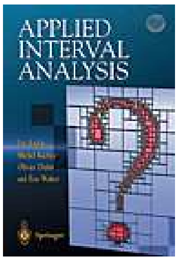
Interval constraint propagation; applications to control, estimation and robotics



Luc Jaulin,
ENSIETA, Brest, France

Belo Horizonte, October 27, 2008, 10h00.

1 Motivations

Model : $y_{\mathsf{m}}(\mathbf{p}, t) = p_1 e^{-p_2 t}$.
Parameters : p_1, p_2.
Measurement times : $t_1, t_2, \ldots, t_m$
Data bars : $[y_1^-, y_1^+], [y_2^-, y_2^+], \ldots, [y_m^-, y_m^+]$
Feasible set :

$$\begin{aligned} \mathbb{S} &= \left\{ \mathbf{p} \in \mathbb{R}^2, \forall i \in \{1, \ldots, m\}, y_{\mathsf{m}}(\mathbf{p}, t_i) \in [y_i^-, y_i^+] \right\}. \\ &= \mathbf{y}_{\mathsf{m}}^{-1}([\mathbf{y}]). \end{aligned}$$

Illustration with SetDemo

2 Interval constraint propagation

2.1 Interval arithmetic

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \,.$$

For instance,

$$\begin{aligned}
[-1, 3] + [2, 5] &= [1, 8], \\
[-1, 3].[2, 5] &= [-5, 15], \\
[-1, 3]/[2, 5] &= [-\tfrac{1}{2}, \tfrac{3}{2}], \\
[-1, 3] \vee [2, 5] &= [2, 5].
\end{aligned}$$

$$
\begin{aligned}
[x^-, x^+] + [y^-, y^+] = \; & [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] = \; & [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
& x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+], \\
[x^-, x^+] \vee [y^-, y^+] = \; & [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}
$$

If $f \in \{\text{cos, sin,sqr, sqrt, log, exp, } \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

For instance,

$$\begin{array}{rcl} \text{sin}([0,\pi]) & = & [0,1], \\ \text{sqr}([-1,3]) & = & [-1,3]^2 = [0,9], \\ \text{abs}([-7,1]) & = & [0,7], \\ \text{sqrt}([-10,4]) & = & \sqrt{[-10,4]} = [0,2], \\ \text{log}([-2,-1]) & = & \emptyset. \end{array}$$

2.2 Constraint projection

Let x, y, z be 3 variables such that

$$\begin{aligned} x &\in [-\infty, 5], \\ y &\in [-\infty, 4], \\ z &\in [6, \infty], \\ z &= x + y. \end{aligned}$$

The values < 2 for x, < 1 for y and > 9 for z are inconsistent.

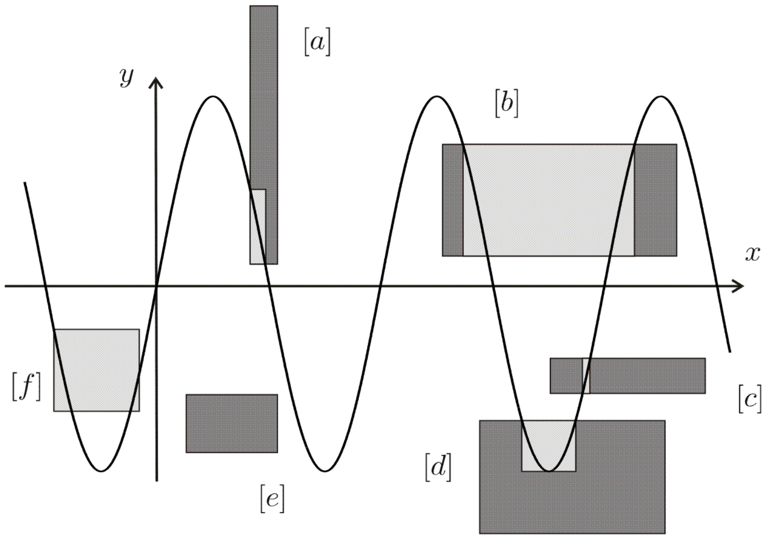
To project a constraint (here, $z = x + y$), is to compute the smallest intervals which contains all consistent values for the variables. For our example, this amounts to project 3 times (following x, y and z) the subset of $\mathbb{R}^3$ given by

$$\mathbb{S} = \{(x, y, z) \in [-\infty, 5] \times [-\infty, 4] \times [6, \infty] \mid z = x + y\}.$$

2.3 Numerical method for projection

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$
\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}
$$



[a]
y
[b]
x
[f]
[c]
[d]
[e]

2.4 Constraint propagation

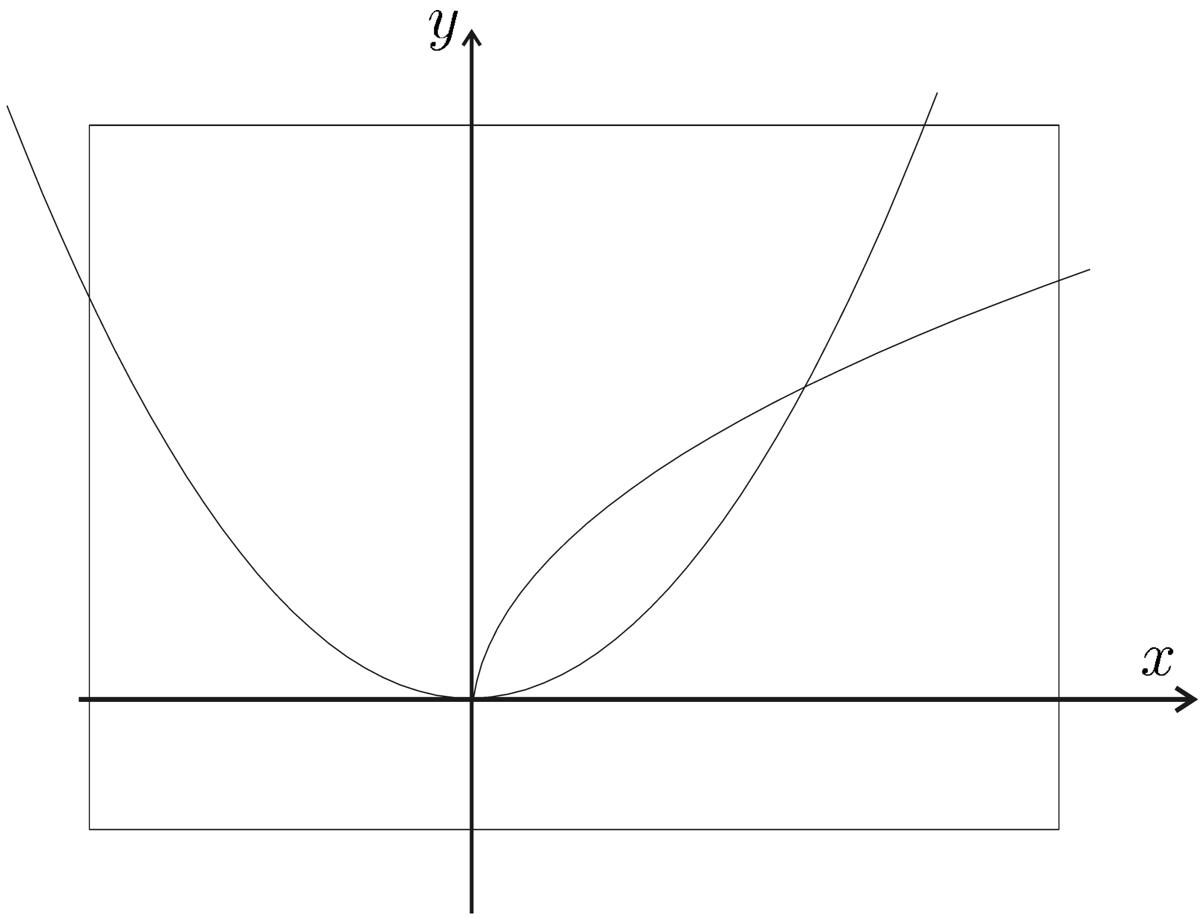
Consider the system of two equations.

