Estimation and navigation of marine robots in underwater exploration applications

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ECC Workshop on Control, Estimation and Modeling Practice for Robotic Applications in Challenging Environments
London, July 12, 2022

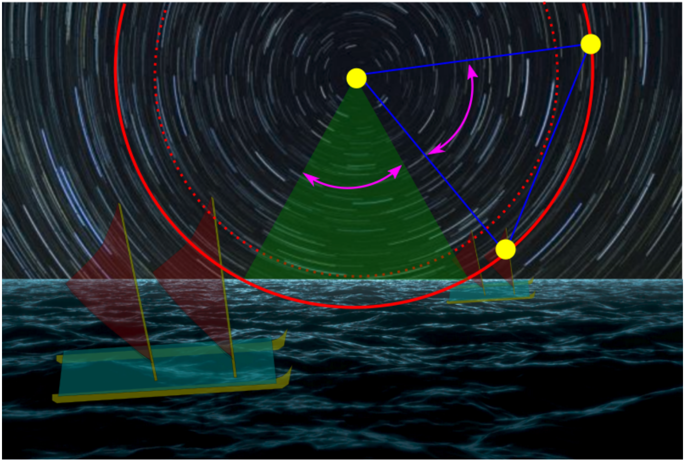
Ancestral method of navigation

Submeeting 2018

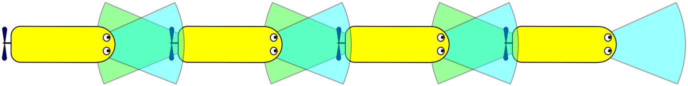
Polynesian navigation

Find the route without GPS, compass and clocks with *wa'a kaulua*[3]

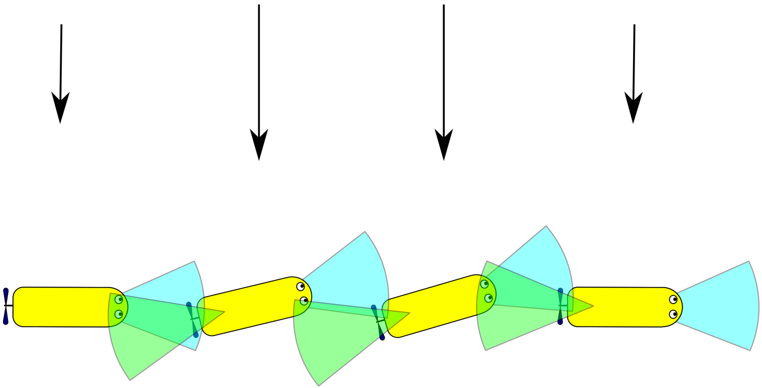
A Challenging environment!



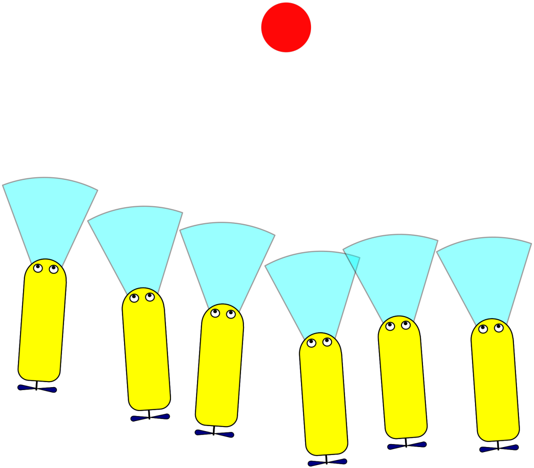
Alignment to keep the heading in case of clouds



