Intervals analysis for sea robotics

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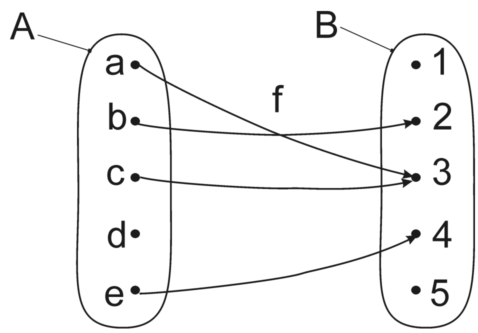
1 Interval approach

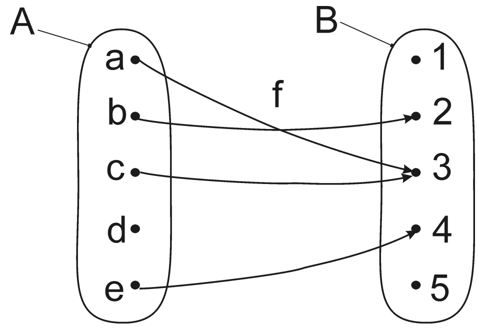
1.1 Basic notions on set theory

Exercise: If f is defined as follows



$$
\begin{aligned}
f(A) &= ?. \\
f^{-1}(B) &= ?. \\
f^{-1}(f(A)) &= ? \\
f^{-1}(f(\{b,c\})) &= ?.
\end{aligned}
$$

Exercise: If f is defined as follows



$$\begin{aligned} f(A) &= \{2,3,4\} = \mathsf{Im}(f). \\ f^{-1}(B) &= \{a,b,c,e\} = \mathsf{dom}(f). \\ f^{-1}(f(A)) &= \{a,b,c,e\} \subset A \\ f^{-1}(f(\{b,c\})) &= \{a,b,c\}. \end{aligned}$$

Exercise: If $f(x) = x^2$, then

$$\begin{aligned} f([2,3]) &= \text{?} \\ f^{-1}([4,9]) &= \text{?}. \end{aligned}$$

Exercise: If $f(x) = x^2$, then

$$\begin{aligned} f([2,3]) &= [4,9] \\ f^{-1}([4,9]) &= [-3,-2] \cup [2,3]. \end{aligned}$$

This is consistent with the property

$$f\left(f^{-1}(\mathbb{Y})\right) \subset \mathbb{Y}.$$

1.2 Interval arithmetic

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \, .$$

For instance,

$$\begin{array}{rl} [-1, 3] + [2, 5] & = [?, ?], \\ [-1, 3].[2, 5] & = [?, ?], \\ [-2, 6]/[2, 5] & = [?, ?]. \end{array}$$

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \, .$$

For instance,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3].[2, 5] &= [-5, 15], \\ [-2, 6]/[2, 5] &= [-1, 3]. \end{aligned}$$

$$
\begin{aligned}
[x^-, x^+] + [y^-, y^+] = \; & [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] = \; & [x^- y^- \wedge x^+ y^- \wedge x^- y^+ \wedge x^+ y^+, \\
& x^- y^- \vee x^+ y^- \vee x^- y^+ \vee x^+ y^+],
\end{aligned}
$$

If $f \in \{\text{cos, sin,sqr, sqrt, log, exp}, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] \,.$$

For instance,

$$\begin{aligned}
\text{sin}([0,\pi]) &= ?, \\
\text{sqr}([-1,3]) &= [-1,3]^2 = ?, \\
\text{abs}([-7,1]) &= ?, \\
\text{sqrt}([-10,4]) &= \sqrt{[-10,4]} = ?, \\
\text{log}([-2,-1]) &= ?.
\end{aligned}$$

If $f \in \{\text{cos}, \text{sin}, \text{sqr}, \text{sqrt}, \text{log}, \text{exp}, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] .$$

For instance,

$$\begin{array}{rcl} \text{sin}([0,\pi]) & = & [0,1], \\ \text{sqr}([-1,3]) & = & [-1,3]^2 = [0,9], \\ \text{abs}([-7,1]) & = & [0,7], \\ \text{sqrt}([-10,4]) & = & \sqrt{[-10,4]} = [0,2], \\ \text{log}([-2,-1]) & = & \emptyset. \end{array}$$

1.3 Boxes

A *box*, or *interval vector* $[\mathbf{x}]$ of $\mathbb{R}^n$ is

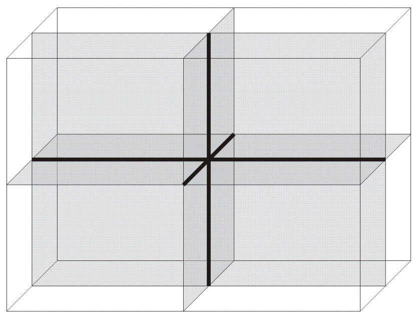
$$[\mathbf{x}] = [x_1^-, x_1^+] \times \cdots \times [x_n^-, x_n^+] = [x_1] \times \cdots \times [x_n].$$

The set of all boxes of $\mathbb{R}^n$ will be denoted by $\mathbb{IR}^n$.

The *width* $w([\mathbf{x}])$ of a box $[\mathbf{x}]$ is the length of its largest side. For instance

$$w([1,2]\times[-1,3]) = 4$$

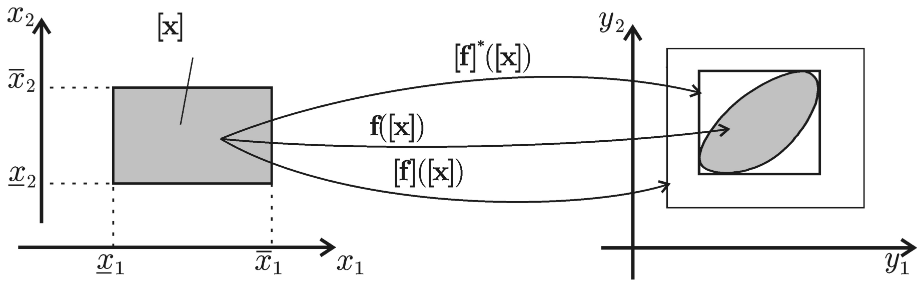
The *principal plane* of $[\mathbf{x}]$ is the symmetric plane $[\mathbf{x}]$ perpendicular to its largest side.



1.4 Inclusion function

The interval function $[\mathbf{f}]$ from $\mathbb{IR}^n$ to $\mathbb{IR}^m$, is an *inclusion function* of $\mathbf{f}$ if

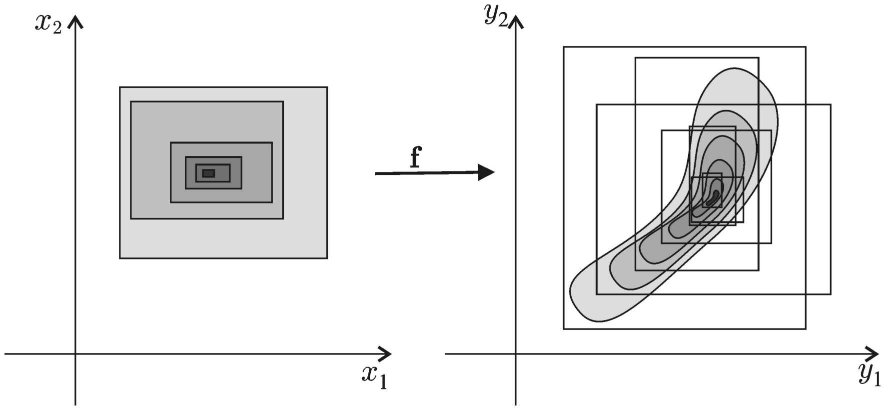
$$\forall\, [\mathbf{x}] \in \mathbb{IR}^n, \quad \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]\,([\mathbf{x}]).$$



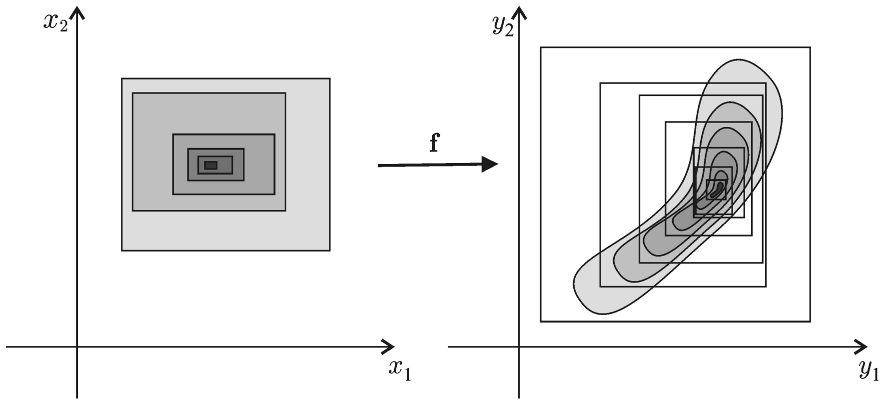
Inclusion functions $[\mathbf{f}]$ and $[\mathbf{f}]^*$; here, $[\mathbf{f}]^*$ is minimal.

The inclusion function $[\mathbf{f}]$ is

monotonic	if	$([\mathbf{x}] \subset [\mathbf{y}]) \Rightarrow ([\mathbf{f}]([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{y}]))$
minimal	if	$\forall [\mathbf{x}] \in \mathbb{IR}^n,\ [\mathbf{f}]([\mathbf{x}]) = [\mathbf{f}([\mathbf{x}])]$
thin	if	$w([\mathbf{x}]) = 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}]) = 0$
convergent	if	$w([\mathbf{x}]) \rightarrow 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}]) \rightarrow 0.$



Convergent but non-monotonic inclusion function



Convergent and monotonic inclusion function

The natural inclusion function for $f(x) = x^2 + 2x + 4$ is

$$[f]([x]) = [x]^2 + 2[x] + 4.$$

If $[x] = [-3, 4]$, we have

$$\begin{aligned}[f]([-3,4]) &= [-3,4]^2 + 2[-3,4] + 4 \\ &= [0,16] + [-6,8] + 4 \\ &= [-2,28].\end{aligned}$$

Note that $f([-3,4]) = [3,28] \subset [f]([-3,4]) = [-2,28]$.

A minimal inclusion function for

$$\mathbf{f}: \begin{array}{rcl} \mathbb{R}^2 & \longrightarrow & \mathbb{R}^3 \\ (x_1, x_2) & \mapsto & \left(x_1 x_2, x_1^2, x_1 - x_2\right). \end{array}$$

is

$$[\mathbf{f}]: \begin{array}{rcl} \mathbb{IR}^2 & \rightarrow & \mathbb{IR}^3 \\ ([x_1], [x_2]) & \rightarrow & \left([x_1] * [x_2], [x_1]^2, [x_1] - [x_2]\right). \end{array}$$

If $\mathbf{f}$ is given by the algorithm

Algorithm $\mathbf{f}$(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: $\mathbf{y} = (y_1, y_2)$)

$z := x_1$;
for $k := 0$ to 100
    $z := x_2(z + kx_3)$;
next;
$y_1 := z$;
$y_2 := \sin(zx_1)$;

Its natural inclusion function is

Algorithm $[\mathbf{f}]$(in: $[\mathbf{x}]$, out: $[\mathbf{y}]$)

```
[z] := [x1];
for k := 0 to 100
    [z] := [x2] * ([z] + k * [x3]);
next;
[y1] := [z] ;
[y2] := sin([z] * [x1]);
```

Here, $[\mathbf{f}]$ is a convergent, thin and monotonic inclusion function for $\mathbf{f}$.

1.5 Subpavings

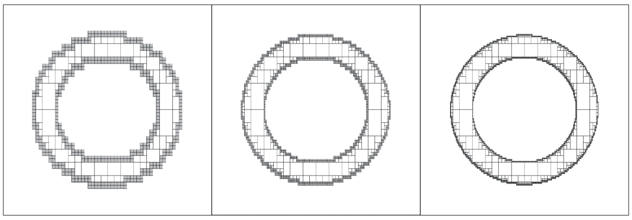
A subpaving of $\mathbb{R}^n$ is a set of non-overlapping boxes of $\mathbb{R}^n$.

Compact sets $\mathbb{X}$ can be bracketed between inner and outer subpavings:

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Example.

$$\mathbb{X} = \{(x_1, x_2) \,\big|\, x_1^2 + x_2^2 \in [1, 2]\}.$$



Set operations such as $\mathbb{Z} := \mathbb{X} + \mathbb{Y}$, $\mathbb{X} := \mathbf{f}^{-1}(\mathbb{Y})$, $\mathbb{Z} := \mathbb{X} \cap \mathbb{Y} \ldots$ can be approximated by subpaving operations.

1.6 Set inversion

If $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ and if $\mathbb{Y}$ is a subset of $\mathbb{R}^m$. Set inversion is the characterization of

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}).$$

We shall use the following tests.

$$\begin{array}{llcl}
\text{(i)} & [\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y} & \Rightarrow & [\mathbf{x}] \subset \mathbb{X} \\
\text{(ii)} & [\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset & \Rightarrow & [\mathbf{x}] \cap \mathbb{X} = \emptyset.
\end{array}$$

Boxes for which these tests failed, will be bisected, except if they are too small.

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Algorithm Sivia(in: $[\mathbf{x}](0), \mathbf{f}, \mathbb{Y}$)

1 $\mathcal{L} := \{[\mathbf{x}](0)\}$;
2 pull $[\mathbf{x}]$ from $\mathcal{L}$;
3 if $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y}$, draw($[\mathbf{x}]$, 'red');
4 elseif $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset$, draw($[\mathbf{x}]$, 'blue');
5 elseif $w([\mathbf{x}]) < \varepsilon$, {draw ($[\mathbf{x}]$, 'yellow')};
6 else bisect $[\mathbf{x}]$ and push into $\mathcal{L}$;
7 if $\mathcal{L} \neq \emptyset$, go to 2

If $\Delta\mathbb{X}$ denotes the union of yellow boxes and if $\mathbb{X}^-$ is the union of red boxes then :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^- \cup \Delta\mathbb{X}.$$

2 Bounded-error estimation

Model: $\phi(\mathbf{p}, t) = p_1 e^{-p_2 t}$.

Prior feasible box for the parameters: $[\mathbf{p}] \subset \mathbb{R}^2$

Measurement times : $t_1, t_2, \ldots, t_m$

Data bars : $[y_1^-, y_1^+], [y_2^-, y_2^+], \ldots, [y_m^-, y_m^+]$

$$\mathbb{S} = \{\mathbf{p} \in [\mathbf{p}], \phi(\mathbf{p}, t_1) \in [y_1^-, y_1^+], \ldots, \phi(\mathbf{p}, t_m) \in [y_m^-, y_m^+]\}.$$

If

$$\boldsymbol{\phi}(\mathbf{p}) = \begin{pmatrix} \phi(\mathbf{p}, t_1) \\ \\ \phi(\mathbf{p}, t_m) \end{pmatrix}$$

and

$$[\mathbf{y}] = [y_1^-, y_1^+] \times \cdots \times [y_m^-, y_m^+]$$

then

$$\mathbb{S} = [\mathbf{p}] \cap \boldsymbol{\phi}^{-1}([\mathbf{y}]).$$

3 Sailboat

3.1 State equations

$$
\begin{cases}
\dot{x} & = & v \cos\theta \\
\dot{y} & = & v \sin\theta - 1 \\
\dot{\theta} & = & \omega \\
\dot{\delta}_s & = & u_1 \\
\dot{\delta}_r & = & u_2 \\
\dot{v} & = & f_s \sin\delta_s - f_r \sin\delta_r - v \\
\dot{\omega} & = & (1 - \cos\delta_s)\, f_s - \cos\delta_r . f_r - \omega \\
f_s & = & \cos(\theta + \delta_s) - v \sin\delta_s \\
f_r & = & v \sin\delta_r .
\end{cases}
$$

In a cruising phase

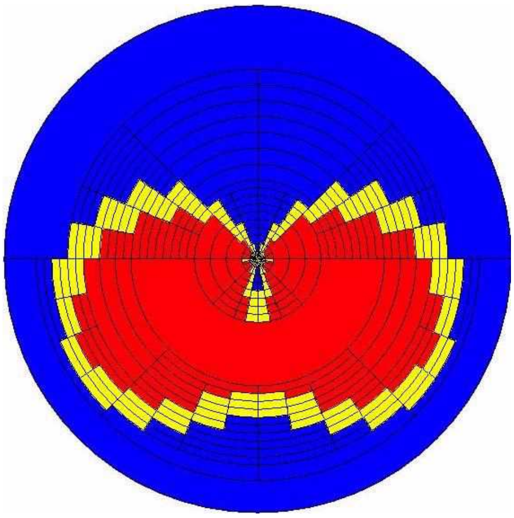
$$\dot{\theta} = 0, \dot{\delta}_s = 0, \dot{\delta}_r = 0, \dot{v} = 0, \dot{\omega} = 0.$$

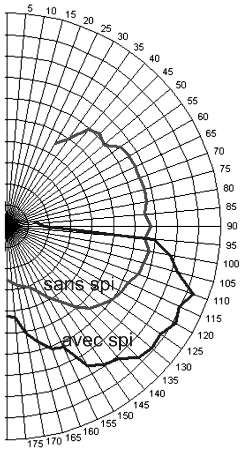
i.e.,

$$\begin{cases} 0 &= \omega \\ 0 &= u_1 \\ 0 &= u_2 \\ 0 &= f_s \sin \delta_s - f_r \sin \delta_r - v \\ 0 &= (1 - \cos \delta_s) f_s - \cos \delta_r . f_r - \omega \\ f_s &= \cos(\theta + \delta_s) - v \sin \delta_s \\ f_r &= v \sin \delta_r . \end{cases}$$

The polar diagram is

$$\begin{aligned}\mathbb{S}_y = \; \{(\theta, v) \quad & \mid \exists f_s, \delta_s, f_r, \delta_r, \\ & f_s \sin\delta_s - f_r \sin\delta_r - v = 0 \\ & (1 - \cos\delta_s)\, f_s - \cos\delta_r f_r = 0 \\ & f_s = \cos(\theta + \delta_s) - v \sin\delta_s \\ & f_r = v \sin\delta_r \;\}\end{aligned}$$

