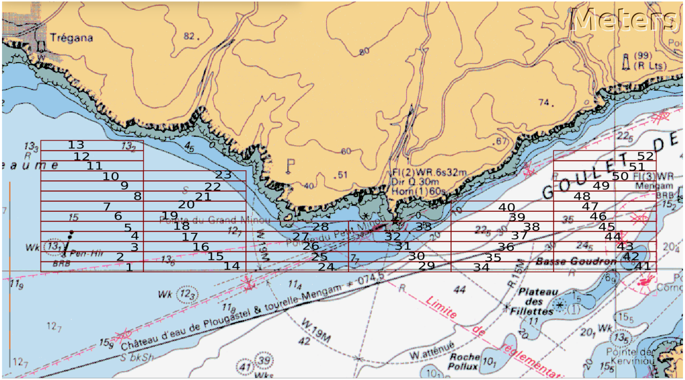
Stable cycles for exploration

L. Jaulin

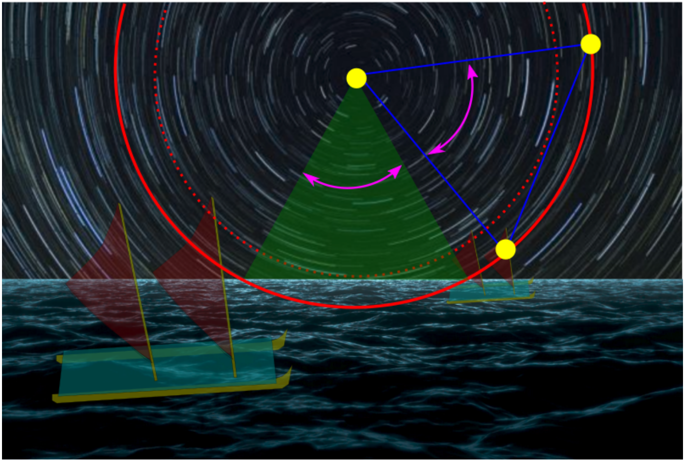
Présentation à contredo
Lundi 2 mars 2020

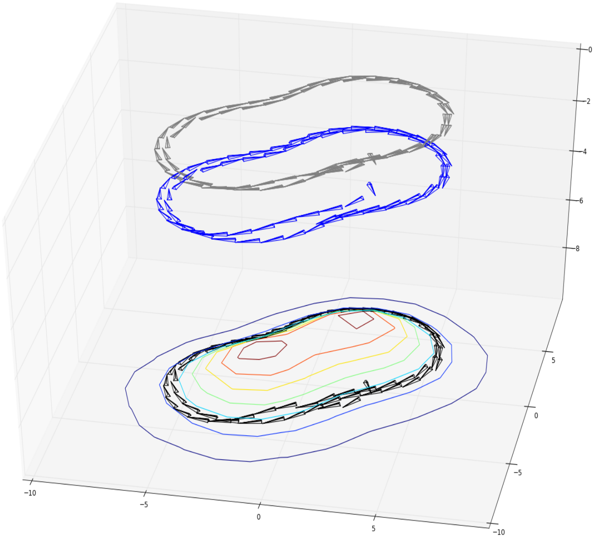
Underwater robots to build maps


Meters
Trégana
Basse Goudron
Plateau des Fillettes
Château d'eau de Plougastel & tourelle-Mengam
Limite de réglementation
Roche Pollux
Pointe du Grand Minou
GOULET DE