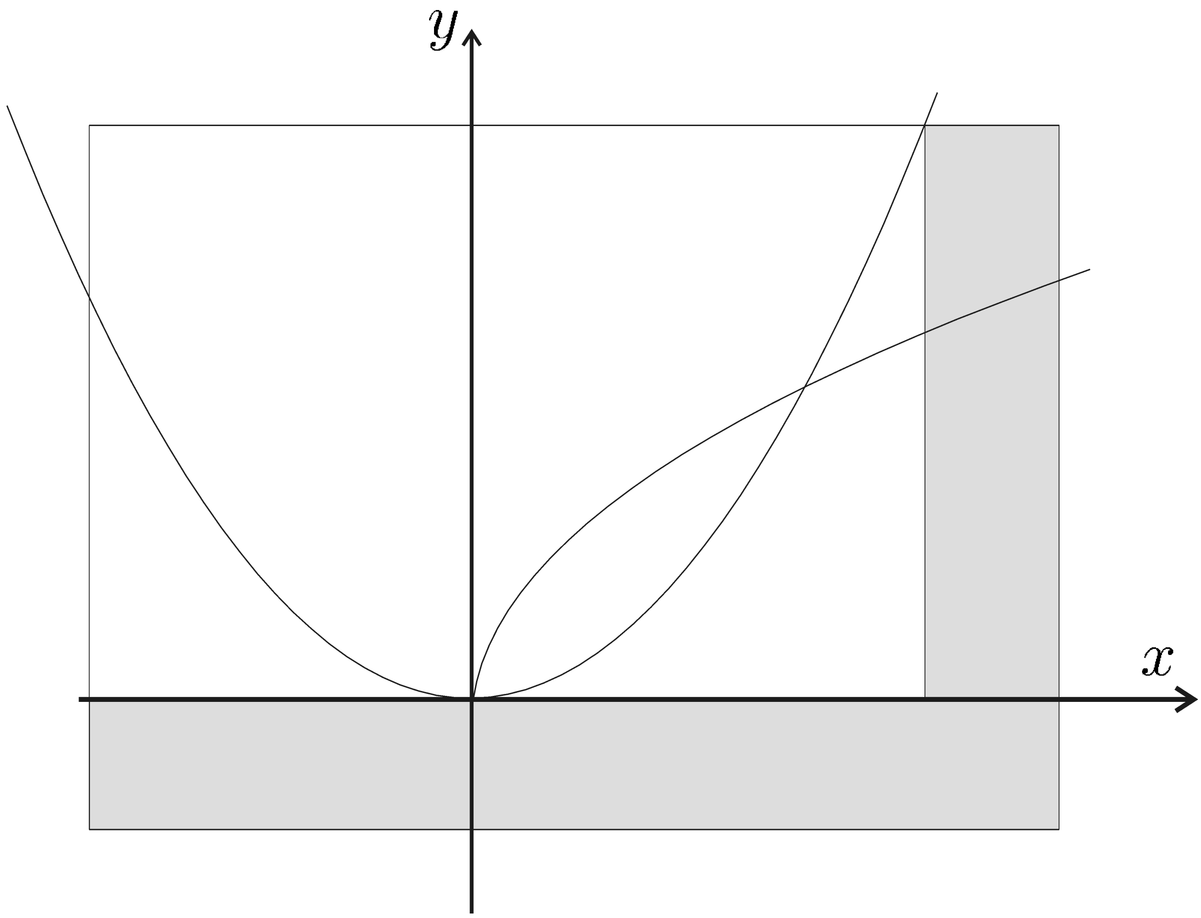
$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

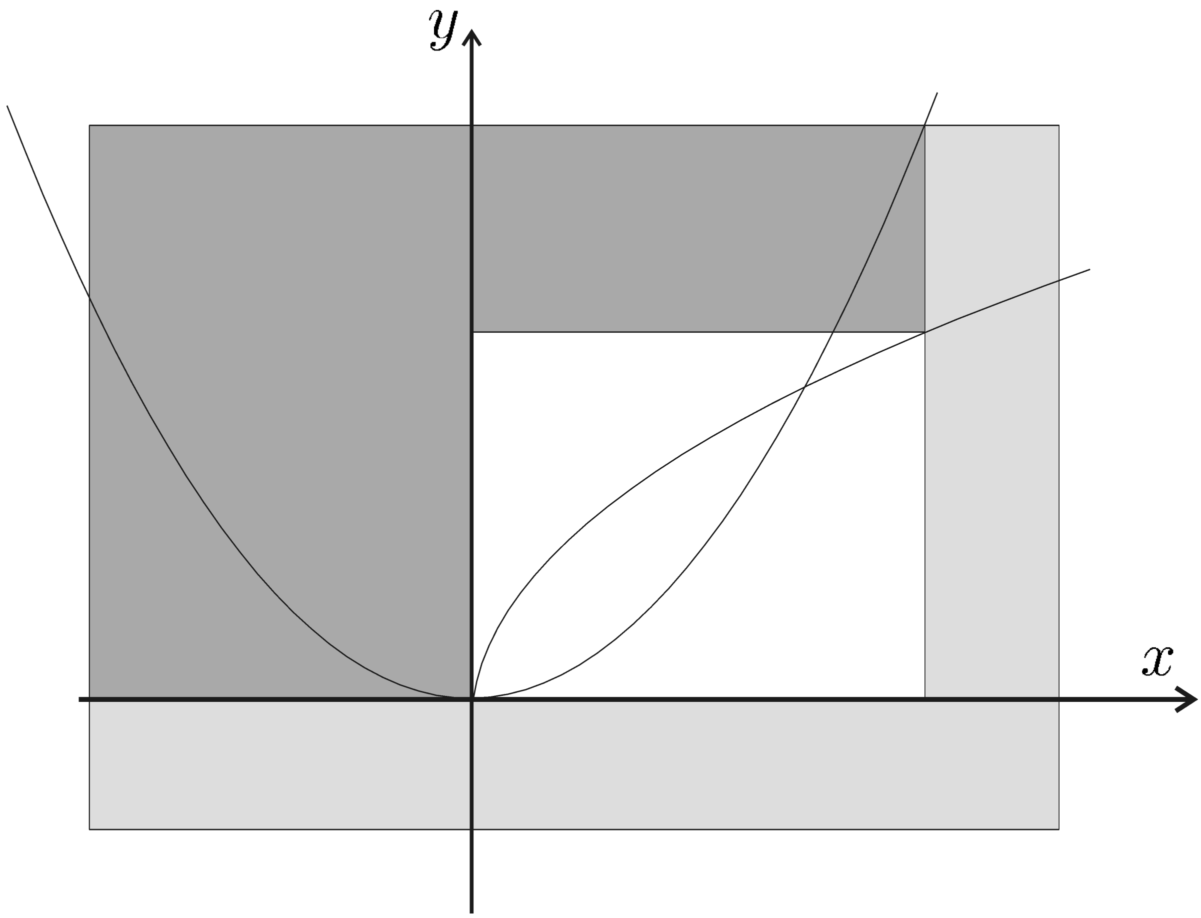
We can build two contractors

$$\mathcal{C}_1 : \left\{ \begin{array}{l} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{array} \right. \quad \text{associated to } y = x^2$$