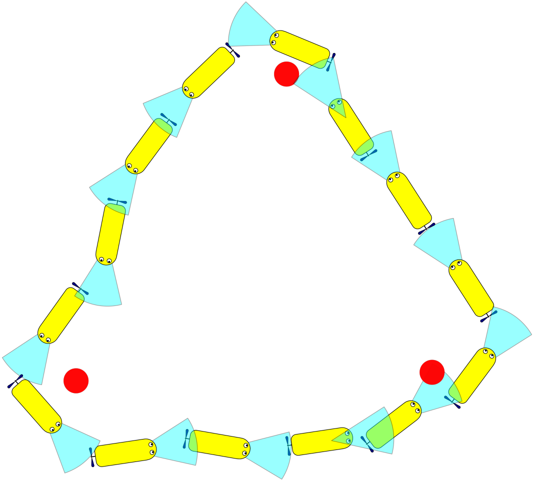
More inertia, more predictable



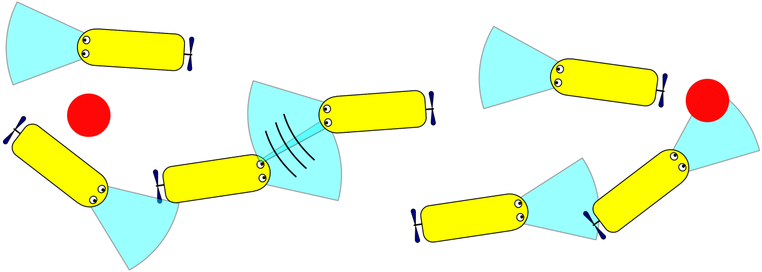
Internal deformations provide information



