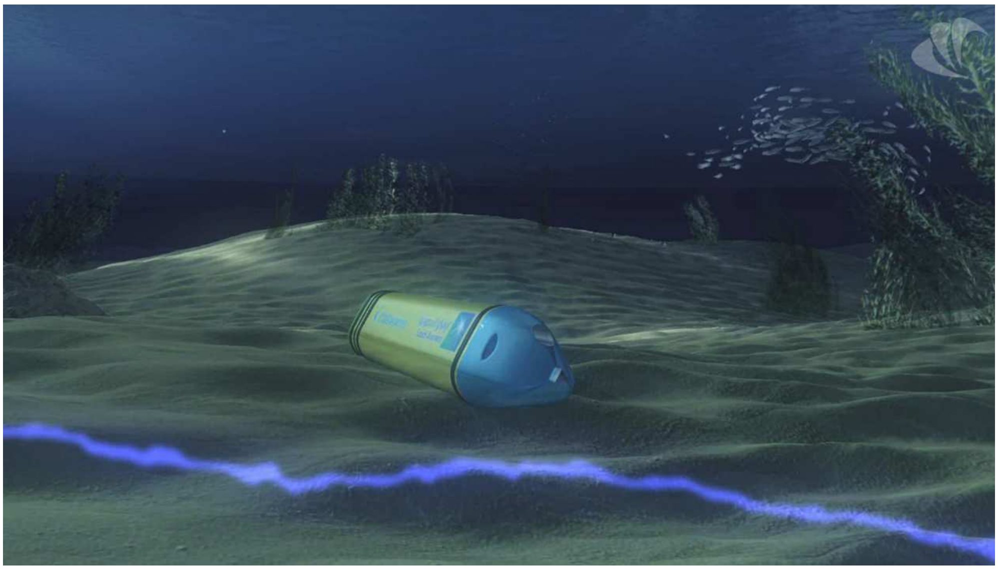
Conséquence.

Il est possible de garantir que le robot restera dans son couloir.

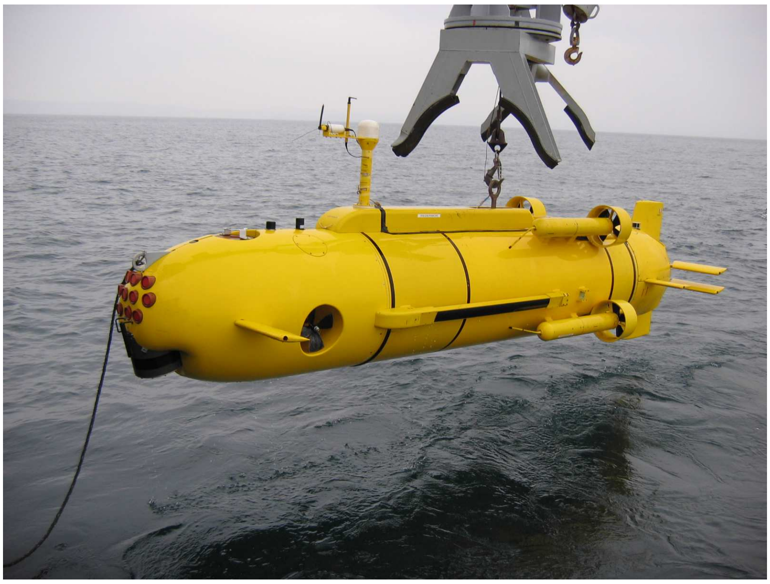
Indispensable pour établir des règles de circulation.

Indispensable pour identifier le responsable en cas d'accident.