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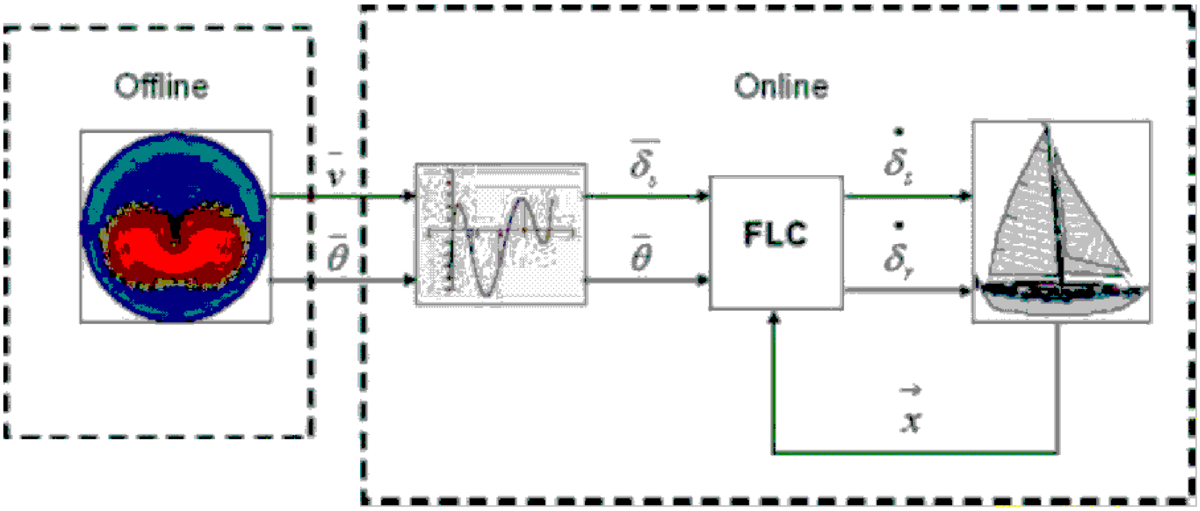
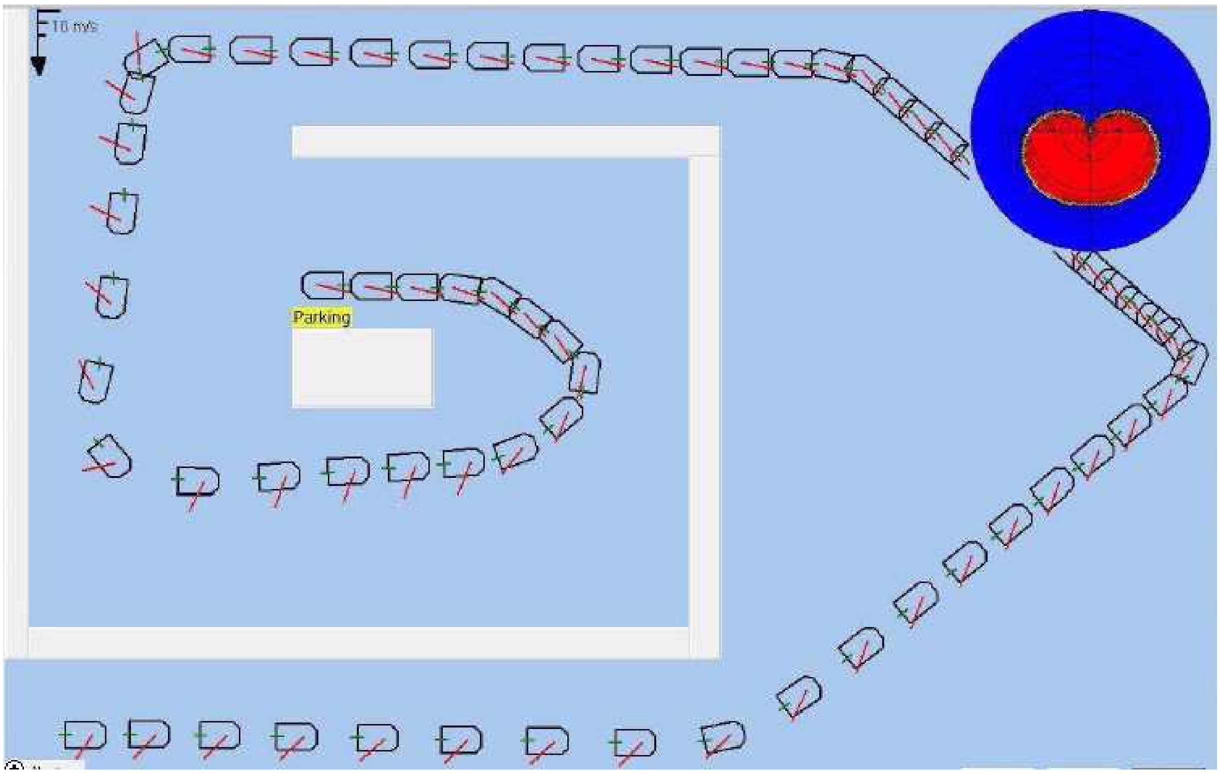
3.2 Control

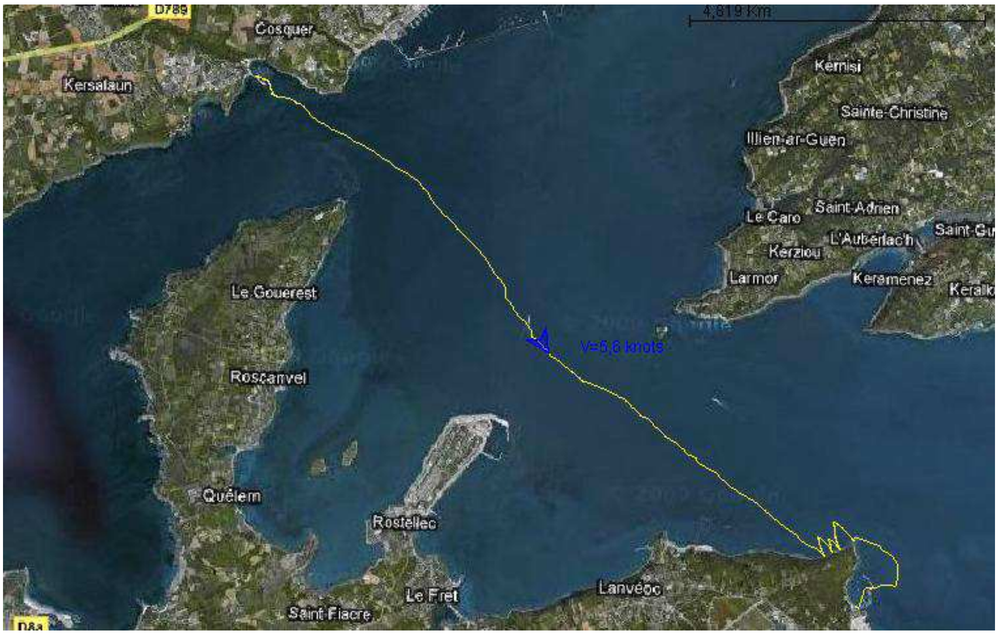




10 m/s
Parking

3.3 Breizh Spirit


BREIZH
ENSIETA
SPIRIT



D789
Cosquer
Kersalaun
4,819 Km
Kernisi
Sainte-Christine
Illien-ar-Guen
Saint-Adrien
Le Caro
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
V=5,6 knots
Roscanvel
Quélem
Rostellec
Le Fret
Saint-Fiacre
Lanvéoc

Voilier autonome. La rade avant la transatlantique

Avant le grand bain, il y a le petit. Le voilier miniature autonome concocté à l'Ensieta a traversé avec succès la rade, en début de semaine. L'idée : réussir un jour une transatlantique.

Une partie de l'équipe : Kostia Poncin, Richard Leloup, Luc Jaulin, Bruno Auzier et Jan Sliwka. Manque Pierre-Henri Reilhac.

Lundi, Breizh-Spirit – c'est son nom – est parti de Saint-Anne-du-Portzic et a rejoint Lanvéoc, soit 12 km en deux heures environ. Il était tout seul, autonome, accompagné à distance, sur un semi-rigide, de ses « parents », une petite équipe d'étudiants et d'enseignants de l'Ensieta. Une traversée réalisée en collaboration avec l'École navale.

De beaucoup, Breizh-Spirit est sans doute resté inaperçu. Il ne fait qu'1,30 m de long pour 10 kg. Mais il a avancé vaillamment, à 3,1 nœuds de moyenne, au près, ce qui n'était pas la configuration idéale. En pointe, il a atteint 5,5 nœuds.

Premier test à la mer près de Porto

L'idée a pris corps en 2005. Luc Jaulin, professeur en automatique-robotique à l'Ensieta, était alors président du jury, à Toulouse, de la première Microtransat. L'objectif, pour une traversée de l'Atlantique, a été fixé à 2010.

Breiz-Spirit a lui-même mûri l'année passée. Richard Leloup, alors en première année, se souvient avoir fabriqué la coque durant les vacances de Noël. D'autres ont apporté leur pierre en électronique, informatique, mécanique, robotique et architecture navale, des compétences qui existent à l'école et que des projets, tels que Breiz-Spirit, permettent de mixer autour d'un objectif à atteindre.

Cet été, le mini-voilier a participé, près de Porto, à la « World robotic sailing championship », premier test à la mer pour lui ; l'occasion aussi de se comparer. Onze bateaux, fort divers, étaient au rendez-vous. Il y avait là aussi des Anglais, des Suisses, des Portugais et des Américains.

Une compétition en septembre 2010

L'équipe de l'Ensieta a en ligne de mire 2010 avec une compétition, en juin, probablement au Canada. Le départ de la fameuse transatlantique pourrait avoir lieu, en septembre, depuis l'Irlande. La traversée risque alors de durer cinq mois...

Pour l'heure, l'équipe de Breizh-Spirit va travailler à améliorer le mini-voilier, rendre plus robuste l'électronique, le gréement et les voiles. Étanchéifier la coque, implanter des panneaux solaires, se passer de la girouette sont aussi au programme. Il est prévu que les bateaux puissent communiquer chaque jour leur position à terre. Normalement, aucun voilier de cette future transat en autonomie ne doit dépasser les 4 m, des « Petits Poucet » comparés aux porte-conteneurs géants...

Montrer une vidéo

4 Contractors

To characterize $\mathbb{X} \subset \mathbb{R}^n$, bisection algorithms bisect all boxes in all directions and become inefficient. Interval methods can still be useful if

- the solution set $\mathbb{X}$ is small (optimization problem, solving equations),

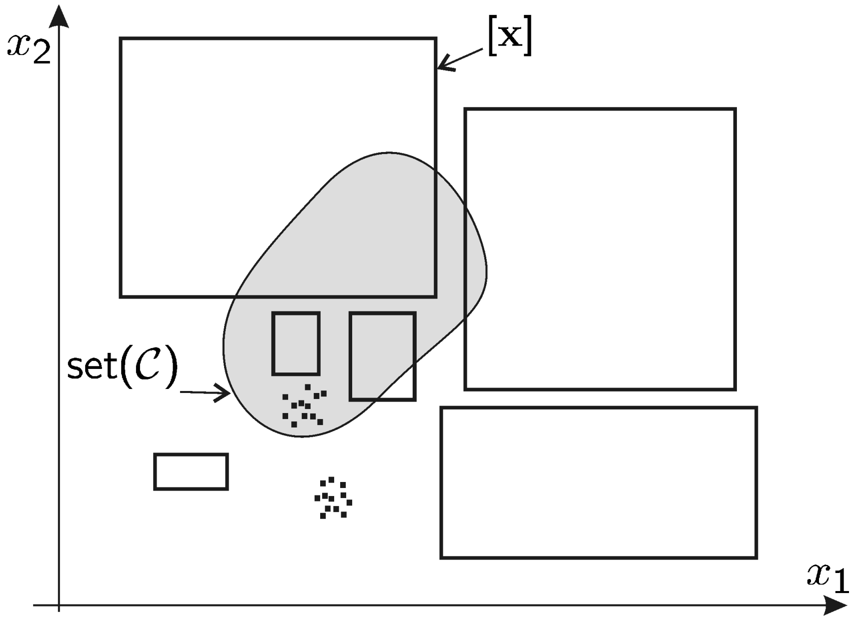
- contraction procedures are used as much as possible,

- bisections are used only as a last resort.

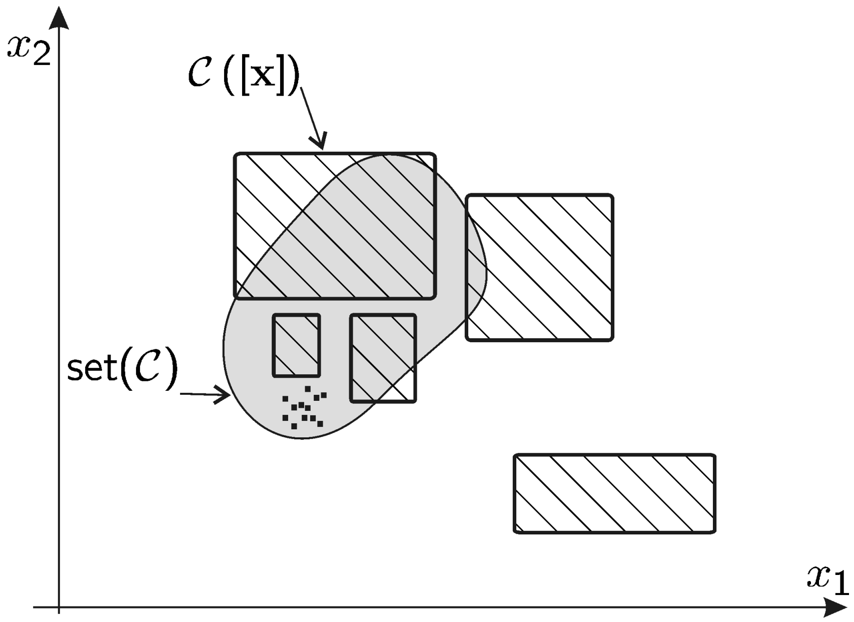
4.1 Definition

The operator $\mathcal{C}_{\mathbb{X}} : \mathbb{IR}^n \to \mathbb{IR}^n$ is a *contractor* for $\mathbb{X} \subset \mathbb{R}^n$ if

$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \left\{ \begin{array}{ll} \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance),} \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(completeness).} \end{array} \right.$$

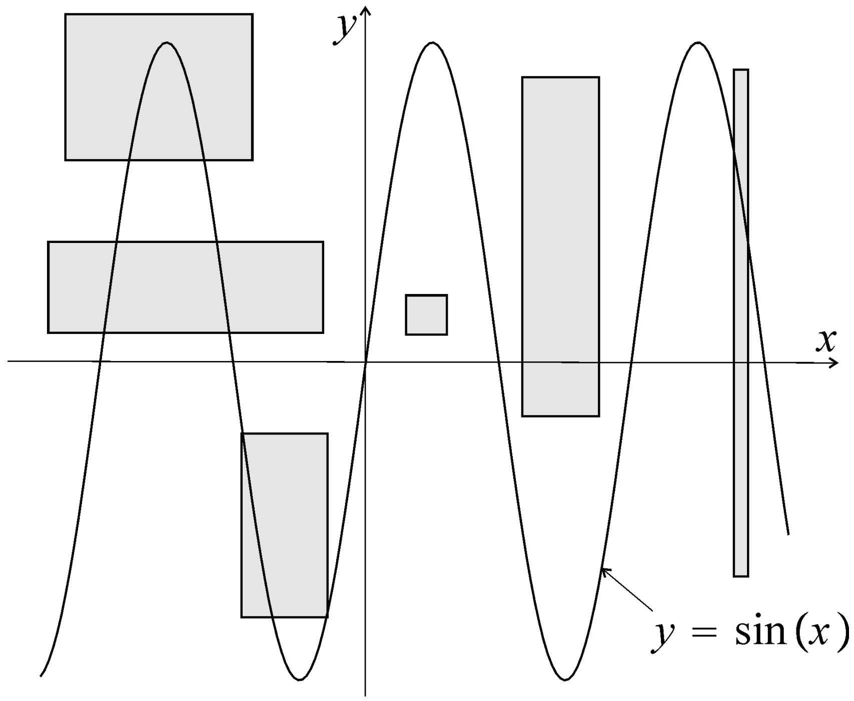


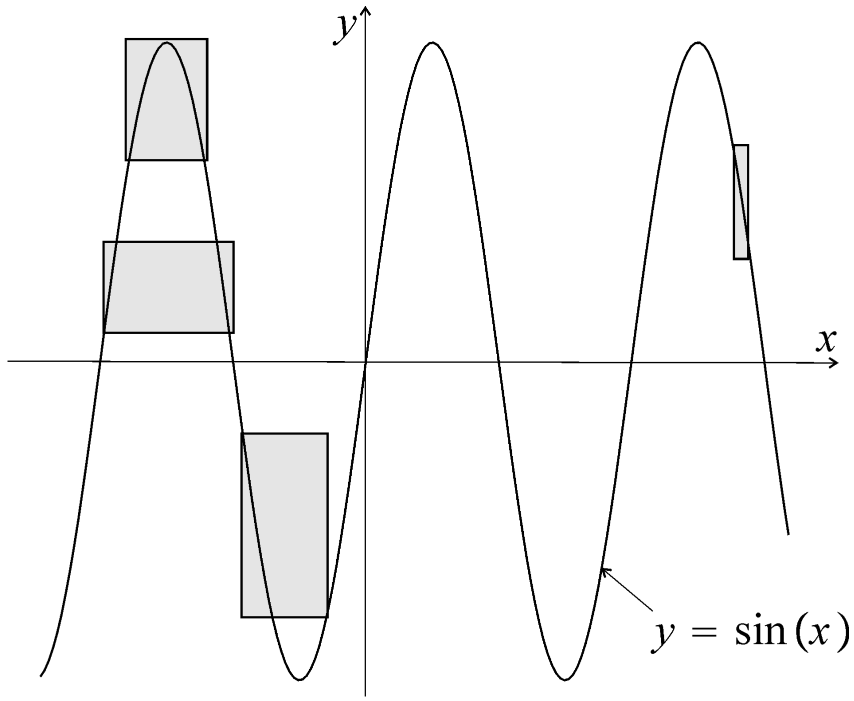
x_2
[x]
set($\mathcal{C}$)
x_1


x_2
$\mathcal{C}([\mathbf{x}])$
set$(\mathcal{C})$
x_1

The operator $\mathcal{C} : \mathbb{IR}^n \rightarrow \mathbb{IR}^n$ is a *contractor* for the equation $f(\mathbf{x}) = 0$, if

$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \left\{ \begin{array}{l} \mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] \\ \mathbf{x} \in [\mathbf{x}] \text{ et } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathcal{C}([\mathbf{x}]) \end{array} \right.$$


y
x
y = sin(x)



y
x
y = sin(x)

$\mathcal{C}_{\mathbb{X}}$ is *monotonic* if	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset \mathcal{C}_{\mathbb{X}}([\mathbf{y}])$
$\mathcal{C}_{\mathbb{X}}$ is *minimal* if	$\forall [\mathbf{x}] \in \mathbb{IR}^n,\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathbb{X}]$
$\mathcal{C}_{\mathbb{X}}$ is *thin* if	$\forall \mathbf{x} \in \mathbb{R}^n,\ \mathcal{C}_{\mathbb{X}}(\{\mathbf{x}\}) = \{\mathbf{x}\} \cap \mathbb{X}$
$\mathcal{C}_{\mathbb{X}}$ is *idempotent* if	$\forall [\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}\left(\mathcal{C}_{\mathbb{X}}([\mathbf{x}])\right) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}])$.

intersection	$(\mathcal{C}_1 \cap \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1([\mathbf{x}]) \cap \mathcal{C}_2([\mathbf{x}])$
union	$(\mathcal{C}_1 \cup \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} [\mathcal{C}_1([\mathbf{x}]) \cup \mathcal{C}_2([\mathbf{x}])]$
composition	$(\mathcal{C}_1 \circ \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1(\mathcal{C}_2([\mathbf{x}]))$
répétition	$\mathcal{C}^\infty \overset{\text{def}}{=} \mathcal{C} \circ \mathcal{C} \circ \mathcal{C} \circ \dots$

$\mathcal{C}_{\mathbb{X}}$ is said to be *convergent* if

$$[\mathbf{x}](k) \to \mathbf{x} \quad \Rightarrow \quad \mathcal{C}_{\mathbb{X}}([\mathbf{x}]\,(k)) \to \{\mathbf{x}\} \cap \mathbb{X}.$$

4.2 Projection of constraints

Let x, y, z be 3 variables such that

$$\begin{aligned} x &\in [-\infty, 5], \\ y &\in [-\infty, 4], \\ z &\in [6, \infty], \\ z &= x + y. \end{aligned}$$

To *project* a constraint (here, $z = x + y$), is to compute the smallest intervals which contains all consistent values.

For our example, this amounts to project onto x, y and z the set

$$\mathbb{S} = \{(x, y, z) \in [-\infty, 5] \times [-\infty, 4] \times [6, \infty] \mid z = x + y\} .$$

4.3 Numerical method for projection

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}$$

The contractor associated with $z = x + y$ is.

Algorithm pplus(inout: $[z], [x], [y]$)

$[z] := [z] \cap ([x] + [y])$;
$[x] := [x] \cap ([z] - [y])$;
$[y] := [y] \cap ([z] - [x])$.

The projection procedure developed for plus can be extended to other ternary constraints such as mult: $z = x*y$, or equivalently

$$\text{mult} \triangleq \left\{(x, y, z) \in \mathbb{R}^3 \mid z = x * y\right\}.$$

The resulting projection procedure becomes

Algorithm pmult(inout: $[z], [x], [y]$)
1 $[z] := [z] \cap ([x] * [y])$;
2 $[x] := [x] \cap ([z] * 1/[y])$;
3 $[y] := [y] \cap ([z] * 1/[x])$.

Consider the binary constraint

$$\exp \triangleq \{(x, y) \in \mathbb{R}^n | y = \exp(x)\}.$$

The associated contractor is

Algorithm pexp(inout: $[y], [x]$)
1 $[y] := [y] \cap \exp([x])$;
2 $[x] := [x] \cap \log([y])$.

