# Navigation sous marine par la méthode des cycles stables

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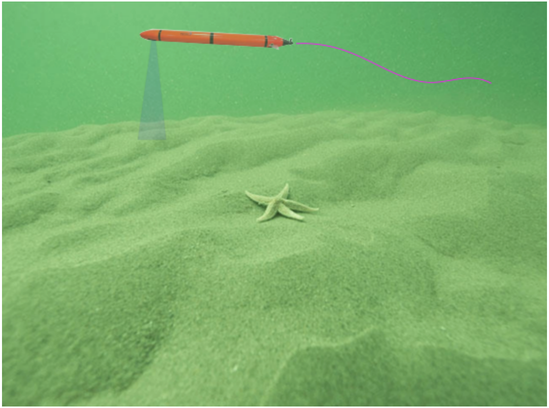
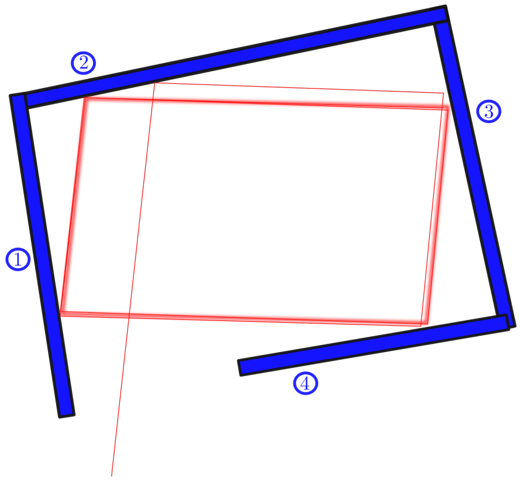
Journées Drone&Caps 27-30 septembre 2021

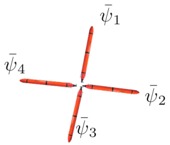
# Stable cycles

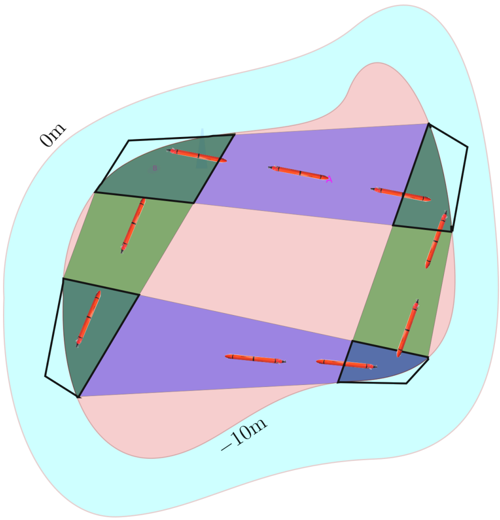
Submeeting 2018

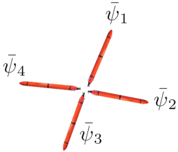
# Cycles

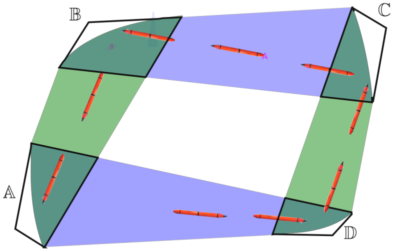


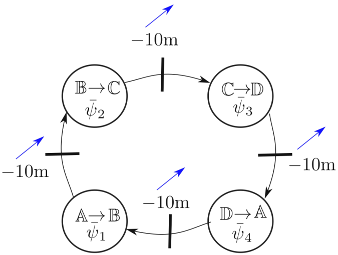

2
3
1
4


$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$


0m
−10m
B
A


$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$


𝔹
ℂ
𝔸
𝔻

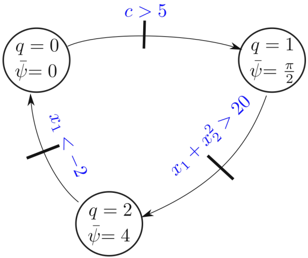

−10m
−10m
−10m
−10m
𝔹→ℂ
$\bar{\psi}_2$
ℂ→𝔻
$\bar{\psi}_3$
𝔸→𝔹
$\bar{\psi}_1$
𝔻→𝔸
$\bar{\psi}_4$