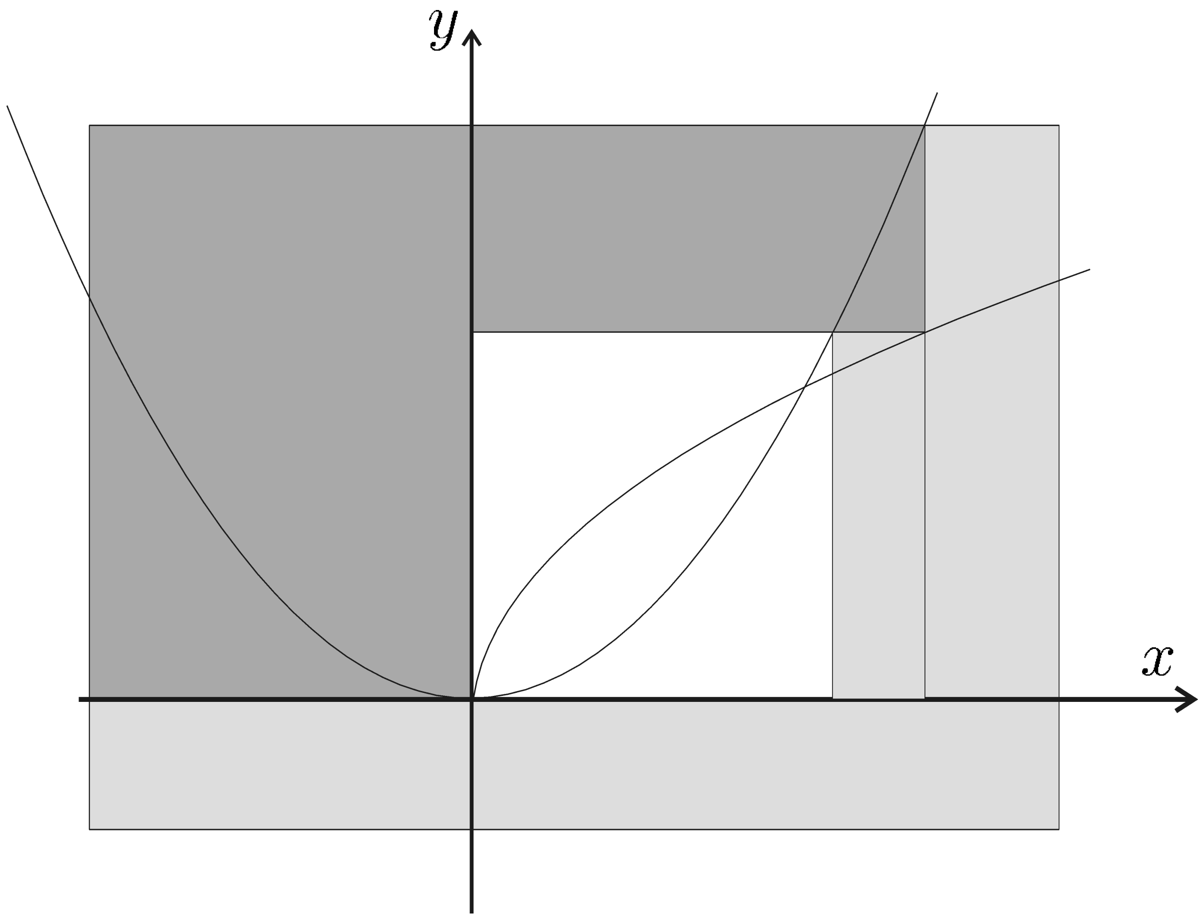
$$\mathcal{C}_2 : \left\{ \begin{array}{l} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{array} \right. \quad \text{associated to } y = \sqrt{x}$$