Explore further

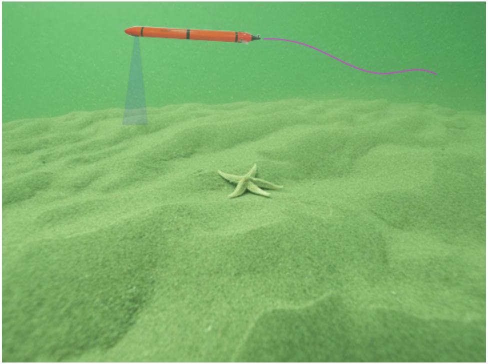


Virtual chain: localization ↔ proprioception

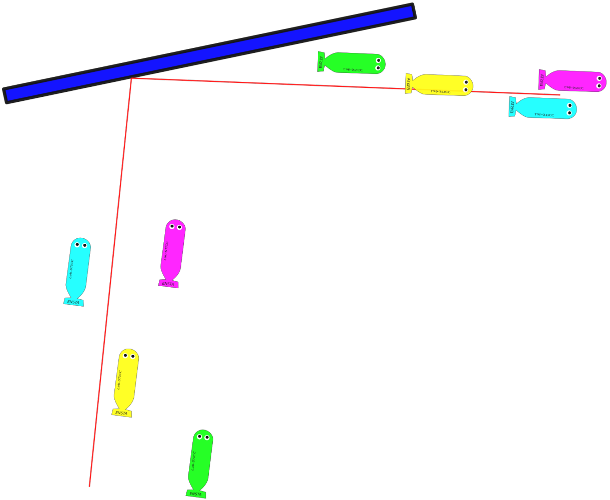


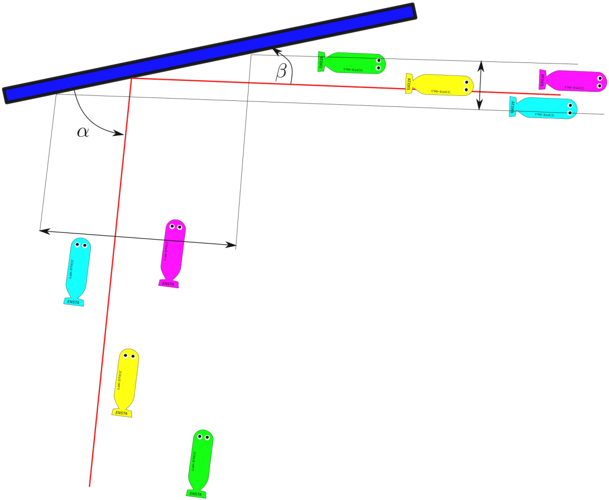
With communication we can do more

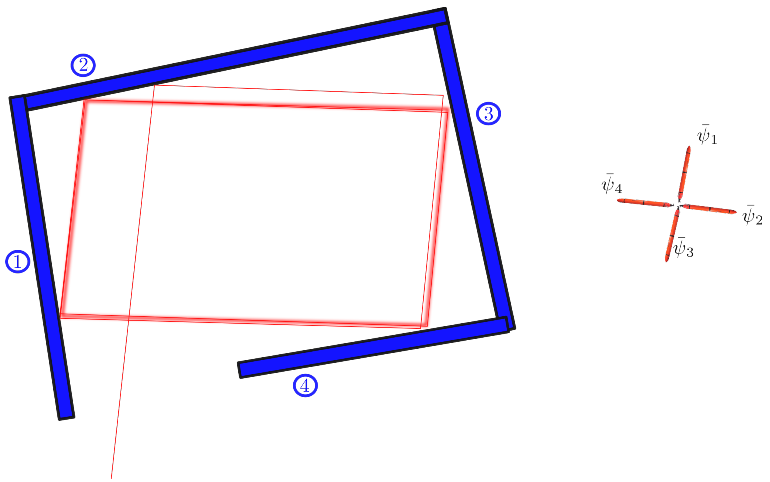
Stable cycles

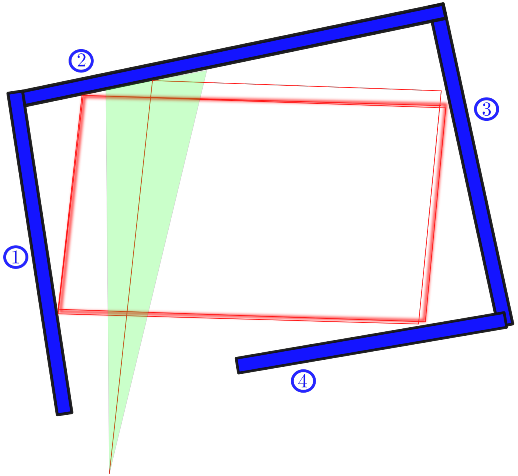


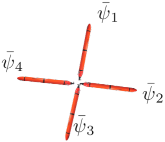
No route exist

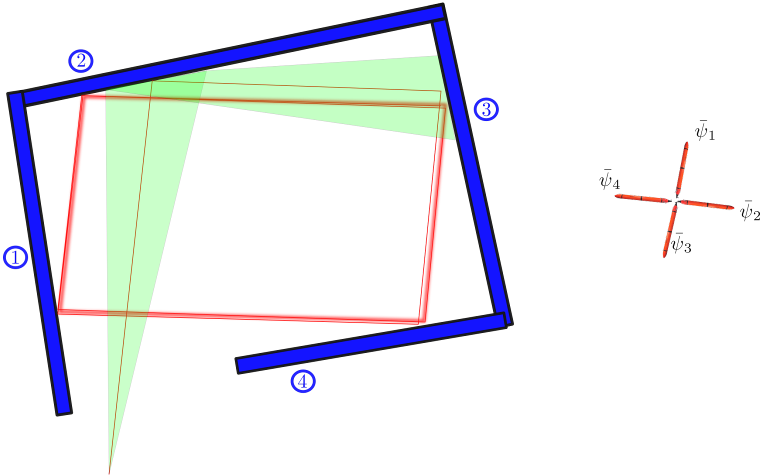

Lab-STICC
ENSTA
Lab-STICC
ENSTA
Lab-STICC
ENSTA
Lab-STICC
ENSTA

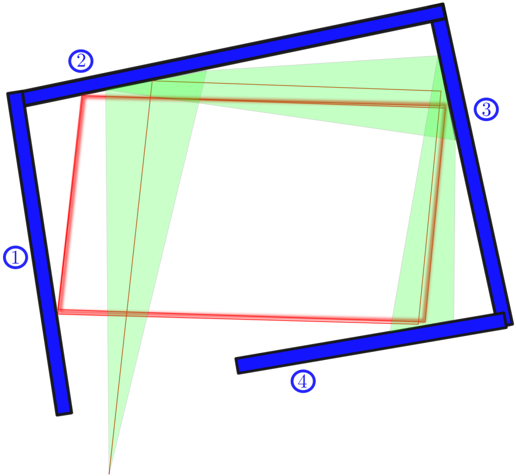

α
β

