Validation a priori d'une mission sous-marine

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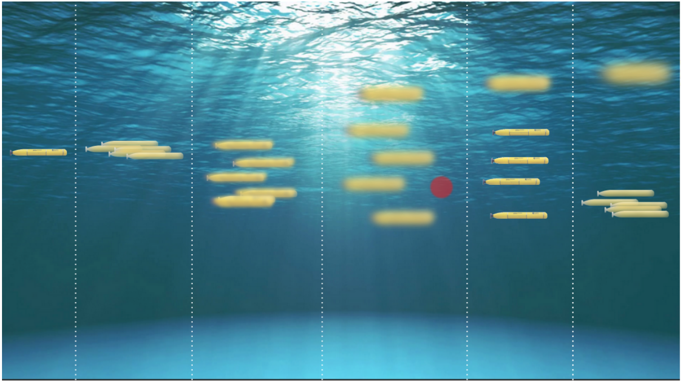
https://drones-cap-2023.sciencesconf.org/

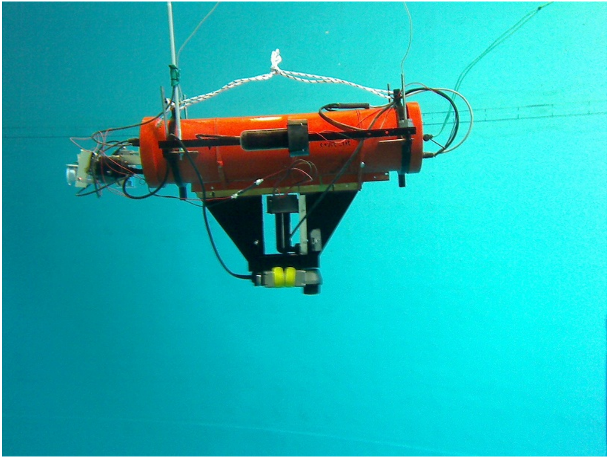
https://youtu.be/hNqIShmMQjA



1. Robots

A robot is a mechanical system equipped with

- actuators
- sensors
- an intelligence
- a memory.



Saucisse (ENSTA Bretagne)

Reachability

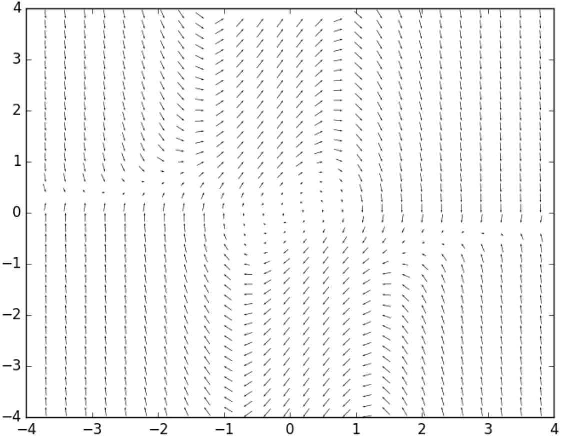
A dynamical system [Newton 1690]

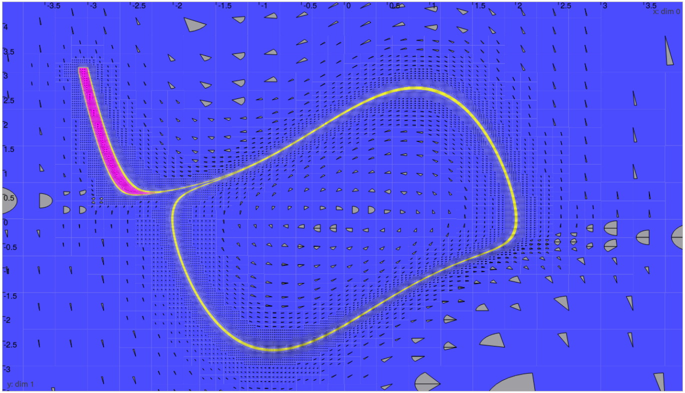
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

Example. Van der Pol oscillator

$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -\left(x_1^2 - 1\right) x_2 - x_1 \end{cases}$$

$$\mathbf{x}(0) \in \mathbb{X}_0$$


4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4


-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
x: dim 0
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
y: dim 1