4.4 Solvers

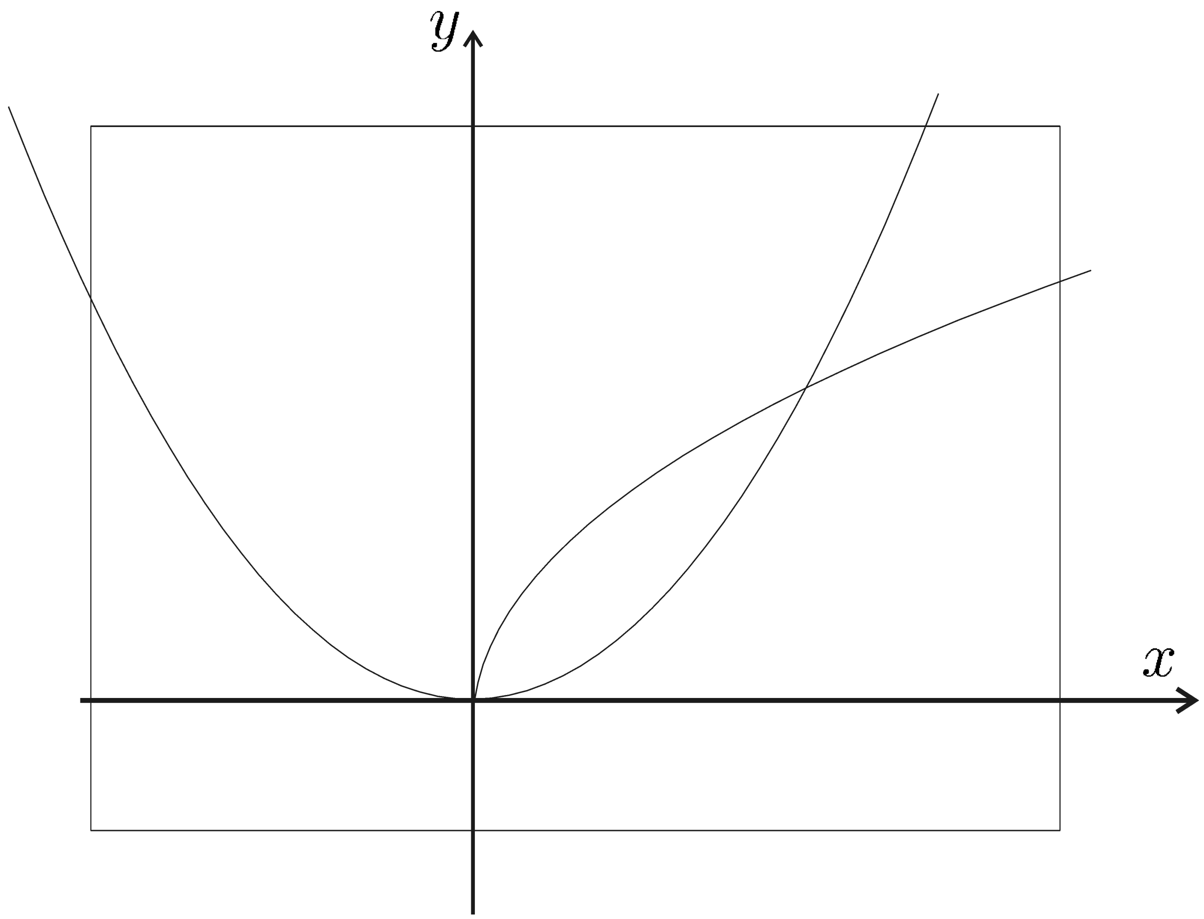
Example. Consider the system.

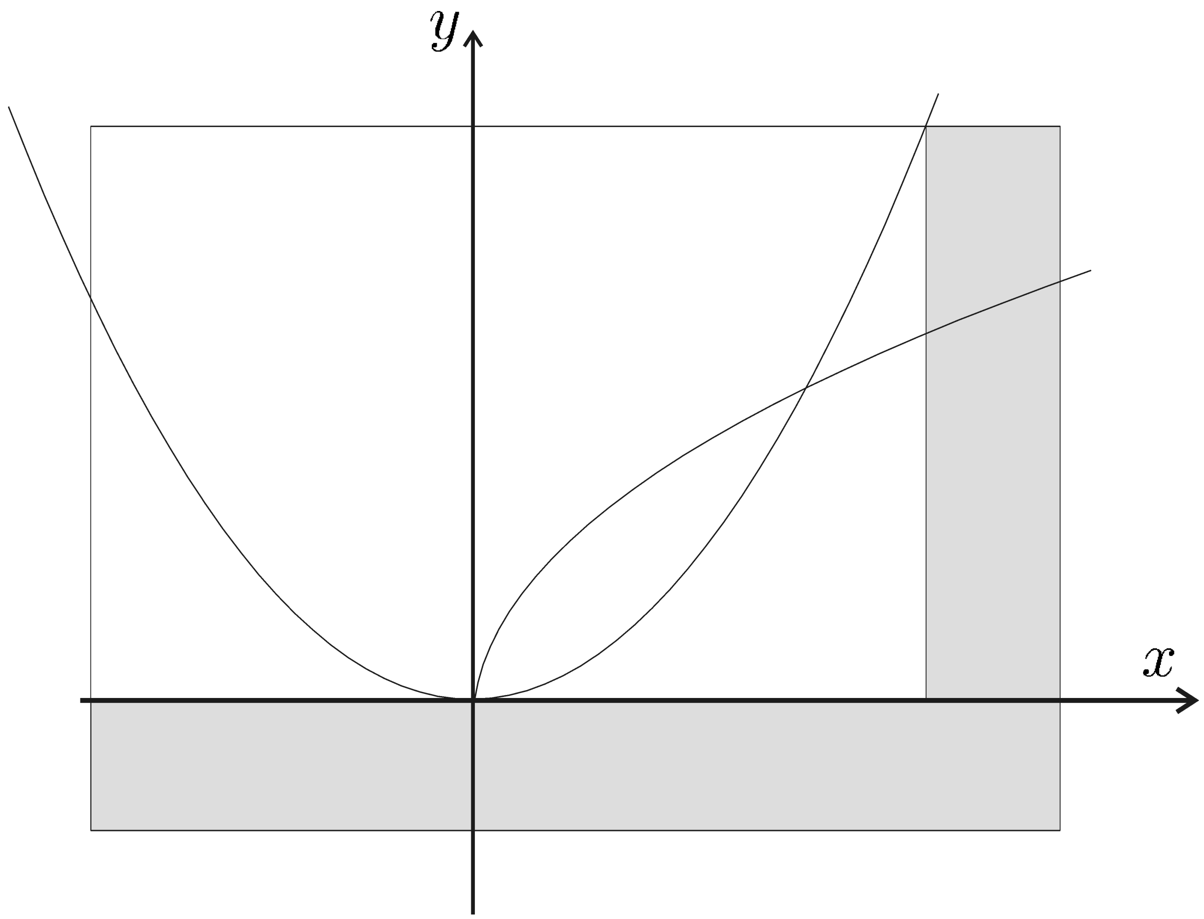
$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

We build two contractors

$$\mathcal{C}_1 : \left\{ \begin{array}{l} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{array} \right. \quad \text{associated to } y = x^2$$

$$\mathcal{C}_2 : \left\{ \begin{array}{l} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{array} \right. \quad \text{associated to } y = \sqrt{x}$$


y
x


y
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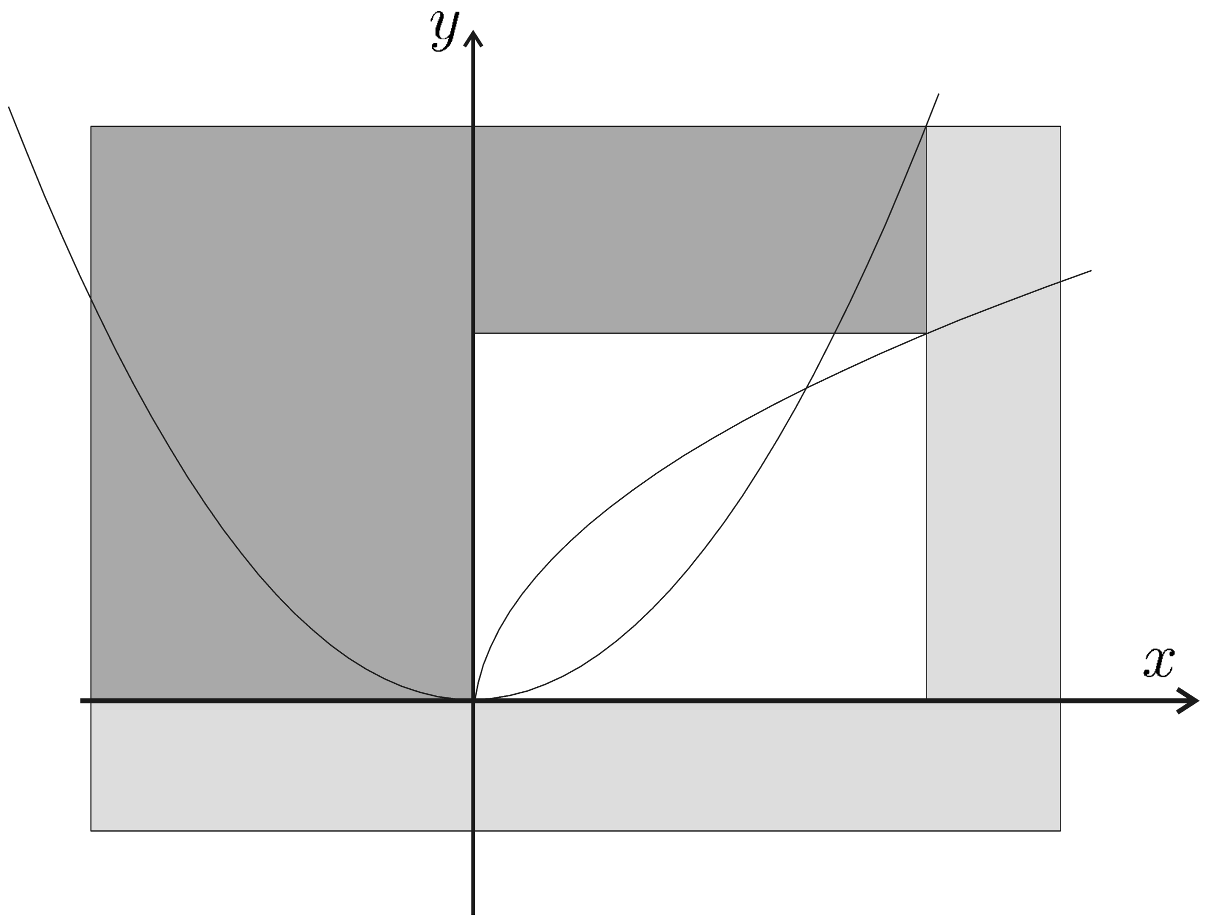

y
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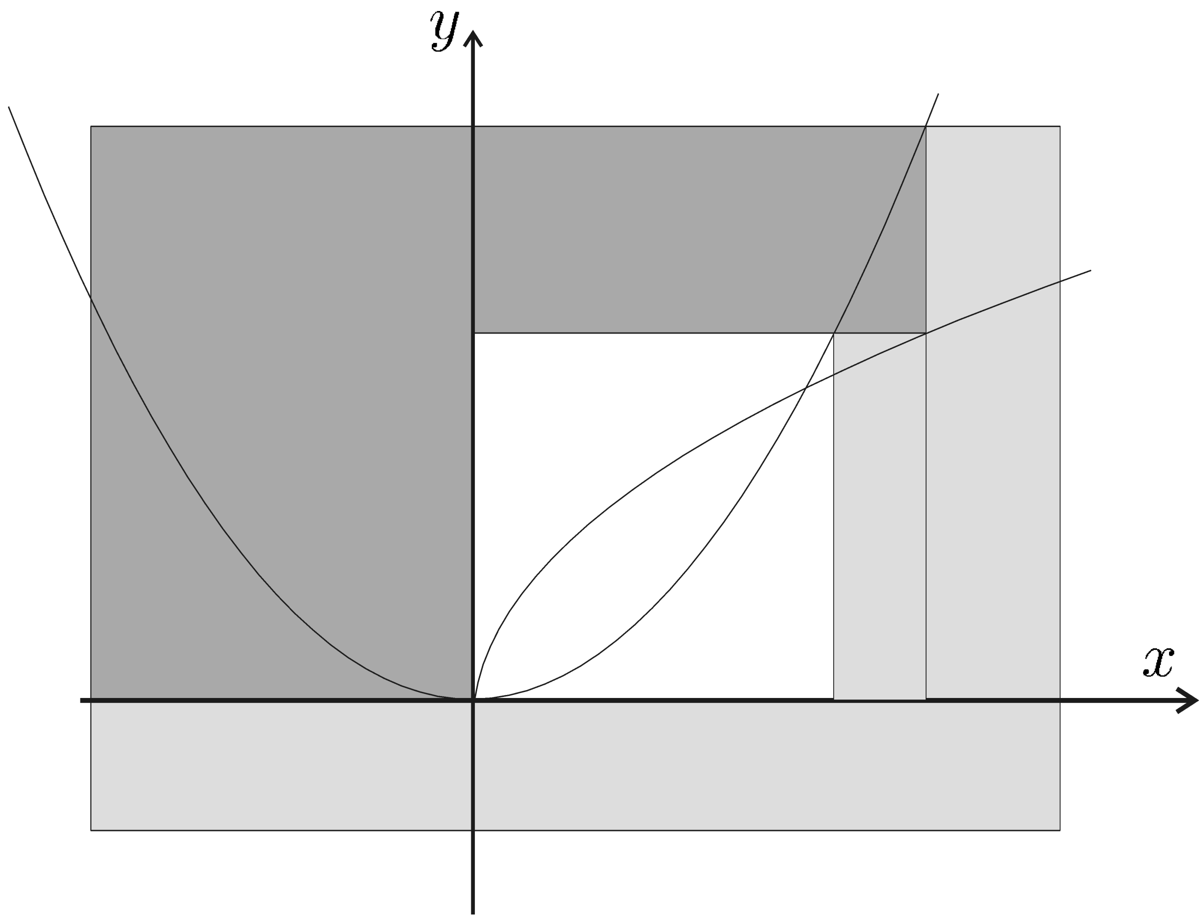
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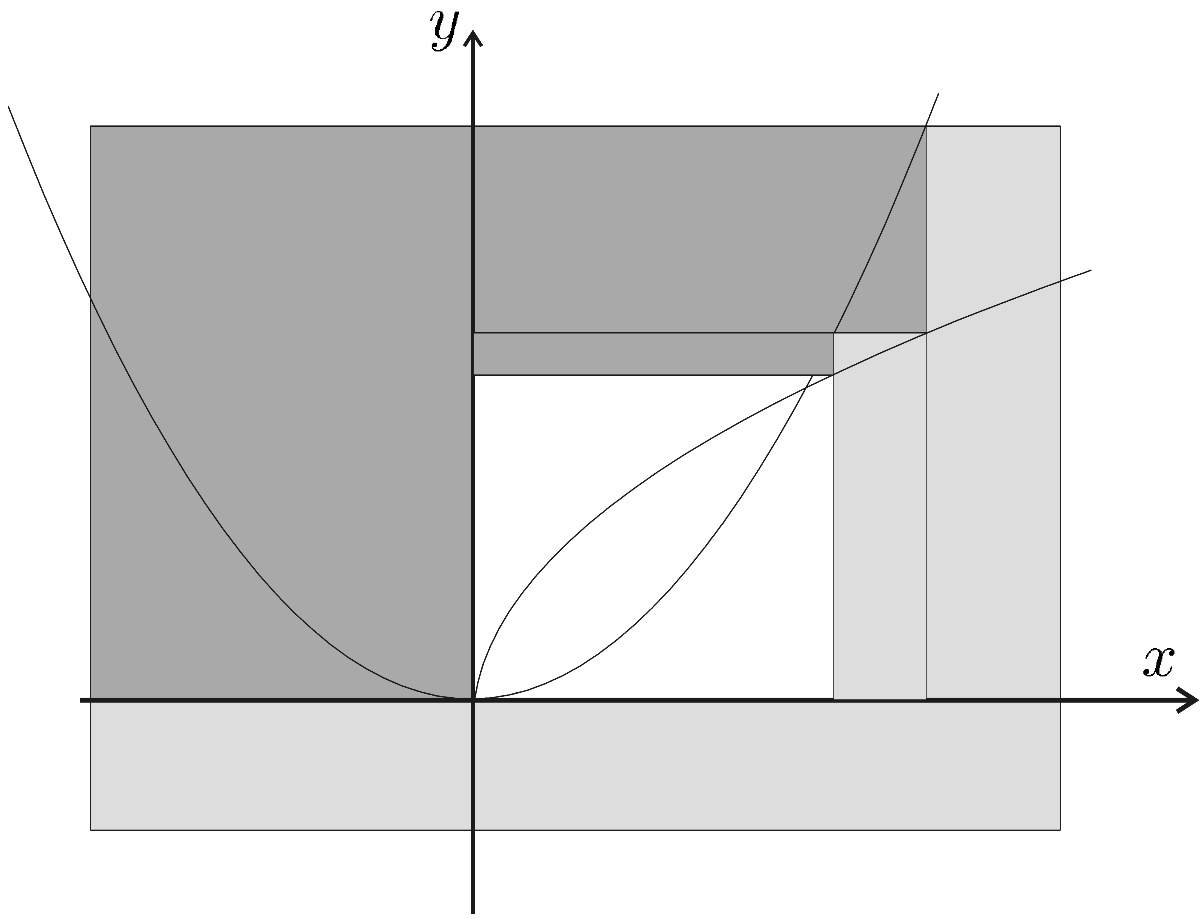