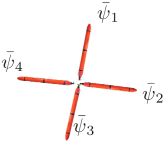

②
③
①
④
$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

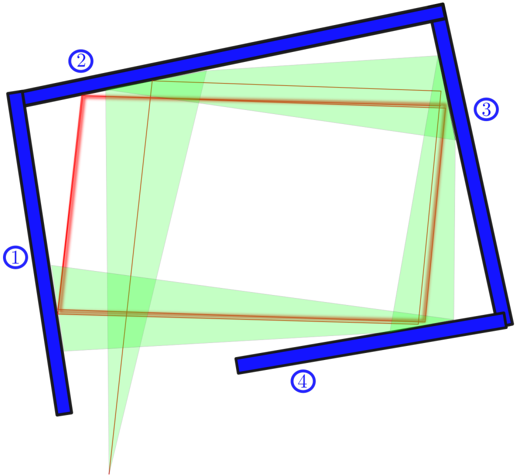

2
3
1
4

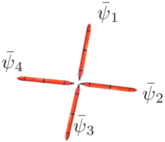

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

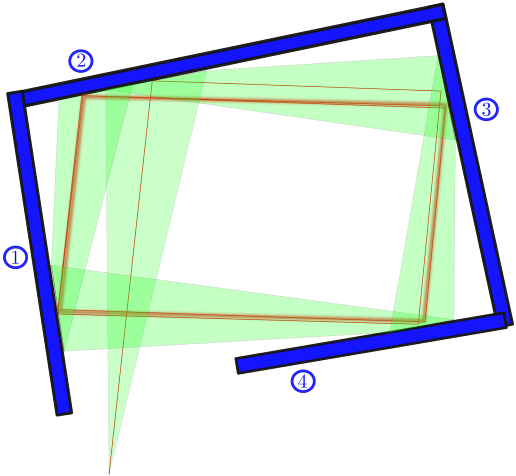

1
2
3
4
$\bar{\psi}_1$
$\bar{\psi}_2$
$\bar{\psi}_3$
$\bar{\psi}_4$

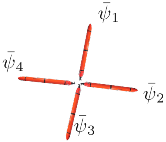

1
2
3
4

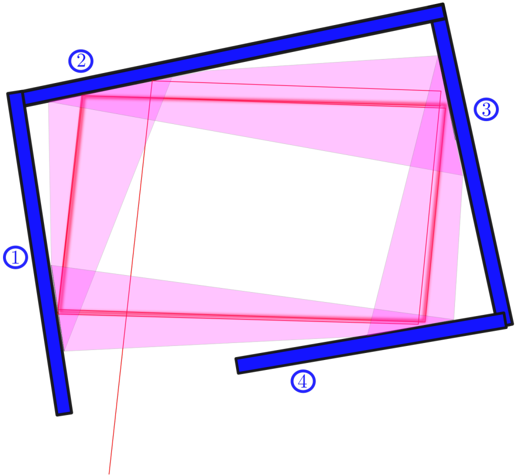

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

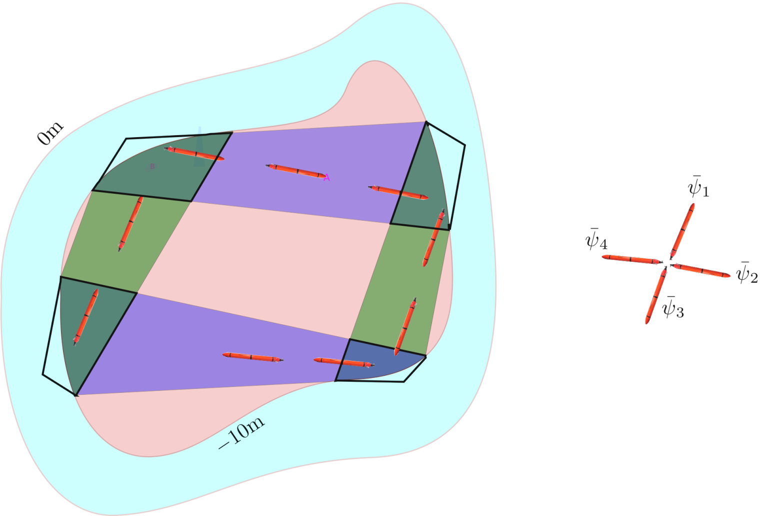

2
3
1
4