y
x

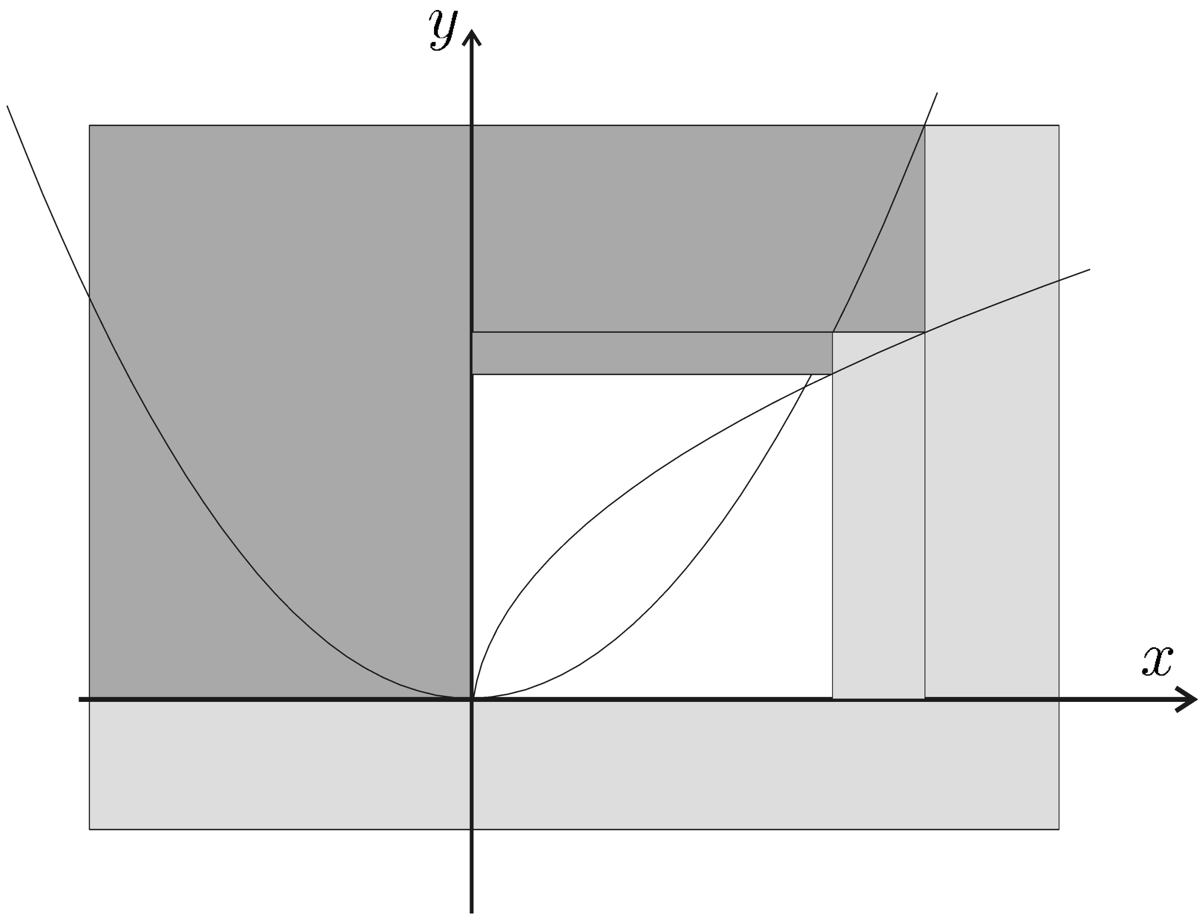

y
x

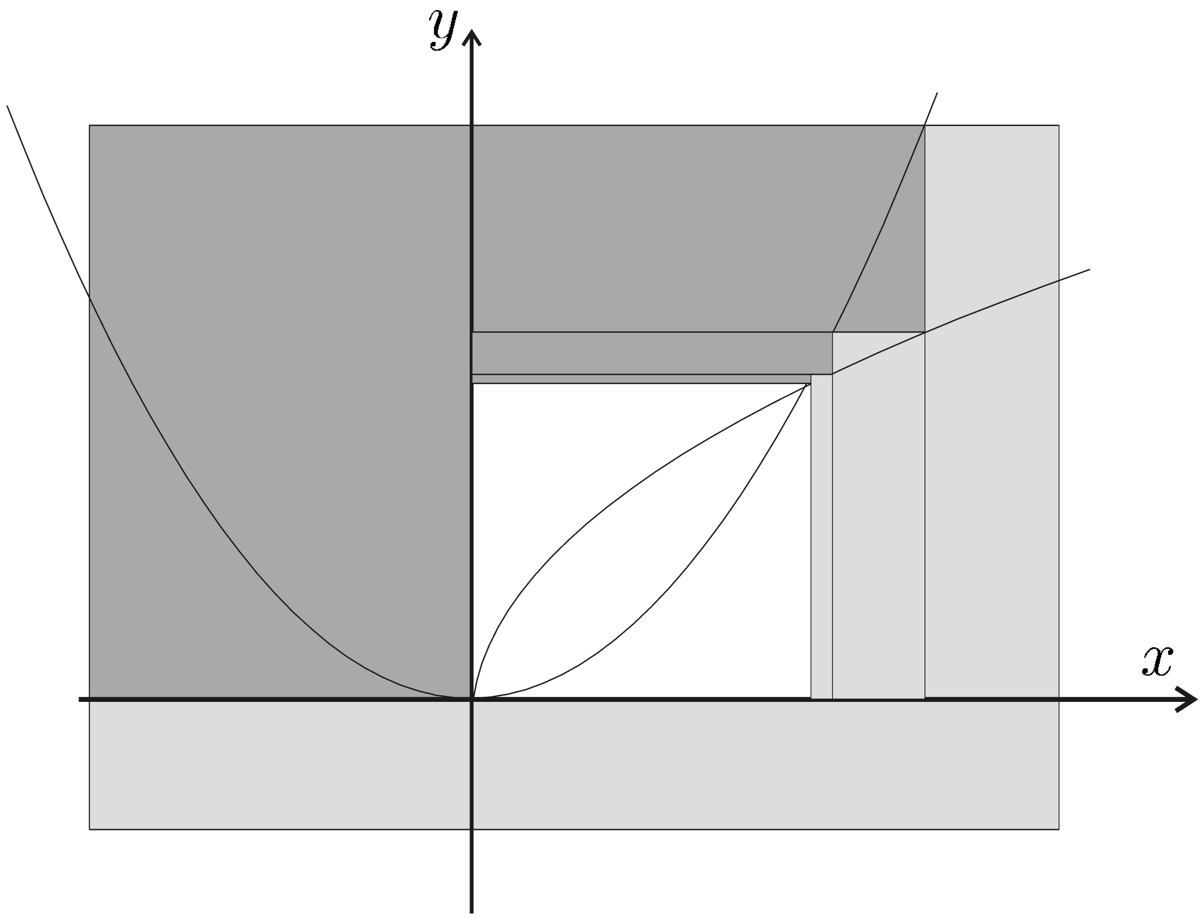


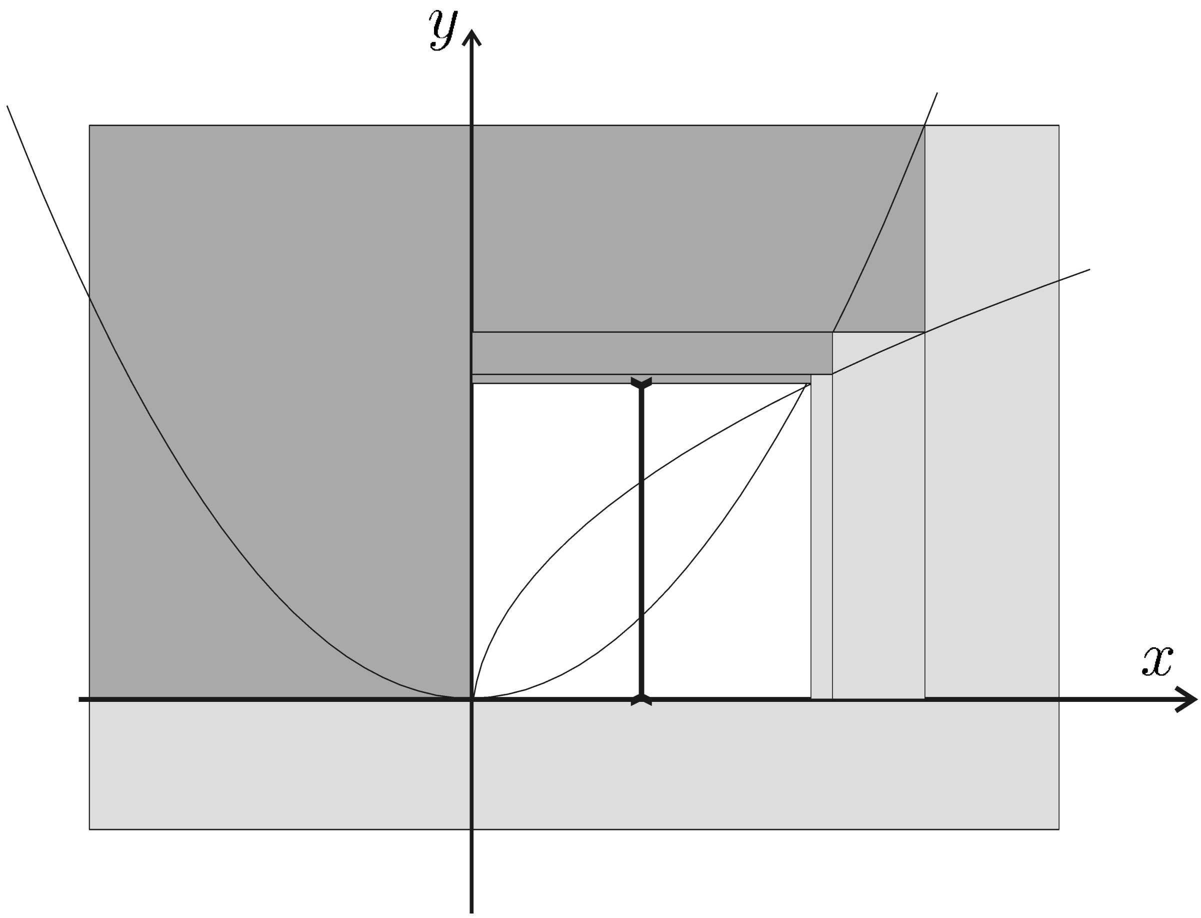
y
x

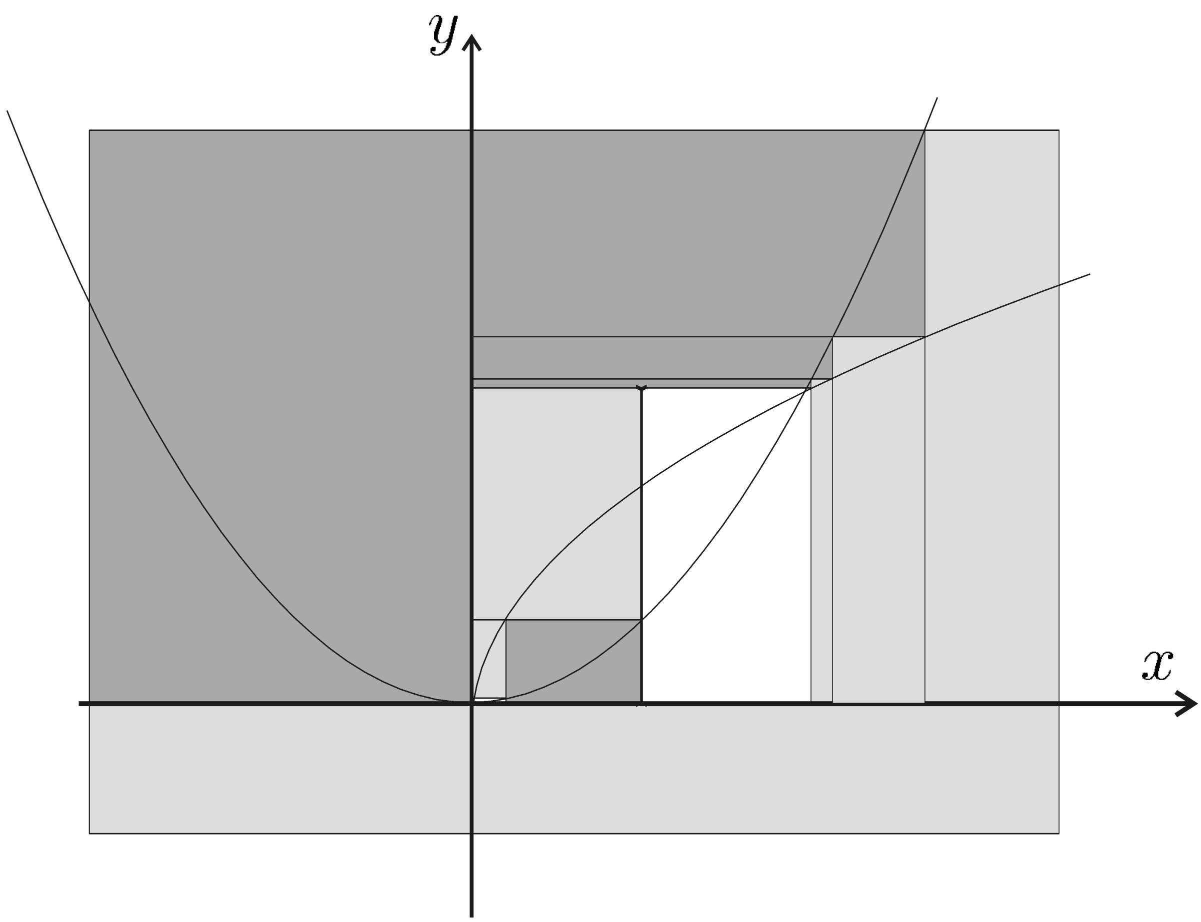


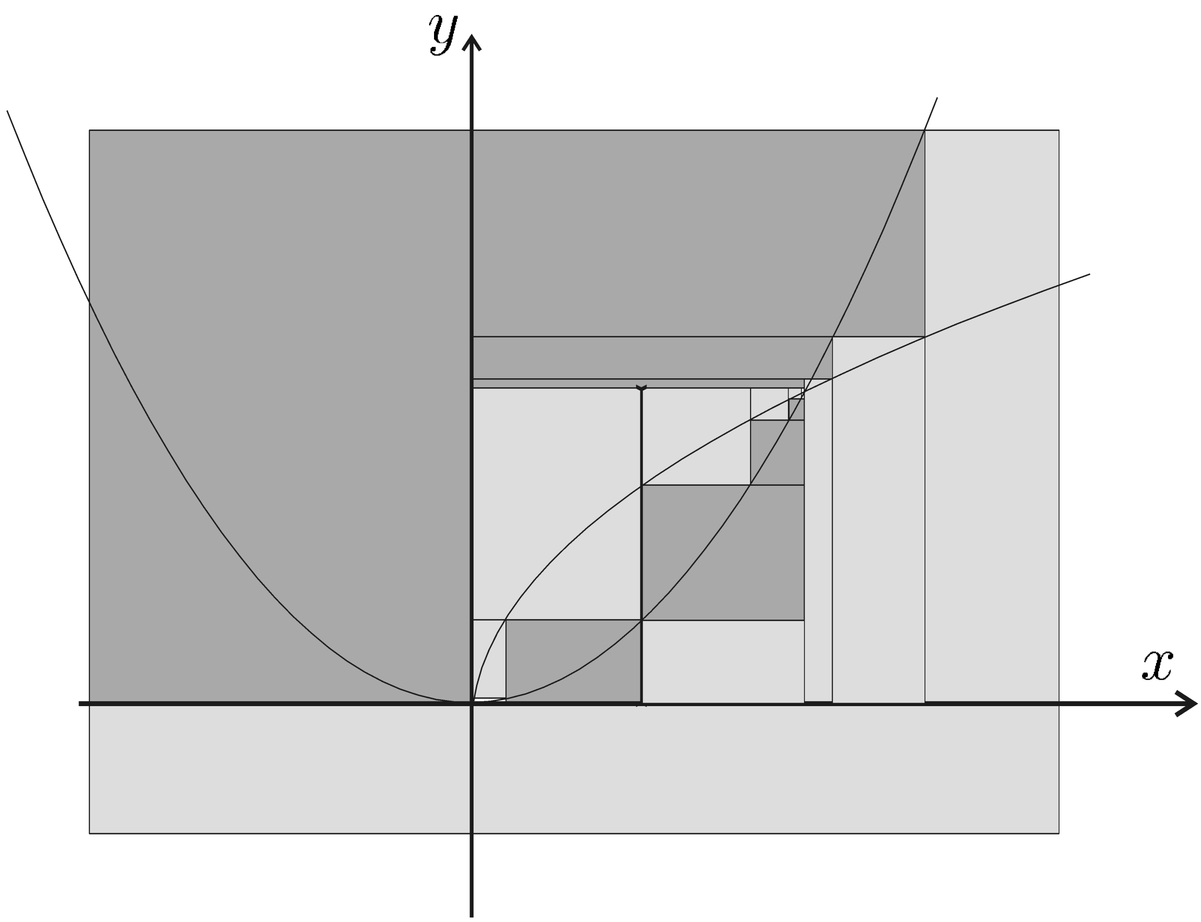
y
x


y
x


y
x


y
x


y
x



y
x

2.5 Decomposition

For complex constraints, we have to perform a decomposition. For instance

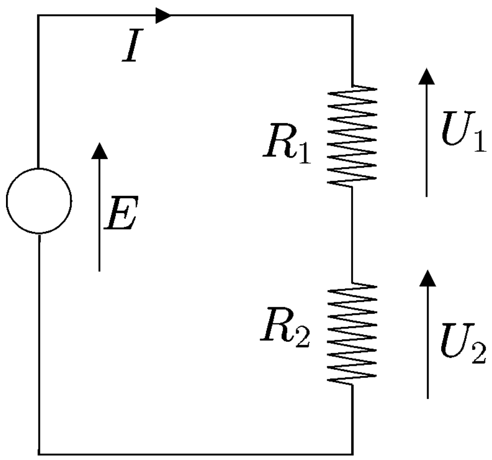
$$x + \mathsf{sin}(xy) \leq \mathsf{0},$$
$$x \in [-1, 1], y \in [-1, 1], z \in [-1, 1]$$

can be decomposed into

$$\begin{cases} a = xy \\ b = \mathsf{sin}(a) \\ c = x + b \end{cases}, \qquad \begin{array}{ll} x \in [-1, 1] & a \in [-\infty, \infty] \\ y \in [-1, 1] & b \in [-\infty, \infty] \\ z \in [-1, 1] & c \in [-\infty, \mathsf{0}] \end{array}$$

3 Applications

