Localization of an AUV using interval analysis

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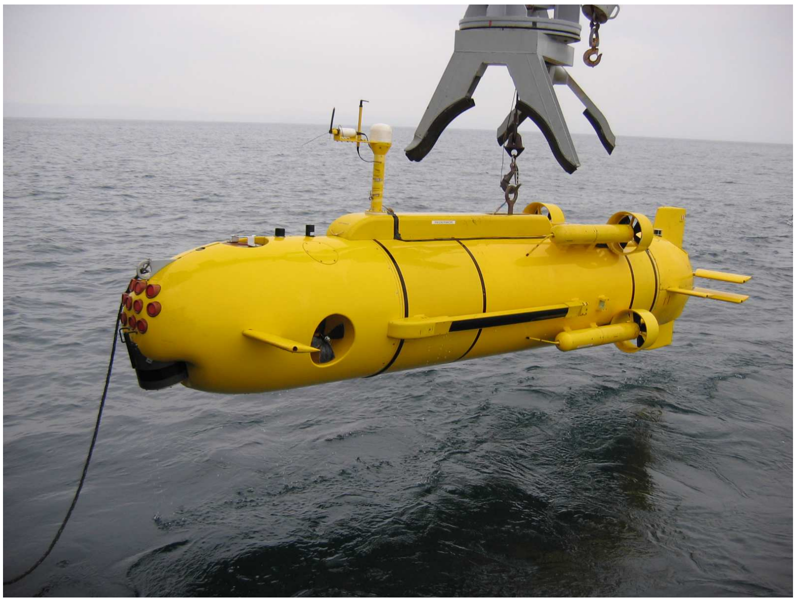
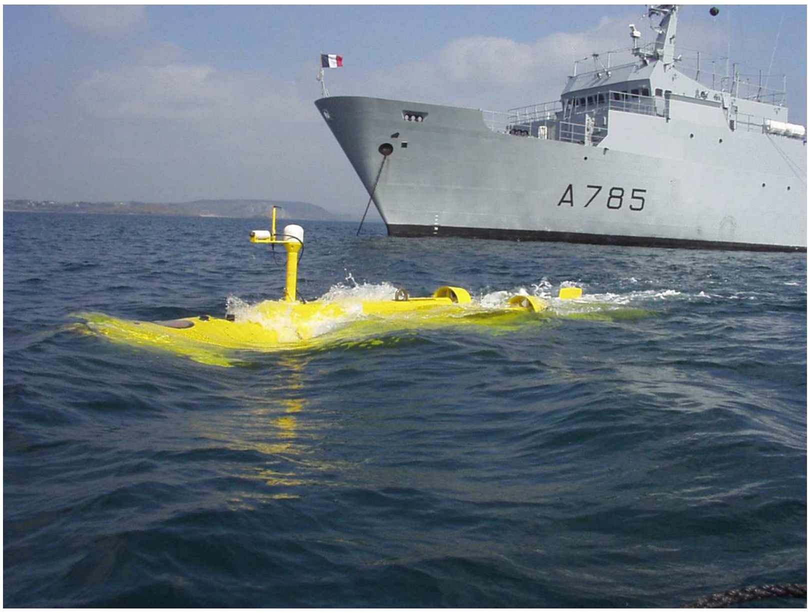
March 7, 2007.

1 The Redermor



The *Redermor*, GESMA



The *Redermor* at the surface

Show simulation

2 SLAM

Localization

When the mark locations are known, the localization problem is a state estimation problem.

The model of the system is

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x}) \end{cases}$$

where $\mathbf{x} = (x, y, z, \phi, \theta, \psi, v)$.

State equations

$$\begin{cases} \dot{p}_x &= v\cos\theta\cos\psi \\ \dot{p}_y &= v\cos\theta\sin\psi \\ \dot{p}_z &= -v\sin\theta \\ \dot{v} &= u_1 \\ \dot{\psi} &= \frac{\sin\varphi}{\cos\theta}.u_2 + \frac{\cos\varphi}{\cos\theta}.u_3 \\ \dot{\theta} &= \cos\varphi.u_2 - \sin\varphi.u_3 \\ \dot{\varphi} &= \tan\theta\left(\sin\varphi.u_2 + \cos\varphi.u_3\right). \end{cases}$$

Controller

$$\mathbf{u} = \frac{1}{v}\begin{pmatrix} v.c_\theta.c_\psi & v.c_\theta.s_\psi & -v.s_\theta \\ -s_\psi.s_\varphi - s_\theta.c_\psi.c_\varphi & c_\psi.s_\varphi - s_\theta.c_\varphi.s_\psi & -c_\theta.c_\varphi \\ -c_\varphi.s_\psi + s_\theta.c_\psi.s_\varphi & c_\psi.c_\varphi + s_\theta.s_\psi.s_\varphi & c_\theta.s_\varphi \end{pmatrix} * \left((\mathbf{w}-\mathbf{p}) + 2\left(\dot{\mathbf{w}} - \begin{pmatrix} v.c_\theta.c_\psi \\ v.c_\theta.s_\psi \\ -v.s_\theta \end{pmatrix}\right) + \ddot{\mathbf{w}}\right).$$

SLAM (simultaneous localization and mapping)

The mark locations are unknown.

Determine the location of the robot as well as the location of the marks.

Why choosing an interval constraint approach ?

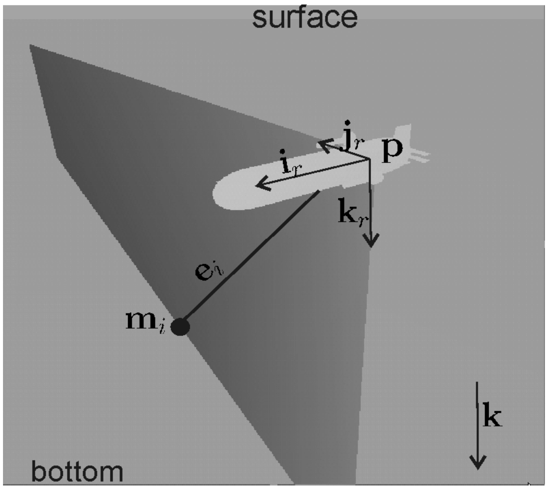
1) A reliable method is needed.
2) The model is nonlinear.
3) The pdf of the noises are unknown.
4) Reliable error bounds are provided by the sensors.
5) A huge number of redundant data are available.