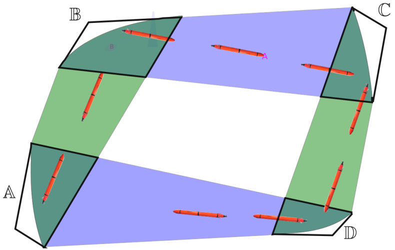

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

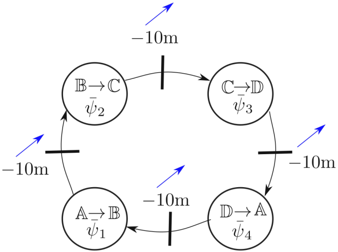

2
3
1
4


$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$


1
2
3
4


0m
−10m
$\bar{\psi}_1$
$\bar{\psi}_2$
$\bar{\psi}_3$
$\bar{\psi}_4$


$\mathbb{B}$
$\mathbb{C}$
$\mathbb{A}$
$\mathbb{D}$

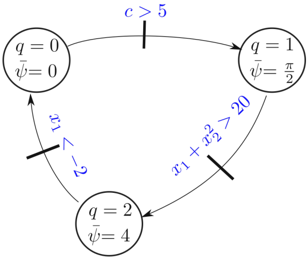

$\mathbb{B}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{D}$ $\bar{\psi}_3$
$\mathbb{A}\to\mathbb{B}$ $\bar{\psi}_1$
$\mathbb{D}\to\mathbb{A}$ $\bar{\psi}_4$
−10m
−10m
−10m
−10m