Vehicle

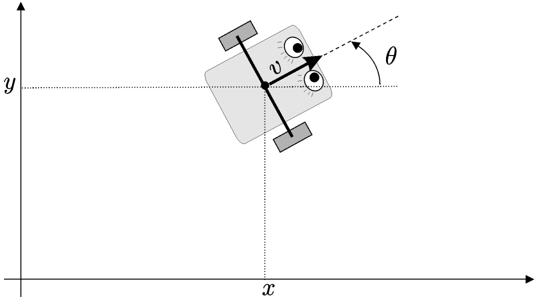
A **vehicle** is a dynamical system with actuators

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) .$$

Example. The Dubin's car (1957).

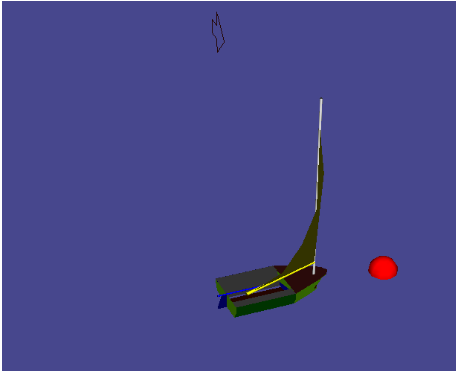
$$\begin{cases} \dot{x} = \cos\theta \\ \dot{y} = \sin\theta \\ \dot{\theta} = u \end{cases}$$

with $u \in [-1, 1]$.



Simulation (formule d'Euler, 1770)

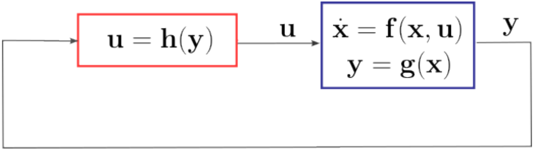
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \quad \sim \quad \mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \delta \cdot \mathbf{f}(\mathbf{x}(t_k),\mathbf{u}(t_k))$$



Intelligence

A robot is a vehicle with sensors, actuators and an intelligence:

$$\begin{array}{rcll} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x},\mathbf{u}) & \text{(evolution)} \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) & \text{(observation)} \\ \mathbf{u} & = & \mathbf{h}(\mathbf{y}). & \text{(control)} \end{array}$$

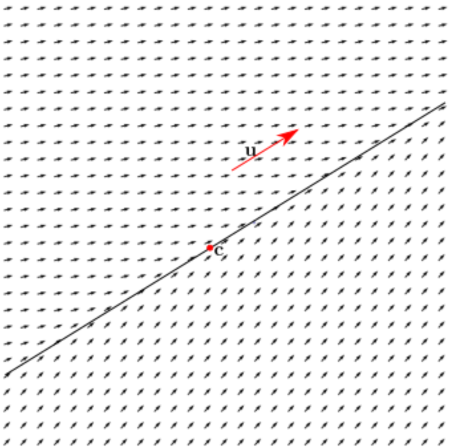


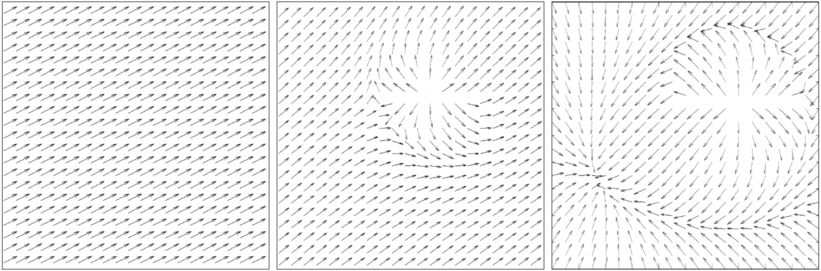
We have

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{h}(\mathbf{g}(\mathbf{x}))) = \psi(\mathbf{x})$$

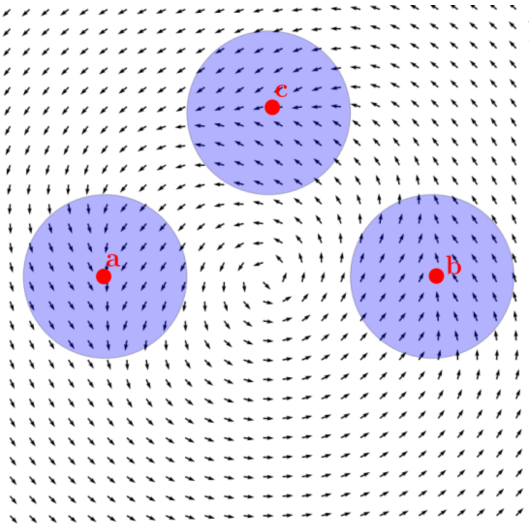
Thus an intelligent vehicle is a dynamical system.