y
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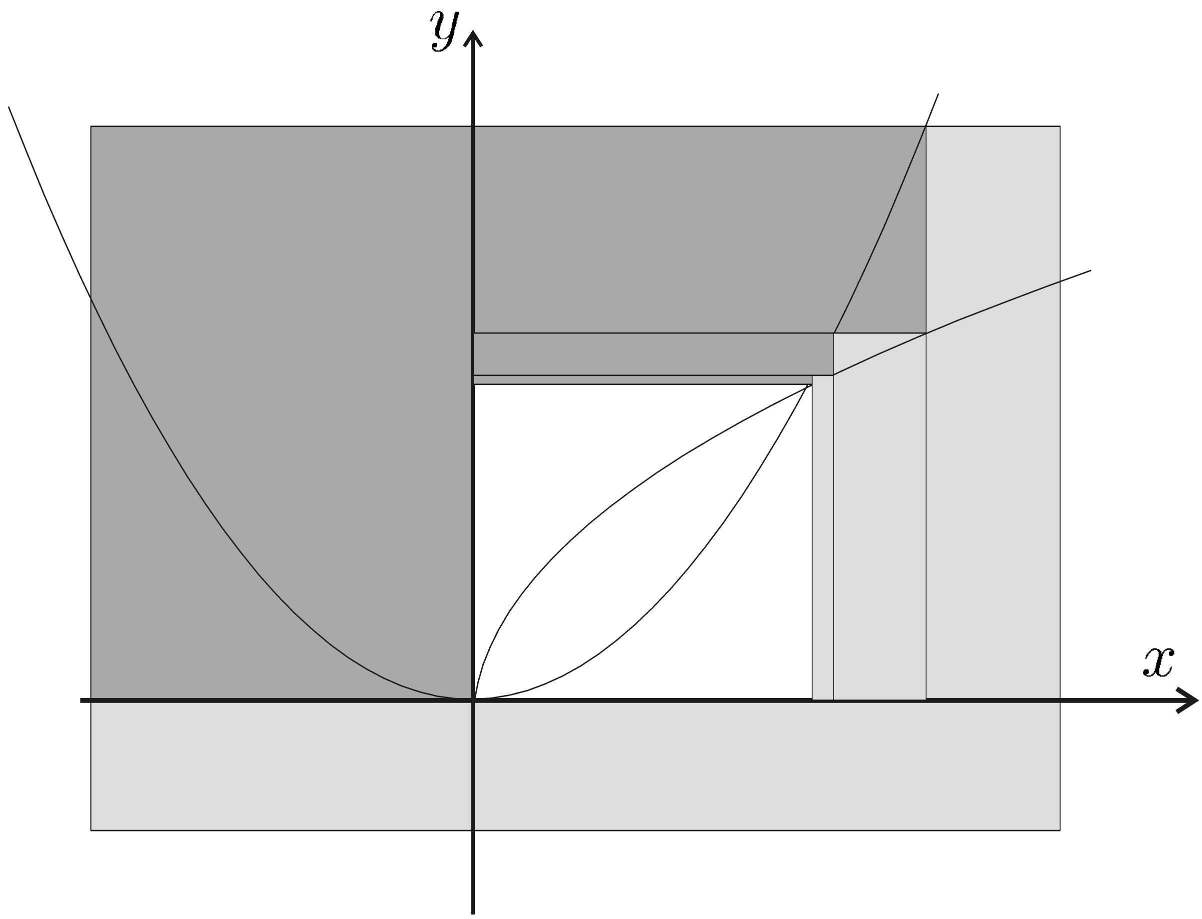
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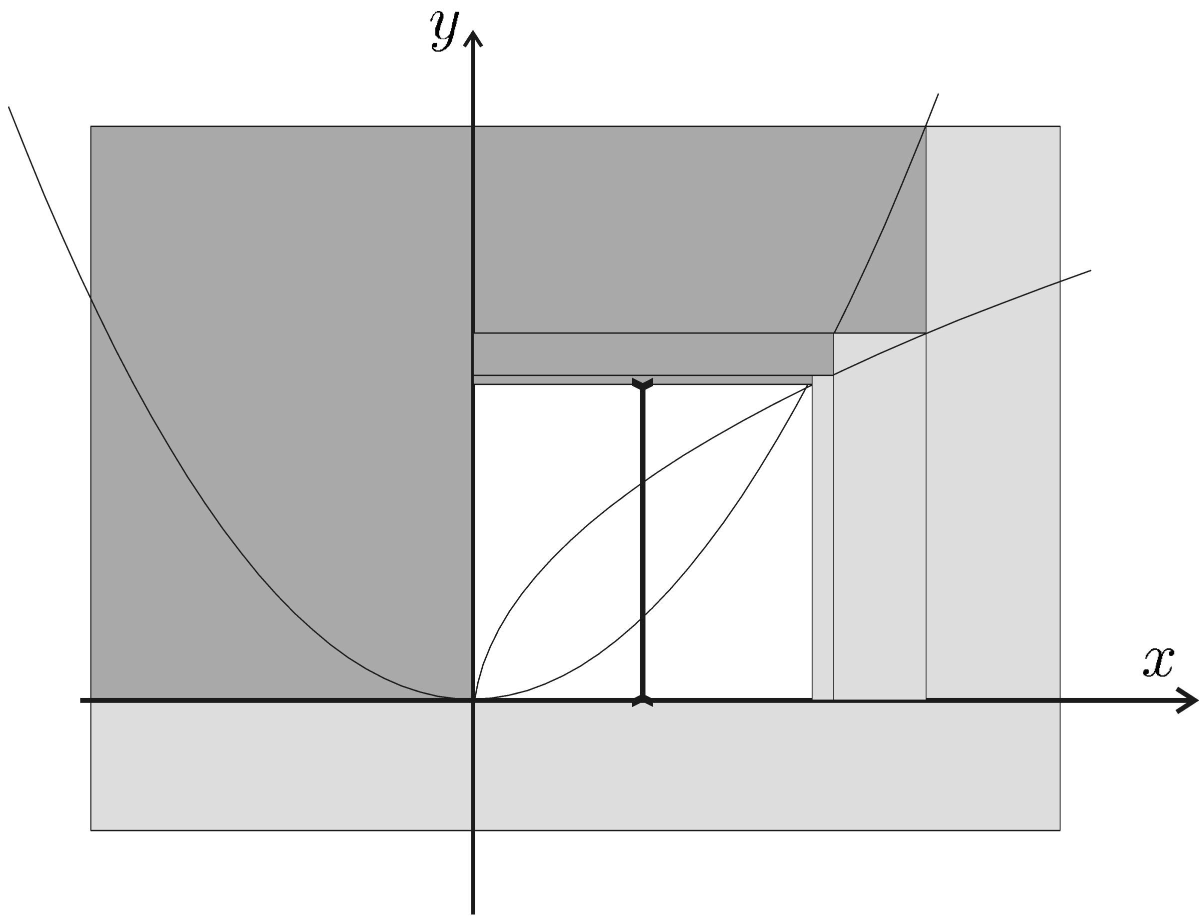

y
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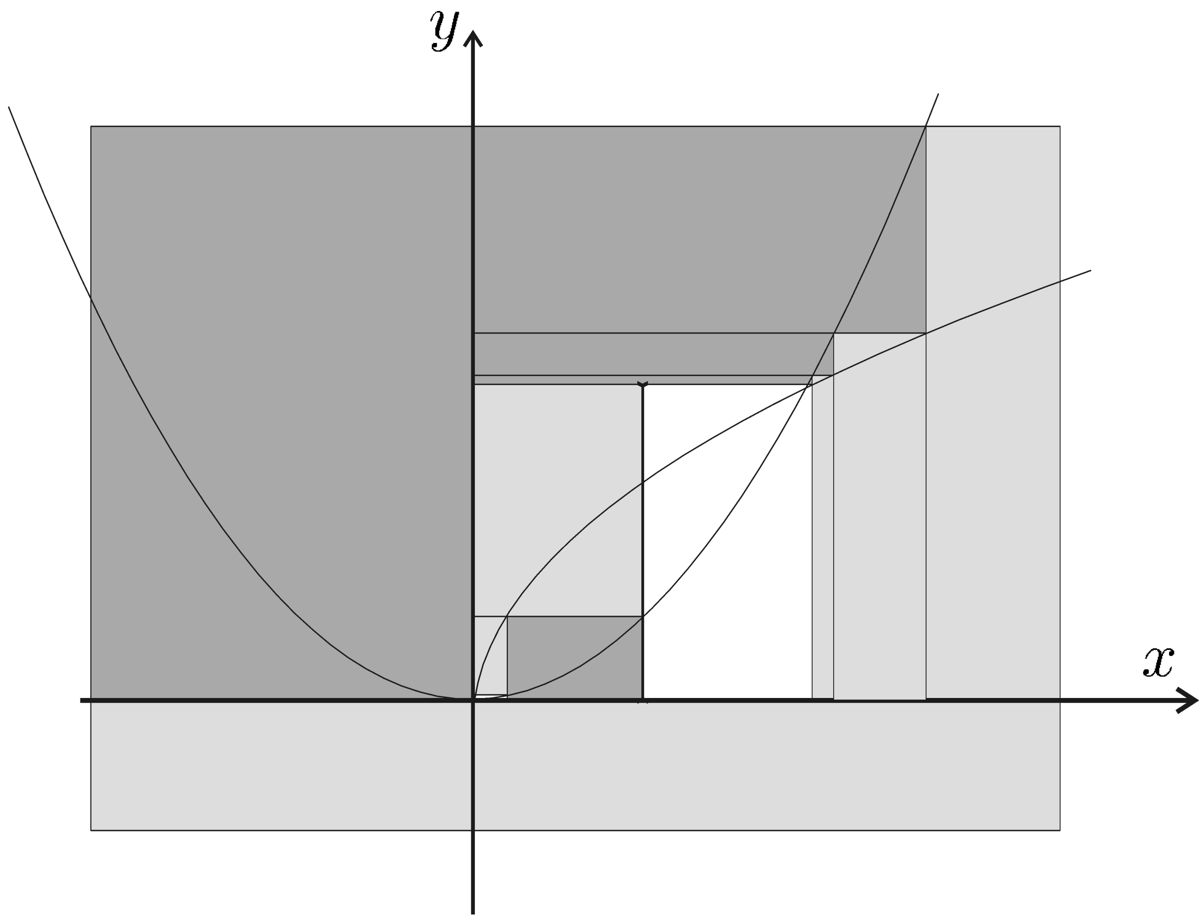
y
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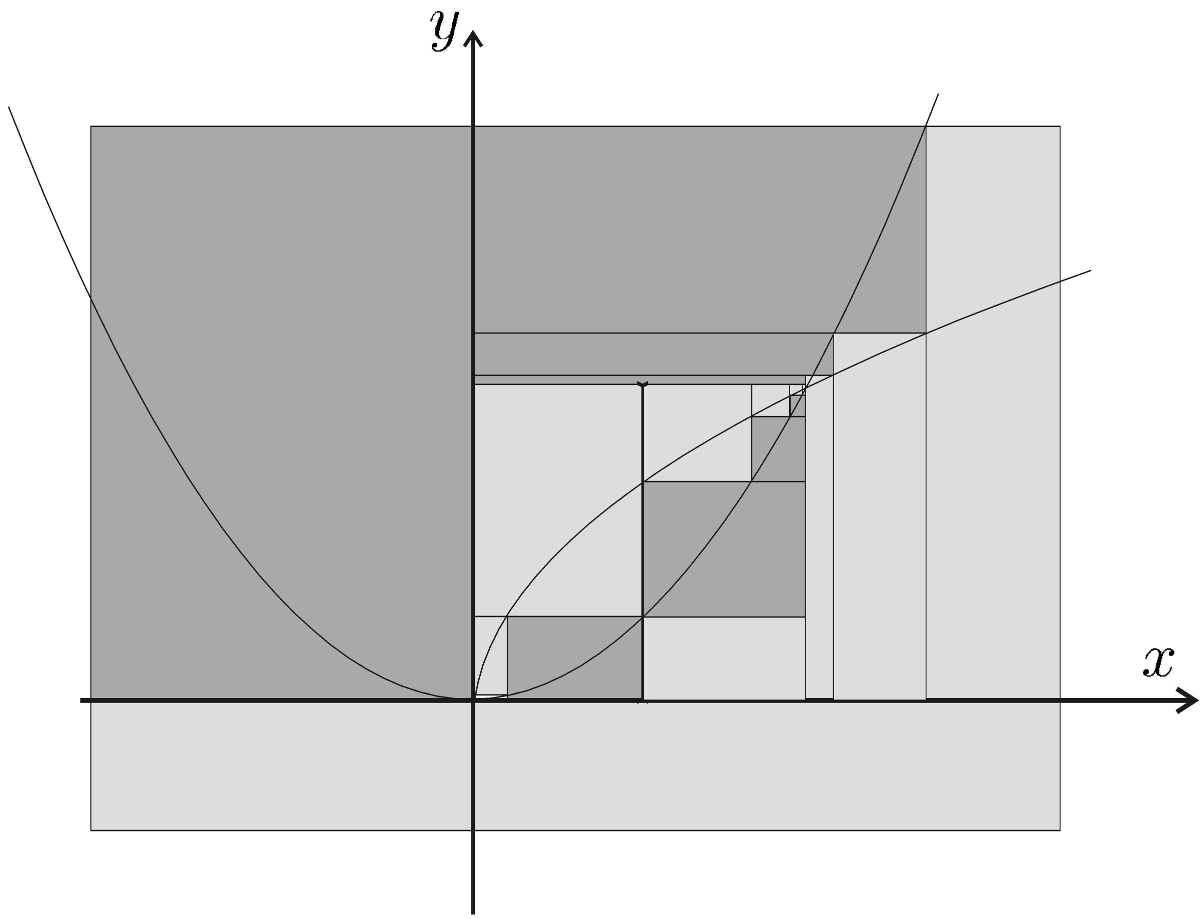
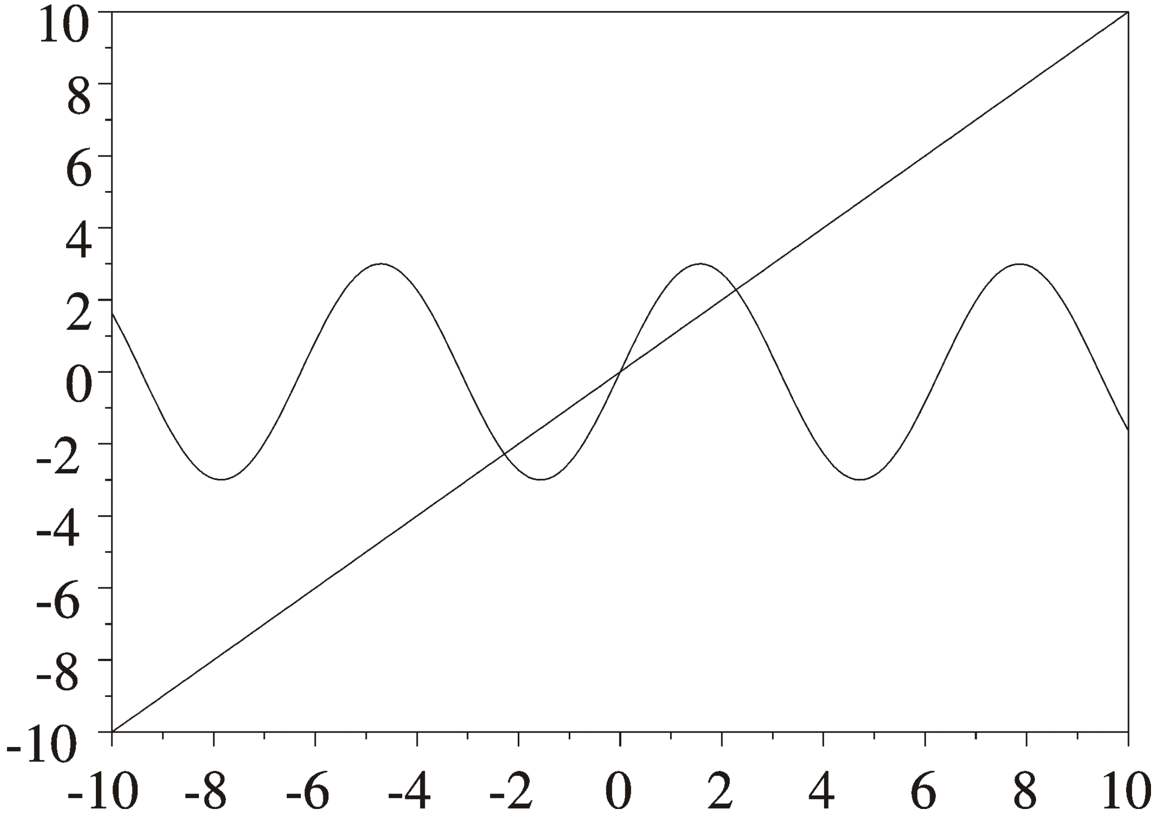


y
x

Exemple. Consider the system

$$\begin{cases} y &= 3\sin(x) \\ y &= x \end{cases} \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$