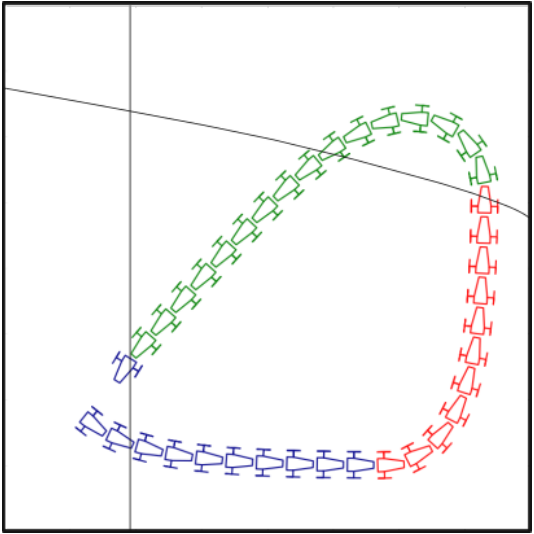
# Test-case

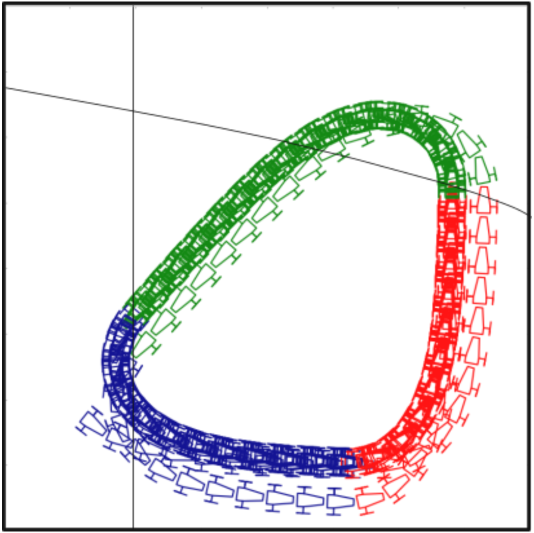
Consider the robot [2]

$$\begin{cases} \dot{x}_1 & = & \cos x_3 \\ \dot{x}_2 & = & \sin x_3 \\ \dot{x}_3 & = & u \end{cases}$$

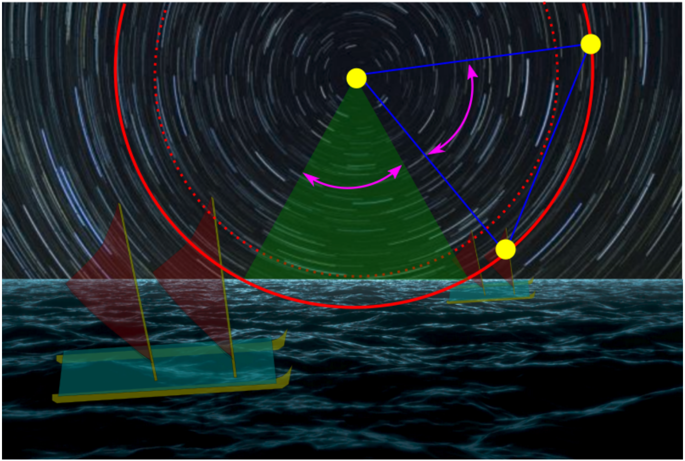
with the heading control $u = \sin(\bar{\psi} - x_3)$.