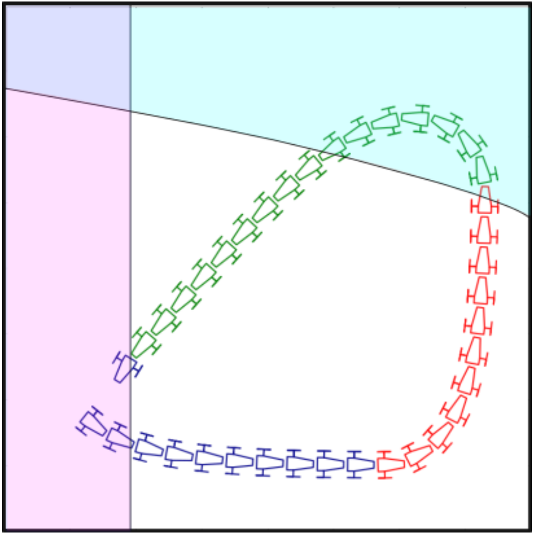
Test-case

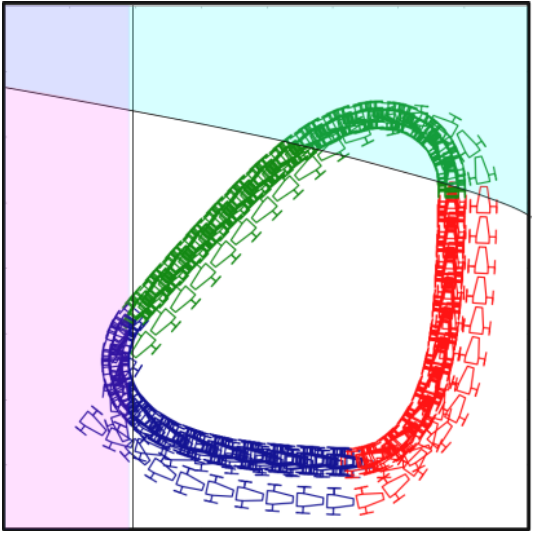
Consider the robot [2]

$$\begin{cases} \dot{x}_1 & = & \cos x_3 \\ \dot{x}_2 & = & \sin x_3 \\ \dot{x}_3 & = & u \end{cases}$$

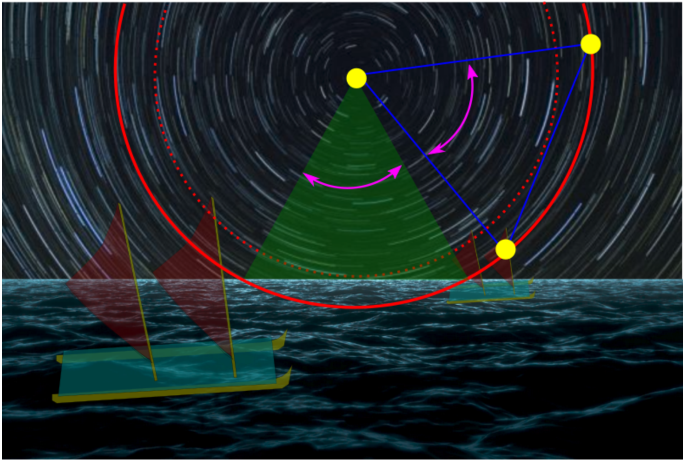
with the heading control $u = \sin(\bar{\psi} - x_3)$.

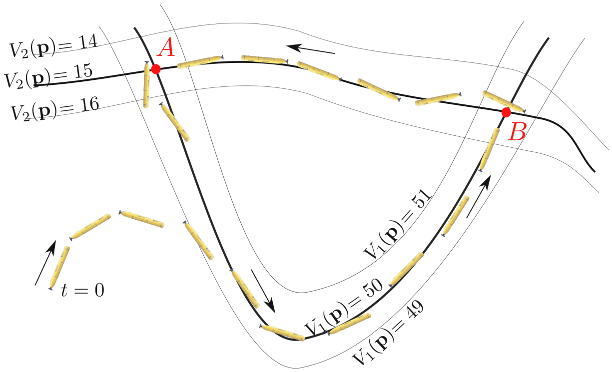


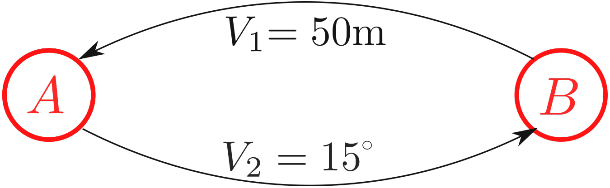




Metric maps ? Topological maps ? Other ?

