Solving set-valued problems;

Application to localisation and mapping with robots

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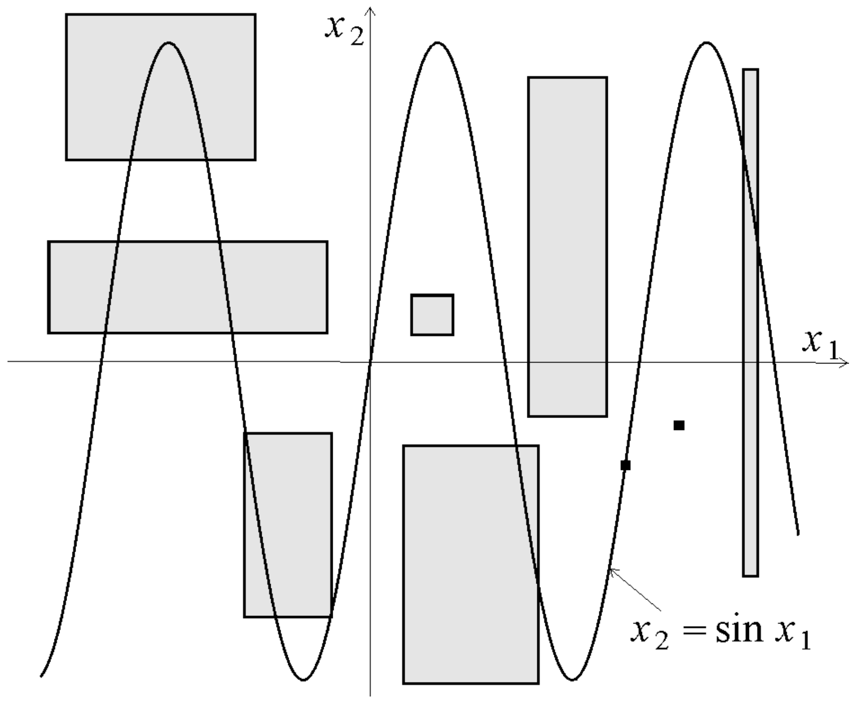
1 Contractors

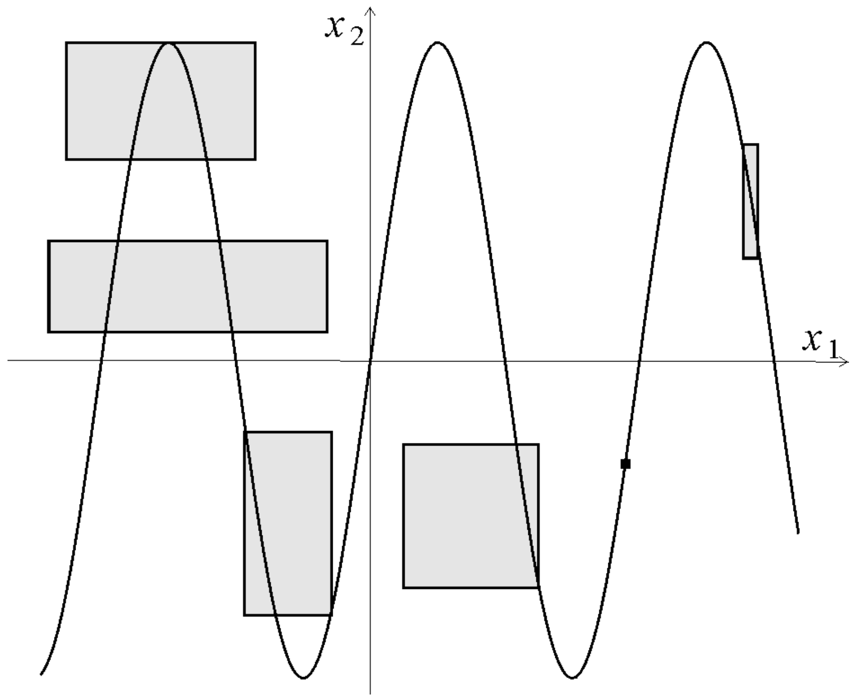
The operator $\mathcal{C} : \mathbb{IR}^n \to \mathbb{IR}^n$ is a *contractor* for the equation $f(\mathbf{x}) = 0$, if

$$\begin{cases} \mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathbf{x} \in [\mathbf{x}] \text{ and } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathcal{C}([\mathbf{x}]) & \text{(consistence)} \end{cases}$$

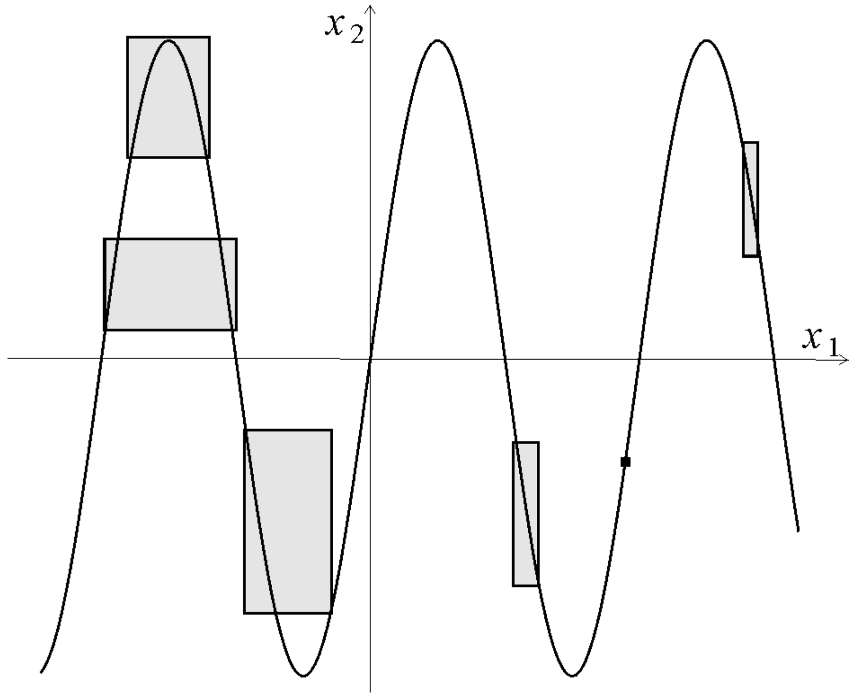
Example. Consider the primitive equation:

$$x_2 = \sin x_1.$$


x_2
x_1
$x_2 = \sin x_1$



Forward contraction



Backward contraction

Building contractors for equations

Consider the primitive equation

$$x_1 + x_2 = x_3$$

with $x_1 \in [x_1]$, $x_2 \in [x_2]$, $x_3 \in [x_3]$.

We have

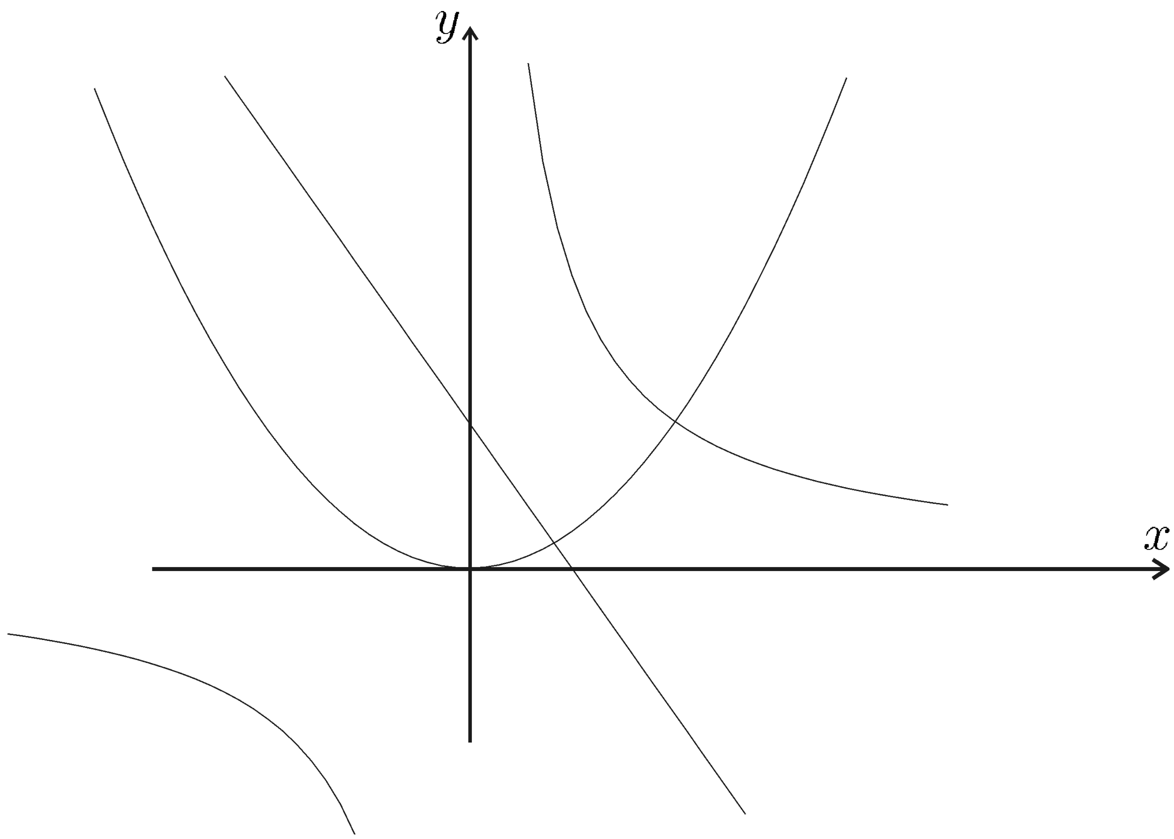
$$
\begin{aligned}
x_3 = x_1 + x_2 \Rightarrow &\quad x_3 \in \quad [x_3] \cap ([x_1] + [x_2]) \quad // \text{ forward} \\
x_1 = x_3 - x_2 \Rightarrow &\quad x_1 \in \quad [x_1] \cap ([x_3] - [x_2]) \quad // \text{ backward} \\
x_2 = x_3 - x_1 \Rightarrow &\quad x_2 \in \quad [x_2] \cap ([x_3] - [x_1]) \quad // \text{ backward}
\end{aligned}
$$