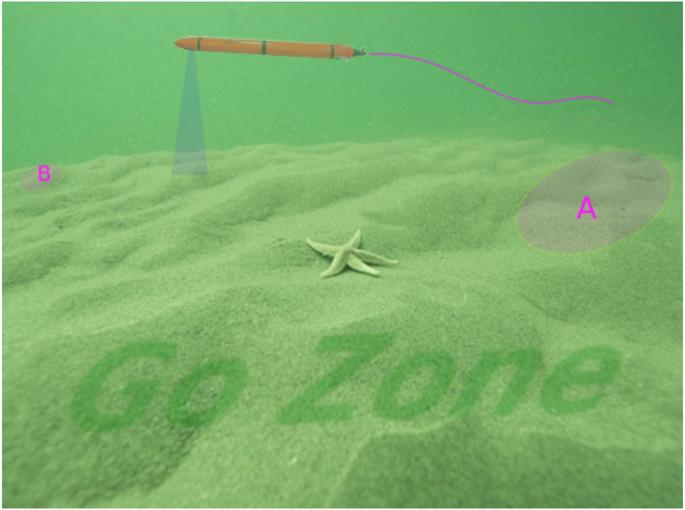
Stable cycles

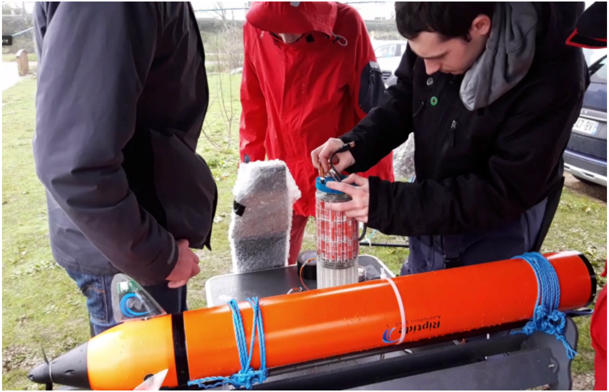
Submeeting 2018




0
2
4
6
8
5
0
−5
−10
−10
−5
0
5

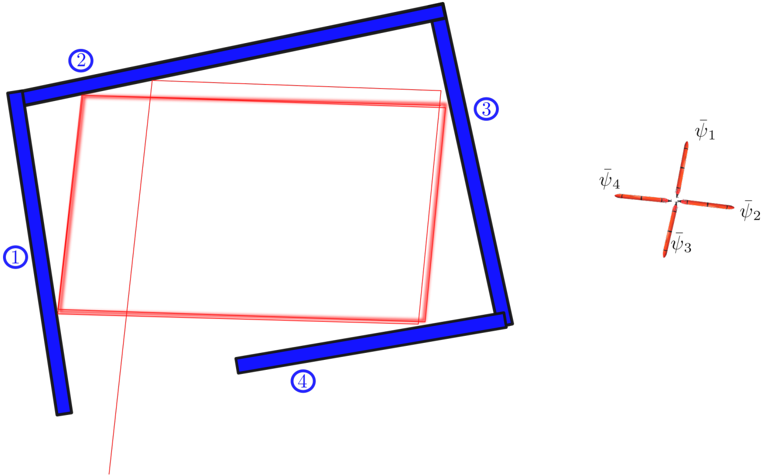
Cycles

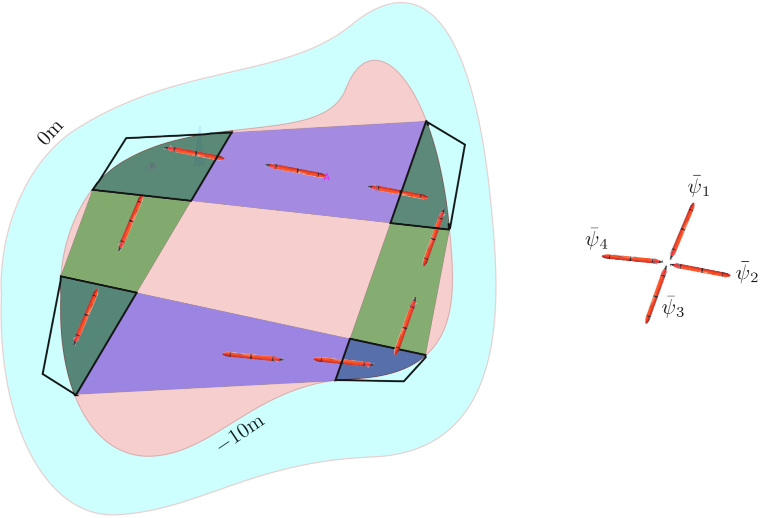

B
A
Go Zone

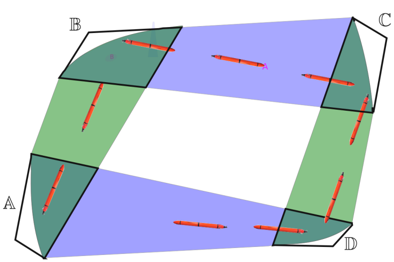


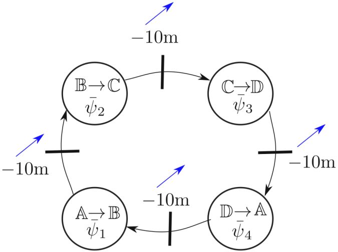
Stable cycles

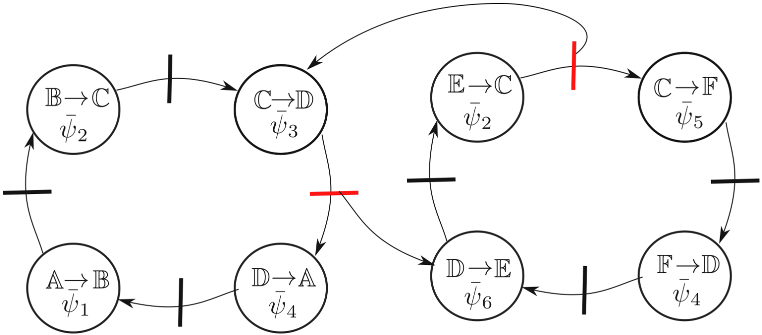
Youtube

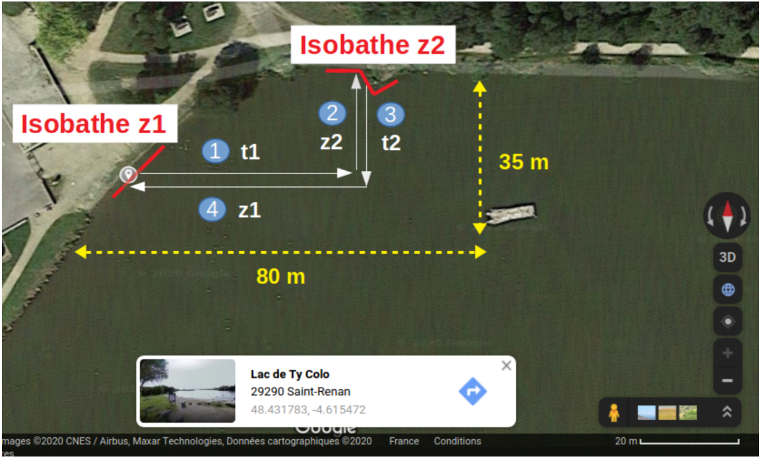

②
③
①
④
$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