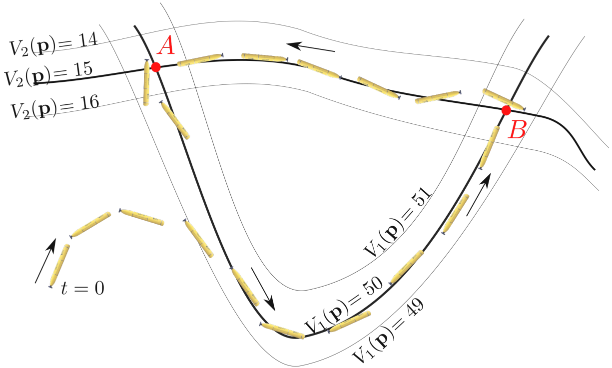


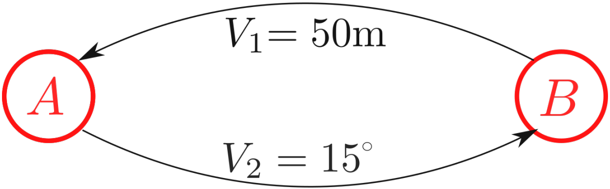


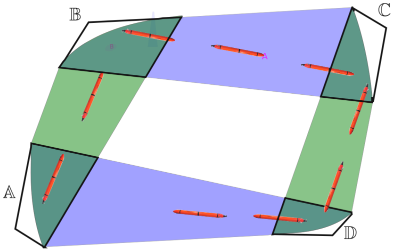


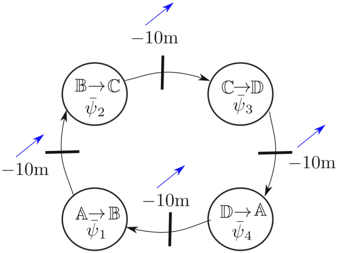
# Metric maps ? Topological maps ? Other ?

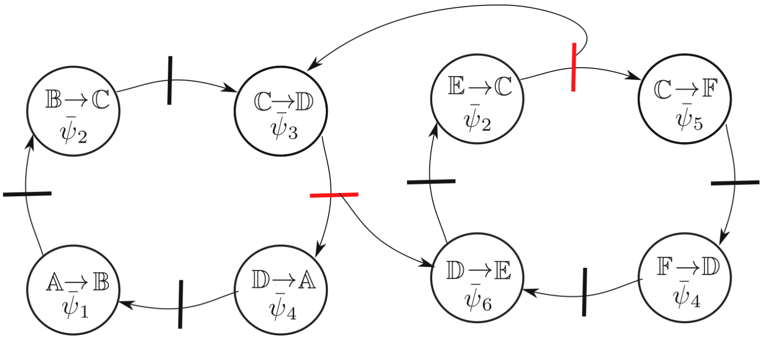



$V_2(\mathbf{p})= 14$
$V_2(\mathbf{p})= 15$
$V_2(\mathbf{p})= 16$
A
B
$V_1(\mathbf{p})= 51$
$V_1(\mathbf{p})= 50$
$V_1(\mathbf{p})= 49$
$t = 0$

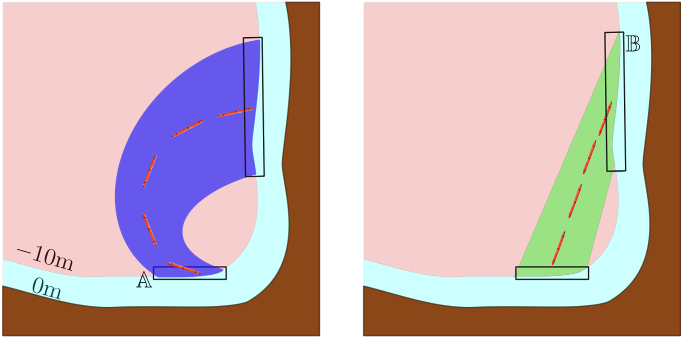

$V_1 = 50\text{m}$
$A$
$B$
$V_2 = 15^\circ$

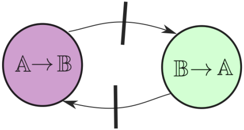

B
C
A
D


−10m
B→C
C→D
−10m
−10m
−10m
A→B
D→A


$\mathbb{B}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{D}$ $\bar{\psi}_3$
$\mathbb{E}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{F}$ $\bar{\psi}_5$
$\mathbb{A}\to\mathbb{B}$ $\bar{\psi}_1$
$\mathbb{D}\to\mathbb{A}$ $\bar{\psi}_4$
$\mathbb{D}\to\mathbb{E}$ $\bar{\psi}_6$
$\mathbb{F}\to\mathbb{D}$ $\bar{\psi}_4$

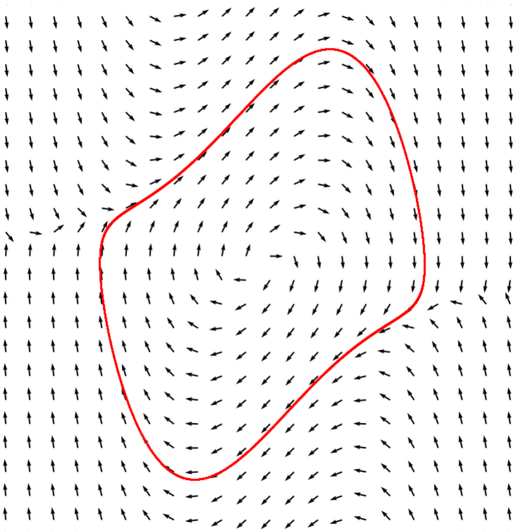
# What to do on the lake ?