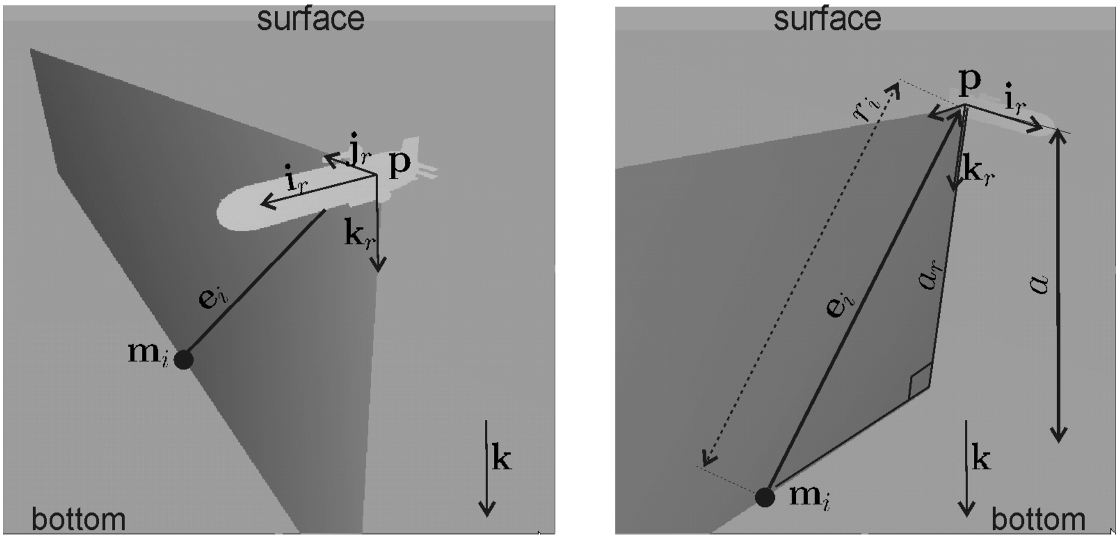
4 Sous la mer



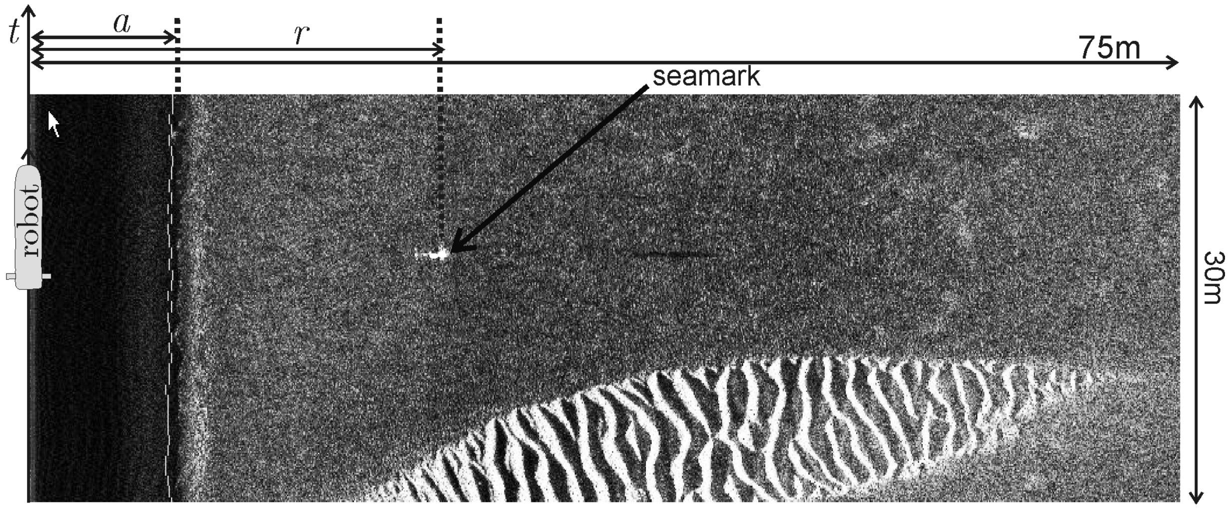
Localisation d'un groupe de robot (avec CGG)