Control


u
c



Artificial potential fields


c
a
b

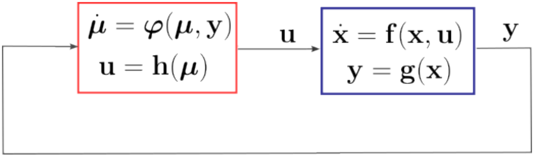
A robot has memory

A robot is a intelligent vehicle with memory

$$\begin{array}{rcll}
\dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & \text{(ontic evolution)} \\
\mathbf{y}(t) & = & \mathbf{g}(\mathbf{x}(t)) & \text{(observation)} \\
\mu_{k+1} & = & \varphi(\mu_k,\mathbf{y}(t_k)) & \text{(epistemic evolution)} \\
\mathbf{u}(t_k) & = & \mathbf{h}(\mu_k) & \text{(control)}
\end{array}$$

With an analog controller

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) && \text{(ontic evolution)} \\
\mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) && \text{(observation)} \\
\dot{\mu}(t) &= \varphi(\mu(t), \mathbf{y}(t)) && \text{(epistemic evolution)} \\
\mathbf{u}(t) &= \mathbf{h}(\mu(t)). && \text{(control)}
\end{aligned}
$$



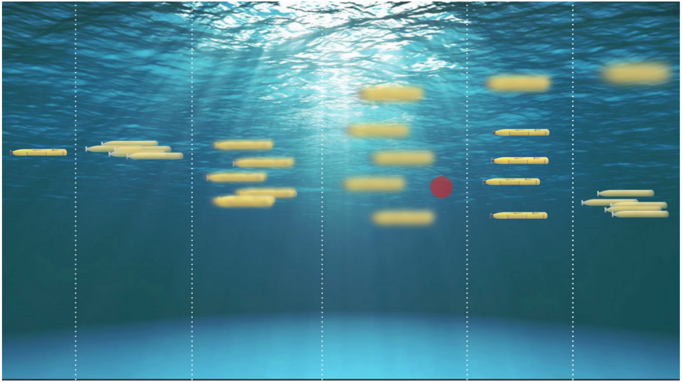
$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{h}(\mu(t))) && \text{(ontic evolution)} \\ \dot{\mu}(t) &= \varphi(\mu(t), \mathbf{g}(\mathbf{x}(t))) && \text{(epistemic evolution)} \end{aligned}$$

The global state is $\mathbf{z} = (\mathbf{x}, \mu)$. We have

$$\dot{\mathbf{z}} = \psi(\mathbf{z})$$

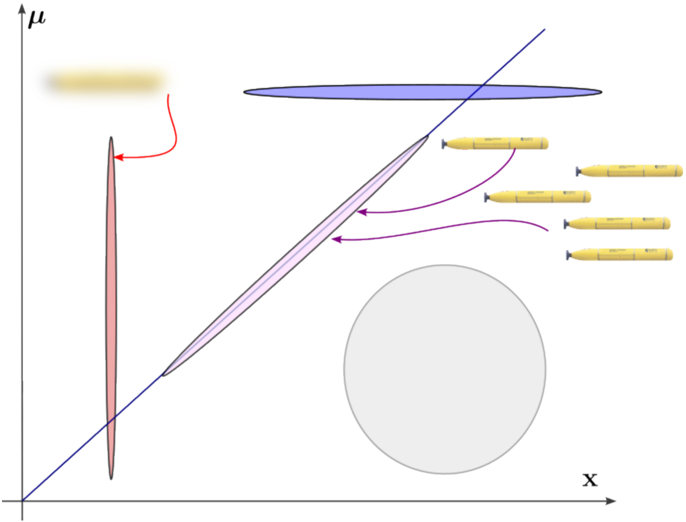
An intelligent vehicle with memory is thus a dynamical system.