−10m
0m
A
B


A→B
B→A

# Stability with Poincaré map

System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
How to prove that the system has a cycle ?
How to prove that the system is stable ? [1][3]

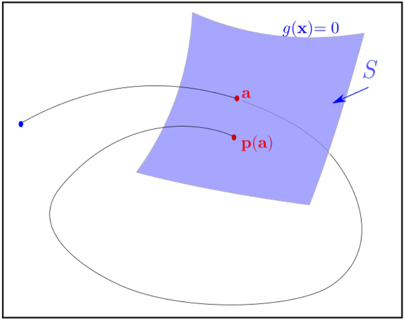


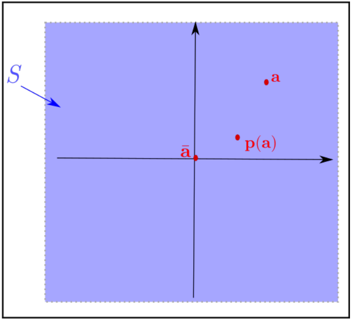
System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
Poincaré section $\mathscr{G}$: $g(\mathbf{x}) = 0$

We define

$$\mathbf{p}: \begin{array}{ccc} \mathscr{G} & \to & \mathscr{G} \\ \mathbf{a} & \mapsto & \mathbf{p}(\mathbf{a}) \end{array}$$

where $\mathbf{p}(\mathbf{a})$ is the point of $\mathscr{G}$ such that the trajectory initialized at $\mathbf{a}$ intersects $\mathscr{G}$ for the first time.


$g(\mathbf{x})=0$
$S$
a
p(a)

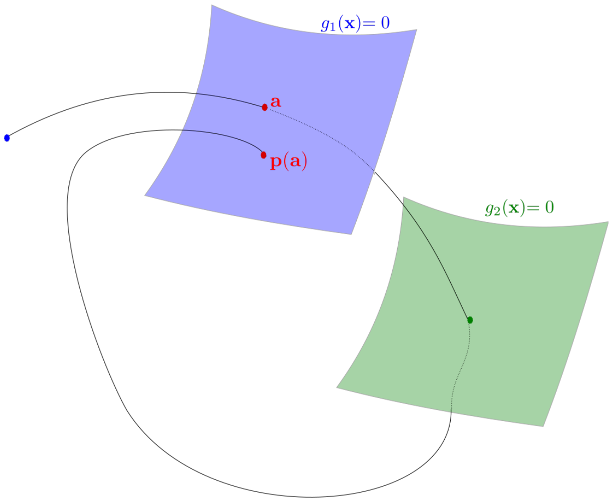

$S$
a
p(a)
ā

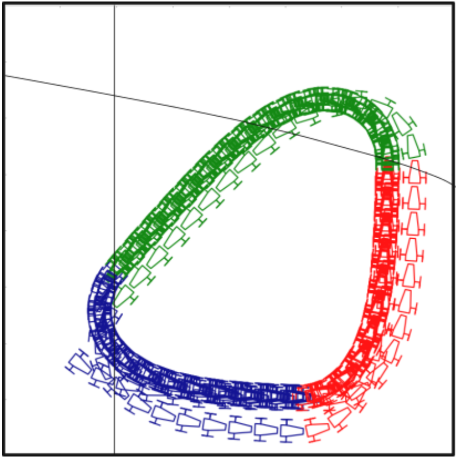
The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$

# With hybrid systems

Systems: $\dot{\mathbf{x}} = \mathbf{f}_i(\mathbf{x}), i \in \{1, \ldots, m\}$
Section $i$: $g_i(\mathbf{x}) = 0$


$g_1(\mathbf{x})=0$
$\mathbf{a}$
$\mathbf{p}(\mathbf{a})$
$g_2(\mathbf{x})=0$



After 10 laps, we get approximately $\bar{\mathbf{a}} = (-2, 2.3, -2.29)$.