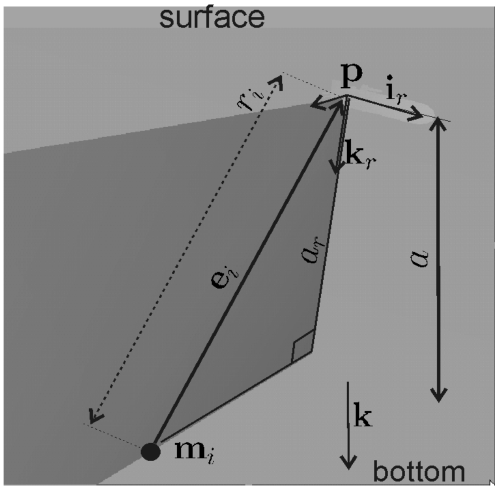
3 Sensors

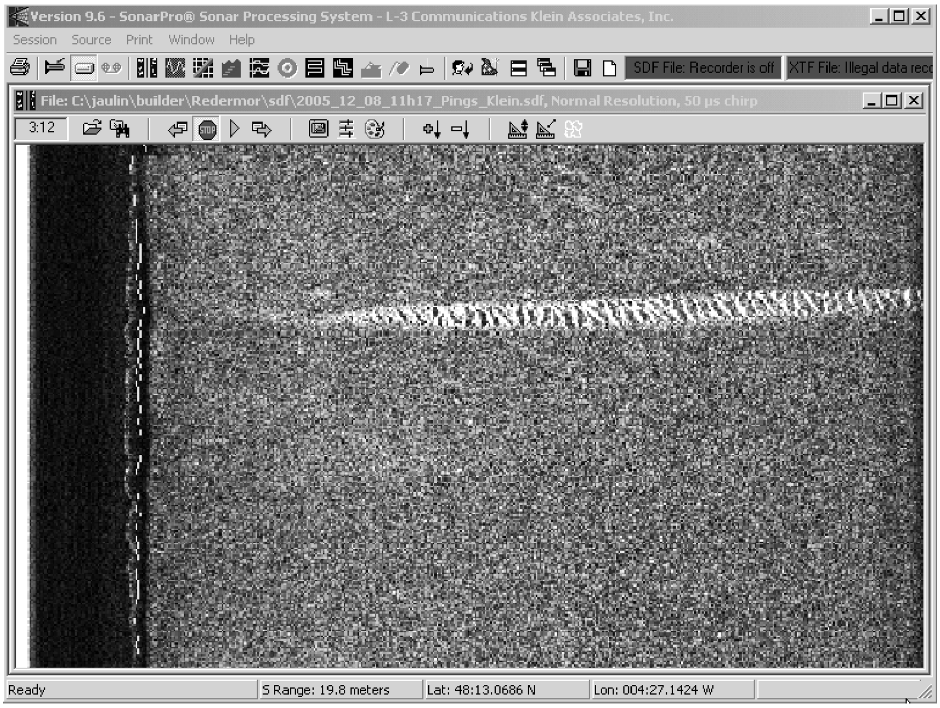
A GPS (Global positioning system), at the surface only.

$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^{\circ}, 48.2129206^{\circ}) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^{\circ}, 48.2191297^{\circ}) \pm 2.5m$$

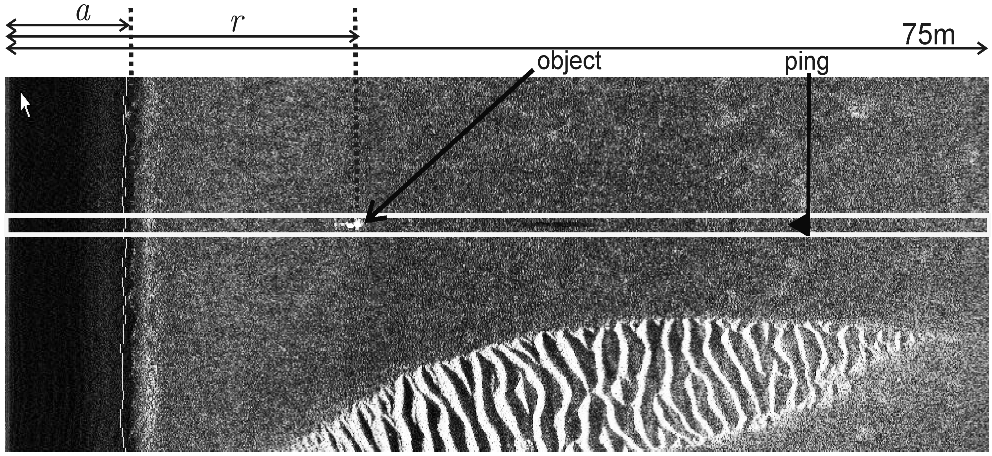
A sonar (KLEIN 5400 side scan sonar). Gives the distance r between the robot to the detected object.







Screenshot of SonarPro



Detection of a mine using SonarPro

A Loch-Doppler. Returns the speed of the robot $\mathbf{v}_r$ and the altitude a of the robot $\pm$ 10cm.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ, the pitch θ and the head ψ.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$

4 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines have been detected by the sonar:

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

5 Constraints satisfaction problem

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_\psi(t).\mathbf{R}_\theta(t).\mathbf{R}_\varphi(t),$$

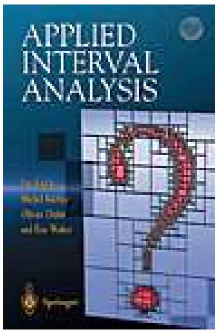
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t)$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^\mathsf{T}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5].$$

6 Interval Constraint Propagation



6.1 Interval arithmetic

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \, .$$

For instance,

$$\begin{aligned} [-1,3] + [2,5] &= [?,?], \\ [-1,3].[2,5] &= [?,?], \\ [-1,3]/[2,5] &= [?,?]. \end{aligned}$$

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] .$$

For instance,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3].[2, 5] &= [-5, 15], \\ [-1, 3]/[2, 5] &= [-\tfrac{1}{2}, \tfrac{3}{2}]. \end{aligned}$$

$$
\begin{aligned}
[x^-, x^+] + [y^-, y^+] &= [?, ?], \\
[x^-, x^+].[y^-, y^+] &= [?, ?].
\end{aligned}
$$

$$
\begin{aligned}
[x^-, x^+] + [y^-, y^+] = \; & [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] = \; & [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
& x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+].
\end{aligned}
$$

If $f \in \{$cos, sin,sqr, sqrt, log, exp, $\dots\}$

$$f\left([x]\right) = \left[\{f(x) \mid x \in [x]\}\right].$$

For instance,

$$\begin{array}{rcl}
\text{sin}\left([0,\pi]\right) & = & [?,?], \\
\text{sqr}\left([-1,3]\right) & = & [?,?], \\
\text{abs}\left([-7,1]\right) & = & [?,?], \\
\text{sqrt}\left([-10,4]\right) & = & [?,?], \\
\text{log}\left([-2,-1]\right) & = & [?,?].
\end{array}$$

If $f \in \{\text{cos, sin,sqr, sqrt, log, exp, } \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] .$$

For instance,

$$\begin{array}{rcl} \text{sin}([0,\pi]) & = & [0,1], \\ \text{sqr}([-1,3]) & = & [-1,3]^2 = [0,9], \\ \text{abs}([-7,1]) & = & [0,7], \\ \text{sqrt}([-10,4]) & = & \sqrt{[-10,4]} = [0,2], \\ \text{log}([-2,-1]) & = & \emptyset. \end{array}$$

6.2 Constraint contraction

Let x, y, z be 3 variables such that

$$\begin{array}{rcl} x & \in & [-\infty, 5], \\ y & \in & [-\infty, 4], \\ z & \in & [6, \infty], \\ z & = & x + y. \end{array}$$

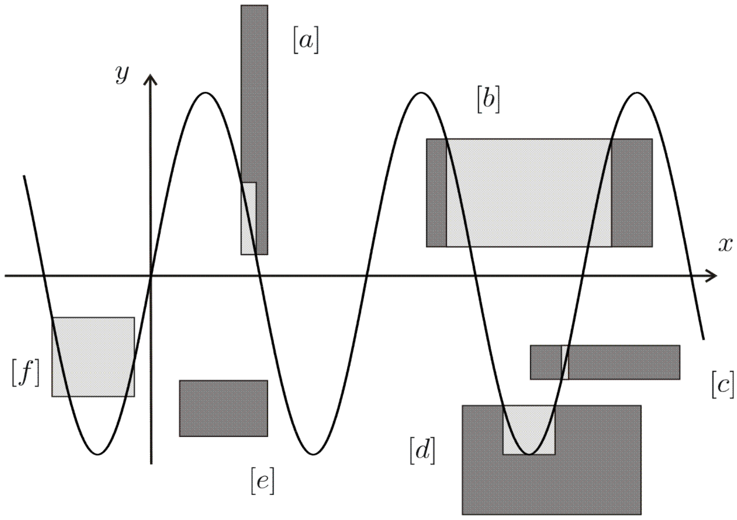
The values < 2 for x, < 1 for y and > 9 for z are inconsistent.