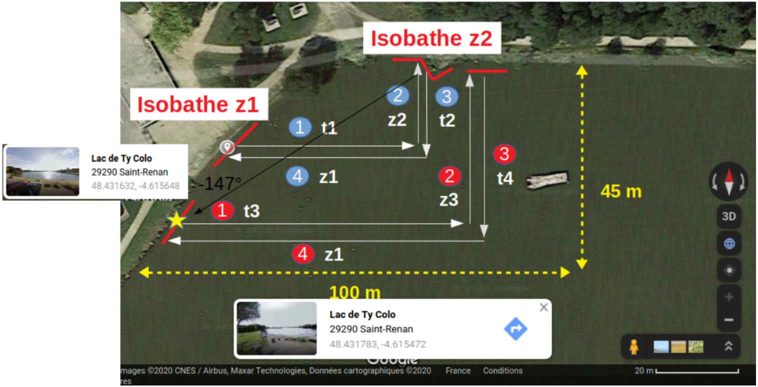

0m
−10m
$\bar{\psi}_1$
$\bar{\psi}_2$
$\bar{\psi}_3$
$\bar{\psi}_4$

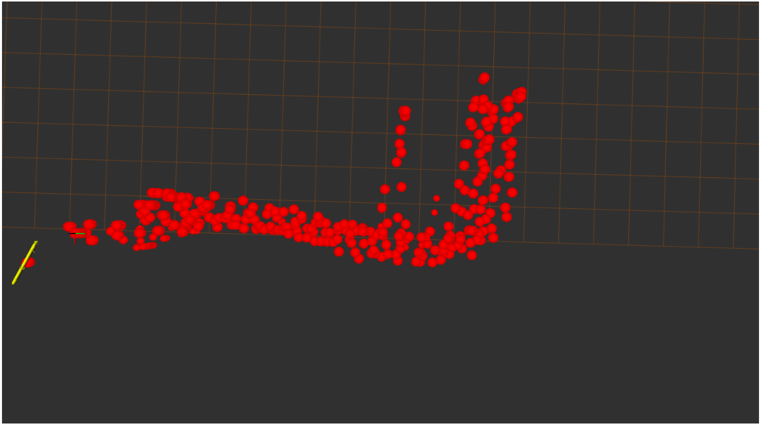

$\mathbb{B}$
$\mathbb{C}$
$\mathbb{A}$
$\mathbb{D}$

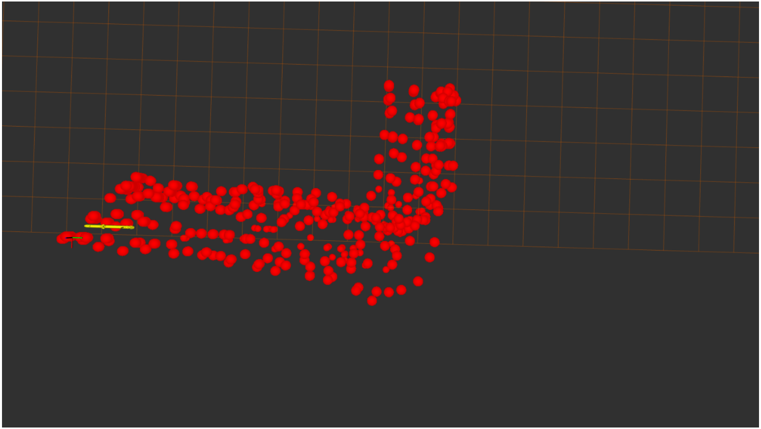

-10m
$\mathbb{B}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{D}$ $\bar{\psi}_3$
-10m
-10m
-10m
$\mathbb{A}\to\mathbb{B}$ $\bar{\psi}_1$
$\mathbb{D}\to\mathbb{A}$ $\bar{\psi}_4$

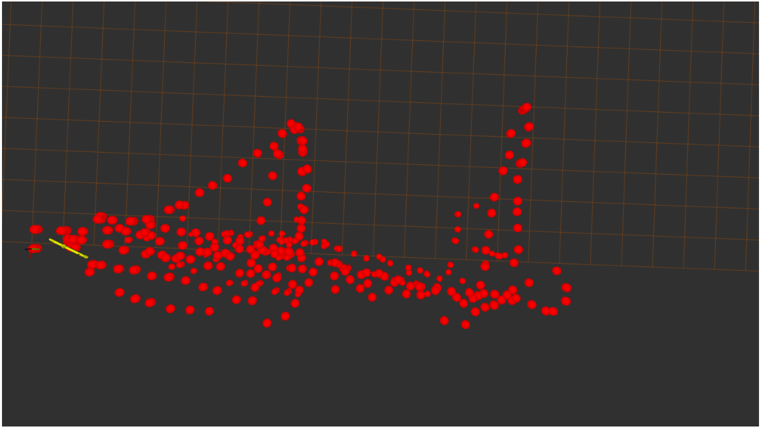

$\mathbb{B}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{D}$ $\bar{\psi}_3$
$\mathbb{A}\to\mathbb{B}$ $\bar{\psi}_1$
$\mathbb{D}\to\mathbb{A}$ $\bar{\psi}_4$
$\mathbb{E}\to\mathbb{C}$ $\bar{\psi}_2$
$\mathbb{C}\to\mathbb{F}$ $\bar{\psi}_5$
$\mathbb{D}\to\mathbb{E}$ $\bar{\psi}_6$
$\mathbb{F}\to\mathbb{D}$ $\bar{\psi}_4$


Isobathe z2
Isobathe z1
1 t1
2 z2
3 t2
4 z1
35 m
80 m
Lac de Ty Colo
29290 Saint-Renan
48.431783, -4.615472
3D
mages ©2020 CNES / Airbus, Maxar Technologies, Données cartographiques ©2020
France
Conditions
20 m