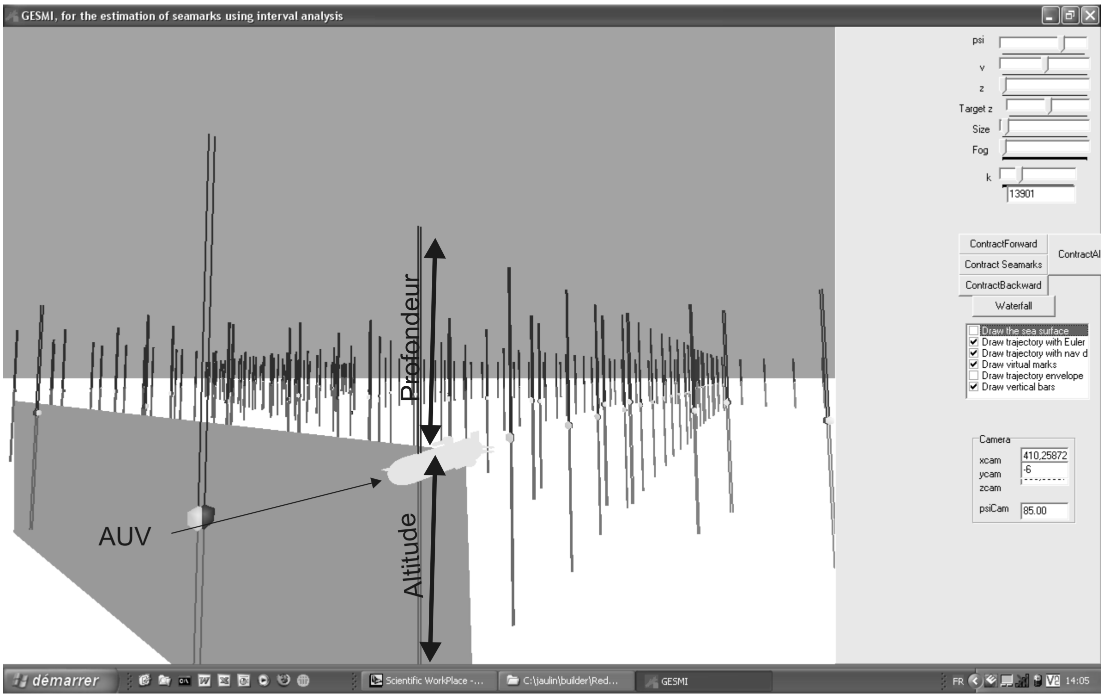
SLAM (avec le GESMA)