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$\begin{aligned} z = x + y \Rightarrow \quad z \in \; & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ & = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\ x = z - y \Rightarrow \quad x \in \; & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ & = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\ y = z - x \Rightarrow \quad y \in \; & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ & = [-\infty, 4] \cap [1, \infty] = [1, 4]. \end{aligned}$$

sinus

$$y = \mathsf{sin}(x),\ x \in [x],\ y \in [y]$$



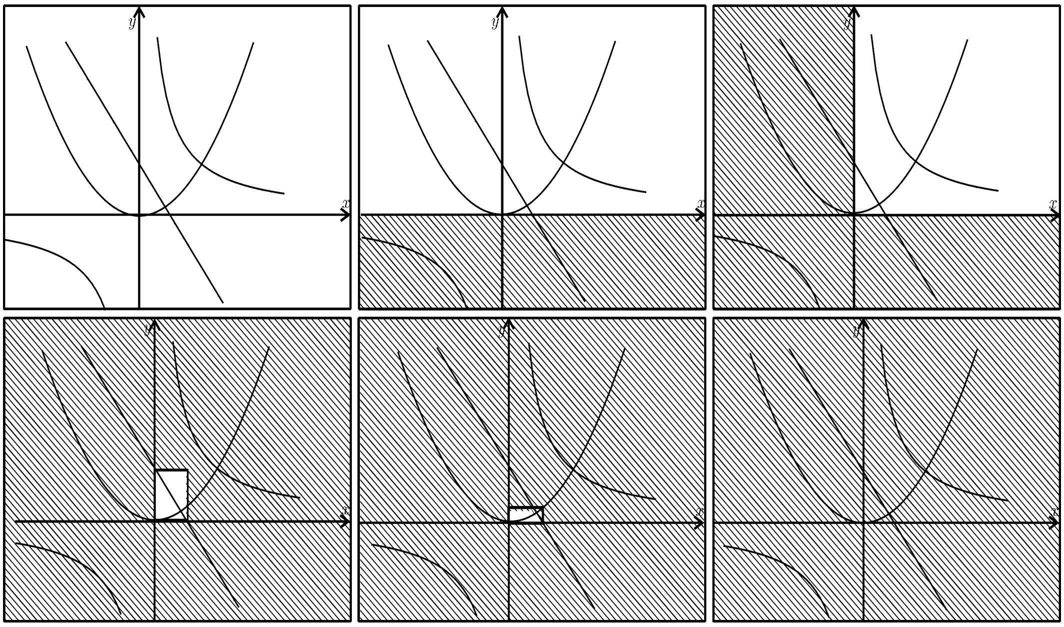
6.3 Constraint propagation

Consider the three constraints

$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$

To each variable we assign the domain $[-\infty, \infty]$.

Constraint propagation amounts to contract all constraints until equilibrium.



y
x

$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty, \infty]^2 \ = \ [0, \infty] \\
(C_2) \Rightarrow & x \in & 1/[0, \infty] \ = [0, \infty] \\
(C_3) \Rightarrow & y \in & [0, \infty] \ \cap \ ((-2)\,.[0, \infty] + 1) \\
 & & = [0, \infty] \ \cap \ ([-\infty, 1]) = [0, 1] \\
 & x \in & [0, \infty] \ \cap \ (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0, 1] \cap [0, 1/2]^2 = [0, 1/4] \\
(C_2) \Rightarrow & x \in & [0, 1/2] \ \cap \ 1/[0, 1/4] = \emptyset \\
 & y \in & [0, 1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

6.4 Decomposition

For more complex constraints, we have to perform a decomposition.

Example 1

$$x + \mathsf{sin}(y) - xz \le 0,$$
$$x \in [-1, 1], y \in [-1, 1], z \in [-1, 1]$$

can be decompose into

$$\begin{cases} a = \mathsf{sin}(y) \\ b = x + a \\ c = xz \\ b - c = d \end{cases}, \qquad \begin{array}{ll} x \in [-1, 1] & a \in [-\infty, \infty] \\ y \in [-1, 1] & b \in [-\infty, \infty] \\ z \in [-1, 1] & c \in [-\infty, \infty] \\ & d \in [-\infty, 0] \end{array}$$

Example 2. Matrix constraint

$$\left\{ \begin{array}{c} \mathbf{y} = \mathbf{A}\mathbf{x} \\ \mathbf{x} \in [\mathbf{x}] \subset \mathbb{R}^2, \mathbf{y} \in [\mathbf{y}] \subset \mathbb{R}^2 \\ \mathbf{A} \in [\mathbf{A}] \end{array} \right.$$

Decomposition

$$\left\{ \begin{array}{ccc} y_1 & = & a_{11}x_1 + a_{12}x_2 \\ y_2 & = & a_{21}x_1 + a_{22}x_2 \end{array} \right.$$

i.e.

$$\left\{ \begin{array}{c} z_1 = a_{11}x_1,\ z_2 = a_{12}x_2,\ y_1 = z_1 + z_2 \\ z_3 = a_{21}x_1,\ z_4 = a_{22}x_2,\ y_2 = z_3 + z_4 \\ z_1 \in [-\infty, \infty], \ldots, z_4 \in [-\infty, \infty] \end{array} \right. .$$

Example 3

$$\begin{array}{c} \mathbf{y} = \mathbf{R}\mathbf{x} \\ \mathrm{Rot}(\mathbf{R}) \end{array}, \qquad \begin{array}{c} \mathbf{x} \in [\mathbf{x}]\,, \mathbf{y} \in [\mathbf{y}] \\ \mathbf{R} \in [\mathbf{R}] \end{array}$$

Contractions

$$\begin{aligned} [\mathbf{y}] &: = [\mathbf{y}] \cap [\mathbf{R}] * [\mathbf{x}], \\ [\mathbf{x}] &: = [\mathbf{x}] \cap [\mathbf{R}]^{\mathsf{T}} * [\mathbf{y}], \end{aligned}$$

Example 4. Differential constraint.

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t)$$
$$\forall t \in [t_0, t_1], \mathbf{R}(t) \in [\mathbf{R}], \mathbf{v}_r(t) \in [\mathbf{v}_r]$$

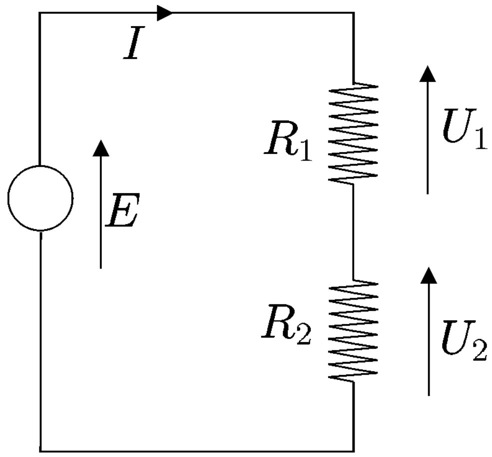
Since

$$\mathbf{p}(t_1) = \mathbf{p}(t_0) + \int_{t_0}^{t_1} \mathbf{R}(t).\mathbf{v}_r(t) \in \mathbf{p}(t_0) + (t_1 - t_0).\left[\mathbf{R}\right].\left[\mathbf{v}_r\right],$$

the domains for $\mathbf{p}(t_0)$ and $\mathbf{p}(t_1)$ can be contracted as follows

$$\begin{array}{rcl} \left[\mathbf{p}\right](t_1) & = & \left[\mathbf{p}\right](t_1) \cap \left(\left[\mathbf{p}\right](t_0) + (t_1 - t_0).\left[\mathbf{R}\right].\left[\mathbf{v}_r\right]\right), \\ \left[\mathbf{p}\right](t_0) & = & \left[\mathbf{p}\right](t_0) \cap \left(\left[\mathbf{p}\right](t_1) + (t_0 - t_1).\left[\mathbf{R}\right].\left[\mathbf{v}_r\right]\right). \end{array}$$

6.5 Estimation problem



Constraints

$$\begin{aligned} P &= EI;\; E = (R_1 + R_2)\, I; \\ U_1 &= R_1 I;\; U_2 = R_2 I;\; E = U_1 + U_2. \end{aligned}$$