10
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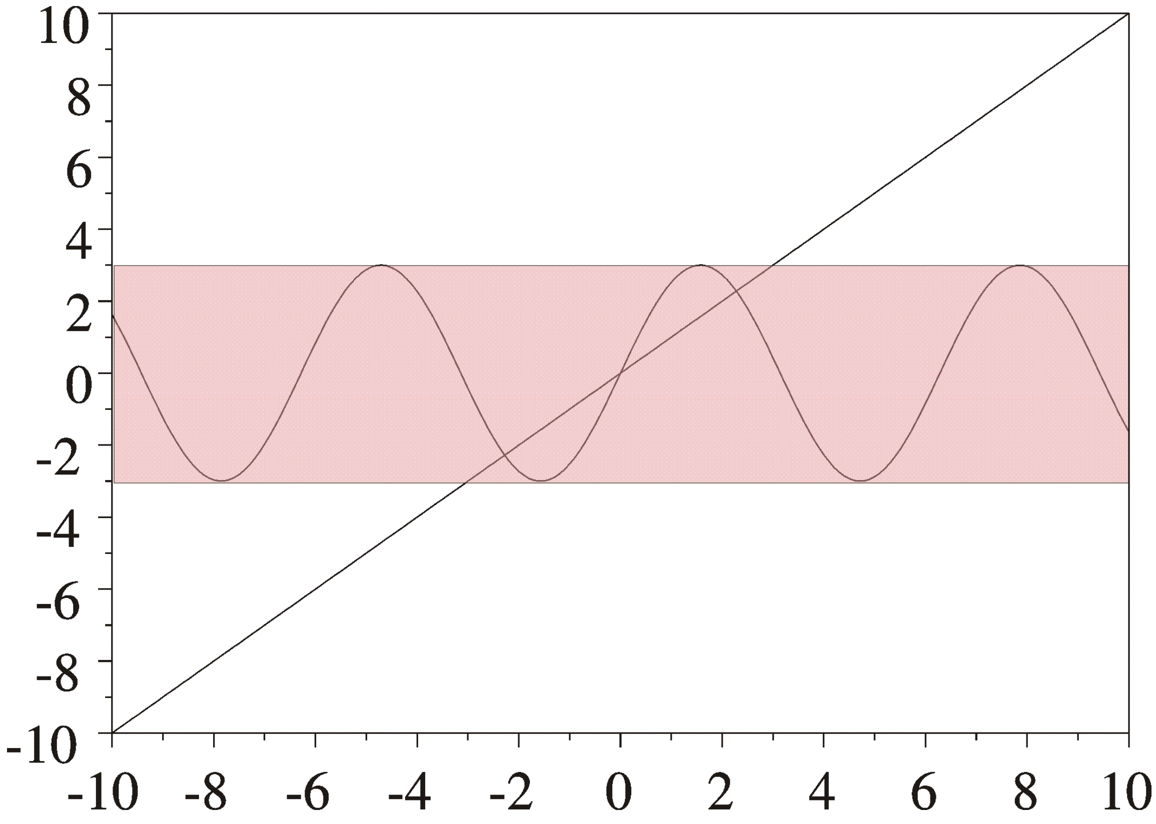
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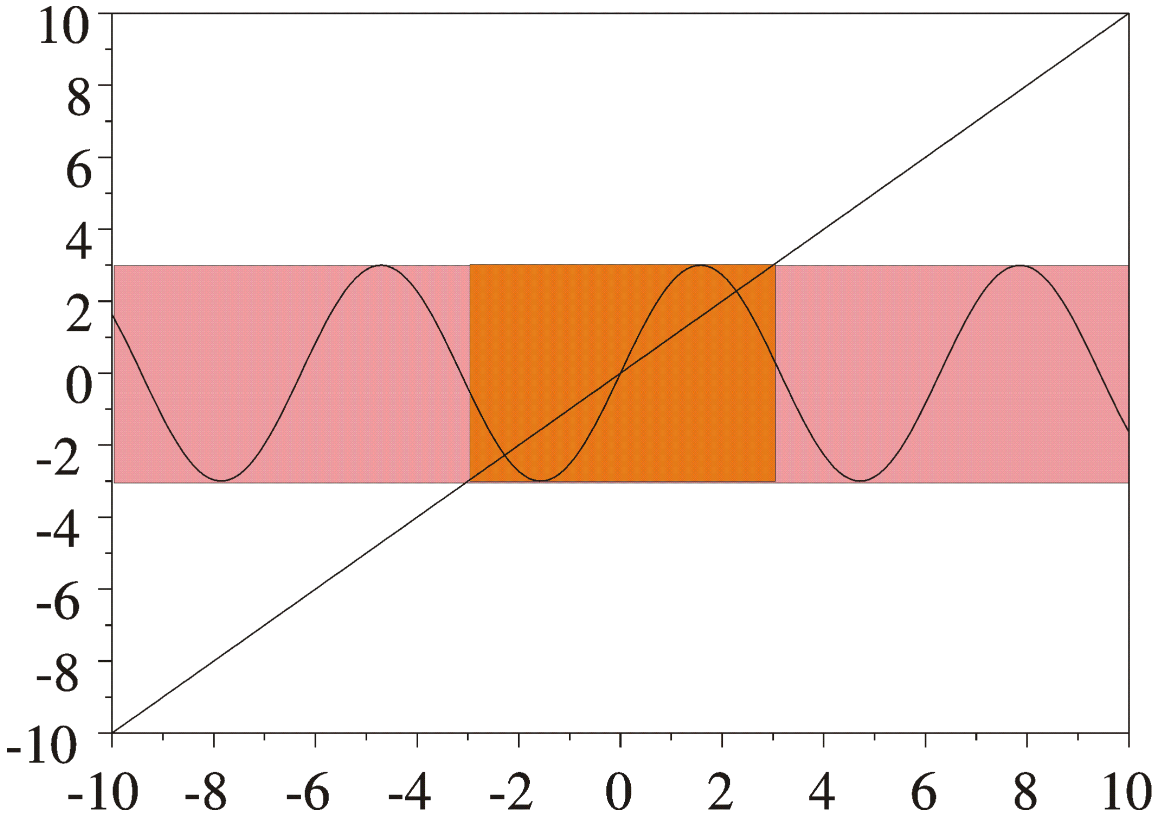
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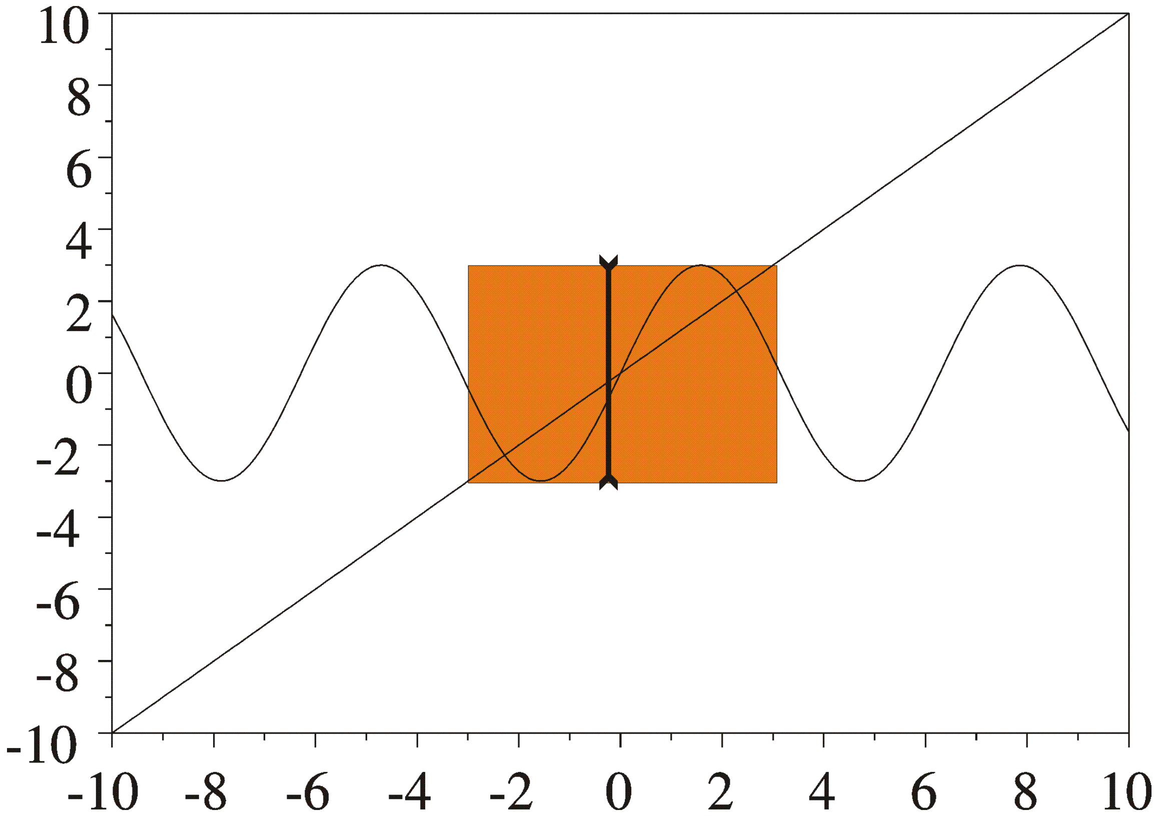
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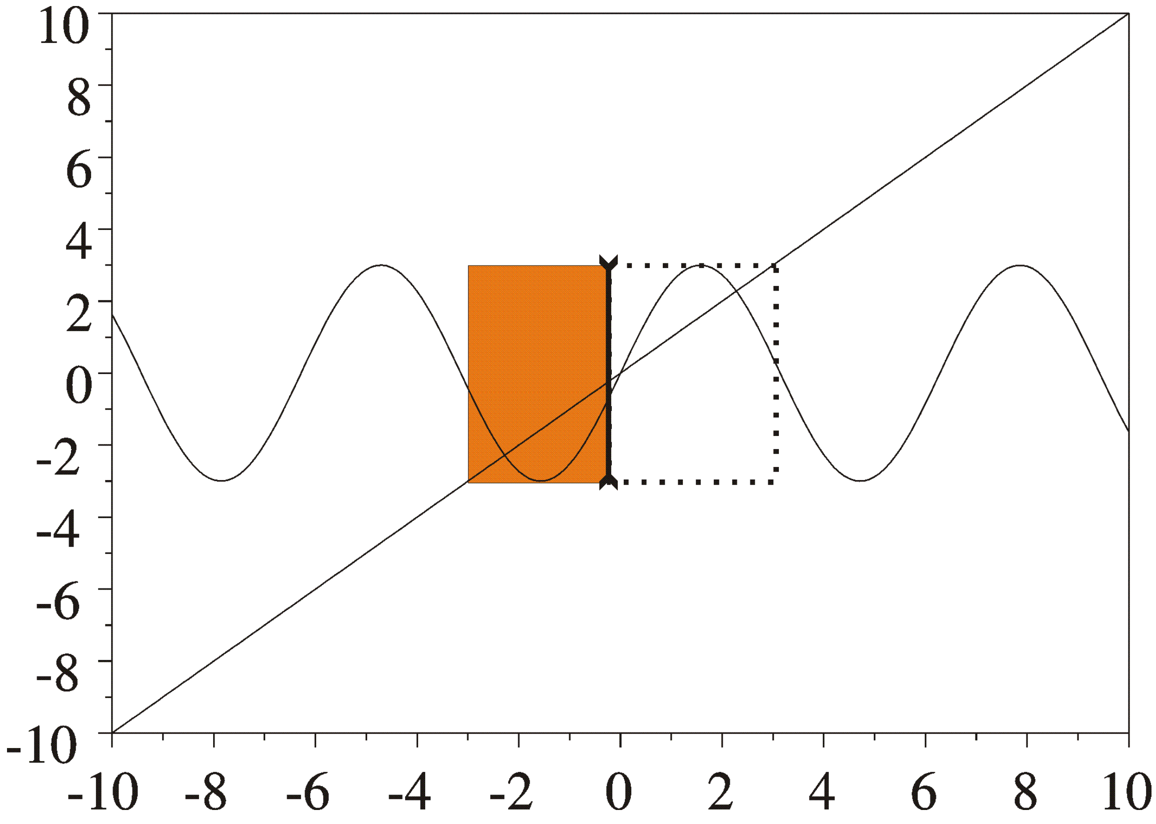
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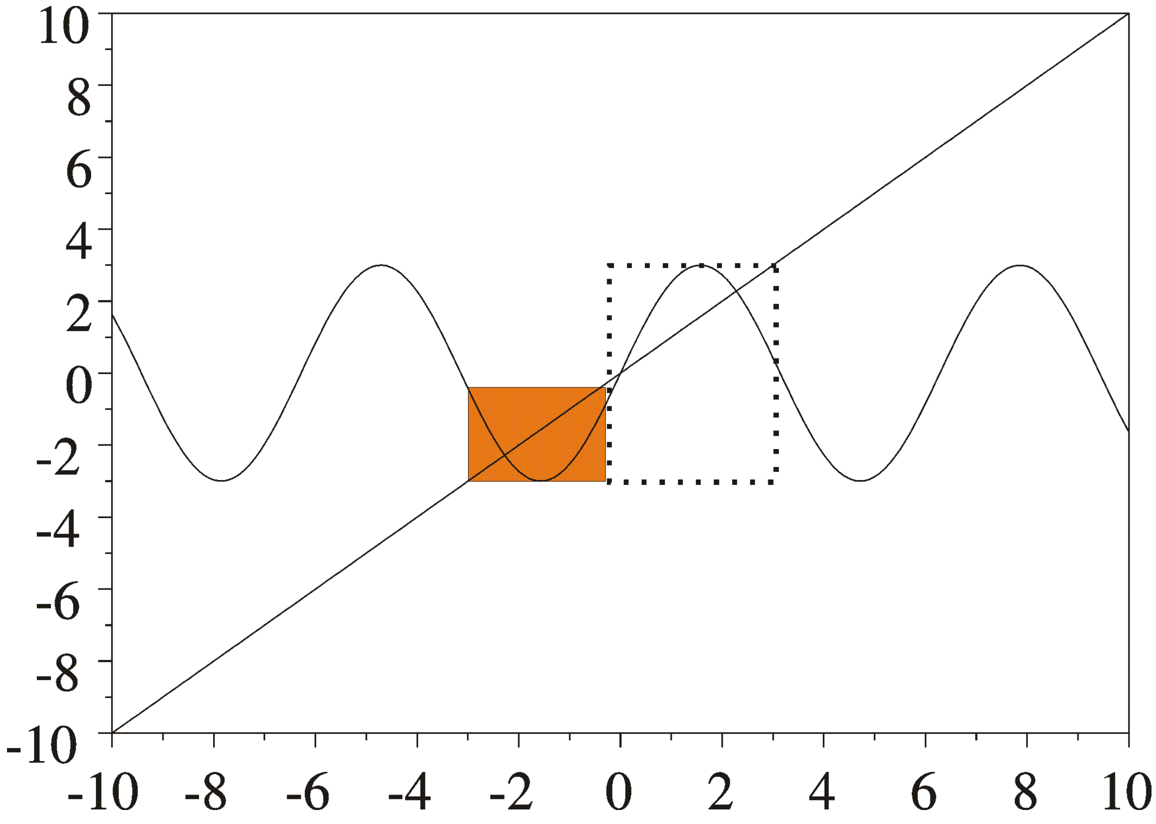
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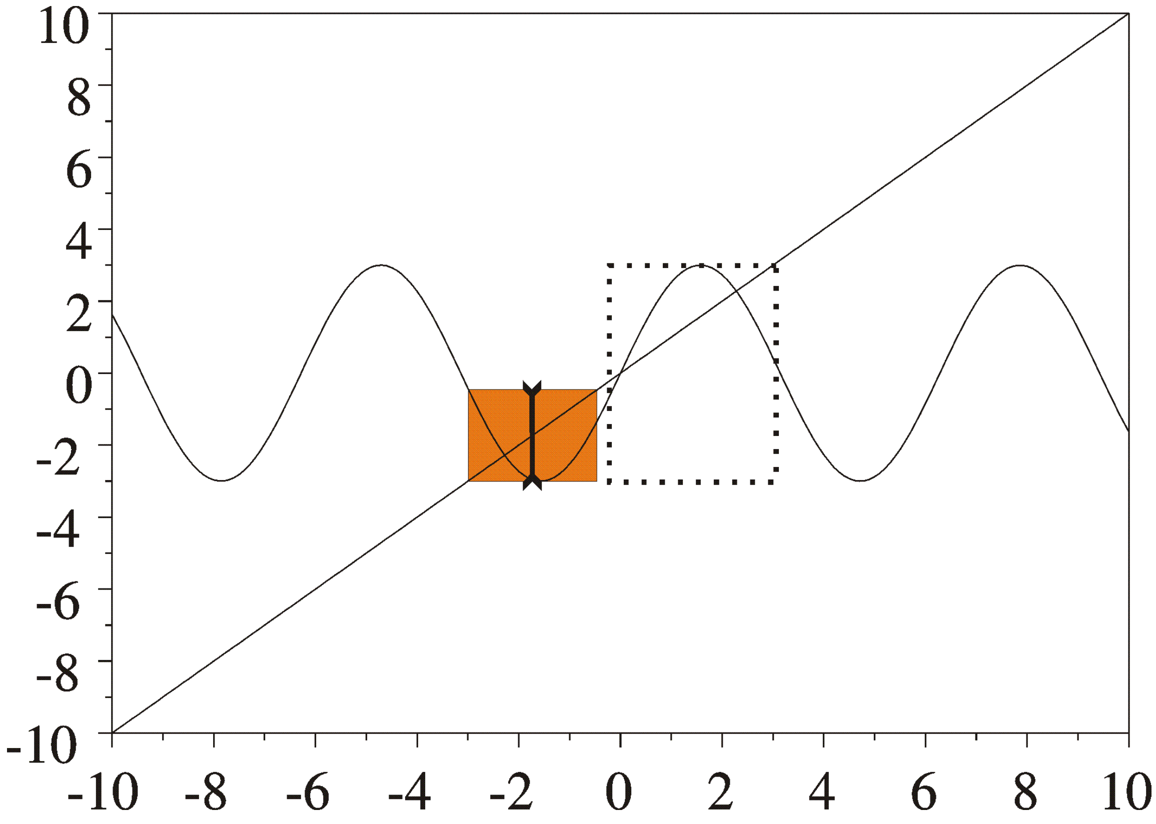

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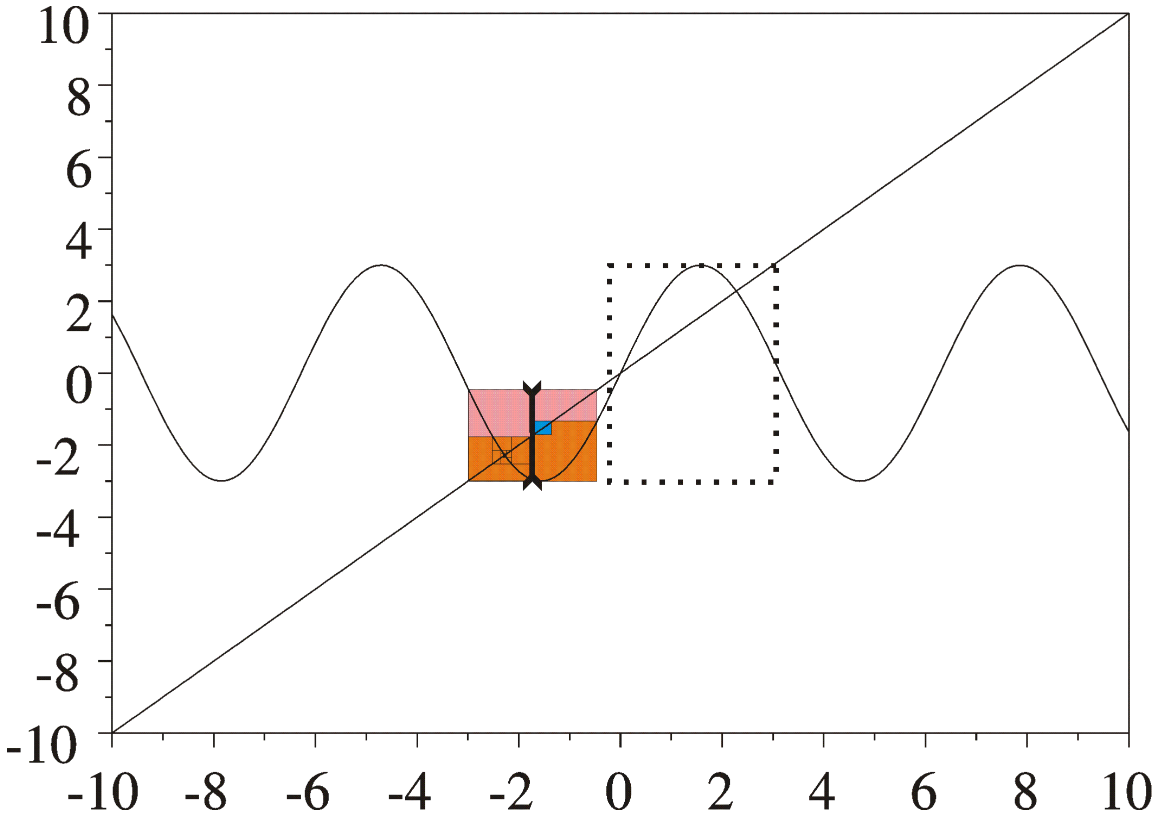
-10 -8 -6 -4 -2 0 2 4 6 8 10



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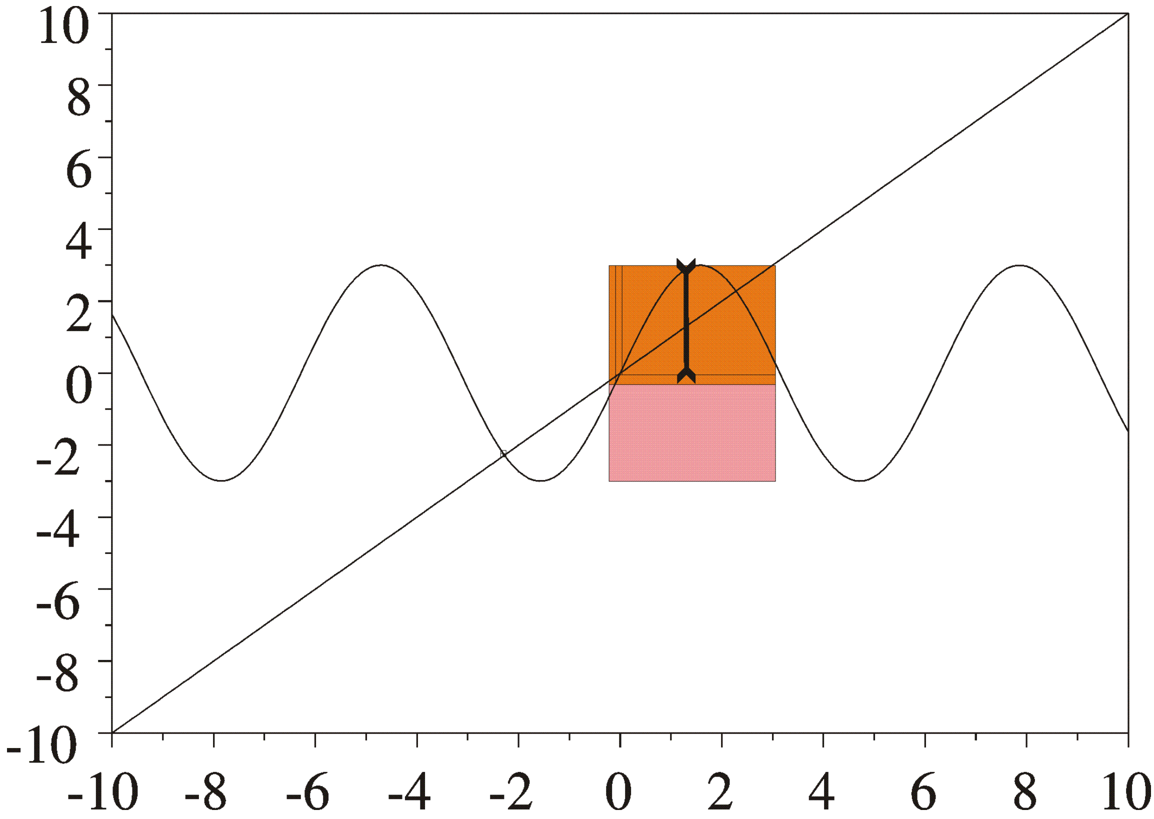
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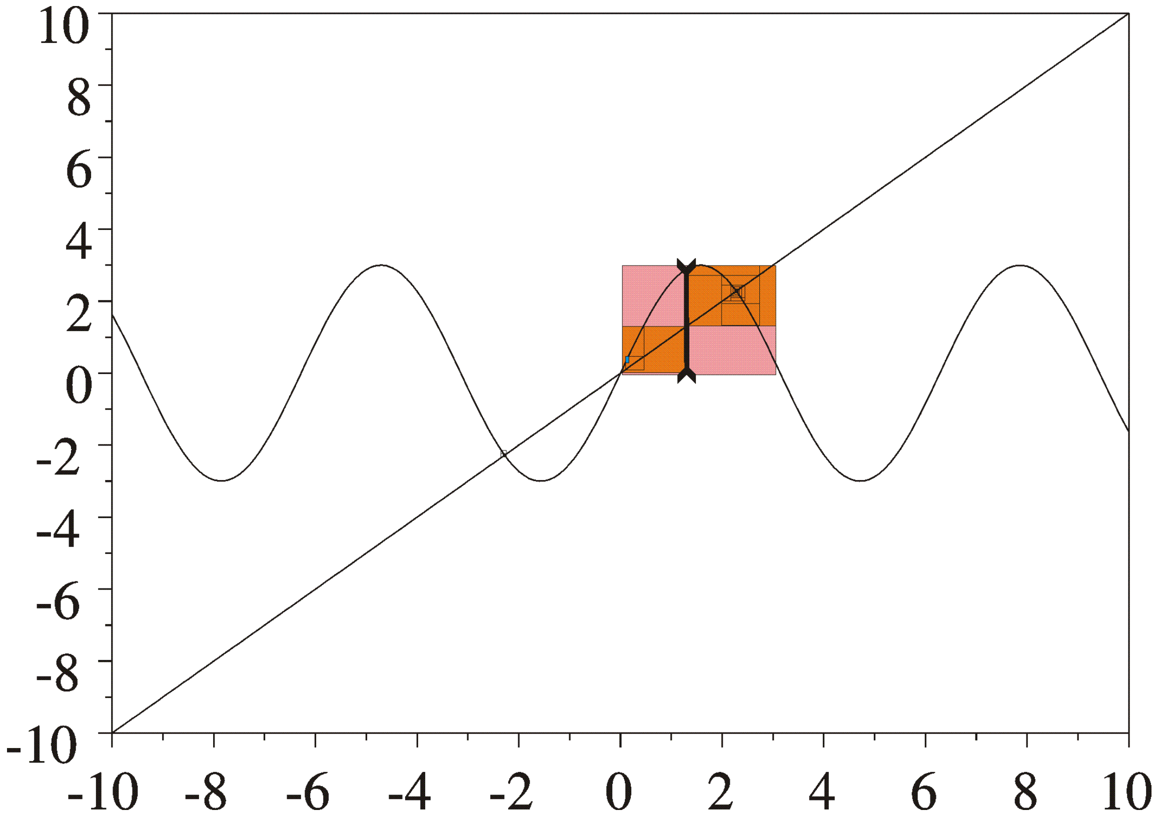