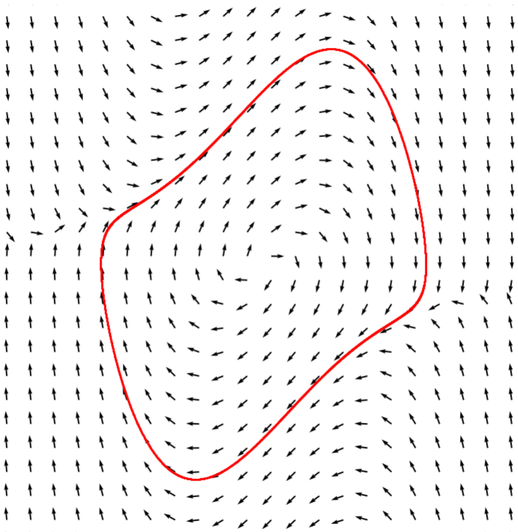



$V_2(\mathbf{p}) = 14$
$V_2(\mathbf{p}) = 15$
$V_2(\mathbf{p}) = 16$
A
B
$V_1(\mathbf{p}) = 51$
$V_1(\mathbf{p}) = 50$
$V_1(\mathbf{p}) = 49$
$t = 0$


$V_1 = 50\text{m}$
A
B
$V_2 = 15^\circ$

Stability with Poincaré map

System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
How to prove that the system has a cycle ?
How to prove that the system is stable ? [1][5]

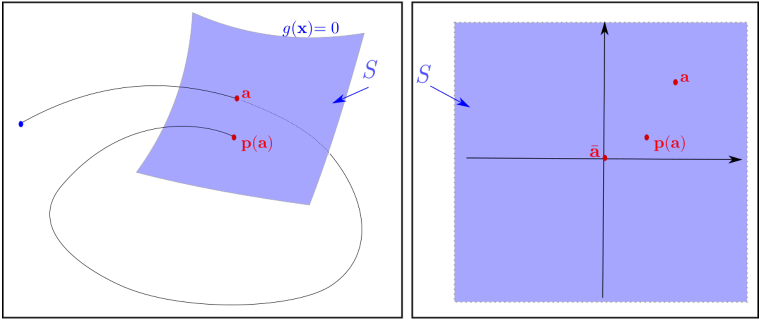


System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
Poincaré section $\mathscr{G}$: $g(\mathbf{x}) = 0$

We define

$$\mathbf{p} : \begin{array}{ccc} \mathscr{G} & \to & \mathscr{G} \\ \mathbf{a} & \mapsto & \mathbf{p}(\mathbf{a}) \end{array}$$

where $\mathbf{p}(\mathbf{a})$ is the point of $\mathscr{G}$ such that the trajectory initialized at $\mathbf{a}$ intersects $\mathscr{G}$ for the first time.

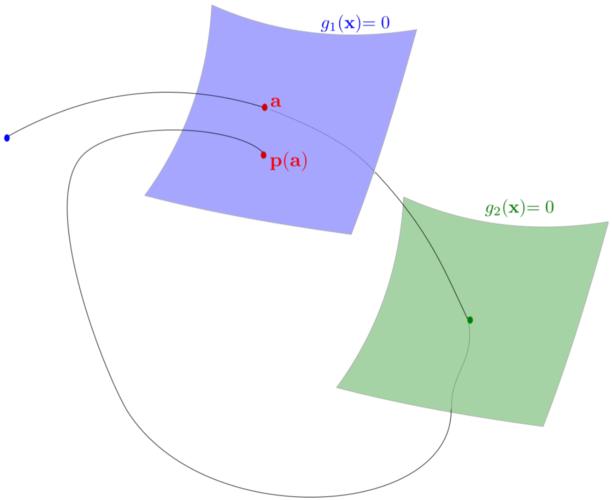

g(x)= 0
S
a
p(a)
S
a
p(a)
ā

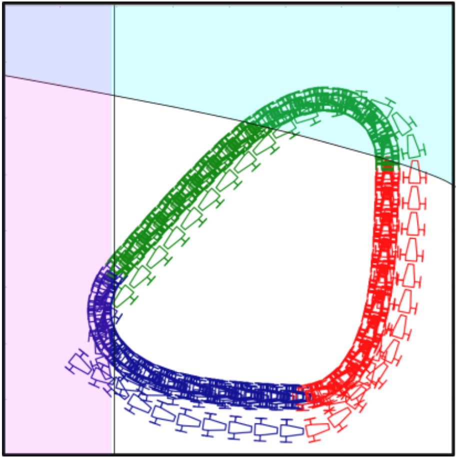
The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$