Initial domains

$$\begin{array}{cc} R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\ E \in [23, 26]\mathsf{V}, & I \in [4, 8]\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, 17]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}, & \end{array}$$

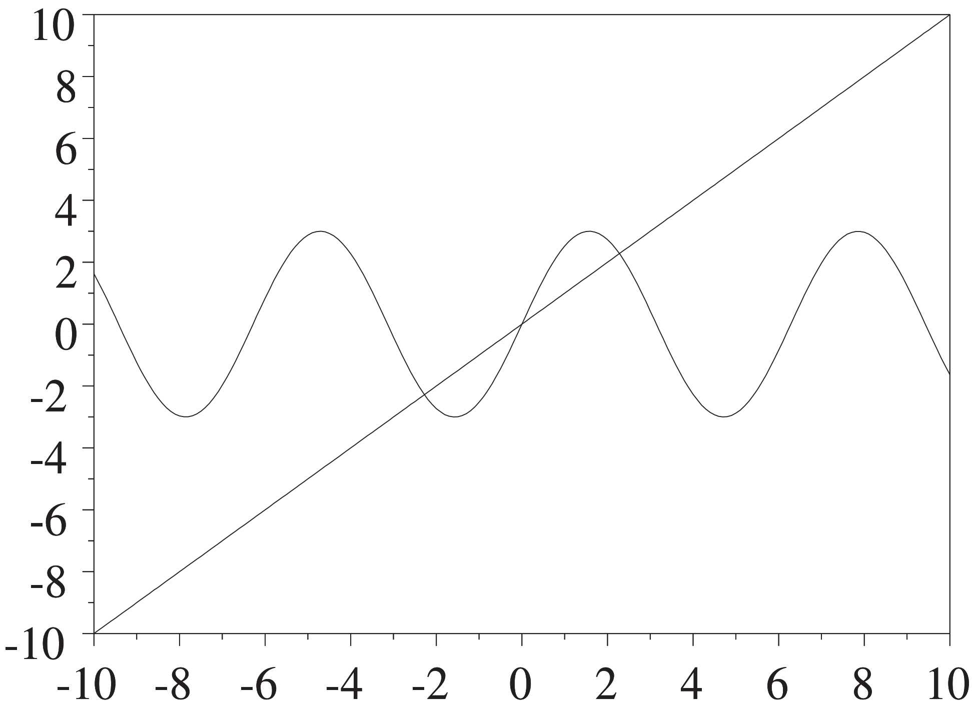
We get the contracted domains

$$\begin{array}{cc} R_1 \in [\mathbf{1.84}, \mathbf{2.31}]\,\Omega, & R_2 \in [\mathbf{2.58}, \mathbf{3.35}]\Omega, \\ E \in [\mathbf{24}, 26]\mathsf{V}, & I \in [\mathbf{4.769}, \mathbf{5.417}]\,\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, \mathbf{16}]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}, & \end{array}$$

7 Resolution of equations

Consider the system

$$\begin{cases} y = 3\sin(x) \\ y = x \end{cases} \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$



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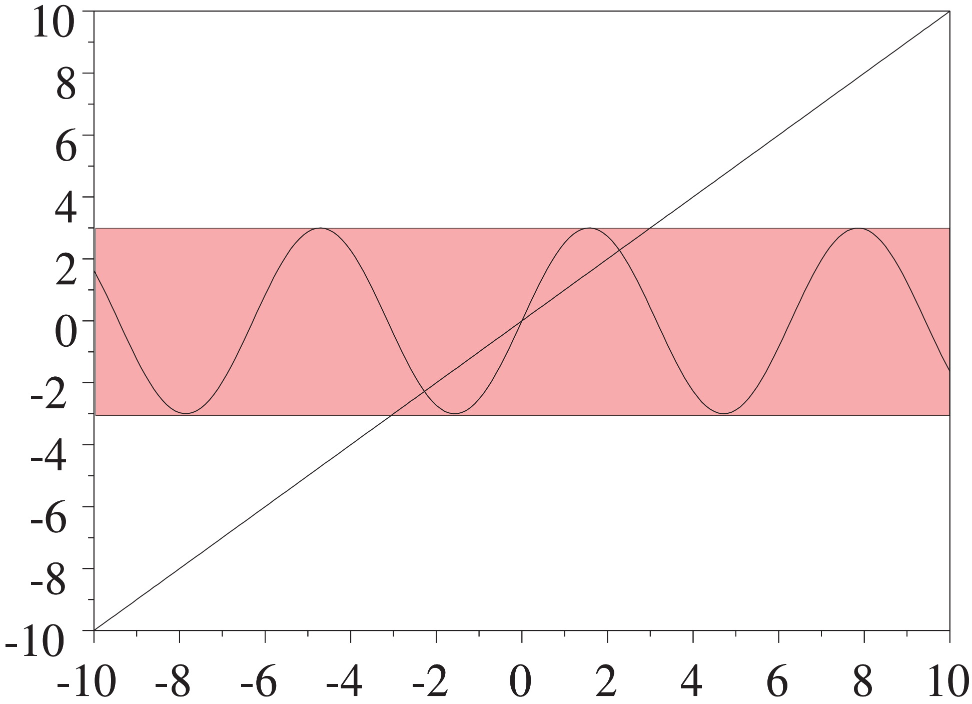
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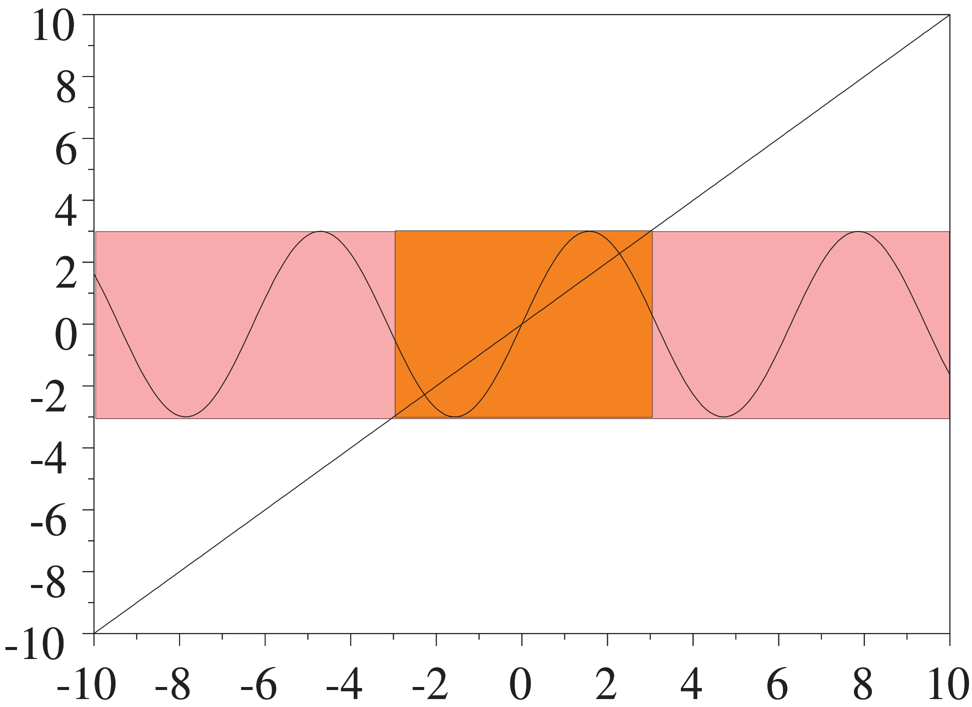