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-8
-6
-4
-2
0
2
4
6
8
10

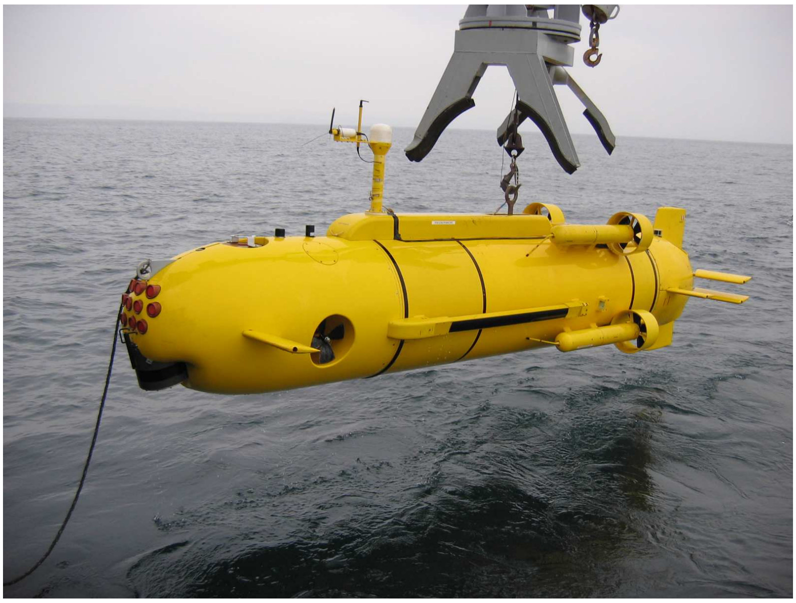
4.5 Decomposition into primitive constraints

$$x + \mathsf{sin}(xy) \leq 0,$$
$$x \in [-1, 1], y \in [-1, 1]$$

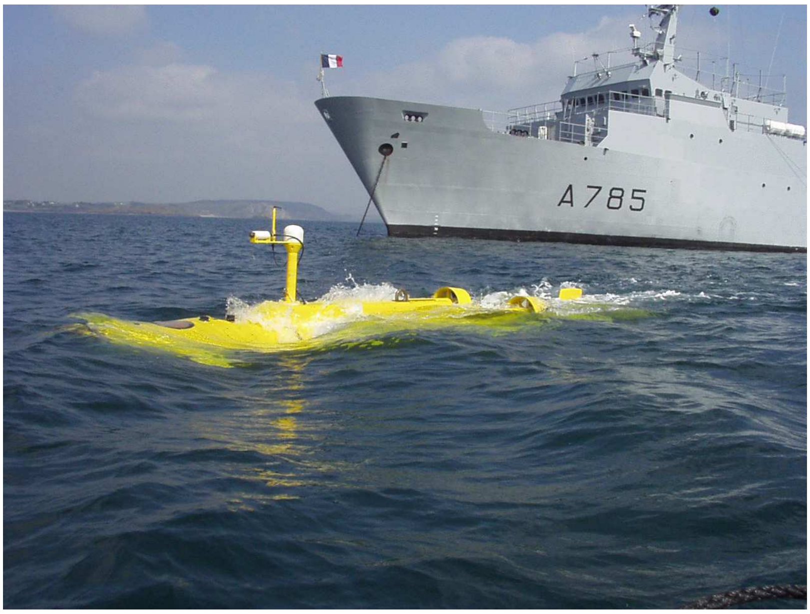
can be decomposed into

$$\begin{cases} a = xy \\ b = \mathsf{sin}(a) \\ c = x + b \end{cases}, \qquad \begin{matrix} x \in [-1, 1] & a \in [-\infty, \infty] \\ y \in [-1, 1] & b \in [-\infty, \infty] \\ & c \in [-\infty, 0] \end{matrix}$$

5 Redermor



The *Redermor*, GESMA



The *Redermor* at the surface

Show simulation

Why choosing an interval constraint approach for SLAM ?

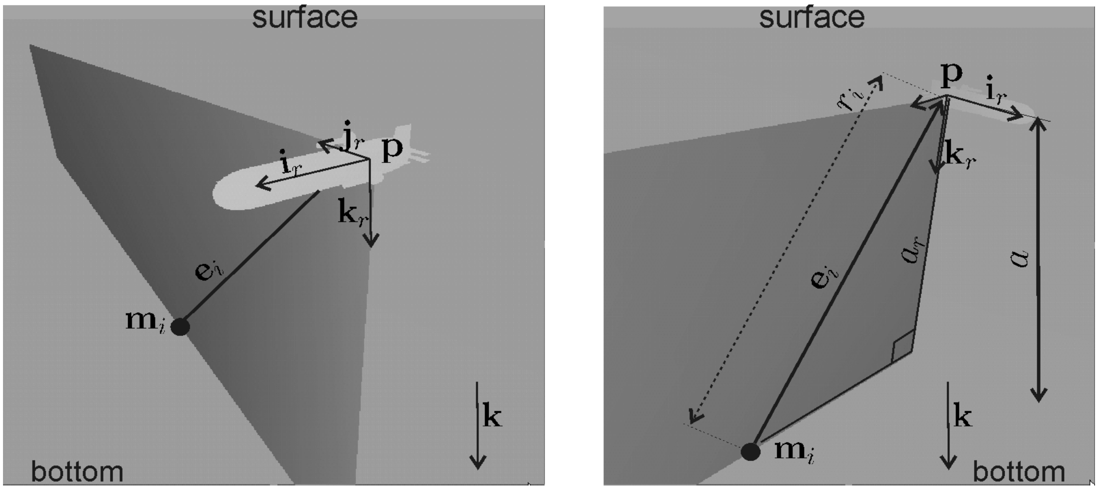
1) A reliable method is needed.
2) The model is nonlinear.
3) The pdf of the noises are unknown.
4) Reliable error bounds are provided by the sensors.
5) A huge number of redundant data are available.

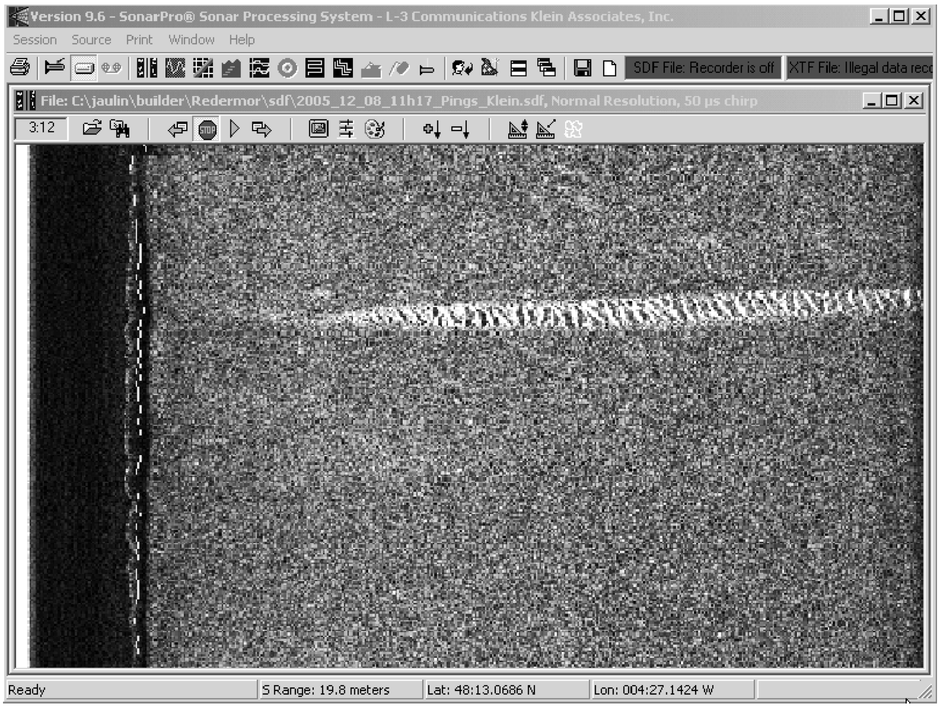
5.1 Sensors

A GPS (Global positioning system) at the surface only.

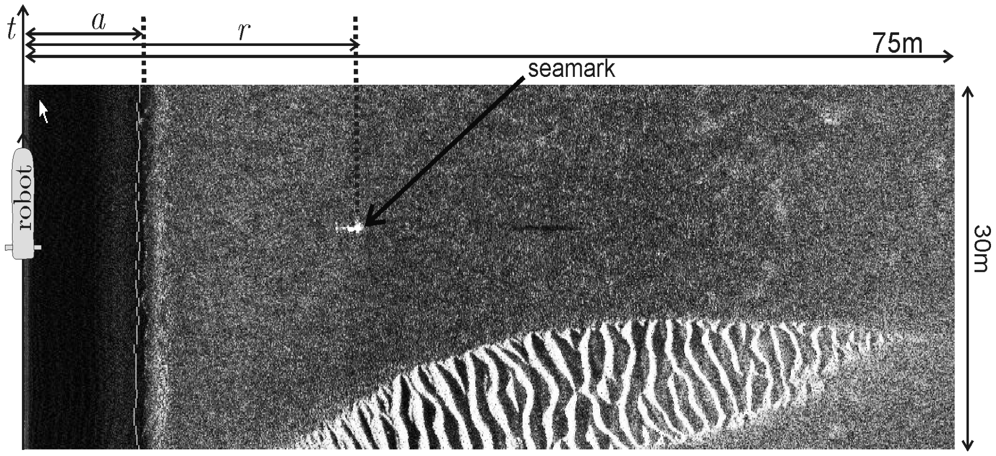
$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^o, 48.2129206^o) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^o, 48.2191297^o) \pm 2.5m$$

A sonar (KLEIN 5400 side scan sonar). Gives the distance r between the robot to the detected object.





Screenshot of SonarPro



Detection of a mine using SonarPro

A Loch-Doppler. Returns the speed of the robot $\mathbf{v}_r$ and the altitude a of the robot $\pm$ 10cm.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ, the pitch θ and the head ψ.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$

5.2 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines have been detected by the sonar:

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

5.3 Constraints satisfaction problem

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t).\mathbf{R}_{\theta}(t).\mathbf{R}_{\varphi}(t),$$

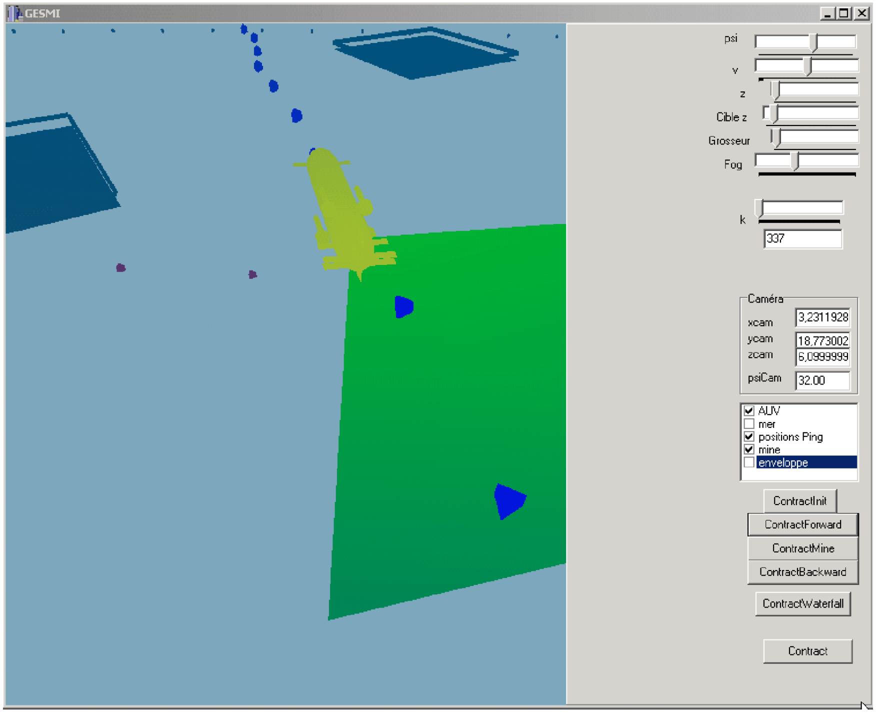
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t)$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

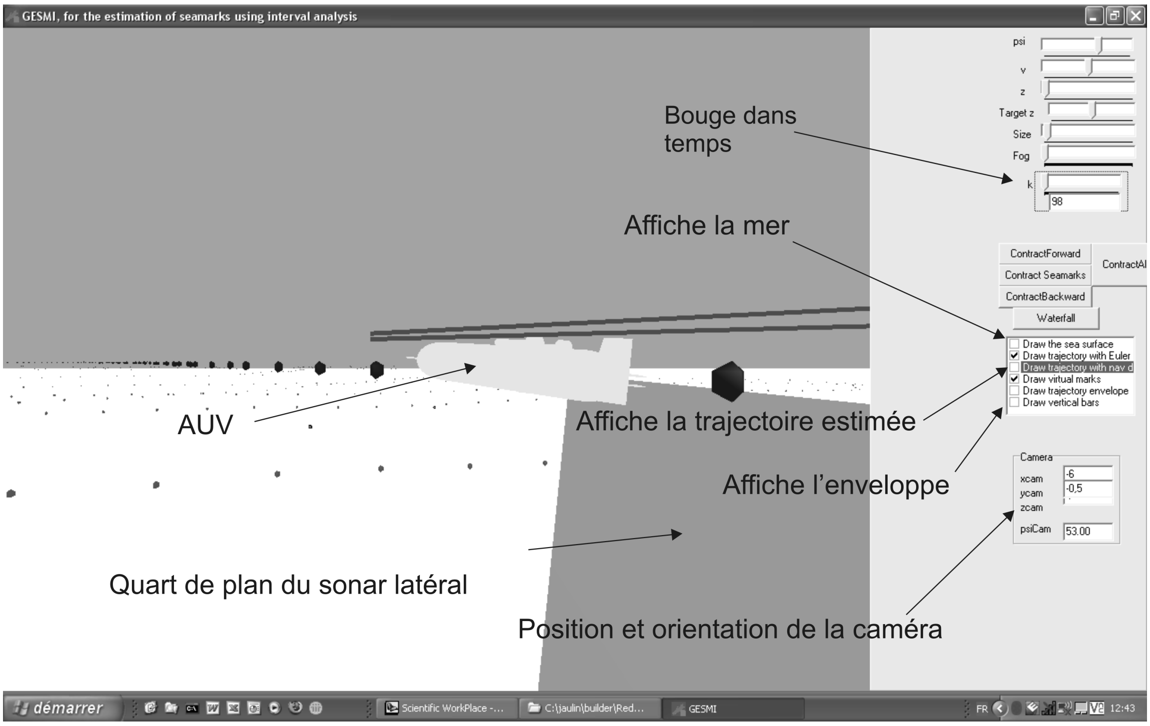
$$\mathbf{R}^{\mathsf{T}}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

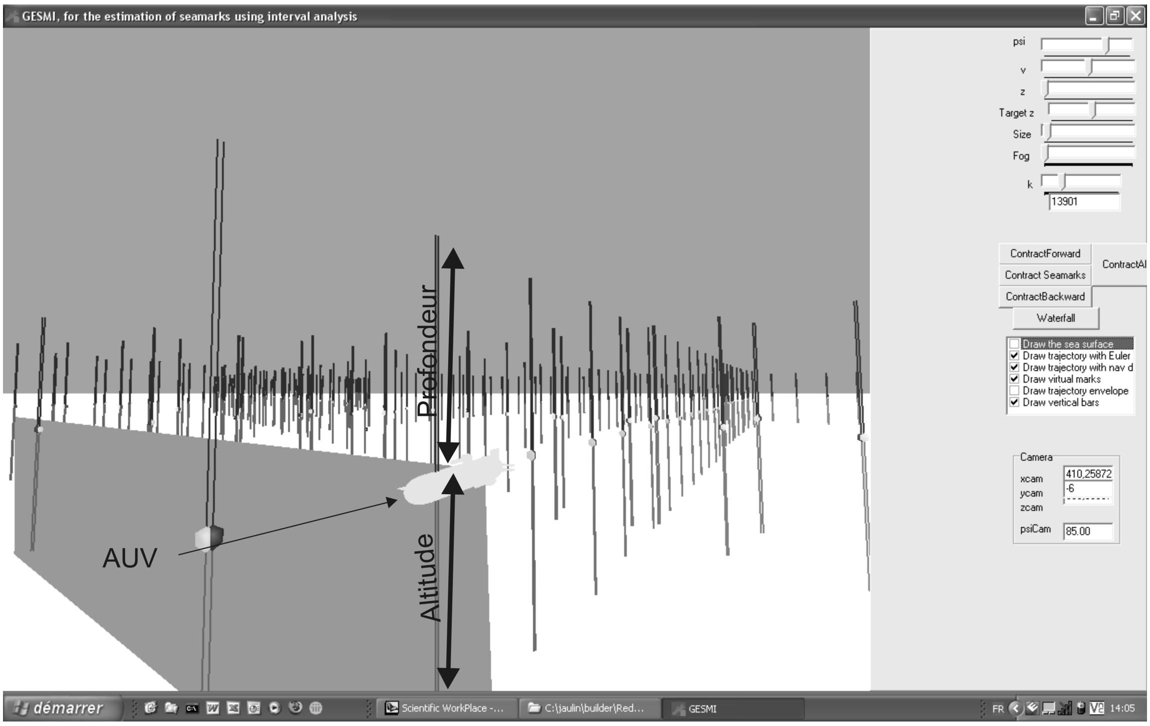
$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5].$$