2. Uncertainties



State of the robot : $\mathbf{z} = (\mathbf{x}, \mu)$, with

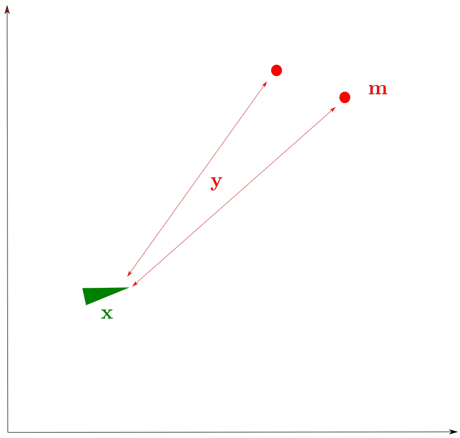
- $\mathbf{x}$: the ontic state
- μ: the epistemic state


μ
x

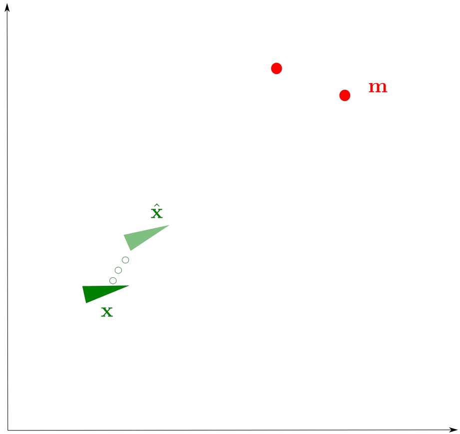
Perception : We measure $\mathbf{x}$
Communication : we measure μ

Swarm

https://youtu.be/xlgp9P0SY1Y
t=1:40


m
y
x

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x}) \end{aligned}$$

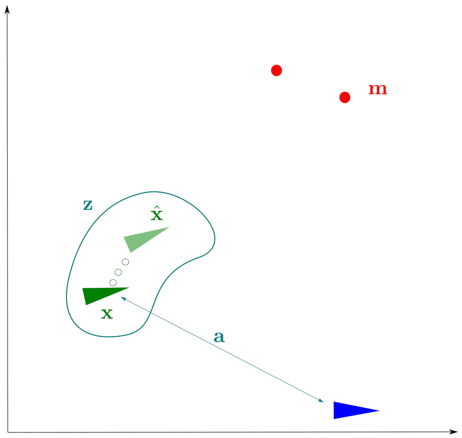

m
$\hat{\mathbf{x}}$
$\mathbf{x}$

If we set $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$, we get

$$\dot{\mathbf{z}} = \psi(\mathbf{z})$$

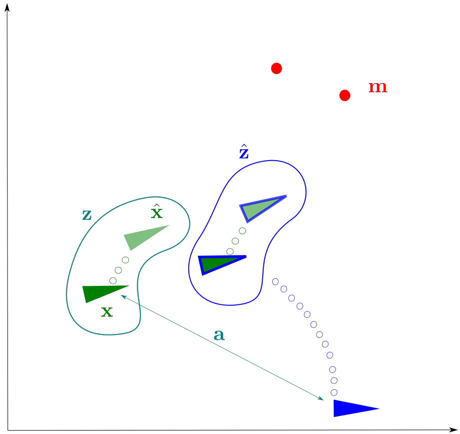
Assume that we can observe the motion of the robot

$$\begin{aligned}\dot{\mathbf{z}} &= \psi(\mathbf{z})\\ \mathbf{a} &= \eta(\mathbf{x})\end{aligned}$$