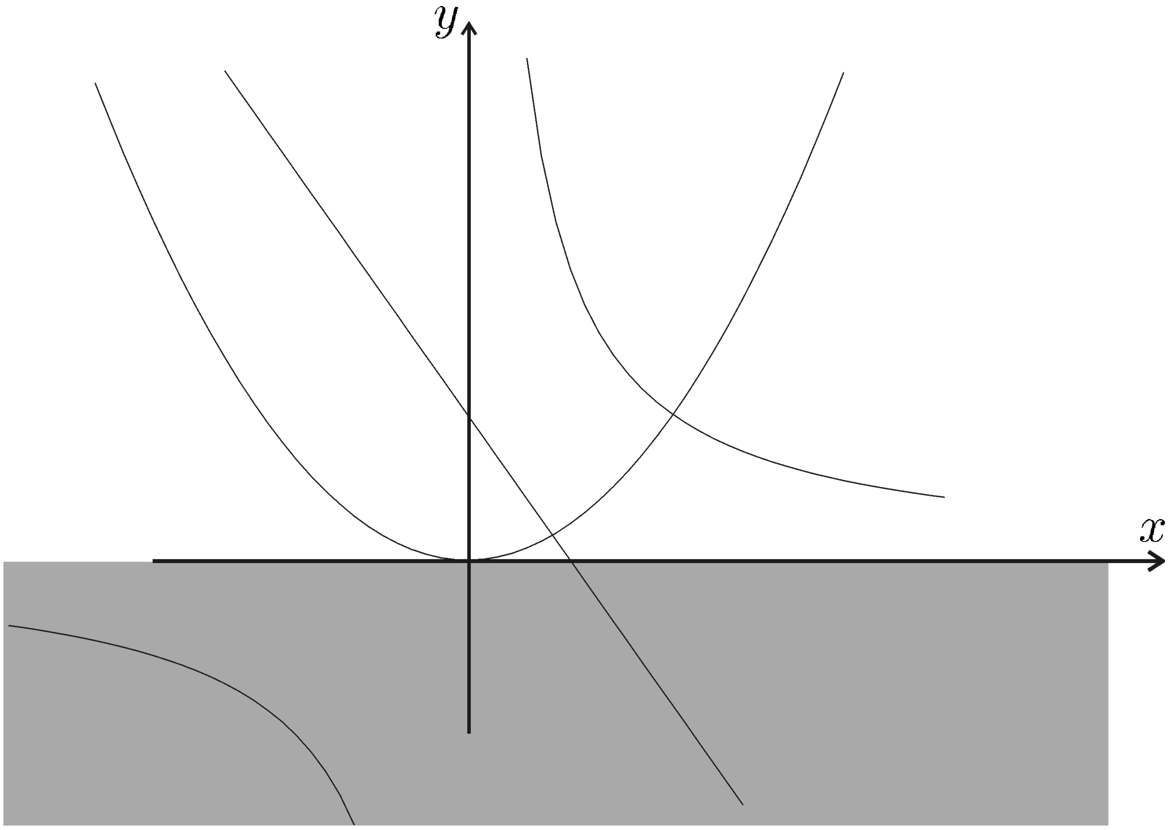
The contractor associated with $x_1 + x_2 = x_3$ is thus

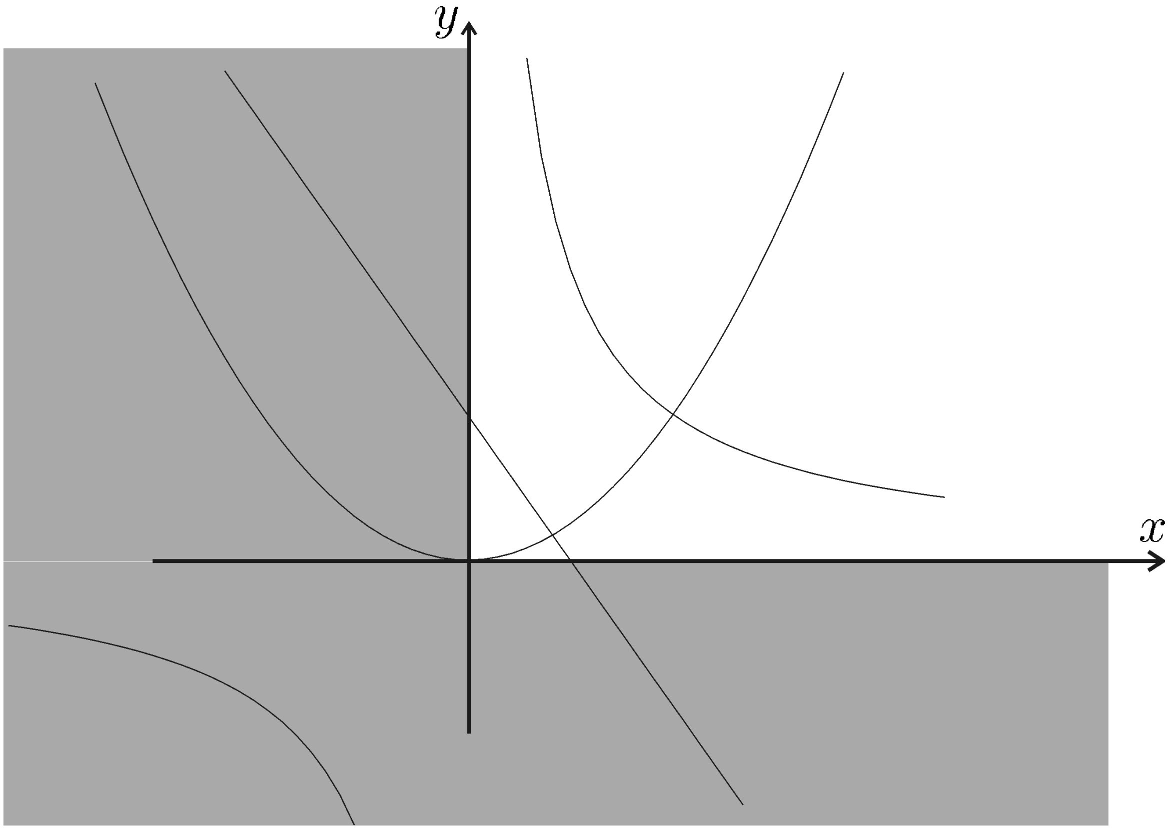
$$\mathcal{C}\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} = \begin{pmatrix} [x_1] \cap ([x_3] - [x_2]) \\ [x_2] \cap ([x_3] - [x_1]) \\ [x_3] \cap ([x_1] + [x_2]) \end{pmatrix}$$