3.1 Estimation problem



Constraints

$$\begin{aligned} P &= EI;\ E = (R_1 + R_2)\, I; \\ U_1 &= R_1 I;\ U_2 = R_2 I;\ E = U_1 + U_2. \end{aligned}$$

Initial domains

$$\begin{array}{cc} R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\ E \in [23, 26]\mathsf{V}, & I \in [4, 8]\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, 17]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}, & \end{array}$$

Constraints

$$
\begin{aligned}
P &= EI;\; E = (R_1 + R_2)\, I; \\
U_1 &= R_1 I;\; U_2 = R_2 I;\; E = U_1 + U_2.
\end{aligned}
$$

We get the contracted domains

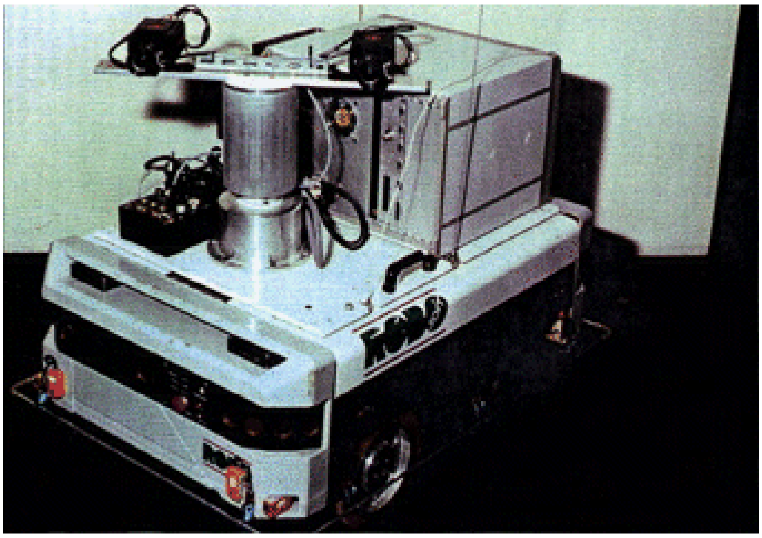
$$\begin{array}{rr} R_1 \in [\mathbf{1.84}, \mathbf{2.31}]\,\Omega, & R_2 \in [\mathbf{2.58}, \mathbf{3.35}]\Omega, \\ E \in [\mathbf{24}, 