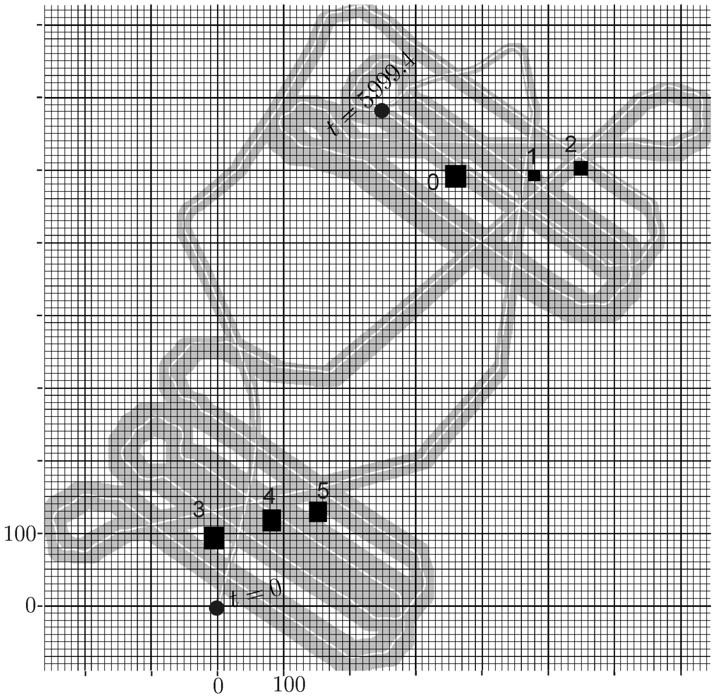


surface
$\mathbf{j}_r$
$\mathbf{i}_r$
$\mathbf{p}$
$\mathbf{k}_r$
$\mathbf{e}_i$
$\mathbf{m}_i$
$\mathbf{k}$
bottom
surface
$\mathbf{p}$
$\mathbf{i}_r$
r_i
$\mathbf{k}_r$
a_r
a
$\mathbf{e}_i$
$\mathbf{k}$
$\mathbf{m}_i$
bottom


t
a
r
75m
seamark
robot
30m

4.1 GESMI

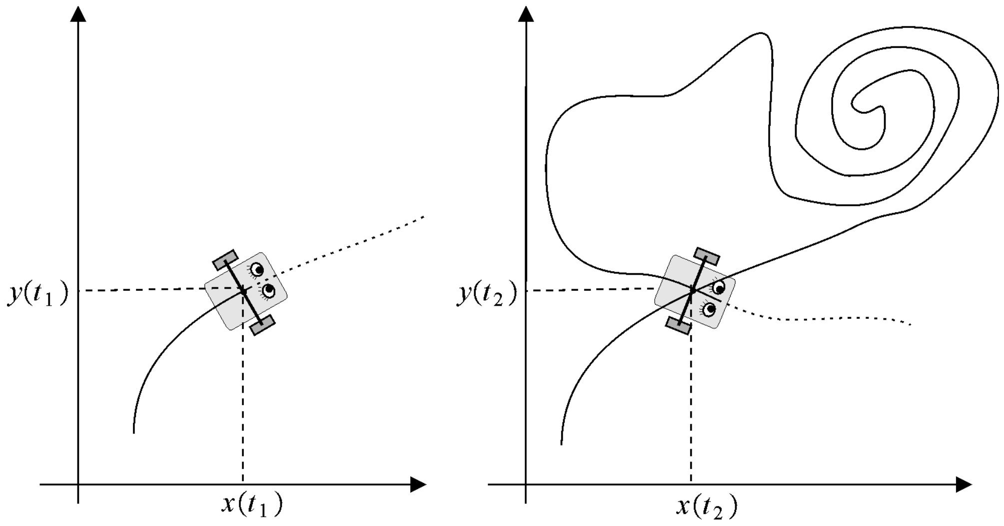

GESMI, for the estimation of seamarks using interval analysis
Profondeur
Altitude
AUV
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
ContractAl
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw trajectory with nav d
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
ycam
zcam
410,25872
-6
psiCam
85.00
démarrer
Scientific WorkPlace -...
C:\jaulin\builder\Red...
GESMI
FR
14:05