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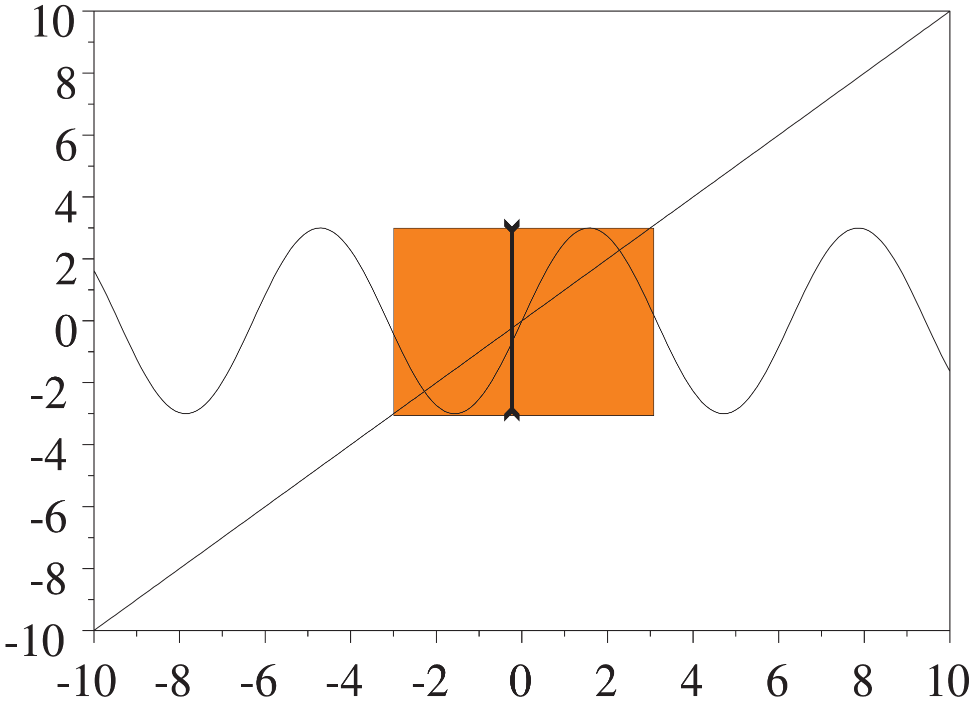
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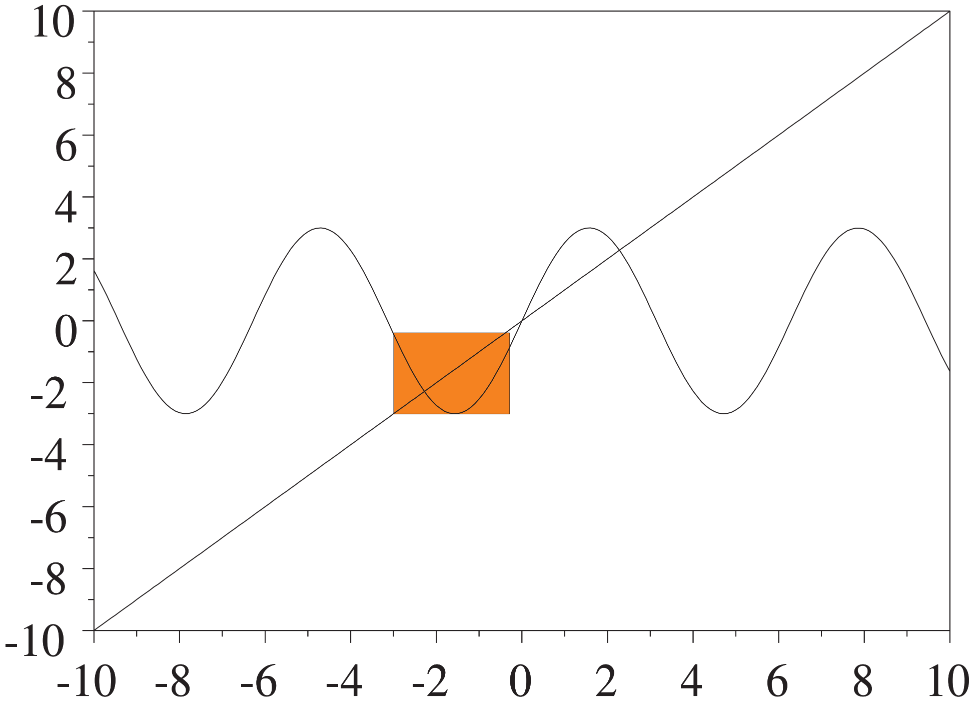
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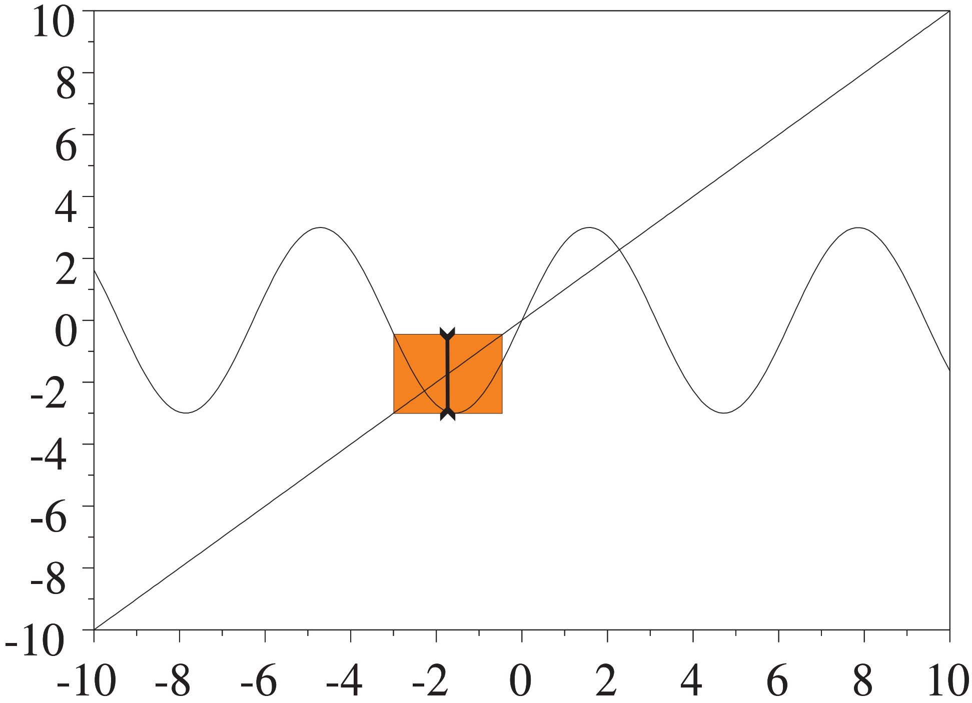
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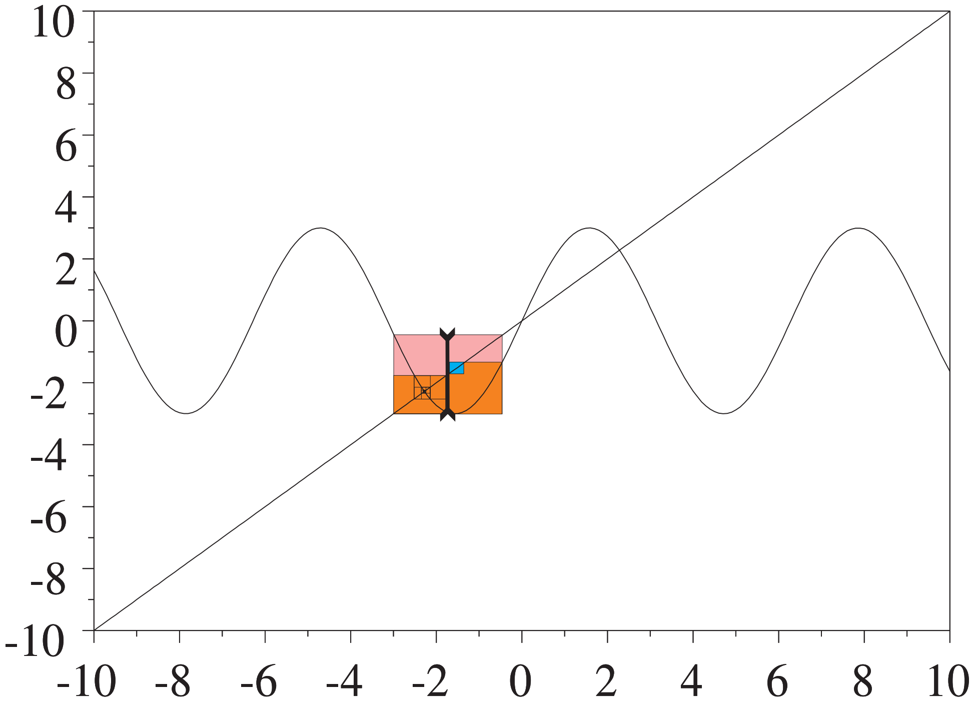
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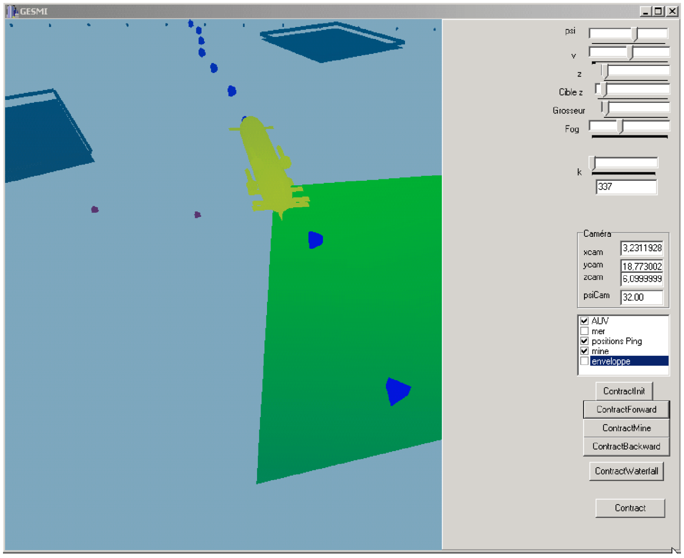
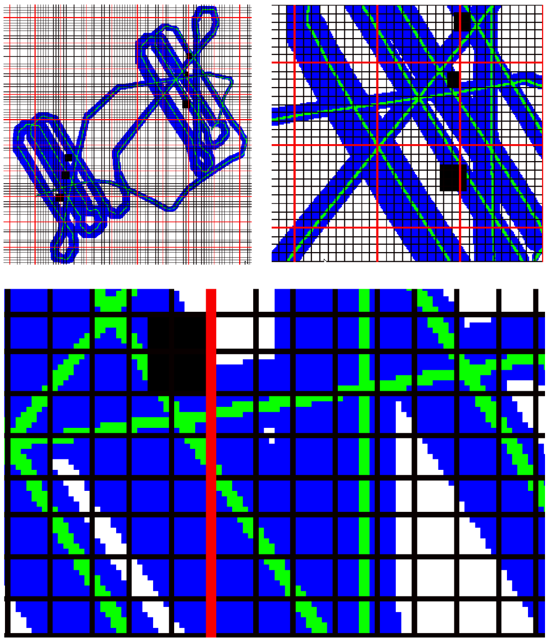
(Illustration with Proj2D)