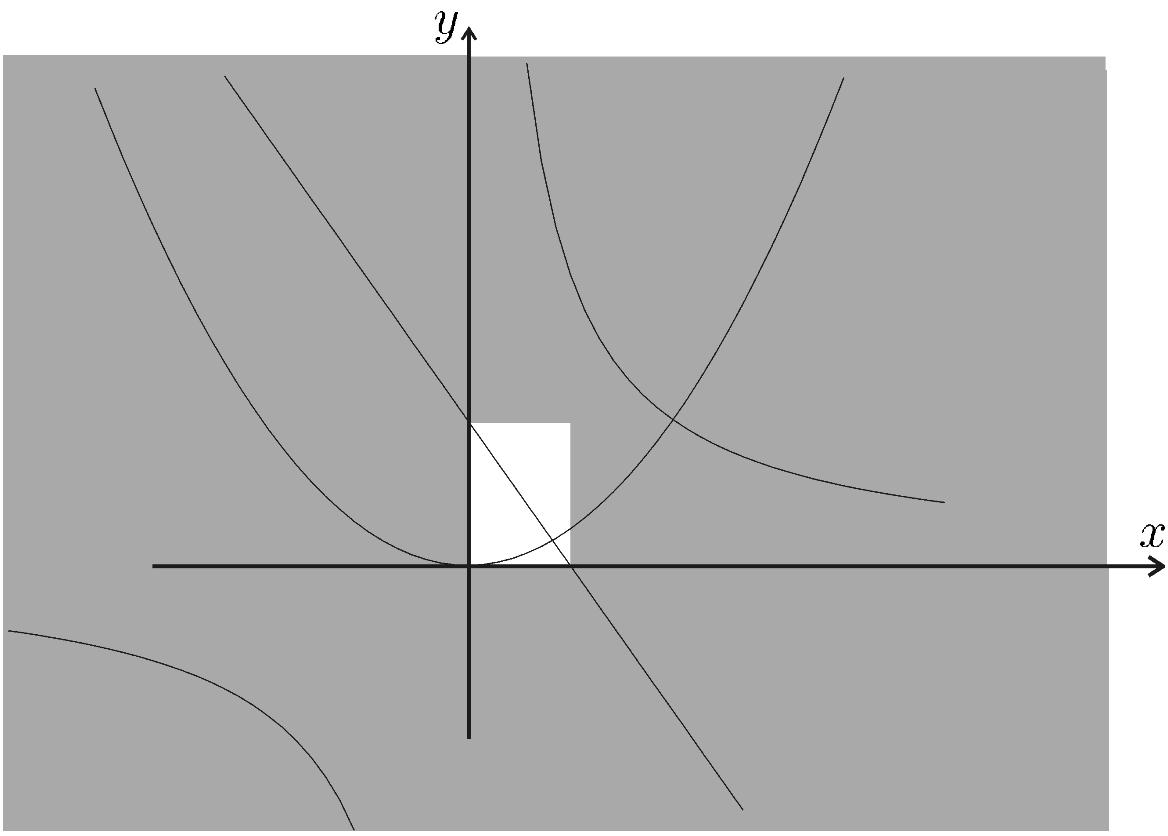
Consider the following problem

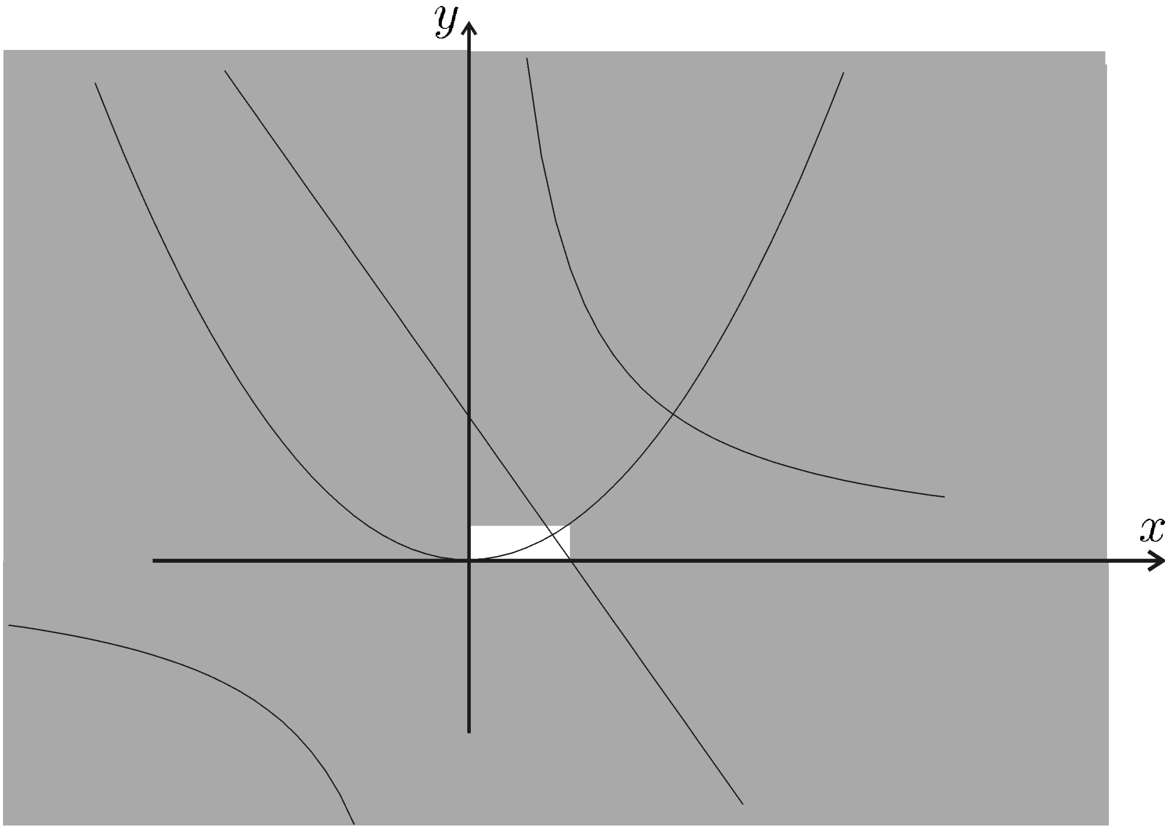
$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$


y
x

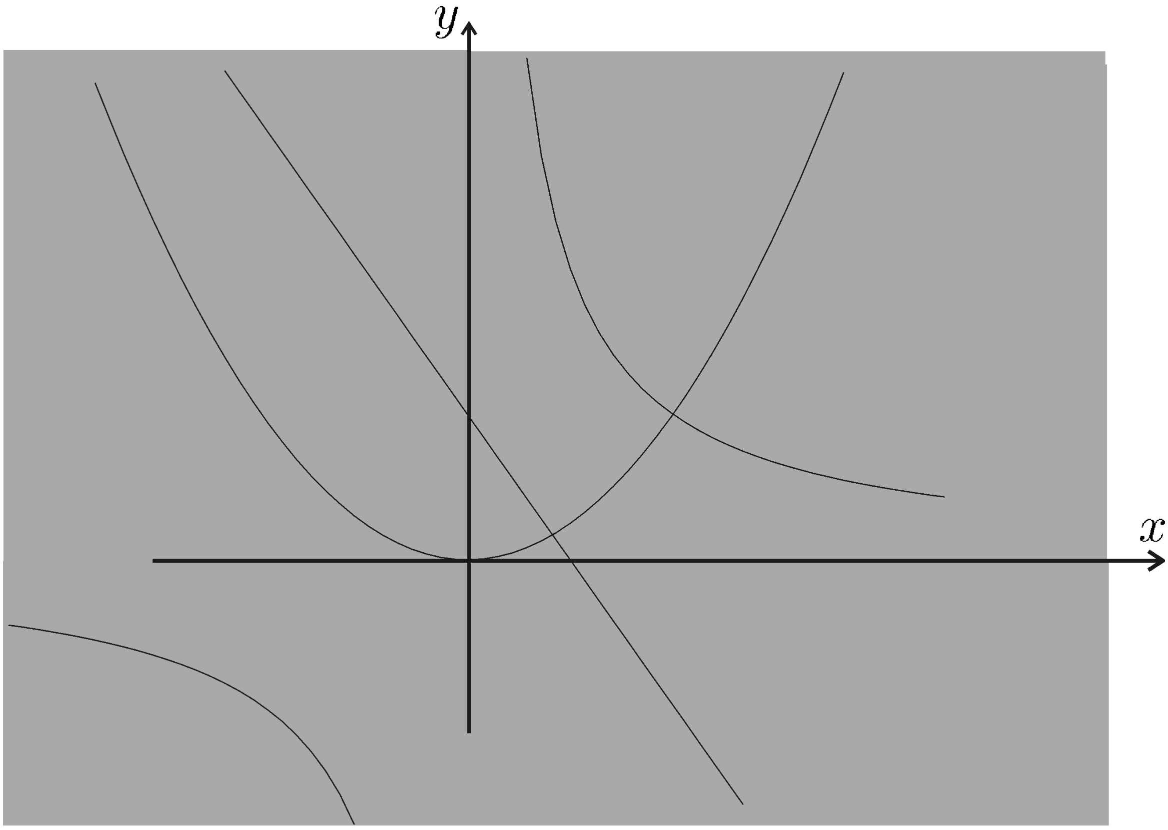

y
x


y
x


y
x



y
x


y
x

$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty, \infty]^2 \ = \ [0, \infty] \\
(C_2) \Rightarrow & x \in & 1/[0, \infty] \ = [0, \infty] \\
(C_3) \Rightarrow & y \in & [0, \infty] \ \cap \ ((-2)\,.[0, \infty] + 1) \\
 & & = [0, \infty] \ \cap \ ([-\infty, 1]) = [0, 1] \\
 & x \in & [0, \infty] \ \cap \ (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0, 1] \cap [0, 1/2]^2 = [0, 1/4] \\
(C_2) \Rightarrow & x \in & [0, 1/2] \ \cap \ 1/[0, 1/4] = \emptyset \\
 & y \in & [0, 1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

2 Interval trajectories

A trajectory is a function $f : \mathbb{R} \to \mathbb{R}^n$. For instance

$$\mathbf{f}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

is a trajectory and

$$[\mathbf{f}](t) = \begin{pmatrix} \cos t + \left[0, t^2\right] \\ \sin t + [-1, 1] \end{pmatrix}$$

is an interval trajectory (or tube).

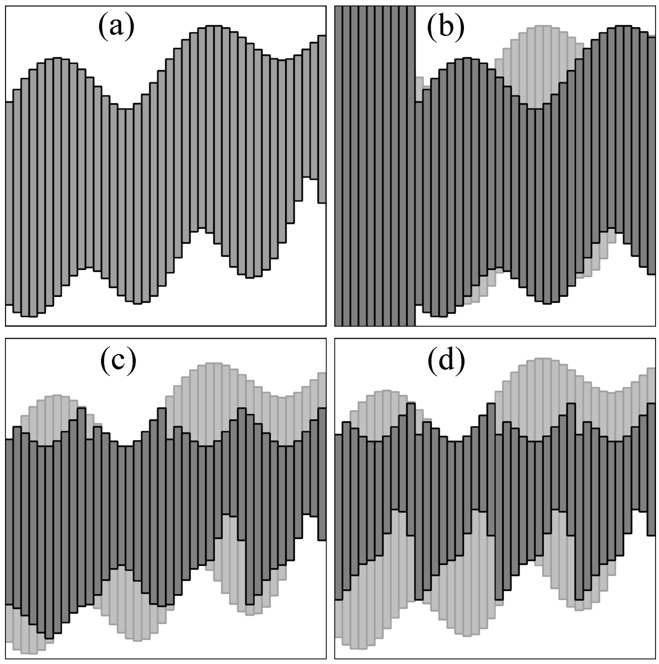
Consider $x(t) \in [x](t)$ with the constraint

$$\forall t,\ x(t) = x(t+1)$$

Contract the tube $[x](t)$.