$t = 5999.4$
0
1
2
3
4
5
100
0
$t = 0$
0
100

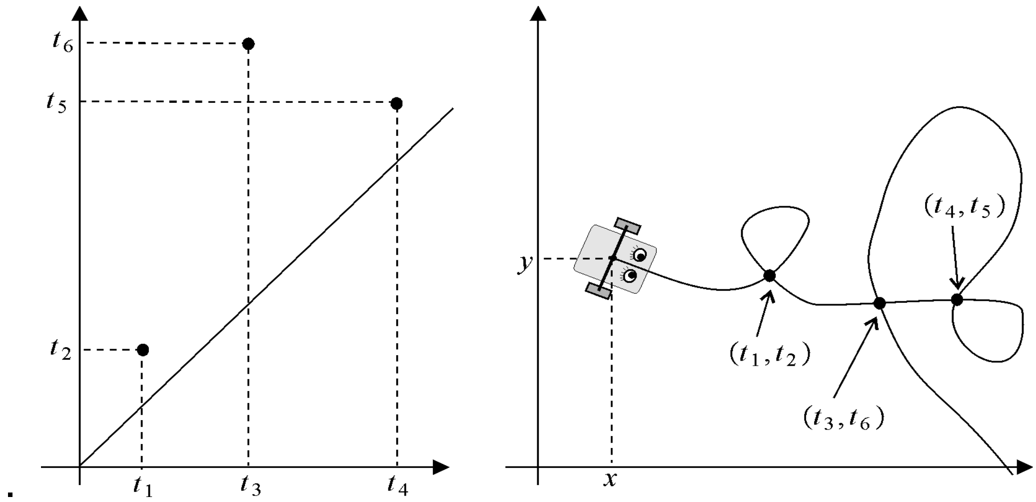
4.2 Détection de boucles

Collaboration ENSTA/Ecole Navale (C. Aubry R. Desmare)

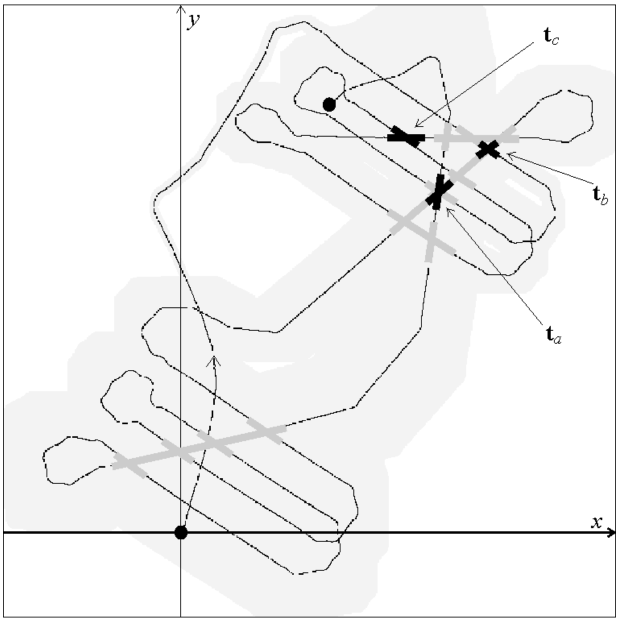


.

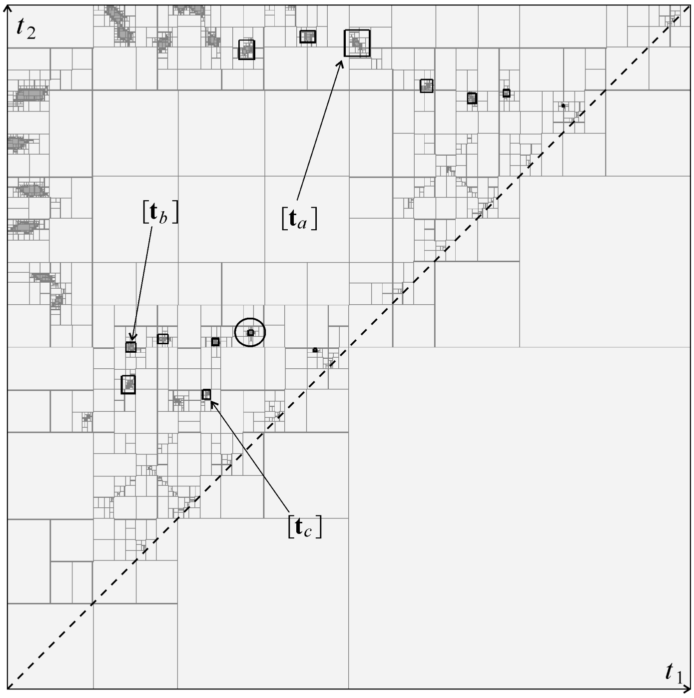
Un robot effectuant une boucle



t-plane



Tube enfermant la trajectoire du robot. On devine 28 boucles.



t_2
t_1
$[\mathbf{t}_a]$
$[\mathbf{t}_b]$
$[\mathbf{t}_c]$

Références.

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