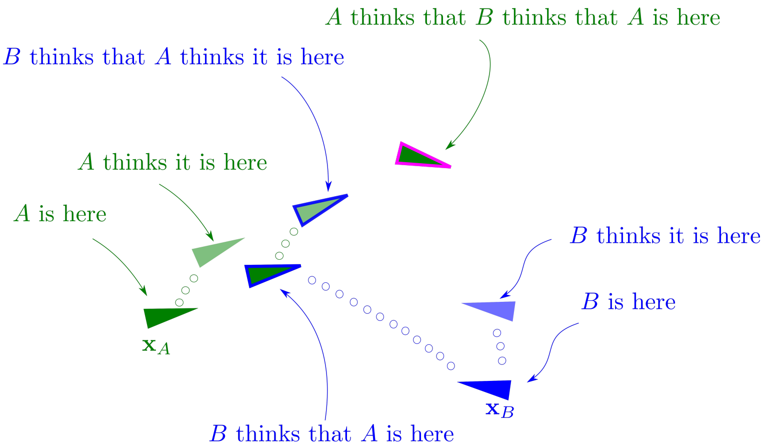

m
z
x̂
x
a

We can build an observer for $\mathbf{z}$:

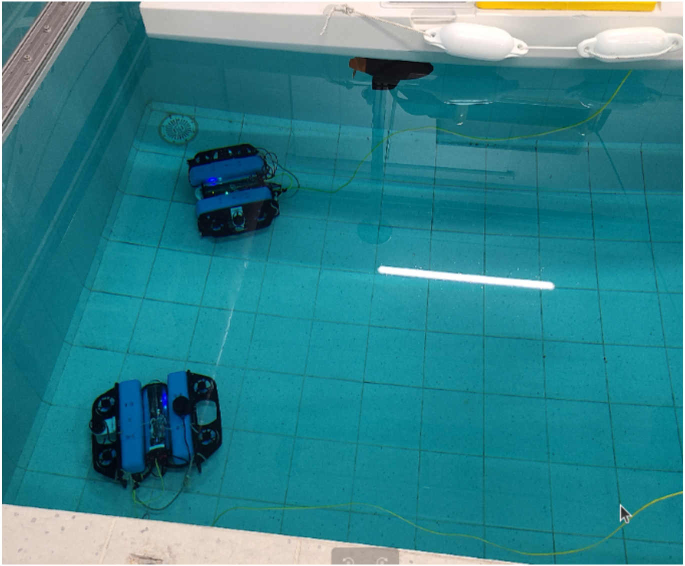
$$\begin{aligned}\dot{\mathbf{z}} &= \psi(\mathbf{z}) \\ \mathbf{a} &= \eta(\mathbf{x}) \\ \dot{\hat{\mathbf{z}}} &= \hat{\psi}(\mathbf{a}, \hat{\mathbf{z}})\end{aligned}$$


m
ẑ
z
x̂
x
a

Distributed knowledge


A thinks that B thinks that A is here
B thinks that A thinks it is here
A thinks it is here
A is here
$\mathbf{x}_A$
B thinks it is here
B is here
$\mathbf{x}_B$
B thinks that A is here

Experiments



References

1. Interval analysis [7, 3, 4]
2. Stability with intervals : [6]
3. SLAM with intervals : [2]
4. Interval tubes [8], [1]
5. Reachability [5]

F. L. Bars, J. Sliwka, O. Reynet, and L. Jaulin.
State estimation with fleeting data.
Automatica, 48(2):381–387, 2012.

L. Jaulin.
Range-only SLAM with occupancy maps; A set-membership approach.
IEEE Transaction on Robotics, 27(5):1004–1010, 2011.

L. Jaulin, M. Kieffer, O. Didrit, and E. Walter.
Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control and Robotics.
Springer-Verlag, London, 2001.

L. Jaulin, O. Reynet, B. Desrochers, S. Rohou, and J. Ninin.
IaMOOC, Interval analysis with applications to parameter estimation and robot localization ,
www.ensta-bretagne.fr/iamooc/.

ENSTA-Bretagne, 2019.

T. Le-Mézo, L. Jaulin, and B. Zerr.
An interval approach to solve an initial value problem.
AIP Conference Proceedings, 1776(1), 2016.

M. Louedec, L. Jaulin, and C. Viel.
Computational tractable guaranteed numerical method to study the stability of n-dimensional time-independent nonlinear systems with bounded perturbation.
Automatica, 153:110981, 2023.

R. Moore.
Methods and Applications of Interval Analysis.
Society for Industrial and Applied Mathematics, jan 1979.

S. Rohou, L. Jaulin, M. Mihaylova, F. L. Bars, and S. Veres.
Guaranteed Computation of Robots Trajectories.
Robotics and Autonomous Systems, 93:76–84, 2017.