Method

$$[x](t) := [x](t) \cap [x](t+1)$$
$$[x](t) := [x](t) \cap [x](t-1)$$

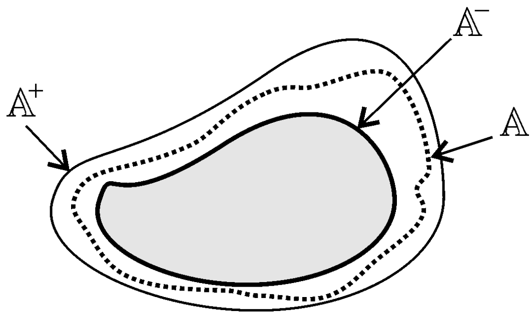

(a)
(b)
(c)
(d)

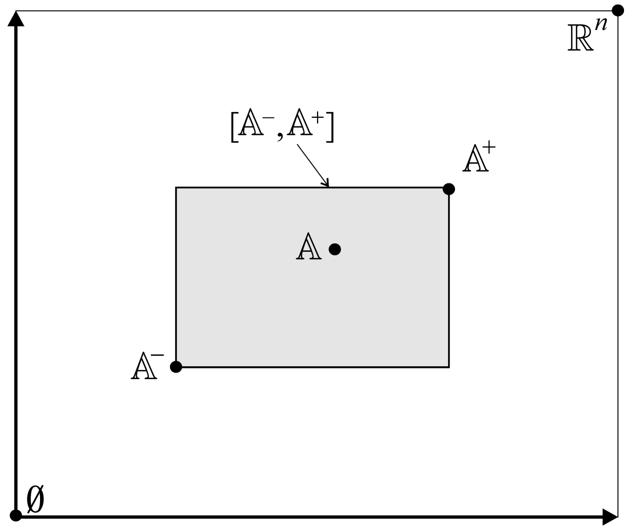
3 Set intervals

Given two sets $\mathbb{A}^-$ and $\mathbb{A}^+$ of $\mathbb{R}^n$, the pair $[\mathbb{A}] = \left[\mathbb{A}^-, \mathbb{A}^+\right]$ which encloses all sets $\mathbb{A}$ such that

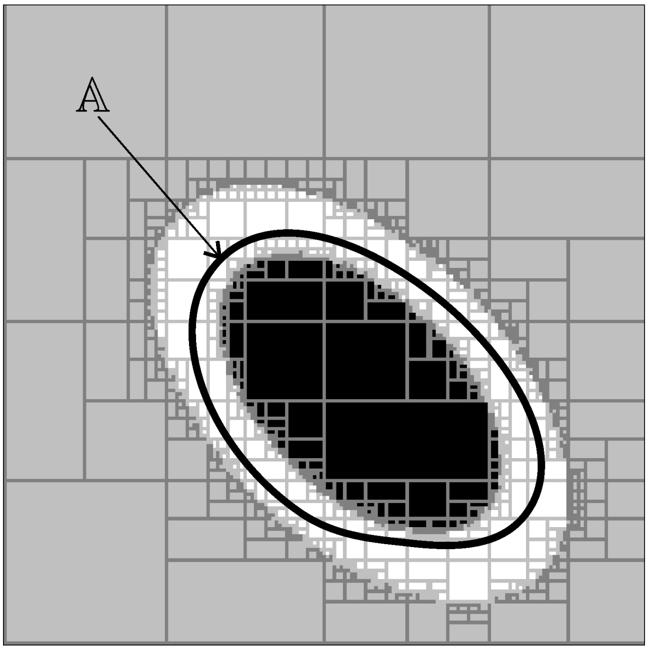
$$\mathbb{A}^- \subset \mathbb{A} \subset \mathbb{A}^+$$

is a *set interval*.





Lattice $(\mathcal{P}(\mathbb{R}^n), \subset)$

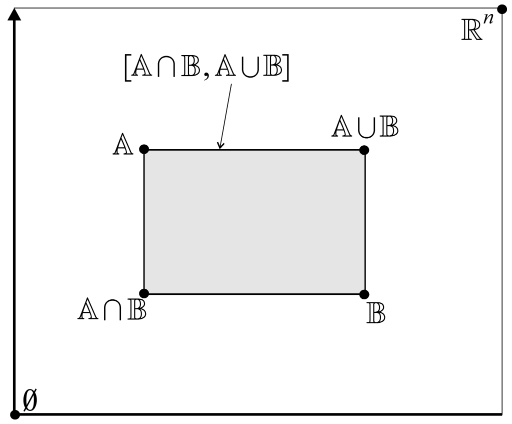


Machine representation of $\left[\mathbb{A}^{-}, \mathbb{A}^{+}\right]$

The set interval $[\emptyset, \emptyset]$ is a singleton : $\emptyset \in [\emptyset, \emptyset]$.
The set interval $[\emptyset, \mathbb{R}^n]$ encloses all sets of $\mathbb{R}^n$.

Given two sets $\mathbb{A}$ and $\mathbb{B}$ of $\mathbb{R}^n$. The smallest set interval which contains $\mathbb{A}$ and $\mathbb{B}$ is

$$\square\{\mathbb{A},\mathbb{B}\} = [\mathbb{A}\cap\mathbb{B}, \mathbb{A}\cup\mathbb{B}].$$



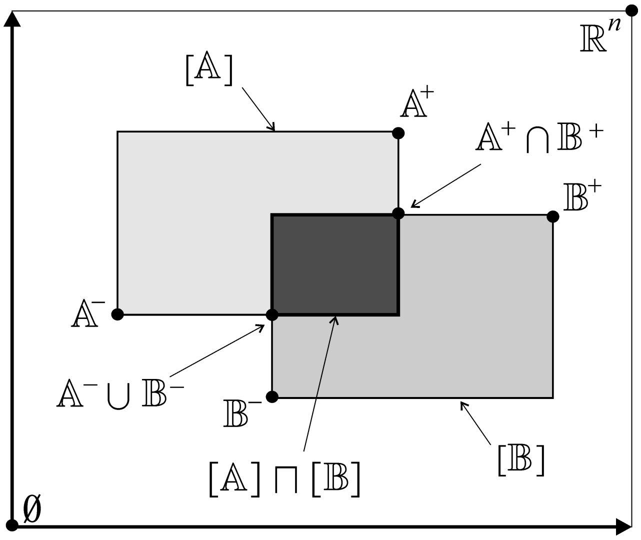
4 Arithmetic

4.1 Specific set interval operations

Set intervals are **sets** (of sets), the intersection, the union, the inclusion can thus be defined.

Intersection.

$$\begin{aligned}[\mathbb{A}] \sqcap [\mathbb{B}] &= \{\mathbb{X}, \mathbb{X} \in [\mathbb{A}] \text{ and } \mathbb{X} \in [\mathbb{B}]\} \\ &= \left[\mathbb{A}^- \cup \mathbb{B}^-, \mathbb{A}^+ \cap \mathbb{B}^+\right].\end{aligned}$$



$\mathbb{R}^n$
$[\mathbb{A}]$
$\mathbb{A}^+$
$\mathbb{A}^+ \cap \mathbb{B}^+$
$\mathbb{B}^+$
$\mathbb{A}^-$
$\mathbb{A}^- \cup \mathbb{B}^-$
$\mathbb{B}^-$
$[\mathbb{A}] \sqcap [\mathbb{B}]$
$[\mathbb{B}]$
$\emptyset$

Inclusion.

$$[\mathbb{A}] \sqsubset [\mathbb{B}] \quad \Leftrightarrow \quad [\mathbb{A}] \sqcap [\mathbb{B}] = [\mathbb{B}] \,.$$

Set interval envelope.

$$\square\left\{\mathbb{A}_i, i\in\mathbb{I}\right\} = \left[\bigcap_{i\in\mathbb{I}}\mathbb{A}_i, \bigcup_{i\in\mathbb{I}}\mathbb{A}_i\right].$$

For instance,

$$\square\left\{[1,4],[3,7],[2,6]\right\} = [[3,4],[1,7]].$$

Union. We have

$$\begin{aligned}[\mathbb{A}] \sqcup [\mathbb{B}] &= \square\{\mathbb{X}, \mathbb{X} \in [\mathbb{A}] \text{ or } \mathbb{X} \in [\mathbb{B}]\} \\ &= \left[\mathbb{A}^{-} \cap \mathbb{B}^{-}, \mathbb{A}^{+} \cup \mathbb{B}^{+}\right].\end{aligned}$$

4.2 Set extension

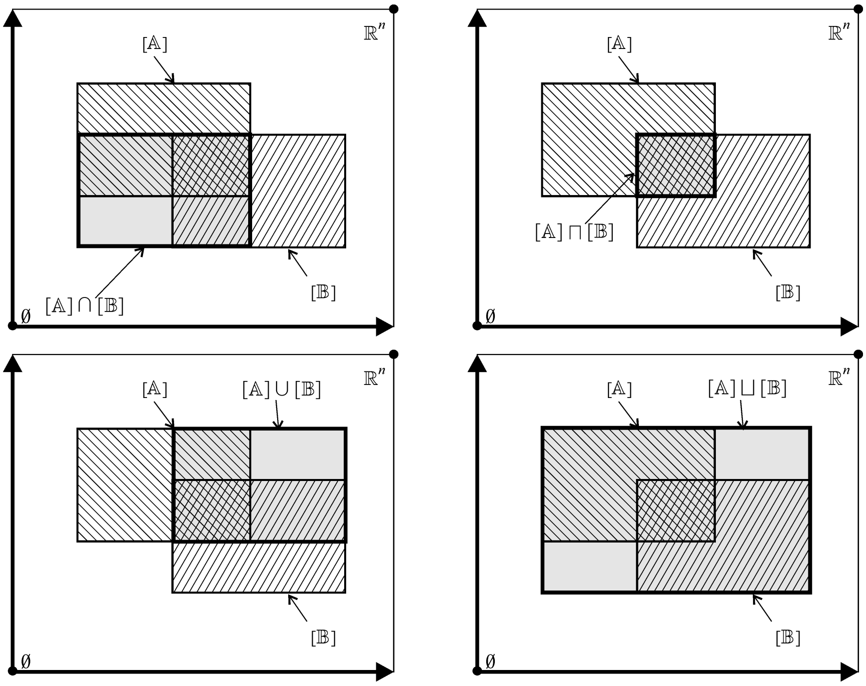
All operations existing for sets such as $\cap, \cup$, reciprocal image, direct image, ... can be extended to set intervals.