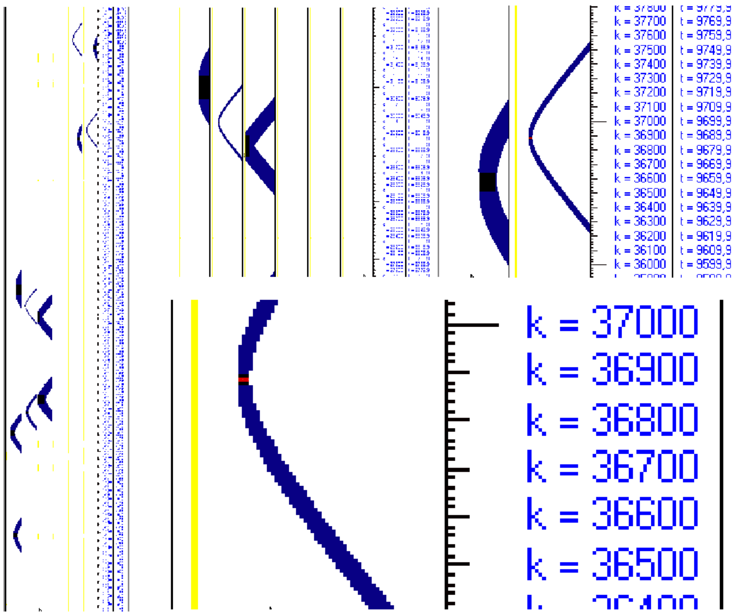
8 GESMI



GESMI (Guaranteed Estimation of Sea Mines with Intervals)



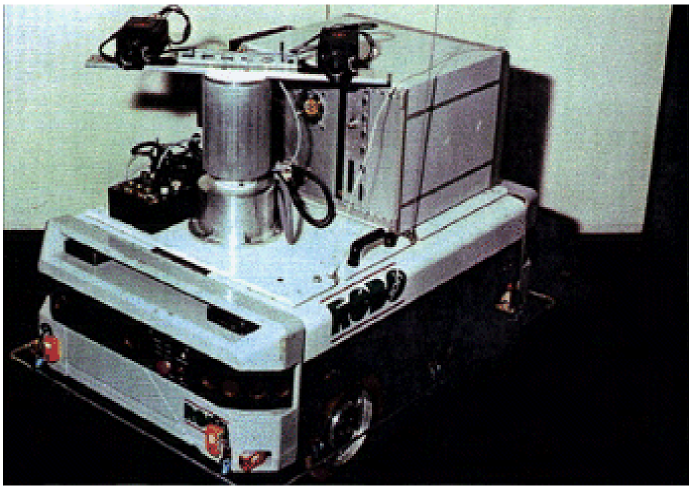
Trajectory reconstructed by GESMI

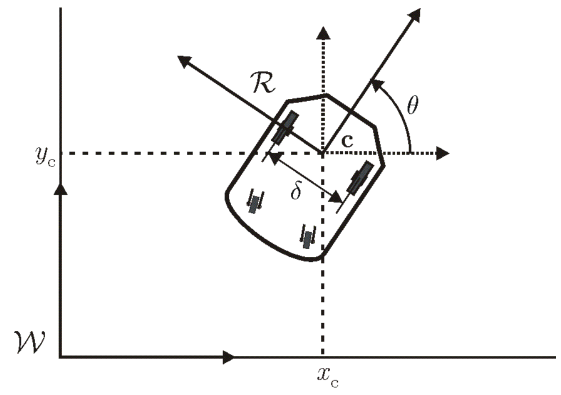


Waterfall reconstructed by GESMI

9 Localization of a wheeled robot

Robot equipped with 24 ultrasonic telemeters





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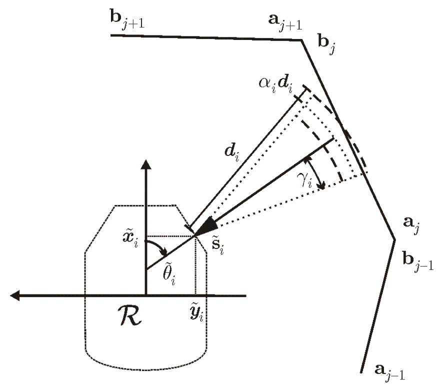