To conclude about the stability, we compute the Jacobian matrix

$$\mathsf{J}(\bar{\mathbf{a}}) = \frac{d\mathbf{p}}{d\mathbf{a}}(\bar{\mathbf{a}}) =$$

$$\begin{pmatrix} \frac{\partial p_1}{\partial x_1} & \frac{\partial p_1}{\partial x_2} & \frac{\partial p_1}{\partial x_3} \\ \frac{\partial p_2}{\partial x_1} & \frac{\partial p_2}{\partial x_2} & \frac{\partial p_2}{\partial x_3} \\ \frac{\partial p_3}{\partial x_1} & \frac{\partial p_3}{\partial x_2} & \frac{\partial p_3}{\partial x_3} \end{pmatrix}$$

We get

$$\begin{pmatrix} \frac{\partial p_2}{\partial x_2}(\bar{\mathbf{a}}) & \frac{\partial p_2}{\partial x_3}(\bar{\mathbf{a}}) \\ \frac{\partial p_3}{\partial x_2}(\bar{\mathbf{a}}) & \frac{\partial p_3}{\partial x_3}(\bar{\mathbf{a}}) \end{pmatrix} \simeq \begin{pmatrix} 0.045 & 2.66 \\ 0.0 & 0.02 \end{pmatrix}.$$

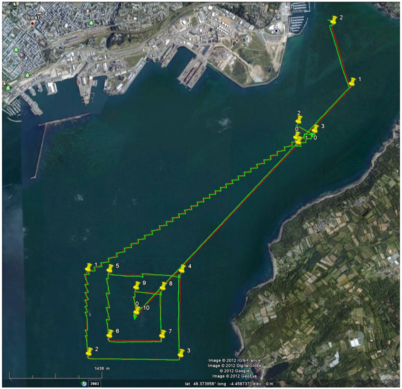
The eigen values are approximately

$$\{0.06, 0.009\}$$

Both are in the unit circle. We conclude that the limit cycle is stable.

# Vaimos

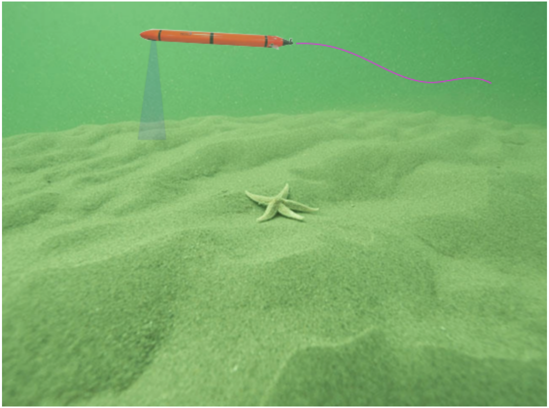
(Fabrice Le Bars, Olivier Ménage, Patrick Rousseaux)


Brest
1438 m
Image © 2012 IGN-France
Image © 2012 DigitalGlobe
© 2012 Google
Image © 2012 GeoEye

When no GPS, a cycle stategy could be considered.
Bounds could be on the coat or on the view of buoys.

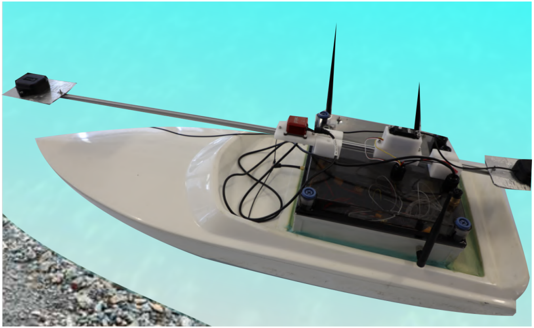
# Robtide

(Nathan Fourniol, Morgan Louédec, Alain Bertholom)



# Robspeed

(Nathan Fourniol, Morgan Louédec, Alain Bertholom)



# Saturne

(Alain Bertholom, Fabrice Le Bars)

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Interval centred form for proving stability of non-linear discrete-time system.
In *SNR, Vienna*, 2020.

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ISTE editions, 2015.

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*Foundations of Computational Mathematics*, 2(1):53–117, 2002.