If $\diamond \in \{\cap, \cup, \times, \backslash, \dots\}$,

$$[\mathbb{A}] \diamond [\mathbb{B}] = \square \left\{\mathbb{C}, \mathbb{A} \in [\mathbb{A}], \mathbb{B} \in [\mathbb{B}], \mathbb{C} = \mathbb{A} \diamond \mathbb{B}\right\}.$$

We have

$$\begin{array}{llll}
\text{(i)} & \left[\mathbb{A}^-, \mathbb{A}^+\right] \cap \left[\mathbb{B}^-, \mathbb{B}^+\right] & = & \left[\mathbb{A}^- \cap \mathbb{B}^-, \mathbb{A}^+ \cap \mathbb{B}^+\right] \\
\text{(ii)} & \left[\mathbb{A}^-, \mathbb{A}^+\right] \cup \left[\mathbb{B}^-, \mathbb{B}^+\right] & = & \left[\mathbb{A}^- \cup \mathbb{B}^-, \mathbb{A}^+ \cup \mathbb{B}^+\right] \\
\text{(iii)} & \left[\mathbb{A}^-, \mathbb{A}^+\right] \times \left[\mathbb{B}^-, \mathbb{B}^+\right] & = & \left[\mathbb{A}^- \times \mathbb{B}^-, \mathbb{A}^+ \times \mathbb{B}^+\right] \\
\text{(iv)} & \left[\mathbb{A}^-, \mathbb{A}^+\right] \backslash \left[\mathbb{B}^-, \mathbb{B}^+\right] & = & \left[\mathbb{A}^- \backslash \mathbb{B}^+, \mathbb{A}^+ \backslash \mathbb{B}^-\right].
\end{array}$$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ℝⁿ
[𝔹]
∅

Extension of functions. A set-valued function f can be extended to set intervals as follows

$$f\left(\left[\mathbb{A}^{-},\mathbb{A}^{+}\right]\right)=\square\left\{f\left(\mathbb{A}\right),\mathbb{A}\in\left[\mathbb{A}^{-},\mathbb{A}^{+}\right]\right\}.$$

When f is inclusion monotonic, we have

$$f\left(\left[\mathbb{A}^{-},\mathbb{A}^{+}\right]\right)=\left[f\left(\mathbb{A}^{-}\right),f\left(\mathbb{A}^{+}\right)\right].$$

5 Interval extension

The interval extension associated with the set-valued expression

$$f(\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3) = \mathbb{X}_1 \cup (\mathbb{X}_2 \cap g(\mathbb{X}_3))$$

is

$$[f]([\mathbb{X}_1], [\mathbb{X}_2], [\mathbb{X}_3]) = [\mathbb{X}_1] \cup ([\mathbb{X}_2] \cap g([\mathbb{X}_3])).$$

Theorem 1. If $\mathbb{X}_1 \in [\mathbb{X}_1], \ldots, \mathbb{X}_n \in [\mathbb{X}_n]$ then

$$f(\mathbb{X}_1, \mathbb{X}_2, \ldots, \mathbb{X}_n) \in [f]([\mathbb{X}_1], [\mathbb{X}_2], \ldots, [\mathbb{X}_n]).$$

Moreover, if in the expression of f, all $\mathbb{X}_i$ occur only once, the set interval evaluation is minimal.

Dependency problem. For instance,

$$\left[\mathbb{A}^{-}, \mathbb{A}^{+}\right] \setminus \left[\mathbb{A}^{-}, \mathbb{A}^{+}\right] = \left[\mathbb{A}^{-} \backslash \mathbb{A}^{+}, \mathbb{A}^{+} \backslash \mathbb{A}^{-}\right] = \left[\emptyset, \mathbb{A}^{+} \backslash \mathbb{A}^{-}\right].$$

Of course, we have the inclusion property

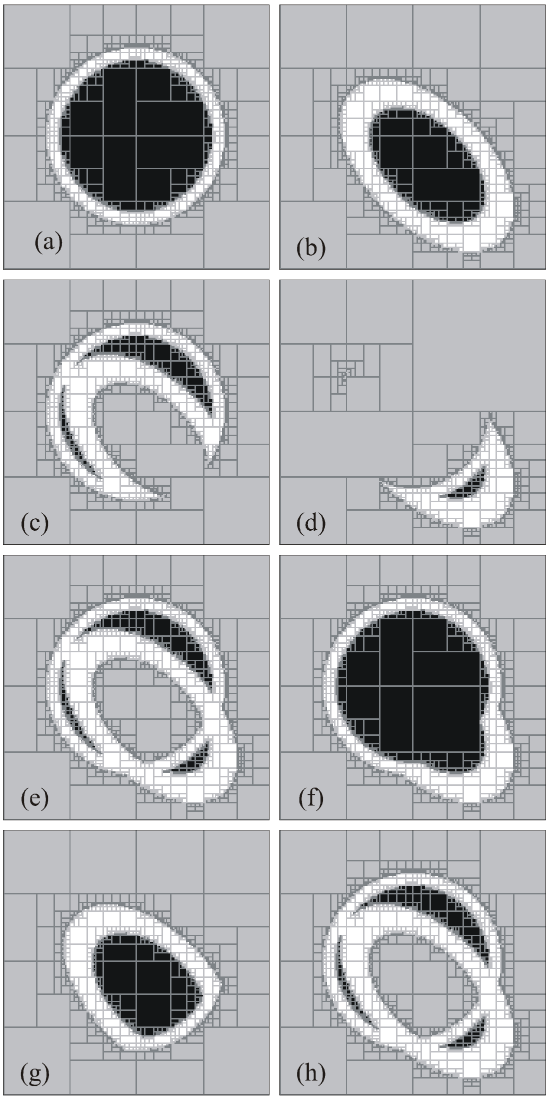
$$\left\{\mathbb{A} \backslash \mathbb{A}, \mathbb{A} \in \left[\mathbb{A}^{-}, \mathbb{A}^{+}\right]\right\} = [\emptyset, \emptyset] \sqsubset \left[\emptyset, \mathbb{A}^{+} \backslash \mathbb{A}^{-}\right].$$

Example. Consider two equivalent expressions of the exclusive union

$$\begin{aligned} f(\mathbb{A},\mathbb{B}) &= (\mathbb{A}\backslash\mathbb{B}) \cup (\mathbb{B}\backslash\mathbb{A}) \\ g(\mathbb{A},\mathbb{B}) &= (\mathbb{A}\cup\mathbb{B}) \setminus (\mathbb{A}\cap\mathbb{B}). \end{aligned}$$

The two natural set interval extensions are given by

$$\begin{aligned} [f]([\mathbb{A}],[\mathbb{B}]) &= ([\mathbb{A}] \setminus [\mathbb{B}]) \cup ([\mathbb{B}] \setminus [\mathbb{A}]) \\ [g]([\mathbb{A}],[\mathbb{B}]) &= ([\mathbb{A}] \cup [\mathbb{B}]) \setminus ([\mathbb{A}] \cap [\mathbb{B}]). \end{aligned}$$



(a) $\mathbb{A} \in \left[\mathbb{A}^{-}, \mathbb{A}^{+}\right]$

(b) $\mathbb{B} \in \left[\mathbb{B}^{-}, \mathbb{B}^{+}\right]$

(c) $[\mathbb{A}] \setminus [\mathbb{B}]$

(d) $[\mathbb{B}] \setminus [\mathbb{A}]$

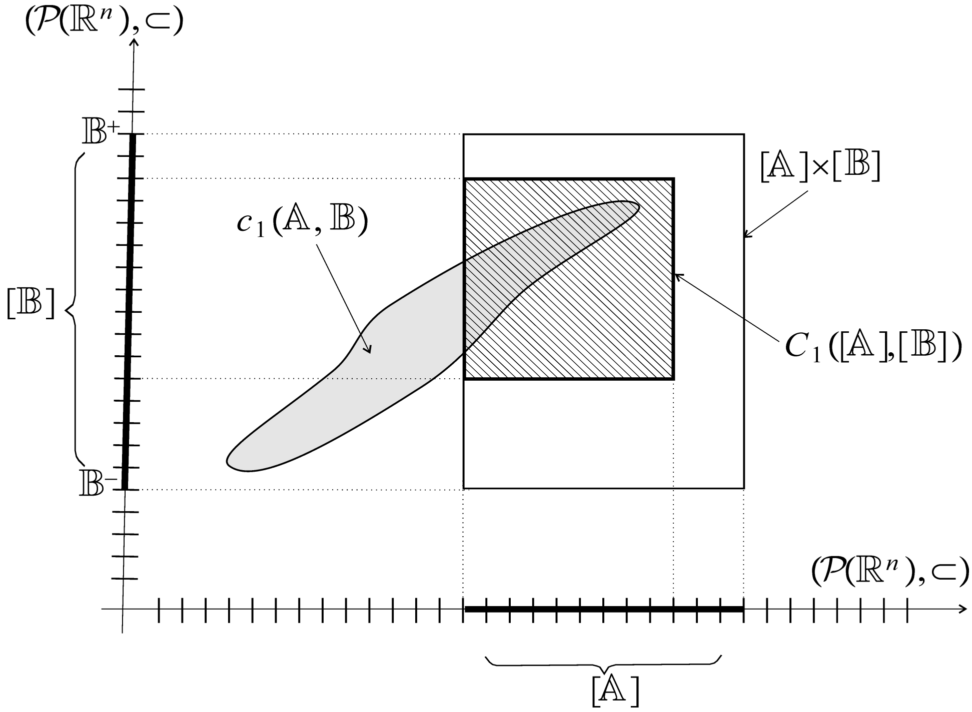
(e) $[\mathbb{A}] \setminus [\mathbb{B}] \cup [\mathbb{B}] \setminus [\mathbb{A}]$

(f) $[\mathbb{A}] \cup [\mathbb{B}]$

(g) $[\mathbb{A}] \cap [\mathbb{B}]$

(h) $([\mathbb{A}] \cup [\mathbb{B}]) \setminus ([\mathbb{A}] \cap [\mathbb{B}])$