26]\mathsf{V}, & I \in [\mathbf{4.769}, \mathbf{5.417}]\,\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, \mathbf{16}]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}, & \end{array}$$

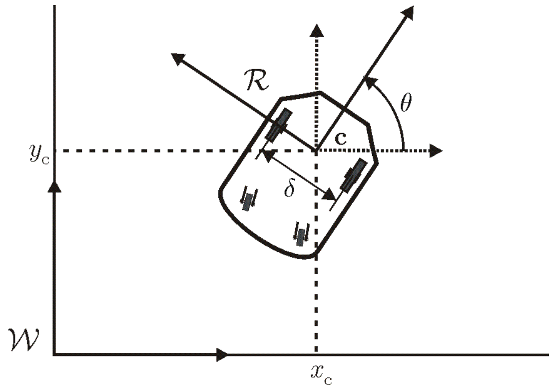
instead of the initial domains

$$\begin{array}{rr} R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\ E \in [23, 26]\mathsf{V}, & I \in [4, 8]\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, 17]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}, & \end{array}$$

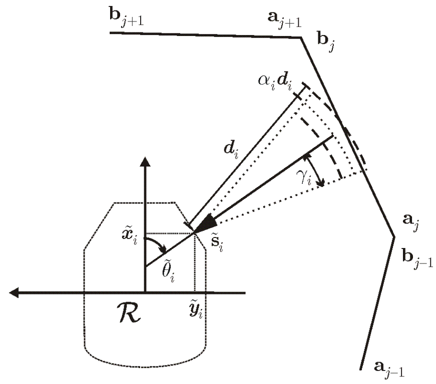
3.2 Static localization

Robot with 24 ultrasonic telemeters



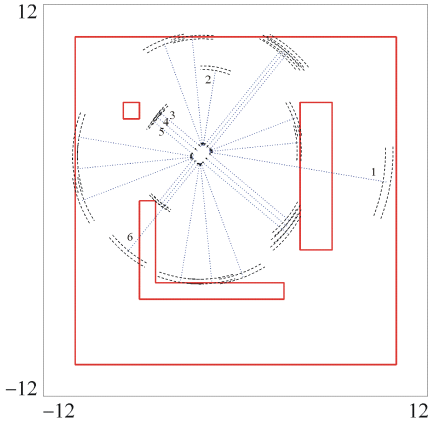


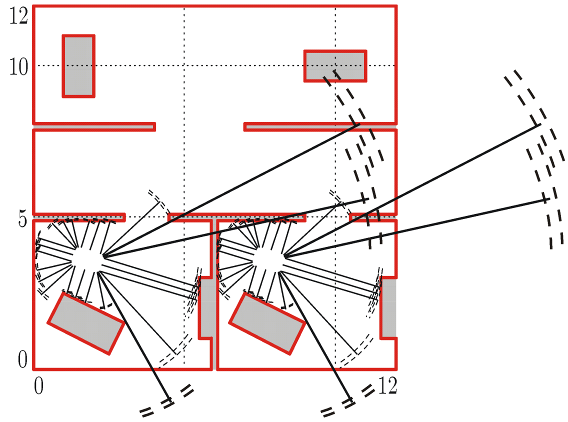
$\mathcal{R}$
θ
$\mathbf{c}$
y_c
δ
$\mathcal{W}$
x_c