$\mathbf{b}_{j+1}$
$\mathbf{a}_{j+1}$
$\mathbf{b}_j$
$\alpha_i \boldsymbol{d}_i$
$\boldsymbol{d}_i$
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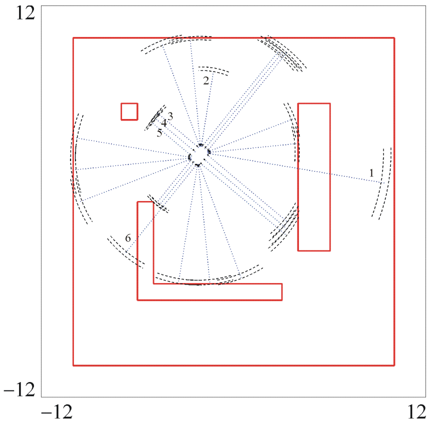
$\tilde{\mathbf{s}}_i$
$\tilde{\theta}_i$
$\mathcal{R}$
$\tilde{\boldsymbol{y}}_i$
$\mathbf{a}_{j-1}$



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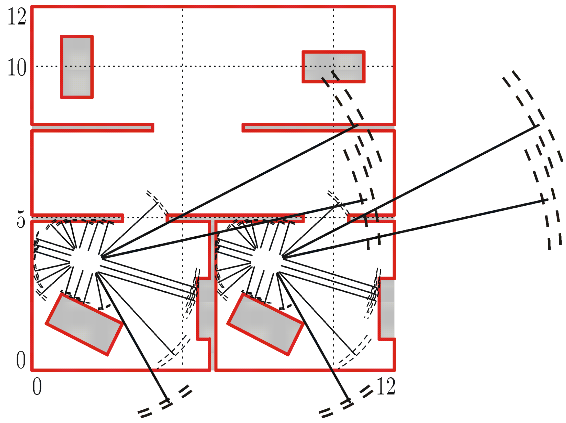

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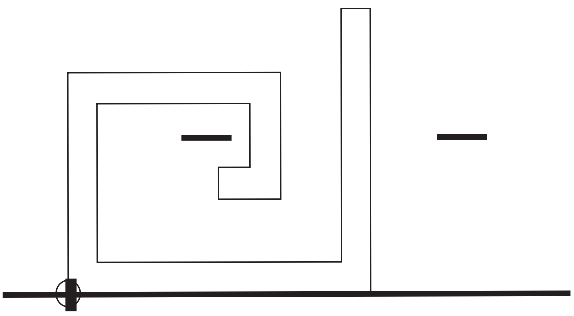
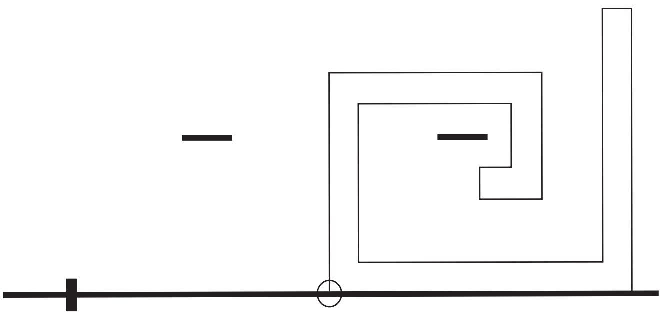
12

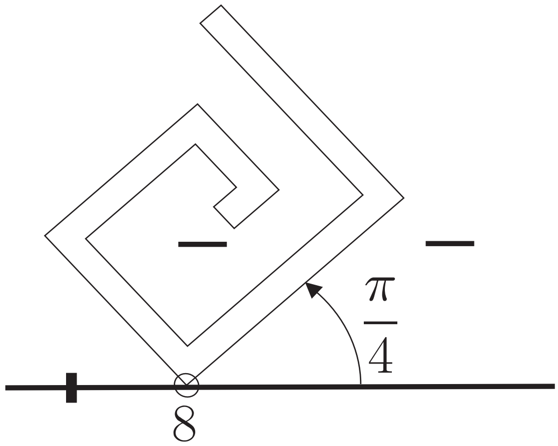
9.1 Path planning



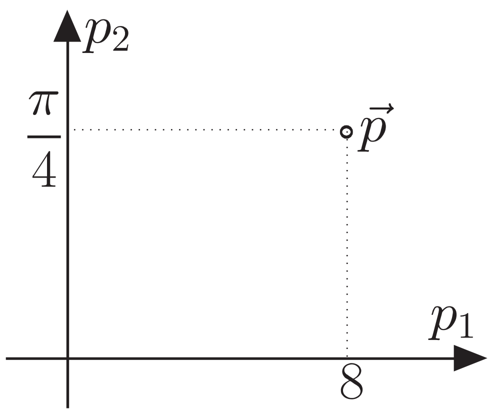
Initial configuration: $\vec{p} = (0\ 0)^{\mathrm{T}}$



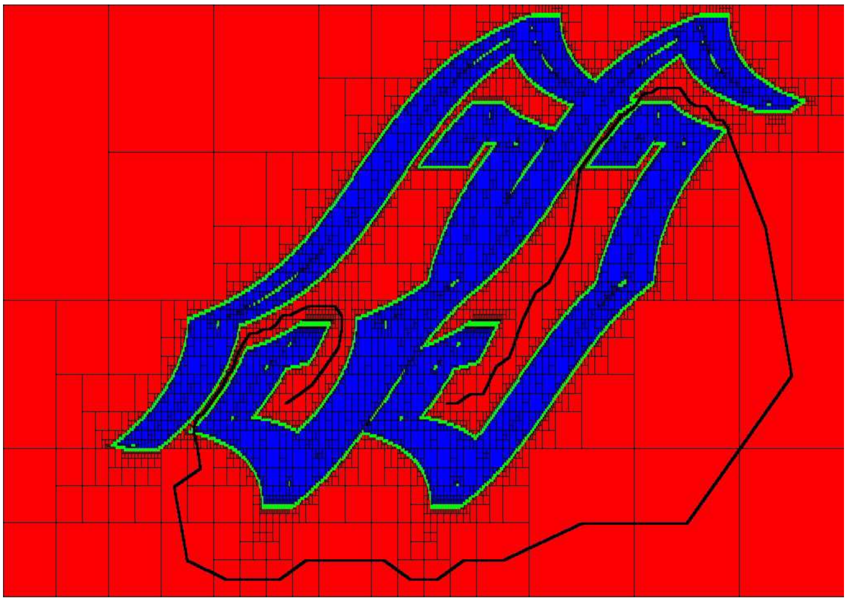
Goal configuration: $\vec{p} = (17\ 0)^{\mathrm{T}}$