6 Contractors



$(\mathcal{P}(\mathbb{R}^n),\subset)$
$\mathbb{B}^+$
$[\mathbb{B}]$
$\mathbb{B}^-$
$c_1(\mathbb{A},\mathbb{B})$
$[\mathbb{A}]\times[\mathbb{B}]$
$C_1([\mathbb{A}],[\mathbb{B}])$
$(\mathcal{P}(\mathbb{R}^n),\subset)$
$[\mathbb{A}]$

Consider the CSP

$$\begin{cases} \mathbb{A} \subset \mathbb{B} \\ \mathbb{A} \in [\mathbb{A}], \mathbb{B} \in [\mathbb{B}]. \end{cases}$$

The optimal contractor is

$$\begin{cases} \text{(i)} & [\mathbb{A}] := [\mathbb{A}] \sqcap ([\mathbb{A}] \cap [\mathbb{B}]) \\ \text{(ii)} & [\mathbb{B}] := [\mathbb{B}] \sqcap ([\mathbb{A}] \cup [\mathbb{B}]) \end{cases}$$

Proof.

$$\mathbb{A} \subset \mathbb{B} \quad \Leftrightarrow \quad \mathbb{A} = \mathbb{A} \cap \mathbb{B} \quad \Leftrightarrow \quad \mathbb{B} = \mathbb{A} \cup \mathbb{B}.$$

Consider the CSP

$$\left\{ \begin{array}{c} \mathbb{A} \cap \mathbb{B} = \emptyset \\ \mathbb{A} \in [\mathbb{A}], \mathbb{B} \in [\mathbb{B}], \end{array} \right.$$

The optimal contractor is

$$\left\{ \begin{array}{ll} \text{(i)} & [\mathbb{A}] := [\mathbb{A}] \sqcap ([\emptyset, \mathbb{R}^n] \setminus [\mathbb{B}]) \\ \text{(ii)} & [\mathbb{B}] := [\mathbb{B}] \sqcap ([\emptyset, \mathbb{R}^n] \setminus [\mathbb{A}]). \end{array} \right.$$

Proof.

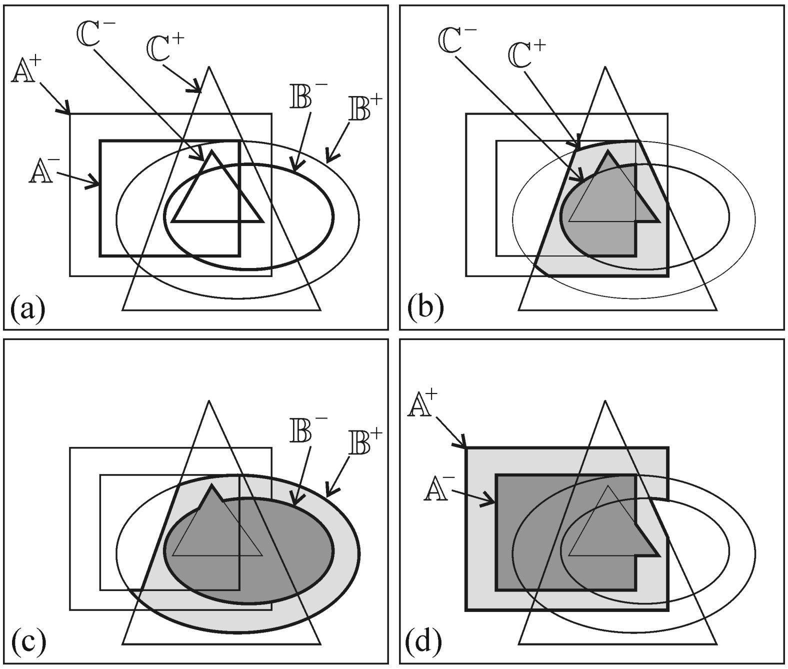
$$\begin{array}{rcl} \mathbb{A} \cap \mathbb{B} = \emptyset & \Leftrightarrow & \exists \mathbb{Z} \in [\emptyset, \mathbb{R}^n] \text{ such that } \mathbb{A} = \mathbb{Z} \backslash \mathbb{B} \\ & \Leftrightarrow & \exists \mathbb{Z} \in [\emptyset, \mathbb{R}^n] \text{ such that } \mathbb{B} = \mathbb{Z} \backslash \mathbb{A}. \end{array}$$

Consider the CSP

$$\begin{cases} \mathbb{A} \cap \mathbb{B} = \mathbb{C} \\ \mathbb{A} \in [\mathbb{A}]\,, \mathbb{B} \in [\mathbb{B}]\,, \mathbb{C} \in [\mathbb{C}]\,. \end{cases}$$

The optimal contractor is

$$\begin{cases} \text{(i)} & [\mathbb{C}] := [\mathbb{C}] \sqcap ([\mathbb{A}] \cap [\mathbb{B}]) \\ \text{(ii)} & [\mathbb{A}] := [\mathbb{A}] \sqcap ([\mathbb{C}] \cup ([\emptyset, \mathbb{R}^n] \setminus ([\mathbb{B}] \setminus [\mathbb{C}]))) \\ \text{(iii)} & [\mathbb{B}] := [\mathbb{B}] \sqcap ([\mathbb{C}] \cup ([\emptyset, \mathbb{R}^n] \setminus ([\mathbb{A}] \setminus [\mathbb{C}])))\,. \end{cases}$$



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(a)
ℂ⁻
ℂ⁺
(b)
𝔹⁻
𝔹⁺
(c)
𝔸⁺
𝔸⁻
(d)

Consider the CSP

$$\begin{cases} f(\mathbb{A}) = \mathbb{B} \\ \mathbb{A} \in [\mathbb{A}], \mathbb{B} \in [\mathbb{B}] \end{cases}$$

where $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is bijective. The optimal contractor is

$$\begin{cases} \text{(i)} & [\mathbb{B}] := [\mathbb{B}] \sqcap f([\mathbb{A}]) \\ \text{(ii)} & [\mathbb{A}] := [\mathbb{A}] \sqcap f^{-1}([\mathbb{B}]). \end{cases}$$

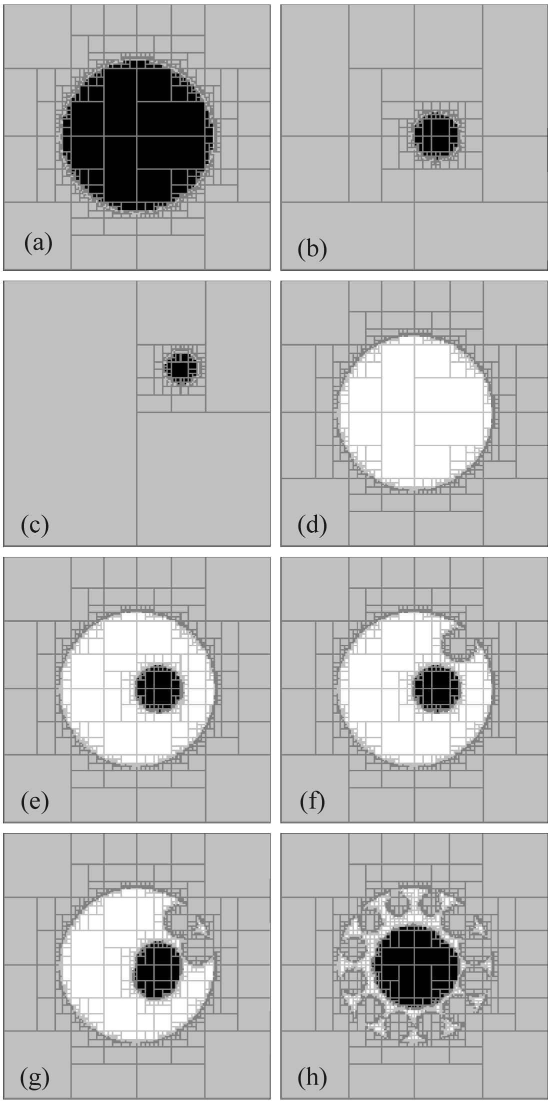
7 Application

Consider the following CSP

$$\begin{cases} \text{(i)} & \mathbb{X} \subset \mathbb{A} \\ \text{(ii)} & \mathbb{B} \subset \mathbb{X} \\ \text{(iii)} & \mathbb{X} \cap \mathbb{C} = \emptyset \\ \text{(iv)} & f(\mathbb{X}) = \mathbb{X}, \end{cases}$$

where $\mathbb{X}$ is an unknown subset of $\mathbb{R}^2$, f is a rotation with an angle of $-\frac{\pi}{6}$, and

$$\begin{cases} \mathbb{A} = \left\{(x_1, x_2), x_1^2 + x_2^2 \leq 3\right\} \\ \mathbb{B} = \left\{(x_1, x_2), (x_1 - 0.5)^2 + x_2^2 \leq 0.3\right\} \\ \mathbb{C} = \left\{(x_1, x_2), (x_1 - 1)^2 + (x_2 - 1)^2 \leq 0.15\right\} \end{cases}$$