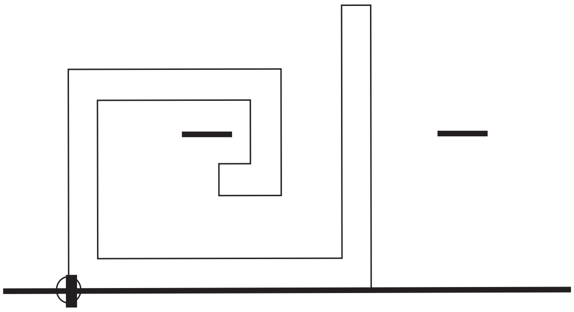
$\mathbf{b}_{j+1}$
$\mathbf{a}_{j+1}$
$\mathbf{b}_j$
$\alpha_i d_i$
d_i
γ_i
$\tilde{x}_i$
$\tilde{s}_i$
$\tilde{\theta}_i$
$\mathbf{a}_j$
$\mathbf{b}_{j-1}$
$\mathcal{R}$
$\tilde{y}_i$
$\mathbf{a}_{j-1}$

After set inversion

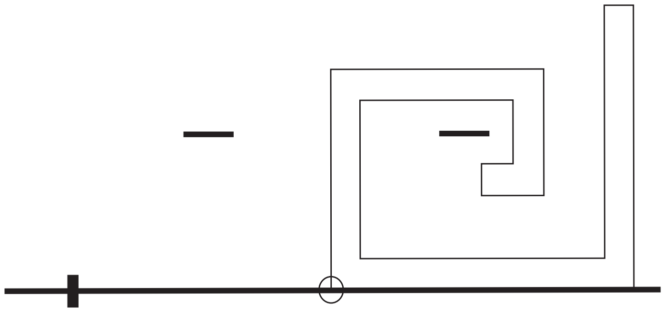




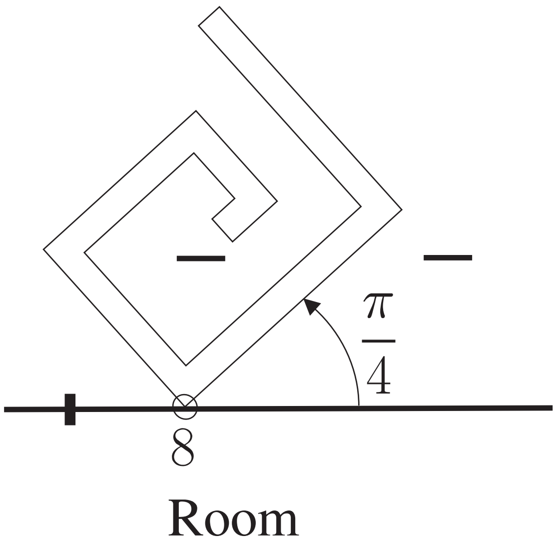
3.3 Path planning

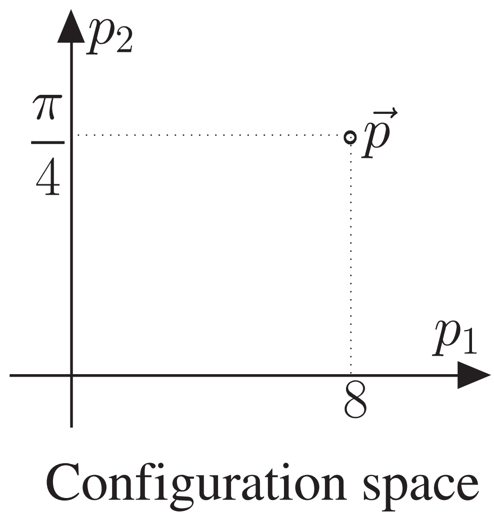


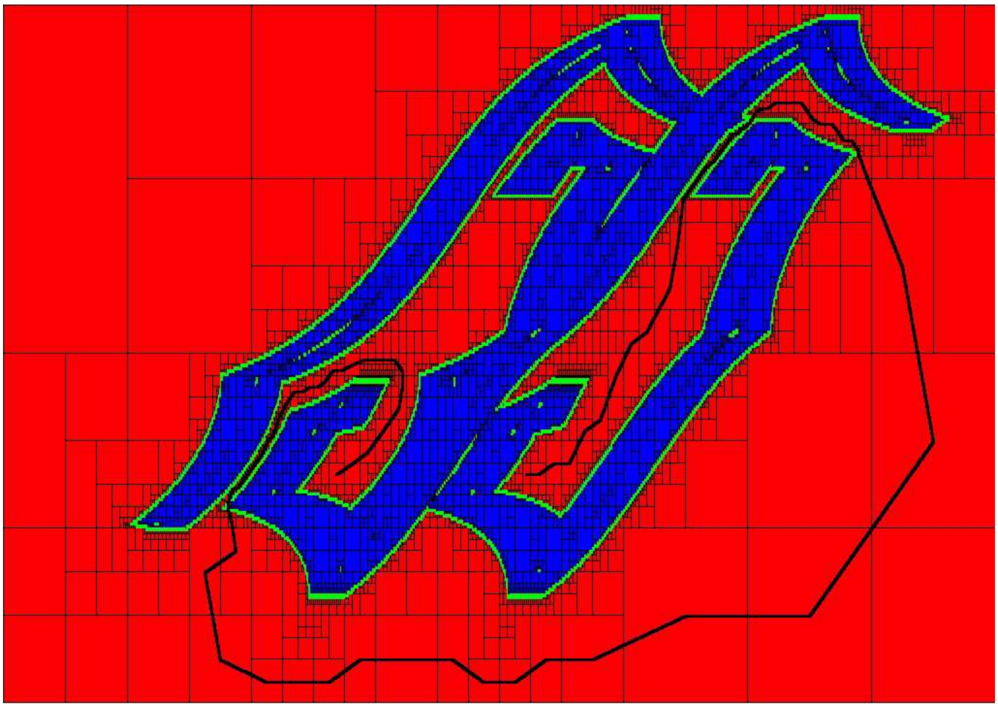
Initial configuration: $\vec{p} = (0\ 0)^{\mathrm{T}}$