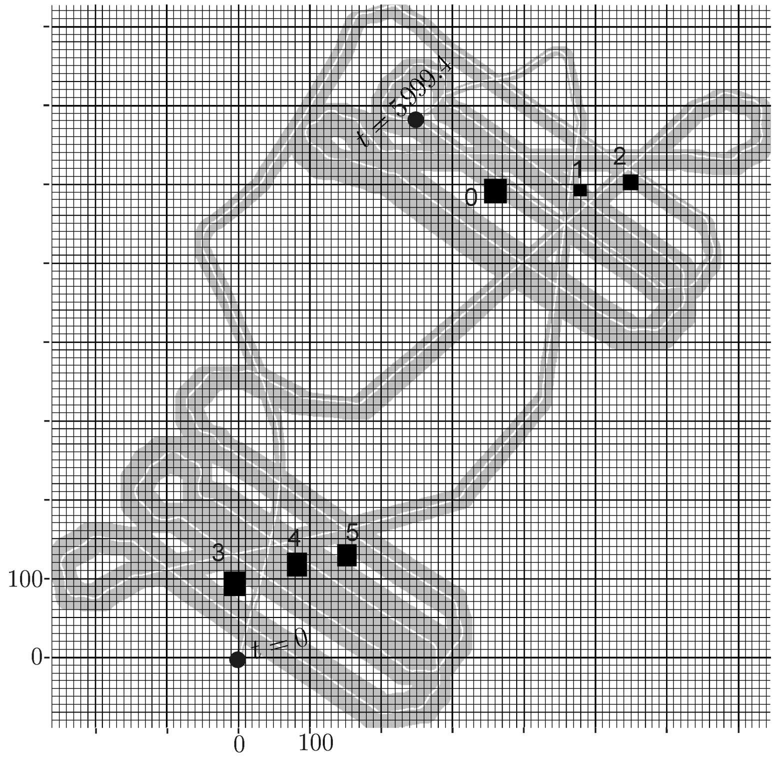
5.4 GESMI

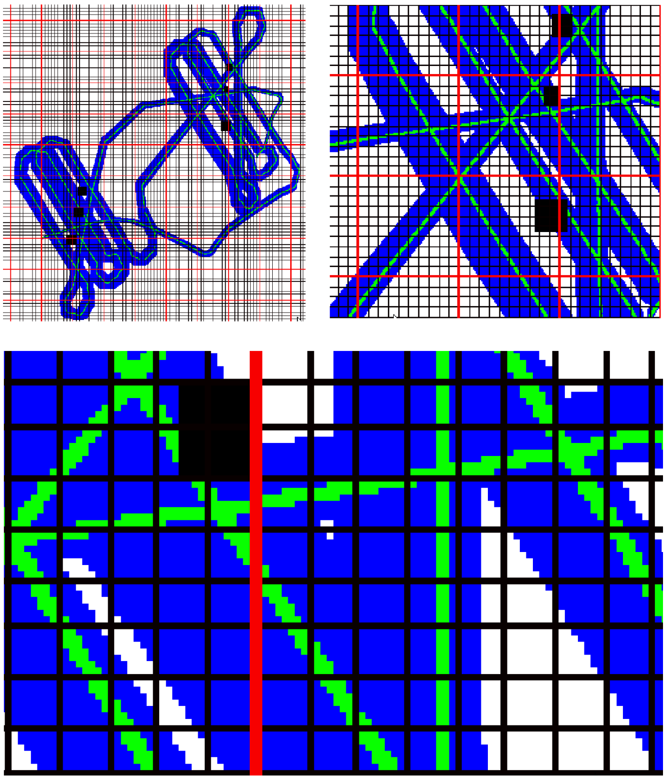


GESMI (Guaranteed Estimation of Sea Mines with Intervals)


GESMI, for the estimation of seamarks using interval analysis
Bouge dans temps
Affiche la mer
AUV
Affiche la trajectoire estimée
Affiche l'enveloppe
Quart de plan du sonar latéral
Position et orientation de la caméra
psi
v
z
Target z
Size
Fog
k
98
ContractForward
Contract Seamarks
ContractBackward
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw trajectory with nav d
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
-6
ycam
-0,5
zcam
psiCam
53.00
démarrer
Scientific WorkPlace -...
GESMI
FR
12:43

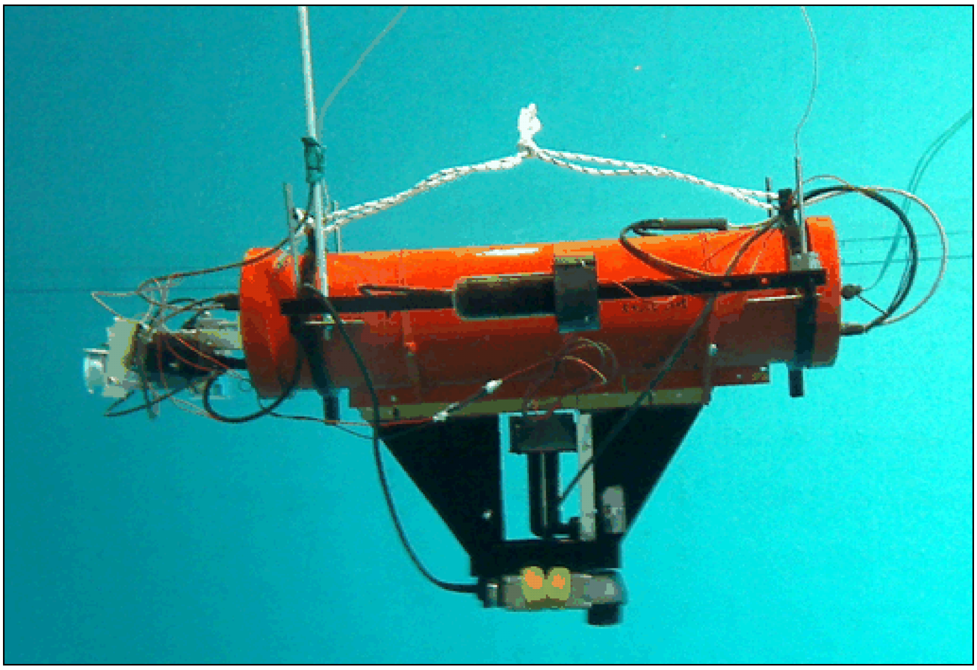

GESMI, for the estimation of seamarks using interval analysis
Profondeur
Altitude
AUV
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
410,25872
ycam
-6
zcam
psiCam
85.00
démarrer
GESMI
14:05


t = 5999.4
0
1
2
3
4
5
t = 0
100
0
0
100



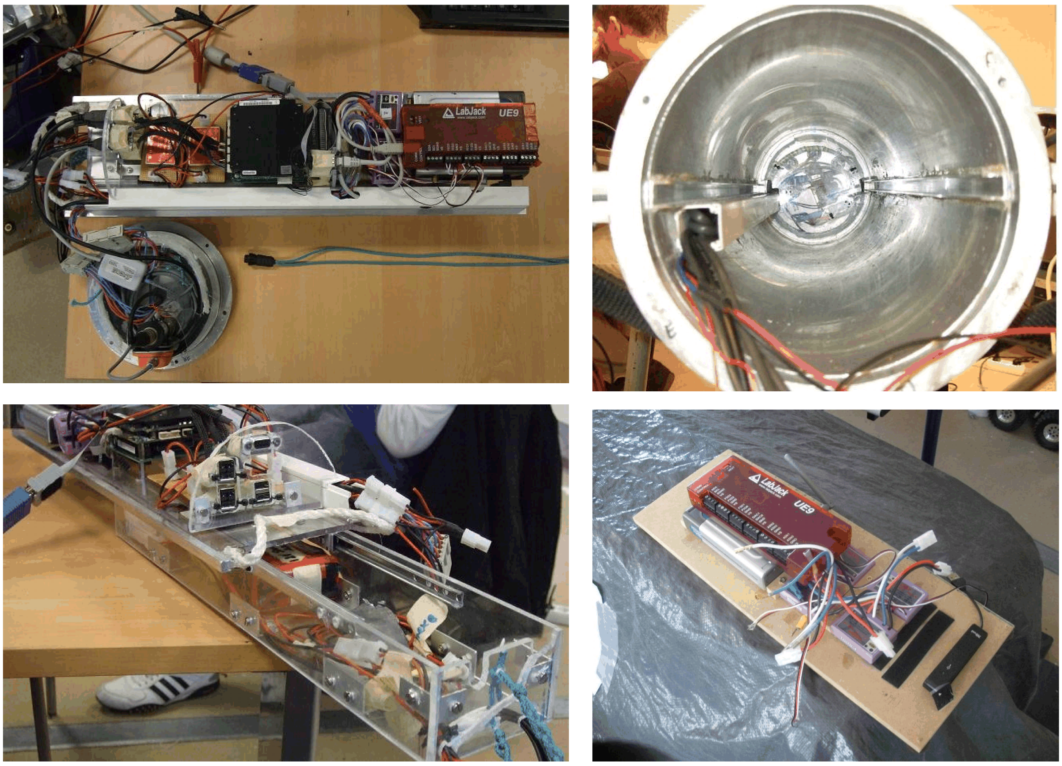
Trajectory reconstructed by GESMI

6 SAUC'ISSE



Sauc'isse robot swimming inside a pool

Covers of the tube



Interval architecture of the robot


1
2
3
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4
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9
LabJack
UE9
6
12V SINCE 2008
11
8
12
10
13

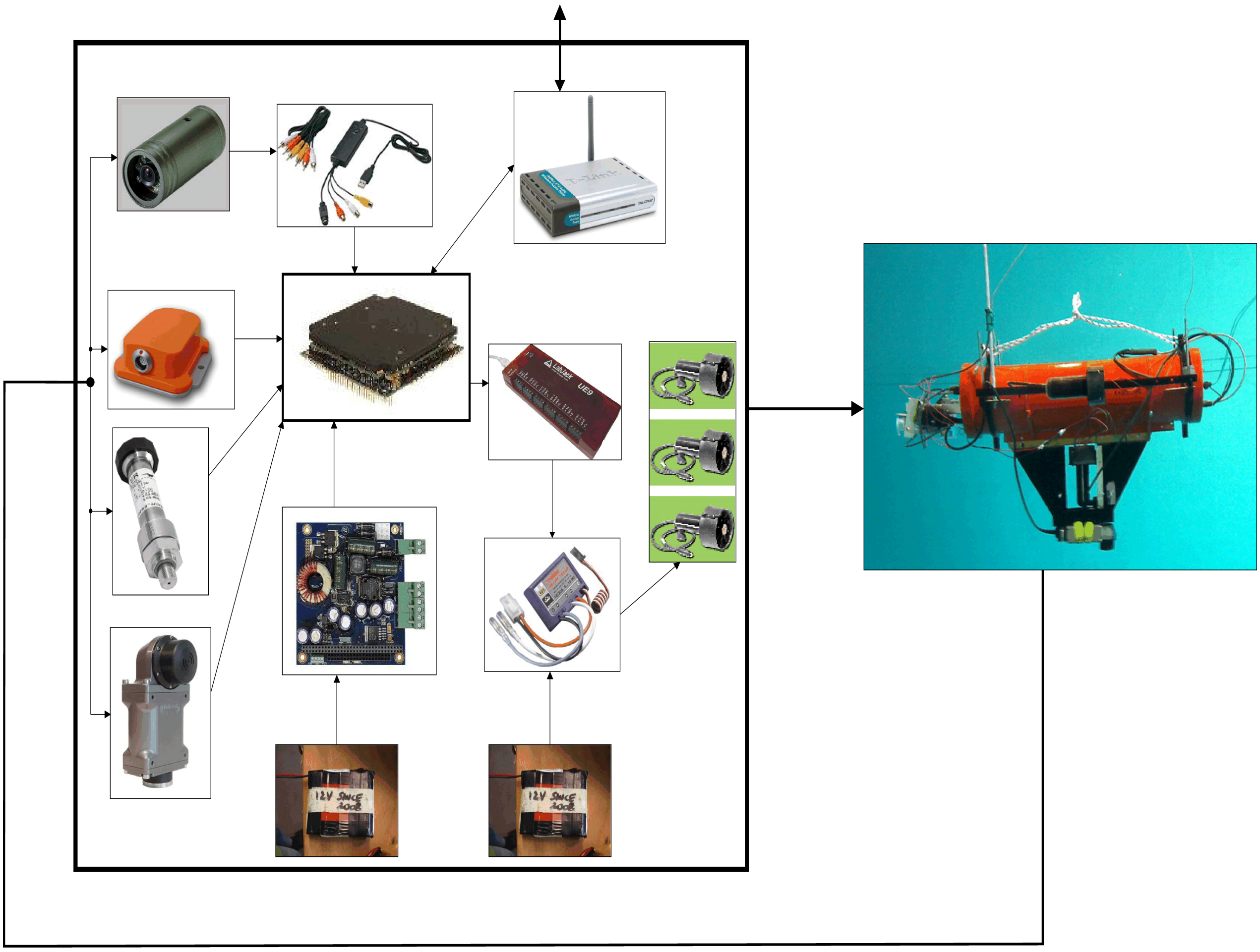
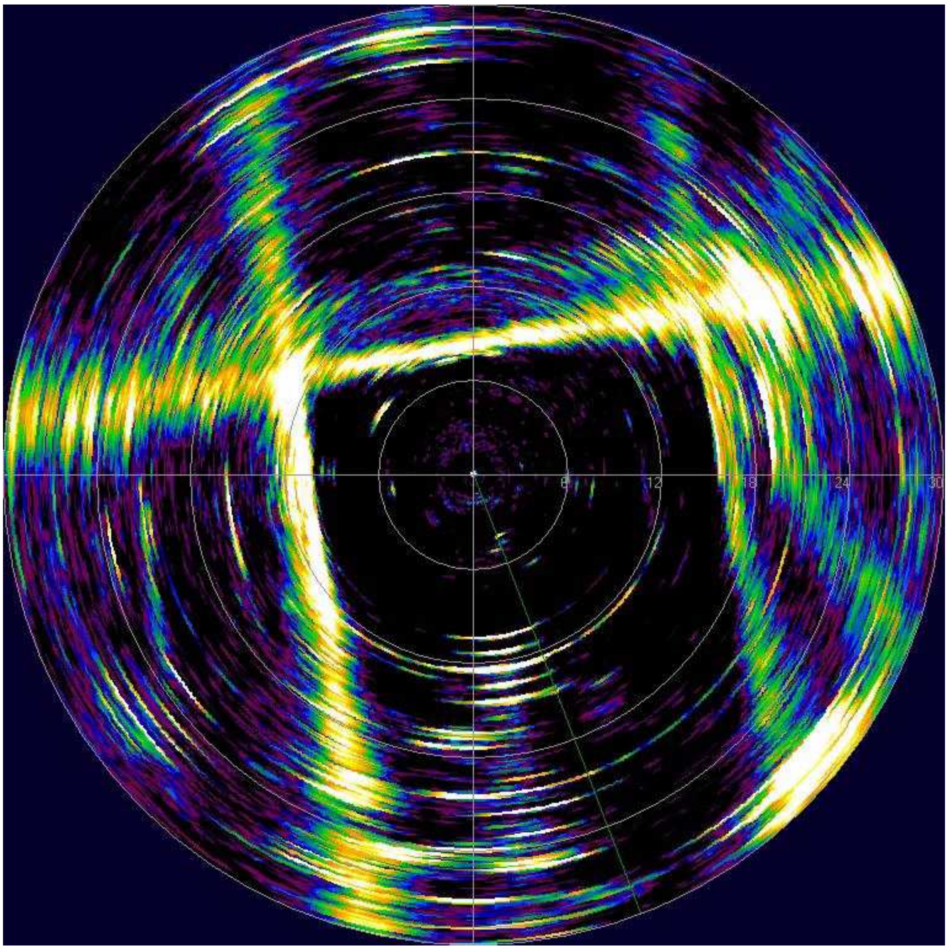


Figure 1: Electronics of the robot

Portsmouth, July 12-15, 2007.

Montrer une vidéo

6.1 Localization with sonar


6
12
18
24
30

6.2 Set-membership approach

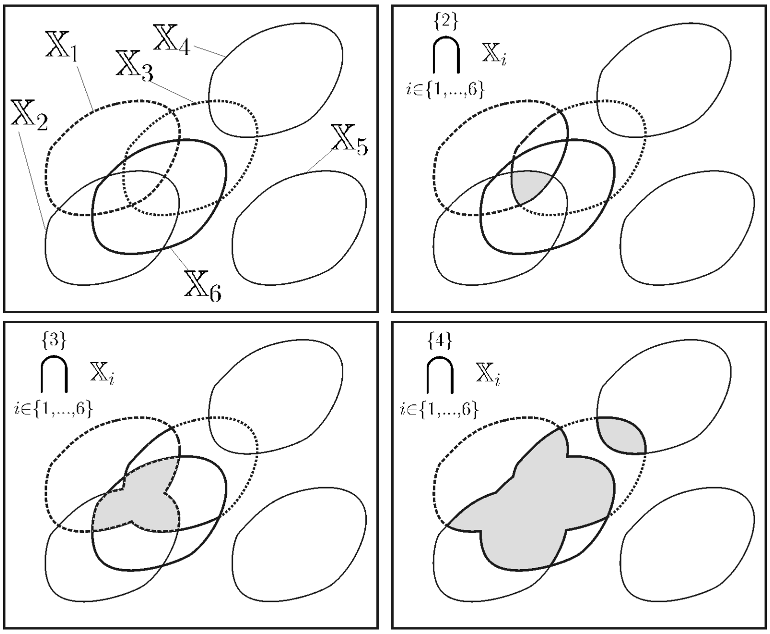
$$\begin{cases} \mathbf{x}(k+1) & = & \mathbf{f}_k(\mathbf{x}(k), \mathbf{n}(k)) \\ \mathbf{y}(k) & = & \mathbf{g}_k(\mathbf{x}(k)), \end{cases}$$

with $\mathbf{n}(k) \in \mathbb{N}(k)$ and $\mathbf{y}(k) \in \mathbb{Y}(k)$.

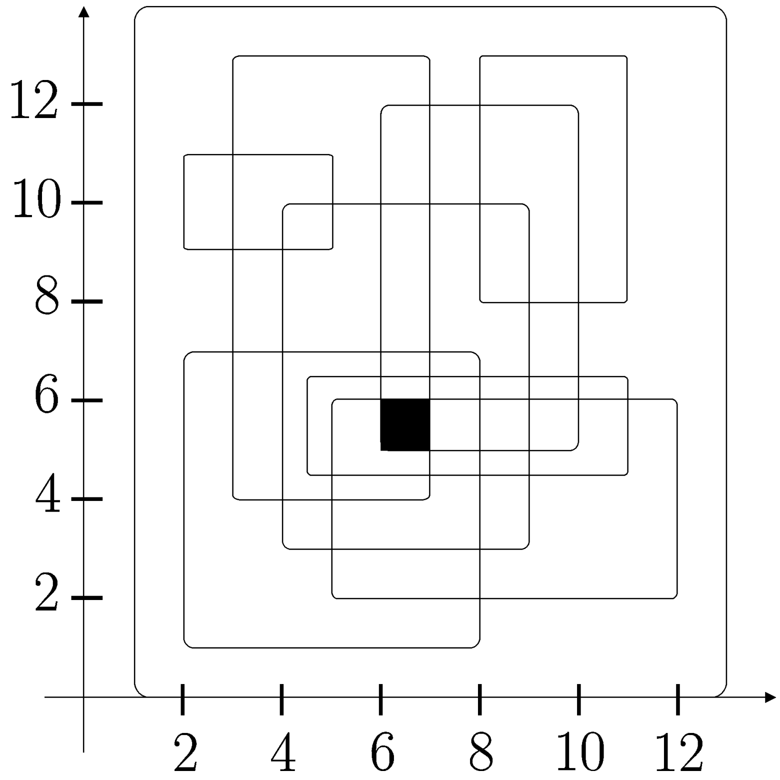
Without outliers

$$\mathbb{X}(k+\mathbf{1}) = \mathbf{f}_k \left(\mathbb{X}(k) \cap \mathbf{g}_k^{-1} \left(\mathbb{Y}(k) \right), \quad \mathbb{N}\left(k\right) \right).$$

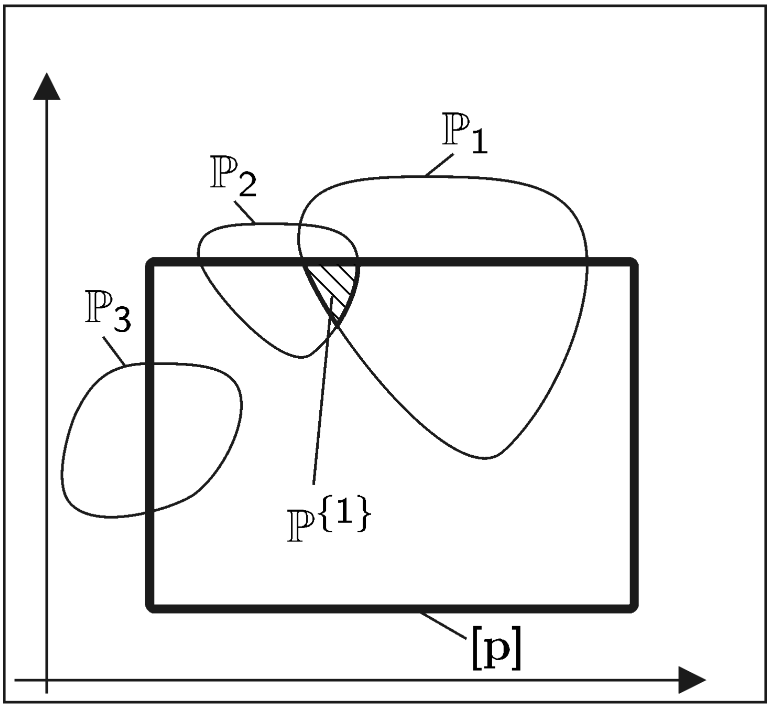
6.3 Relaxed intersection



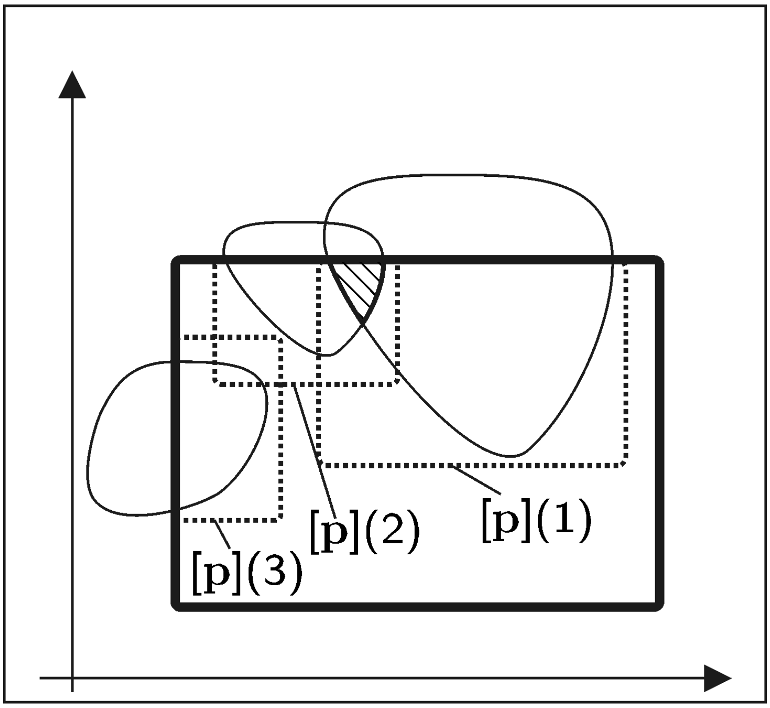
$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$\mathbb{X}_4$
$\mathbb{X}_5$
$\mathbb{X}_6$
$\bigcap_{i\in\{1,\dots,6\}}^{\{2\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\dots,6\}}^{\{3\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\dots,6\}}^{\{4\}} \mathbb{X}_i$