(a) $[\mathbb{A}]$

(b) $[\mathbb{B}]$

(c) $[\mathbb{C}]$

(d) $\mathbb{X} \subset \mathbb{A}$

(e) $\mathbb{B} \subset \mathbb{X}$

(f) $\mathbb{X} \cap \mathbb{C} = \emptyset$

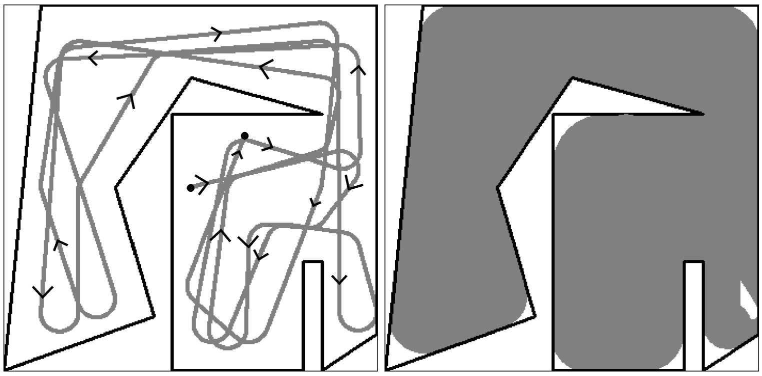
(g) $f(\mathbb{X}) = \mathbb{X}$

(h) $(f(\mathbb{X}) = \mathbb{X})^{\infty}$

8 Range-only SLAM

Range-only SLAM equations

$$\begin{cases} \dot{x}_1(t) & = & u_1(t)\cos\left(u_2(t)\right) \\ \dot{x}_2(t) & = & u_1(t)\sin\left(u_2(t)\right) \\ z\left(t\right) & = & d\left(\mathbf{x}\left(t\right),\mathbb{M}\right). \end{cases}$$



Actual trajectory and dug space

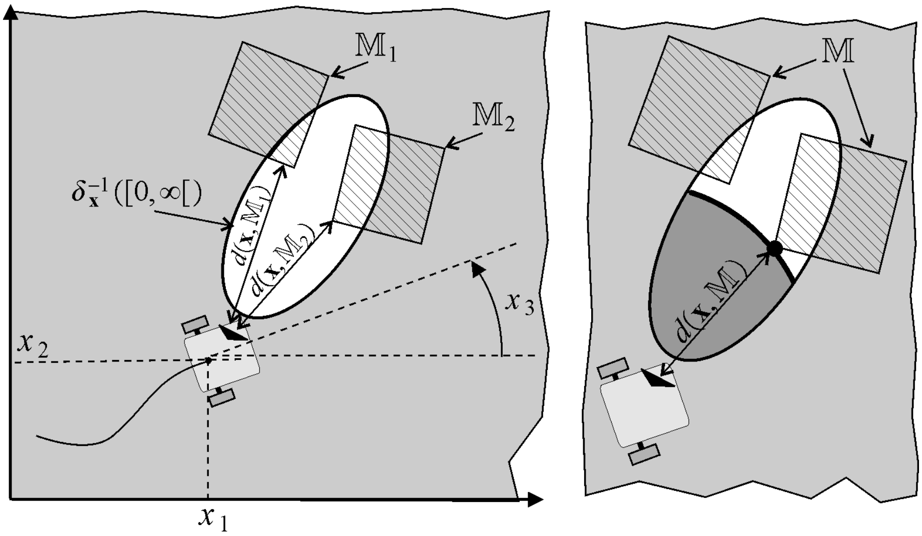
$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) & \text{(evolution equation)} \\ z(t) & = & d(\mathbf{x}(t), \mathbb{M}) & \text{(map equation)} \end{cases}$$

where $t \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, $\mathbb{M} \in \mathcal{C}(\mathbb{R}^q)$ is the occupancy map.

Unknown: the map $\mathbb{M}$ and the trajectory $\mathbf{x}$.

Assumption. d corresponds to a *rangefinder*, i.e.,

$$\begin{cases} d(\mathbf{x}, \mathbb{M}_1 \cup \mathbb{M}_2) = \min\{d(\mathbf{x}, \mathbb{M}_1), d(\mathbf{x}, \mathbb{M}_2)\} \\ d(\mathbf{x}, \emptyset) = +\infty. \end{cases}$$



Impact, covering and dug zones

Define the function $\delta_{\mathbf{x}} : \mathbb{R}^q \longrightarrow \mathbb{R}$ as

$$\delta_{\mathbf{x}}(\mathbf{a}) = d(\mathbf{x}, \{\mathbf{a}\}).$$

For given $\mathbf{x}$ and z, we define

covering zone	$\delta_{\mathbf{x}}^{-1}([0, \infty[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < \infty\}$
impact zone	$\delta_{\mathbf{x}}^{-1}(\{z\}) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) = z\}$
dug zone	$\delta_{\mathbf{x}}^{-1}([0, z[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < z\}$

Theorem 1. The dug zone does not intersect $\mathbb{M}$, i.e.,

$$z = d(\mathbf{x}, \mathbb{M}) \Rightarrow \delta_{\mathbf{x}}^{-1}([0, z[) \cap \mathbb{M} = \emptyset.$$

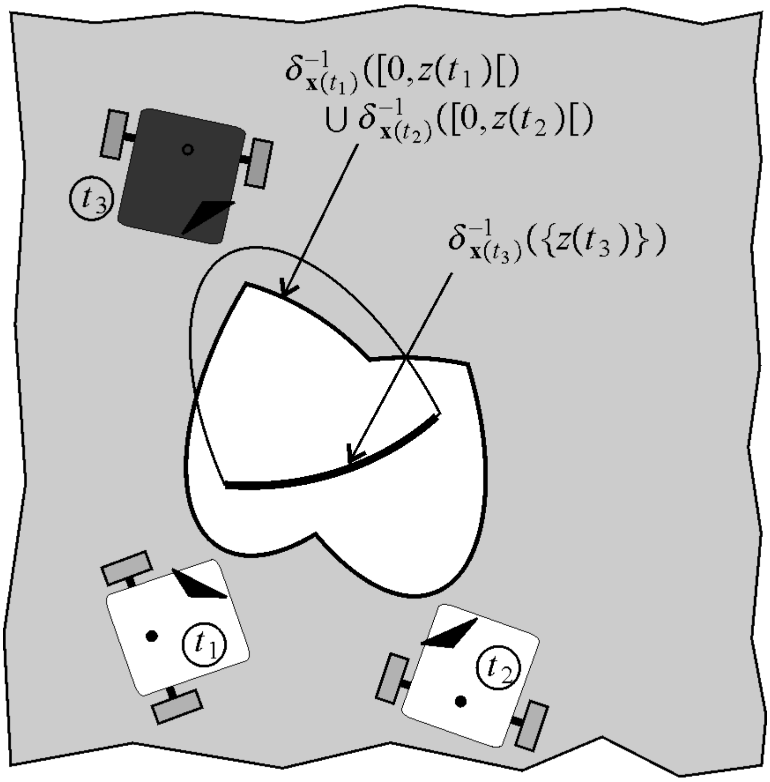
The set $\mathbb{D} = \bigcup_{t\in[t]} \delta_{\mathbf{x}(t)}^{-1}([0, z(t)[)$ is called the *dug space*. We have

$$\mathbb{D} \cap \mathbb{M} = \emptyset.$$

.

Theorem 2. For all $\mathbf{x}$, the impact zone intersects the map, i.e,

$$z = d\left(\mathbf{x}, \mathbb{M}\right) \Rightarrow \delta_{\mathbf{x}}^{-1}\left(\{z\}\right) \cap \mathbb{M} \neq \emptyset.$$



$\delta^{-1}_{\mathbf{x}(t_1)}([0,z(t_1)[)$
$\cup\, \delta^{-1}_{\mathbf{x}(t_2)}([0,z(t_2)[)$
$\delta^{-1}_{\mathbf{x}(t_3)}(\{z(t_3)\})$
t_3
t_1
t_2

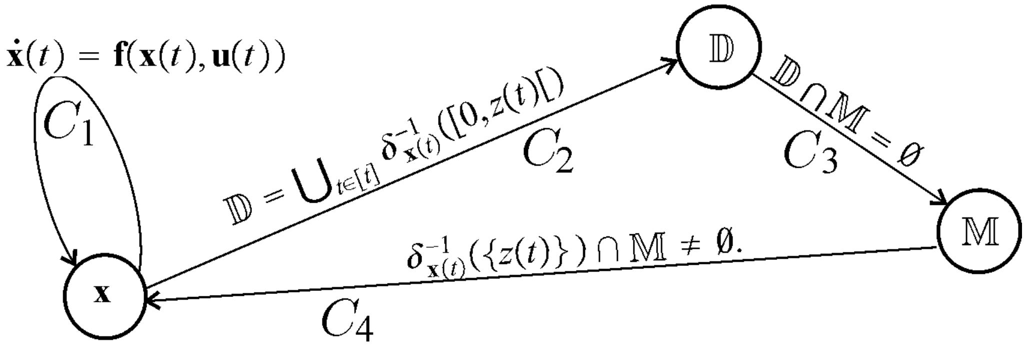
The range-only SLAM problem is a *hybrid CSP*.

Variables: $\mathbf{x}(t)$, $\mathbb{M}$ and $\mathbb{D}$.

Constraints:

$$\left.\begin{array}{ll} (1) & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ (2) & \mathbb{D} = \bigcup_{t \in [t]} \delta^{-1}_{\mathbf{x}(t)}([0, z(t)[) \\ (3) & \mathbb{D} \cap \mathbb{M} = \emptyset \\ (4) & \delta^{-1}_{\mathbf{x}(t)}(\{z(t)\}) \cap \mathbb{M} \neq \emptyset. \end{array}\right\} : z(t) = d(\mathbf{x}(t), \mathbb{M})$$

Domains: $[\mathbb{M}] = [\mathbb{D}] = [\emptyset, \mathbb{R}^q]$, $[\mathbf{x}](t) = \mathbb{R}^n$ for $t > 0$ and $[\mathbf{x}](0) = \mathbf{x}(0)$.