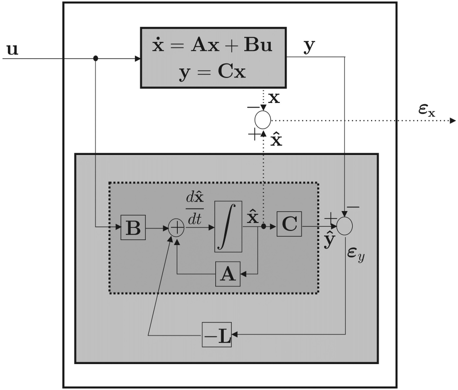

Isobathe z2
Isobathe z1
1 t1
2 z2
3 t2
4 z1
2 z3
3 t4
1 t3
4 z1
-147°
45 m
100 m
Lac de Ty Colo
29290 Saint-Renan
48.431632, -4.615648
Lac de Ty Colo
29290 Saint-Renan
48.431783, -4.615472
3D
mages ©2020 CNES / Airbus, Maxar Technologies, Données cartographiques ©2020 France Conditions
20 m







Stability is an old story



Luenberger observer : $\dot{\varepsilon}_x = (\mathbf{A} - \mathbf{LC})\,\varepsilon_x$

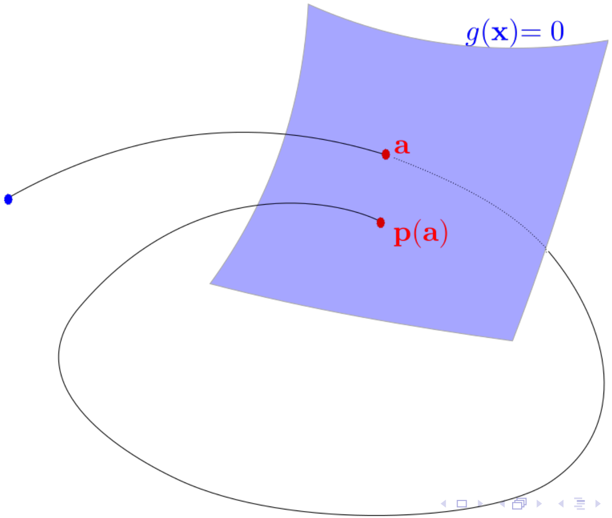
Poincaré map

System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
Poincaré section: $g(\mathbf{x}) = 0$
Transversality: $g(\mathbf{x}) = 0 \Rightarrow \left(\frac{\partial g}{\partial \mathbf{x}} \cdot \mathbf{f}\right)(\mathbf{x}) \neq 0$

Define $\mathscr{G} = g^{-1}(0)$.

$$\mathbf{p}: \begin{array}{ccc} \mathscr{G} & \to & \mathscr{G} \\ \mathbf{x} & \mapsto & \mathbf{p}(\mathbf{x}) \end{array}$$

where $\mathbf{p}(\mathbf{x})$ is the point of $\mathscr{G}$ such that the trajectory initialized at $\mathbf{x}$ intersects $\mathscr{G}$ for the first time.

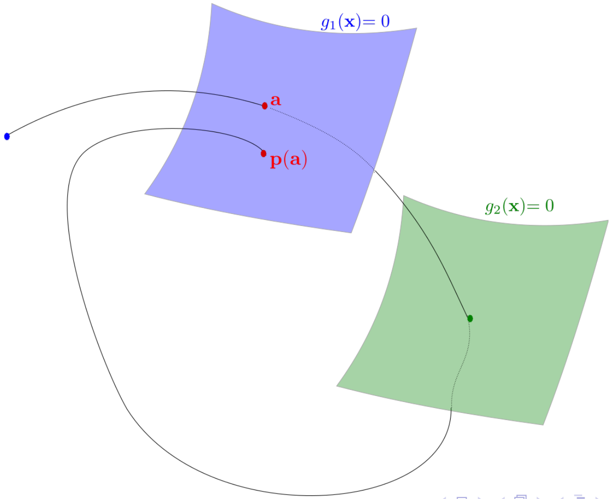

$g(\mathbf{x})=0$
$\mathbf{a}$
$\mathbf{p}(\mathbf{a})$

The Poincaré first recurrence map is defined by

$$\mathbf{x}(k+1) = \mathbf{p}(\mathbf{x}(k))$$

With hybrid systems

Systems: $\dot{\mathbf{x}} = \mathbf{f}_i(\mathbf{x}), i \in \{1, \dots, m\}$
Section i: $g_i(\mathbf{x}) = 0$
Transversality: $g_i(\mathbf{x}) = 0 \Rightarrow \left(\frac{\partial g_i}{\partial \mathbf{x}} \cdot \mathbf{f}_i \right)(\mathbf{x}) \neq 0$
Automaton: $g_i(\mathbf{x}) = 0 \Rightarrow i := \text{mod}(i+1, m)$