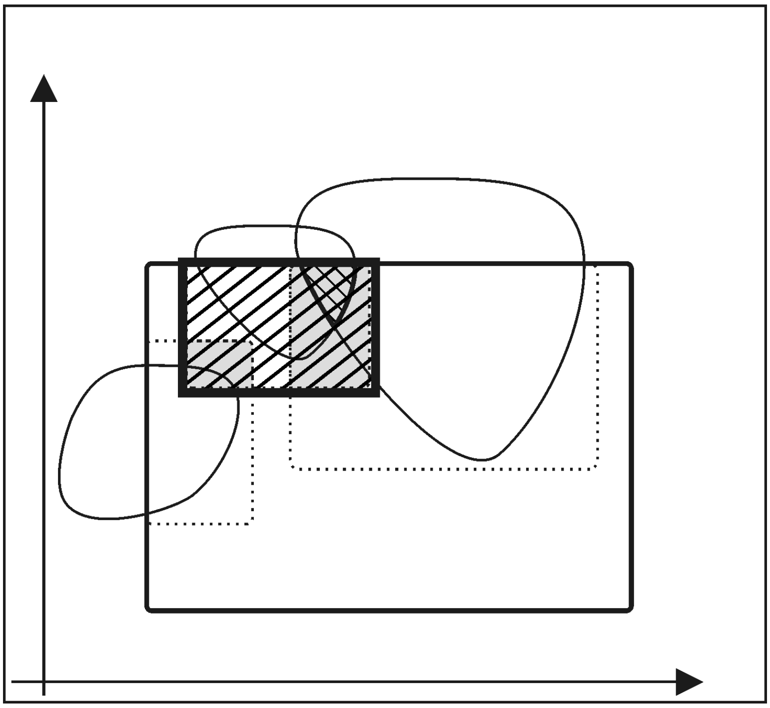


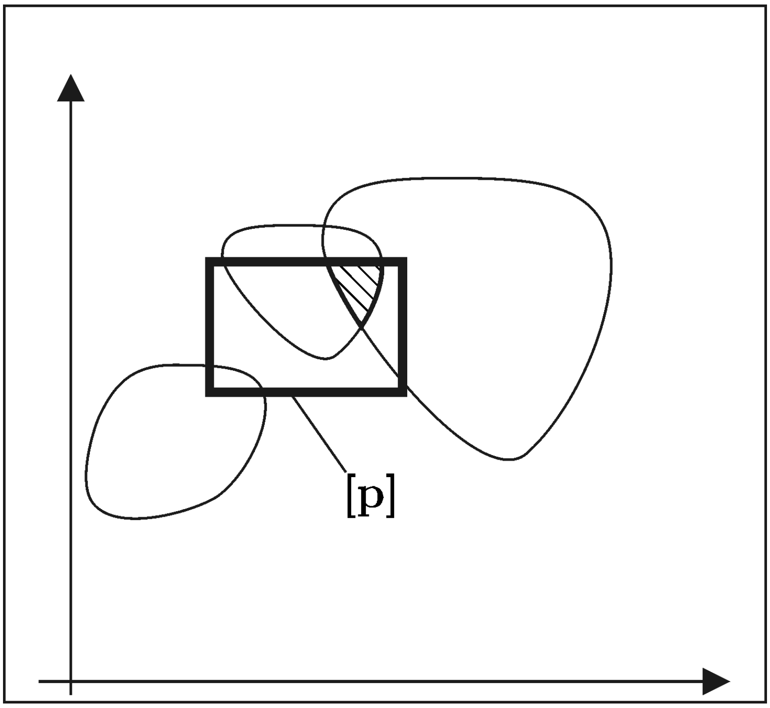
The black box is the 2-intersection of 9 boxes

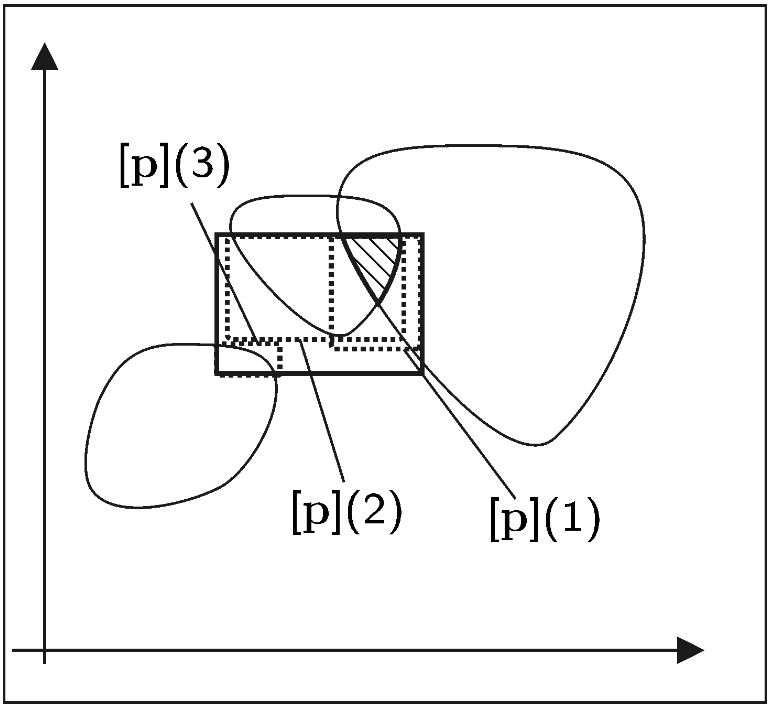


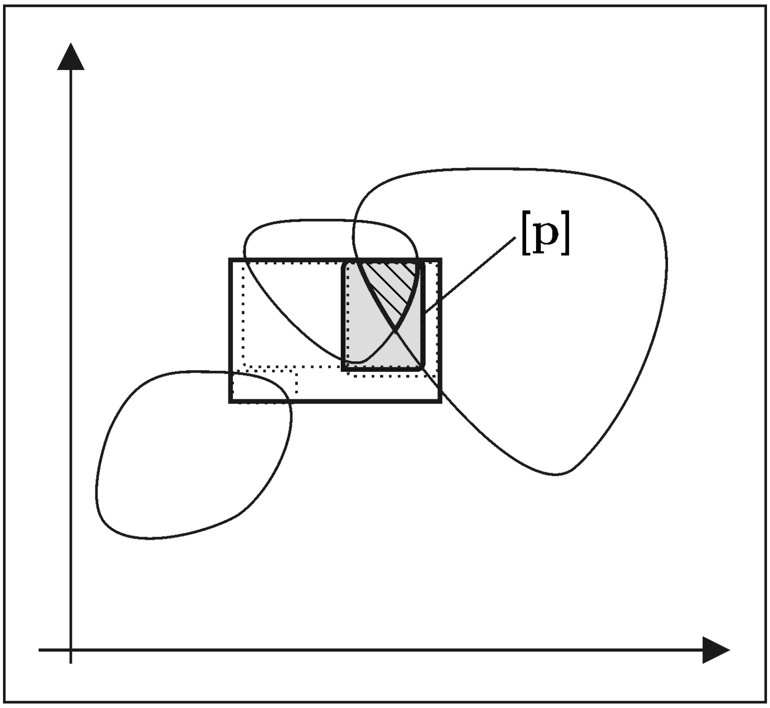
$\mathbb{P}_1$
$\mathbb{P}_2$
$\mathbb{P}_3$
$\mathbb{P}^{\{1\}}$
$[\mathbf{p}]$


[p](1)
[p](2)
[p](3)




[p]


[p](3)
[p](2)
[p](1)


[p]

Show the demo of Jan

6.4 Robust localization

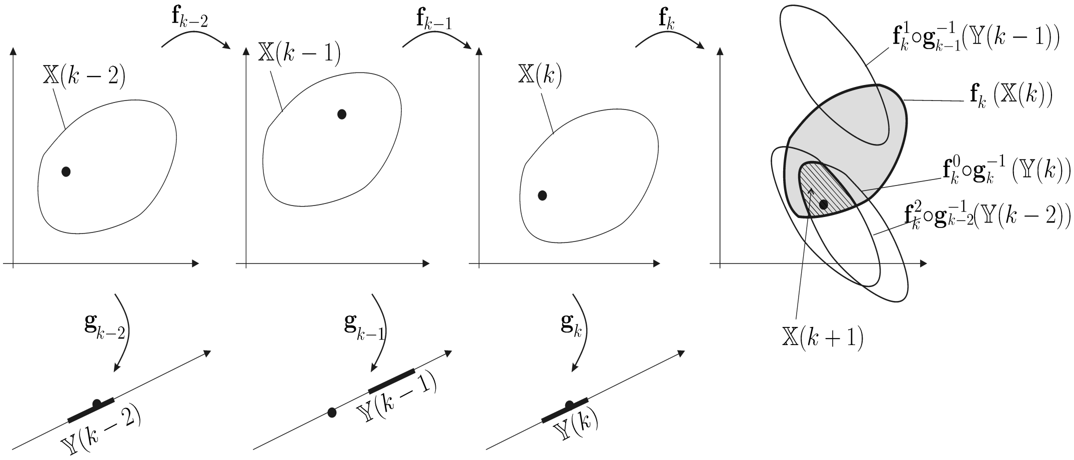
Define

$$\begin{cases} \mathbf{f}_{k:k}(\mathbb{X}) & \stackrel{\text{def}}{=} \quad \mathbb{X} \\ \mathbf{f}_{k_1:k_2+1}(\mathbb{X}) & \stackrel{\text{def}}{=} \quad \mathbf{f}_{k_2}(\mathbf{f}_{k_1:k_2}(\mathbb{X}), \mathbb{N}(k_2)), \ k_1 \leq k_2. \end{cases}$$

The set $\mathbf{f}_{k_1:k_2}(\mathbb{X})$ represents the set of all $\mathbf{x}(k_2)$, consistent with $\mathbf{x}(k_1) \in \mathbb{X}$.

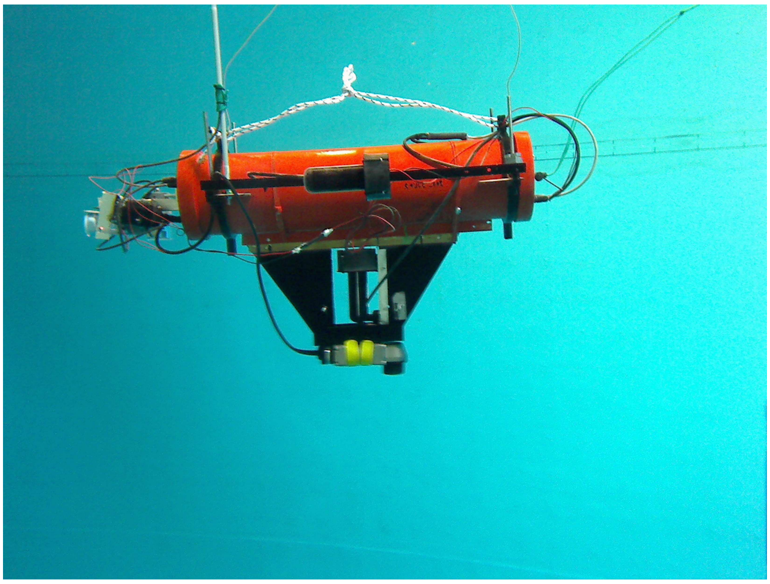
Consider the set state estimator

$$\begin{cases} \mathbb{X}(k) &= \mathbf{f}_{0:k}(\mathbb{X}(0)) \quad \text{if } k < m, \text{ (initialization step)} \\ \mathbb{X}(k) &= \mathbf{f}_{k-m:k}(\mathbb{X}(k-m)) \cap \\ & \bigcap_{i \in \{1,\dots,m\}}^{\{q\}} \mathbf{f}_{k-i:k} \circ \mathbf{g}_{k-i}^{-1}(\mathbb{Y}(k-i)) \quad \text{if } k \geq m \end{cases}$$



$\mathbf{f}_{k-2}$
$\mathbf{f}_{k-1}$
$\mathbf{f}_k$
$\mathbb{X}(k-2)$
$\mathbb{X}(k-1)$
$\mathbb{X}(k)$
$\mathbf{f}_k^1 \circ \mathbf{g}_{k-1}^{-1}(\mathbb{Y}(k-1))$
$\mathbf{f}_k(\mathbb{X}(k))$
$\mathbf{f}_k^0 \circ \mathbf{g}_k^{-1}(\mathbb{Y}(k))$
$\mathbf{f}_k^2 \circ \mathbf{g}_{k-2}^{-1}(\mathbb{Y}(k-2))$
$\mathbb{X}(k+1)$
$\mathbf{g}_{k-2}$
$\mathbf{g}_{k-1}$
$\mathbf{g}_k$
$\mathbb{Y}(k-2)$
$\mathbb{Y}(k-1)$
$\mathbb{Y}(k)$

6.5 Application to localization



Sauc'isse robot inside a swimming pool

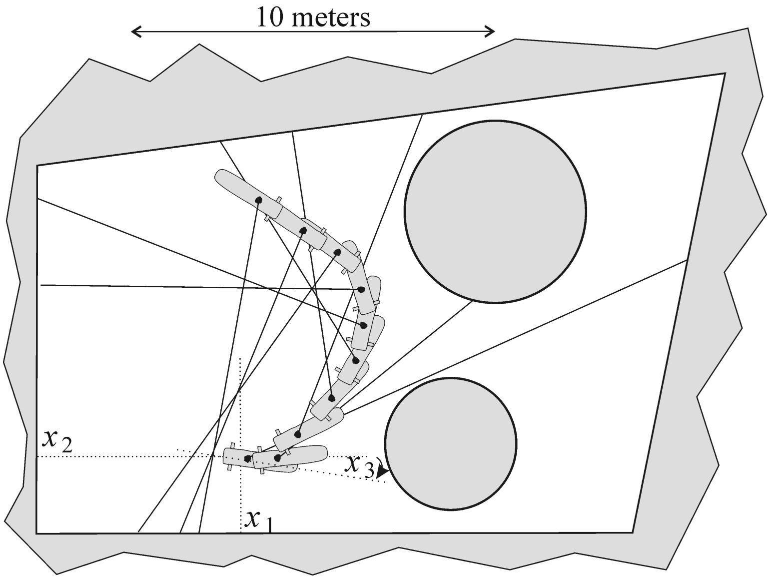
The robot evolution is described by

$$\begin{cases} \dot{x}_1 &= x_4 \cos x_3 \\ \dot{x}_2 &= x_4 \sin x_3 \\ \dot{x}_3 &= u_2 - u_1 \\ \dot{x}_4 &= u_1 + u_2 - x_4, \end{cases}$$

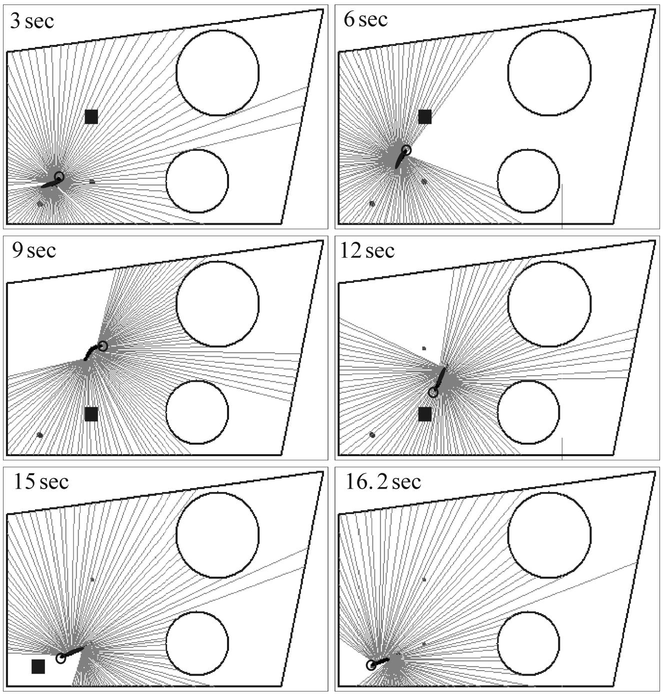
where x_1, x_2 are the coordinates of the robot center, x_3 is its orientation and x_4 is its speed. The inputs u_1 and u_2 are the accelerations provided by the propellers.

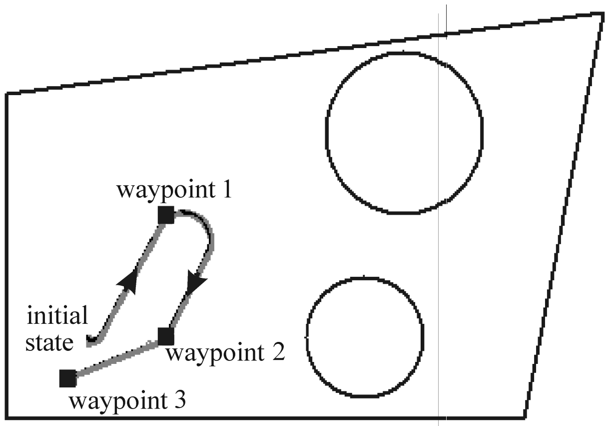
The system can be discretized by $\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k)$, where,

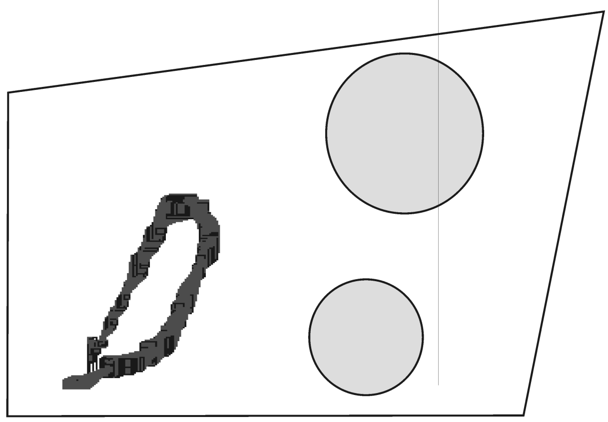
$$\mathbf{f}_k \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + \delta . x_4 . \cos(x_3) \\ x_2 + \delta . x_4 . \sin(x_3) \\ x_3 + \delta . (u_2(k) - u_1(k)) \\ x_4 + \delta . (u_1(k) + u_2(k) - x_4) \end{pmatrix}$$



Underwater robot moving inside a pool


3 sec
6 sec
9 sec
12 sec
15 sec
16. 2 sec


waypoint 1
initial
state
waypoint 2
waypoint 3



Montrer la simu et la vidéo du concours