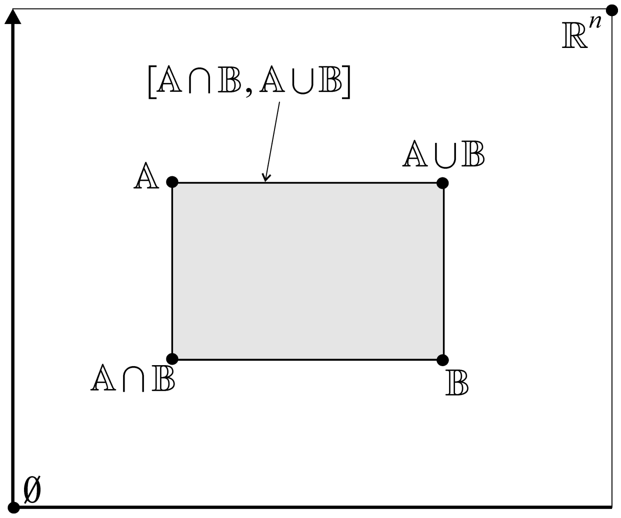
Constraint diagram of the range only SLAM problem

9 Hybrid intervals

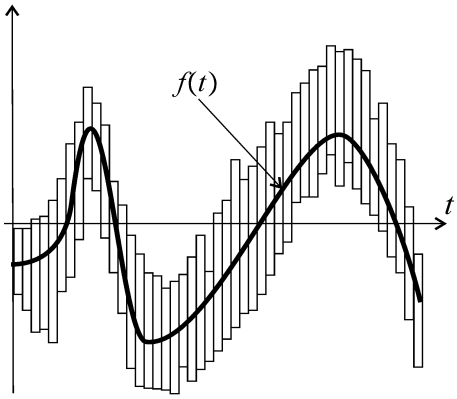
A *closed interval* (or *interval* for short) $[x]$ of a complete lattice $\mathcal{E}$ is a subset of $\mathcal{E}$ which satisfies

$$[x] = \{x \in \mathcal{E} \mid \wedge [x] \leq x \leq \vee[x]\}$$

Both $\emptyset$ and $\mathcal{E}$ are intervals of $\mathcal{E}$.



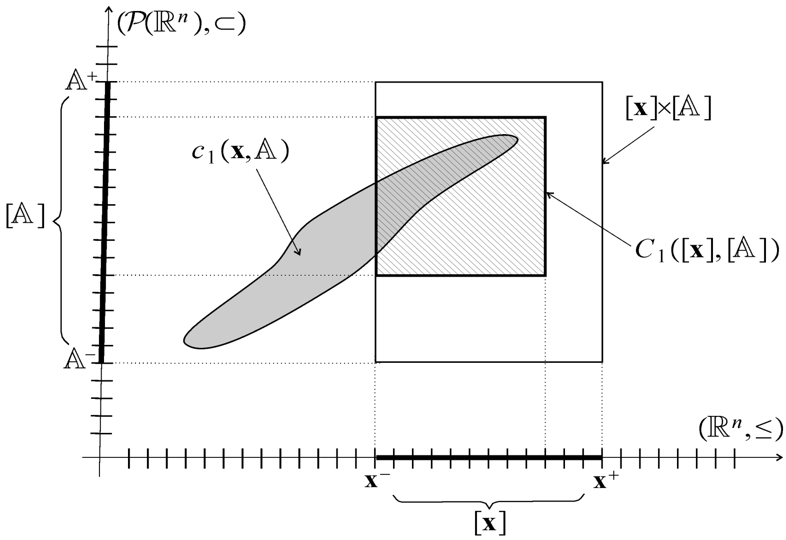
Lattice $(\mathcal{P}(\mathbb{R}^n), \subset)$



An interval function (or tube) and a set interval

Hybrid intervals. If $[x] \in \mathbb{I}\mathcal{E}_x, [y] \in \mathbb{I}\mathcal{E}_y$ then $[x] \times [y]$ is a hybrid interval.

Hybrid contractor

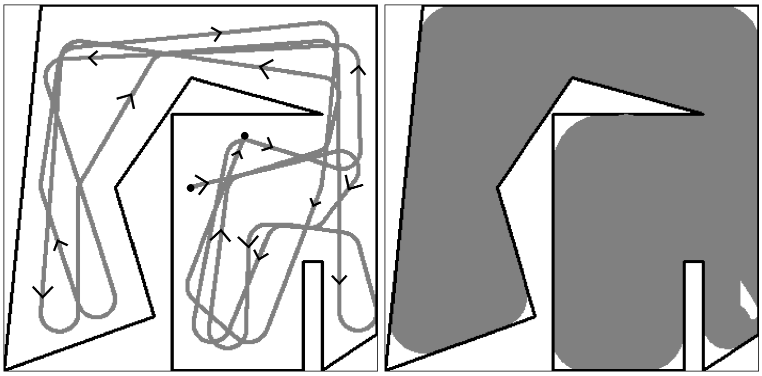


Hybrid contractor $\mathcal{C}_1$

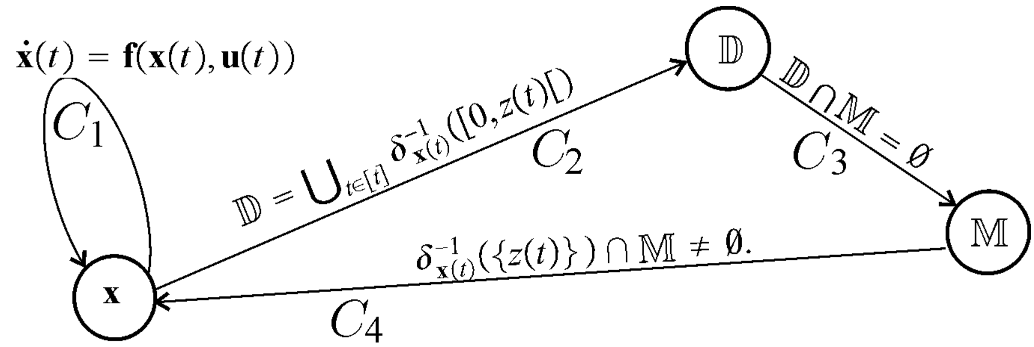
10 SLAM

Range-only SLAM equations

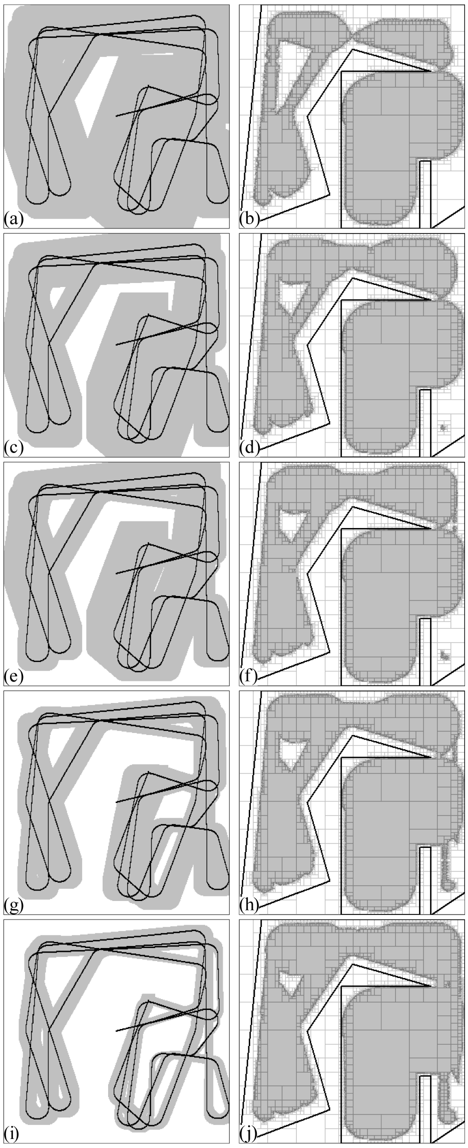
$$\left\{\begin{array}{lll} \dot{x}_1(t) & = & u_1(t)\cos\left(u_2(t)\right) \\ \dot{x}_2(t) & = & u_1(t)\sin\left(u_2(t)\right) \\ z\left(t\right) & = & d\left(\mathbf{x}\left(t\right),\mathbb{M}\right). \end{array}\right.$$

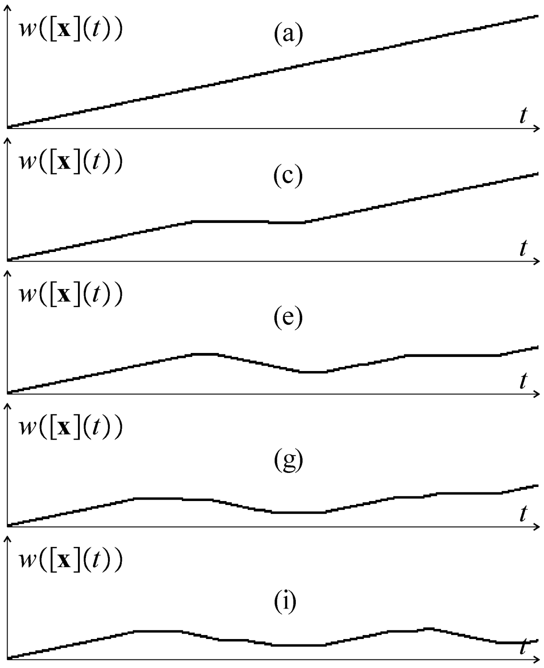


Actual trajectory and dug space



$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$
C_1
$\mathbb{D} = \bigcup_{t \in [t]} \delta^{-1}_{\mathbf{x}(t)}([0, z(t)[)$
C_2
$\mathbb{D}$
$\mathbb{D} \cap \mathbb{M} = \emptyset$
C_3
$\mathbb{M}$
$\delta^{-1}_{\mathbf{x}(t)}(\{z(t)\}) \cap \mathbb{M} \neq \emptyset.$
C_4
$\mathbf{x}$


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)



Width of the tubes $[\mathbf{x}](t)$

References

L. Jaulin (2012). Solving set-valued constraint satisfaction problems. Computing. Volume 94, Issue 2, Page 297-311.

L. Jaulin (2011). Range-only SLAM with occupancy maps; A set-membership approach. IEEE-TRO. Vol 27, Issue 5, pp 1004-101