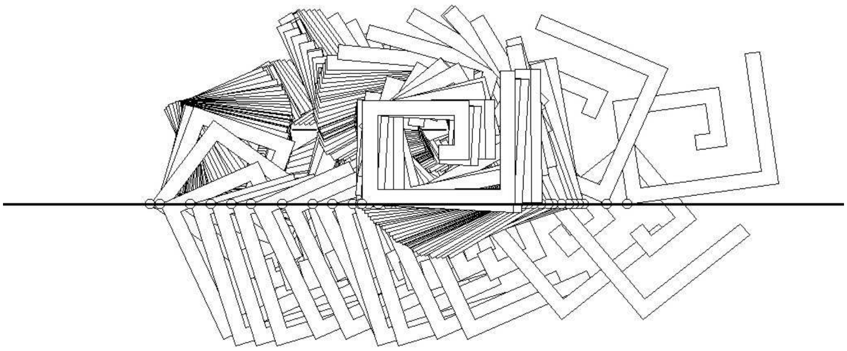


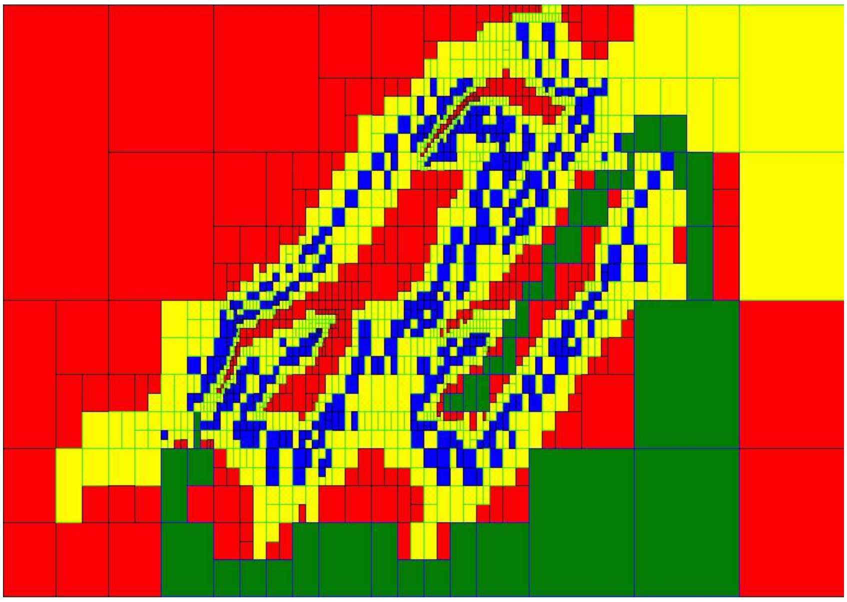
Room

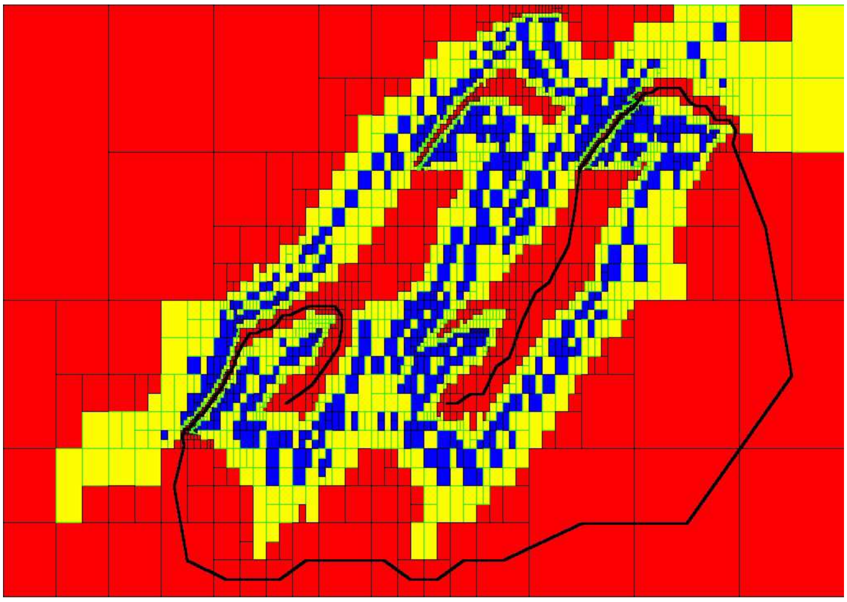


Configuration space









10 Sailboat control

10.1 State equations

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta - \beta V \\ \dot{\theta} = \omega \\ \dot{\delta}_s = u_1 \\ \dot{\delta}_r = u_2 \\ \dot{v} = \frac{f_s \sin\delta_s - f_r \sin\delta_r - \alpha_f v}{m} \\ \dot{\omega} = \frac{(\ell - r_s\cos\delta_s) f_s - r_r \cos\delta_r f_r - \alpha_\theta \omega}{J} \\ f_s = \alpha_s \left(V\cos\left(\theta + \delta_s\right) - v\sin\delta_s\right) \\ f_r = \alpha_r v \sin\delta_r. \end{cases}$$

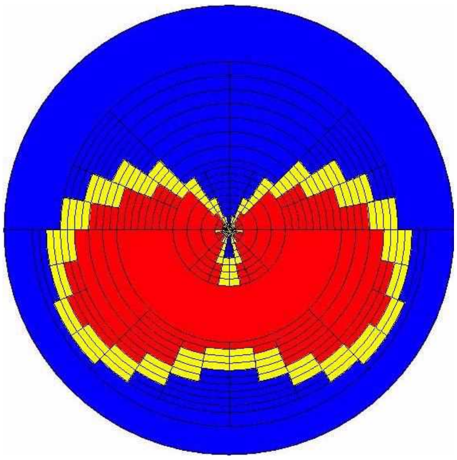
10.2 Is it possible to stop the robot ?

$$
\left\{
\begin{array}{rcl}
0 & = & v\cos\theta \\
0 & = & v\sin\theta - \beta V \\
0 & = & \omega \\
0 & = & u_1 \\
0 & = & u_2 \\
0 & = & \frac{f_s \sin\delta_s - f_r \sin\delta_r - \alpha_f v}{m} \\
0 & = & \frac{(\ell - r_s\cos\delta_s) f_s - r_r \cos\delta_r f_r - \alpha_\theta \omega}{J} \\
f_s & = & \alpha_s \left(V\cos(\theta + \delta_s) - v\sin\delta_s \right) \\
f_r & = & \alpha_r v \sin\delta_r .
\end{array}
\right.
$$

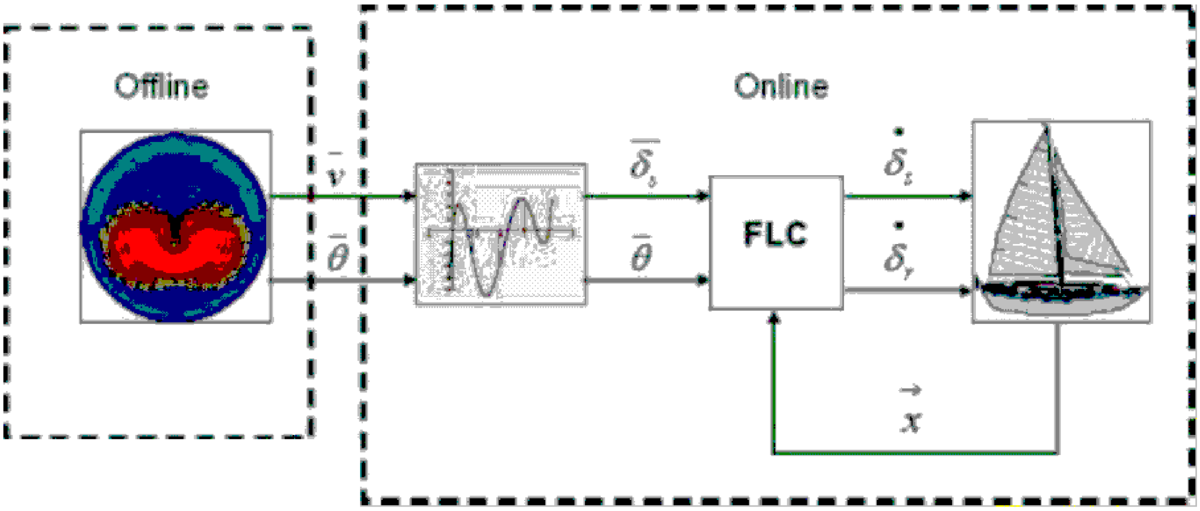
Interval techniques show that for $V \neq 0$, this is not possible

10.3 Polar speed diagram

$$
\begin{aligned}
\mathbb{W} = \{ \quad (\theta, v) \mid \quad & \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\
& \omega = 0, u_1 = 0, u_2 = 0 \\
& \frac{f_s \sin\delta_s - f_r \sin\delta_r - \alpha_f v}{m} = 0 \\
& \frac{(\ell - r_s \cos\delta_s) f_s - r_r \cos\delta_r f_r}{J} = 0 \\
& f_s = \alpha_s \left(V \cos\left(\theta + \delta_s\right) - v \sin\delta_s\right) \\
& f_r = \alpha_r v \sin\delta_r \qquad \}.
\end{aligned}
$$



10.4 Control



Offline
Online
$\bar{v}$
$\bar{\theta}$
$\bar{\delta}_s$
$\bar{\theta}$
FLC
$\dot{\delta}_s$
$\dot{\delta}_r$
$\vec{x}$