$g_1(\mathbf{x}) = 0$
a
p(a)
$g_2(\mathbf{x}) = 0$

Stability contractor

A *stability contractor* $\mathbf{\Psi}:\mathbb{IR}^n \to \mathbb{IR}^n$ of rate $\alpha < 1$ is an operator which satisfies

$$[\mathbf{a}] \subset [\mathbf{b}] \Rightarrow \mathbf{\Psi}([\mathbf{a}]) \subset \mathbf{\Psi}([\mathbf{b}])$$
$$\mathbf{\Psi}([\mathbf{a}]) \subset [\mathbf{a}]$$
$$\mathbf{\Psi}(\mathbf{0}) = \mathbf{0}$$
$$\mathbf{\Psi}([\mathbf{a}]) \subset \alpha \cdot [\mathbf{a}] \Rightarrow \mathbf{\Psi}^2([\mathbf{a}]) \subset \alpha^2 \cdot [\mathbf{a}]$$

If $\mathbf{\Psi}$ is a stability contractor of rate $\alpha < 1$ then we have

$$\mathbf{\Psi}([\mathbf{x}])) \subset \alpha \cdot [\mathbf{x}] \Rightarrow \lim_{k \to \infty} \mathbf{\Psi}^k([\mathbf{x}]) \to \mathbf{0}.$$

Theorem. Consider $\mathbf{f}$ with $\mathbf{f}(\mathbf{0}) = \mathbf{0}$. If $\exists [\mathbf{x}_0] \ni \mathbf{0}$, with $[\mathbf{f}_c]([\mathbf{x}_0]) \subset \alpha \cdot [\mathbf{x}_0]$, $\alpha < 1$ then $[\mathbf{f}_c]([\mathbf{x}])$ is a stability contractor inside $[\mathbf{x}_0]$.

Theorem. If the system is exponentially stable around **0**, then

$$\exists\eta>0,\forall[\mathbf{x}]\in\mathscr{B}_{\eta},\exists k>0,\exists\alpha<1,[\mathbf{f}_{c}^{k}]([\mathbf{x}])\subset\alpha\cdot[\mathbf{x}].$$

Example.

Border: $x_2 = h(x_1) = 20\,(1 - \exp(-0.25 x_1))$

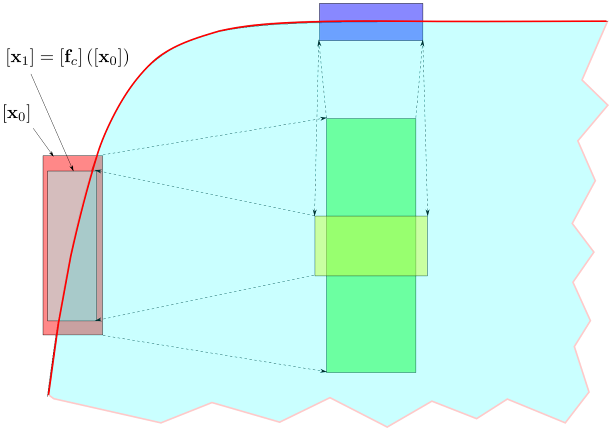
Transition functions :

$$\mathbf{f}_1(\mathbf{x}) = \begin{pmatrix} x_1 + 25 \\ x_2 \end{pmatrix} \qquad \mathbf{f}_2(\mathbf{x}) = \begin{pmatrix} x_1 \\ h(x_1) \end{pmatrix}$$

$$\mathbf{f}_3(\mathbf{x}) = \begin{pmatrix} x_1 \\ x_2 - 7.5 \end{pmatrix} \qquad \mathbf{f}_4(\mathbf{x}) = \begin{pmatrix} h^{-1}(x_2) \\ x_2 \end{pmatrix}$$

Poincaré recurrence:

$$\mathbf{p}(\mathbf{x}) = \mathbf{f}_4 \circ \mathbf{f}_3 \circ \mathbf{f}_2 \circ \mathbf{f}_1(\mathbf{x})$$


$[\mathbf{x}_1] = [\mathbf{f}_c]([\mathbf{x}_0])$
$[\mathbf{x}_0]$