With hybrid systems

Systems: $\dot{\mathbf{x}} = \mathbf{f}_i(\mathbf{x}), i \in \{1, \ldots, m\}$
Section i: $g_i(\mathbf{x}) = 0$


$g_1(\mathbf{x}) = 0$
a
p(a)
$g_2(\mathbf{x}) = 0$

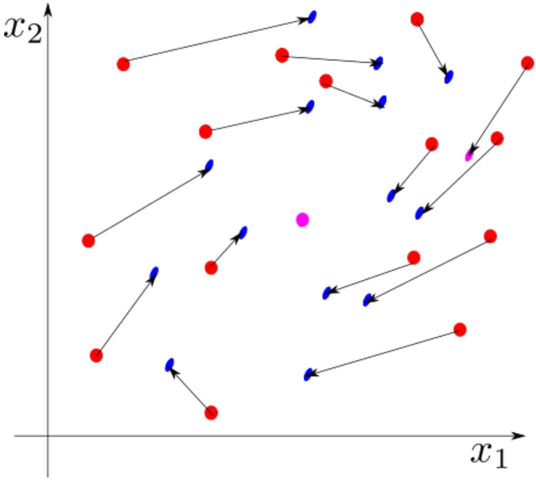


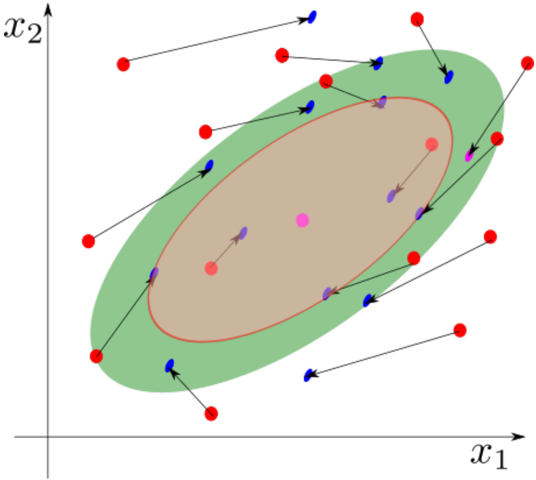
Proving the stability

Consider the discrete time system

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k)$$

with $\mathbf{f}(\mathbf{0}) = \mathbf{0}$.


x_2
x_1


x_2
x_1

We have to find

$$\mathscr{E}_{\mathbf{x}} : \mathbf{x}^{\mathsf{T}} \cdot \mathbf{P} \cdot \mathbf{x} \leq \varepsilon$$

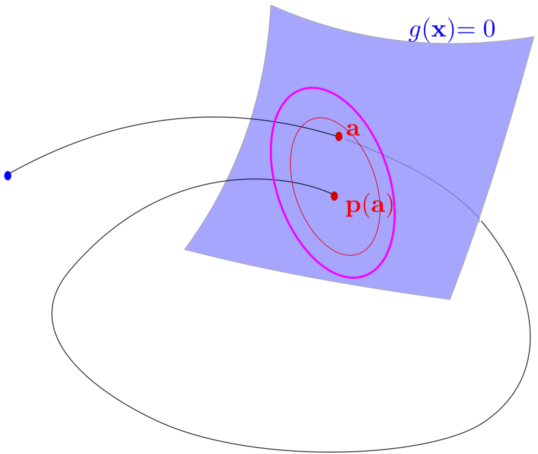
Such that

$$\mathbf{f}(\mathscr{E}_{\mathbf{x}}) \subset \mathscr{E}_{\mathbf{x}}$$

Stability of cycles

The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$



See [4]

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