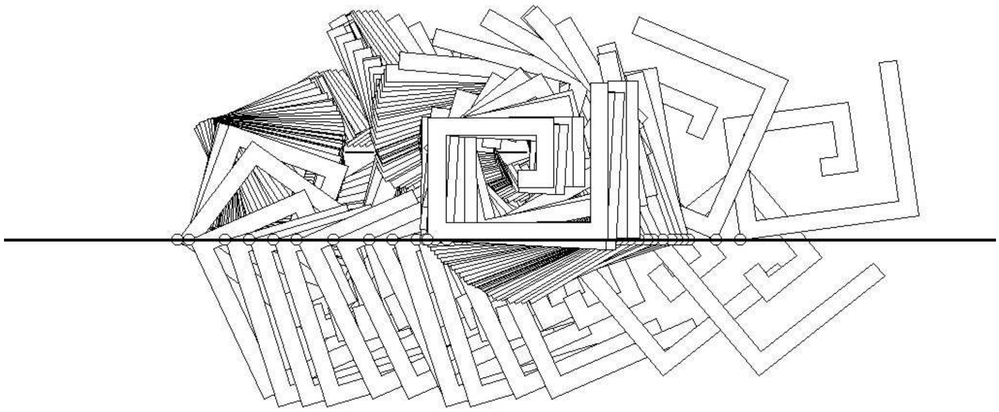


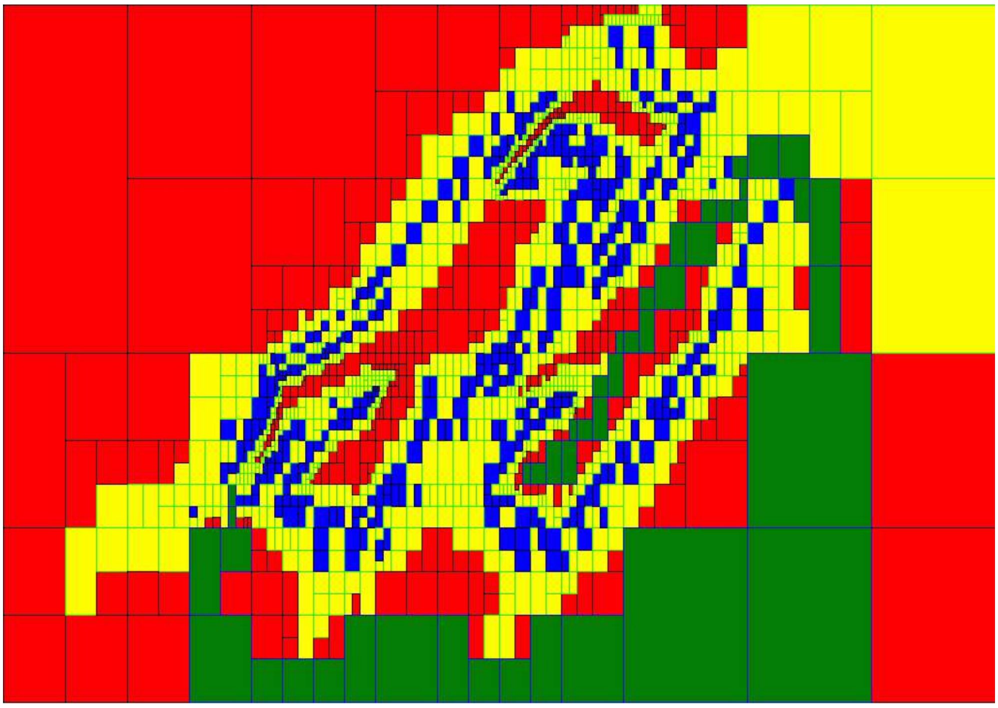
Goal configuration: $\vec{p} = (17\ 0)^{\mathrm{T}}$

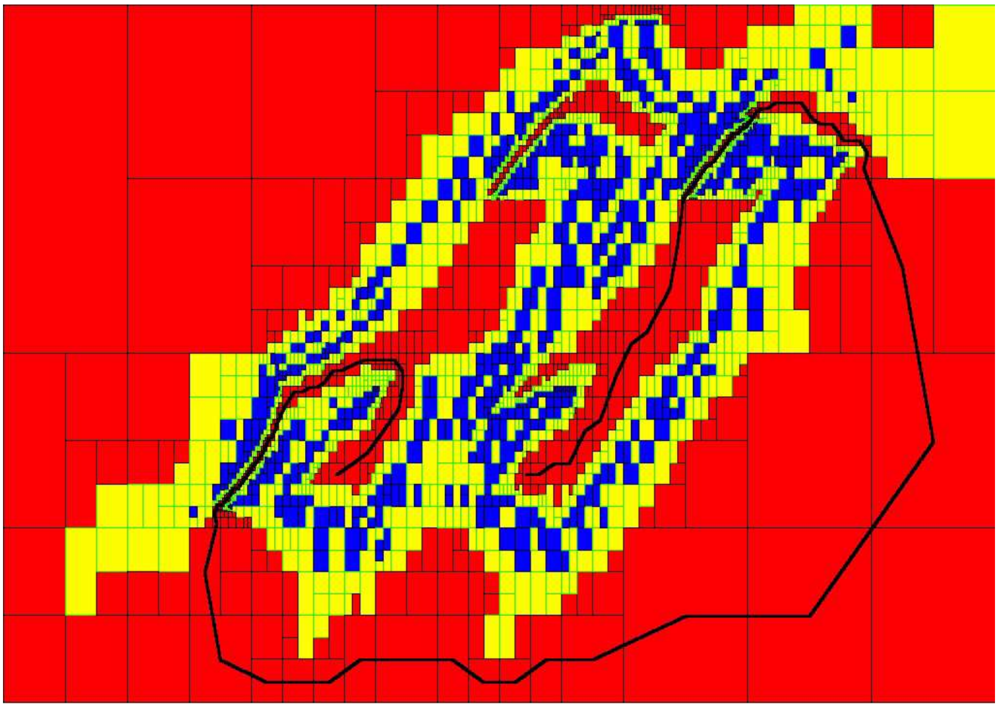

π/4
8
Room


p_2
π/4
$\vec{p}$
p_1
8
Configuration space

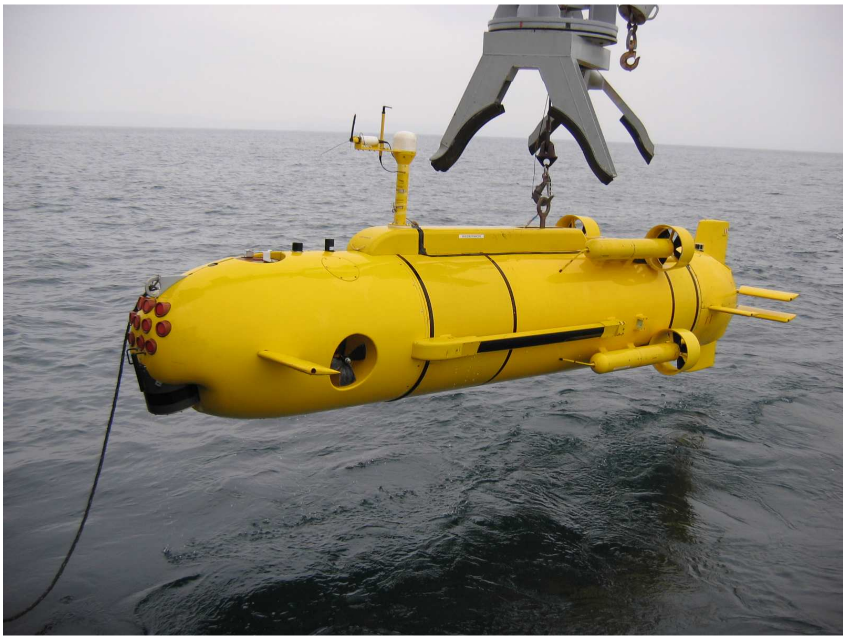




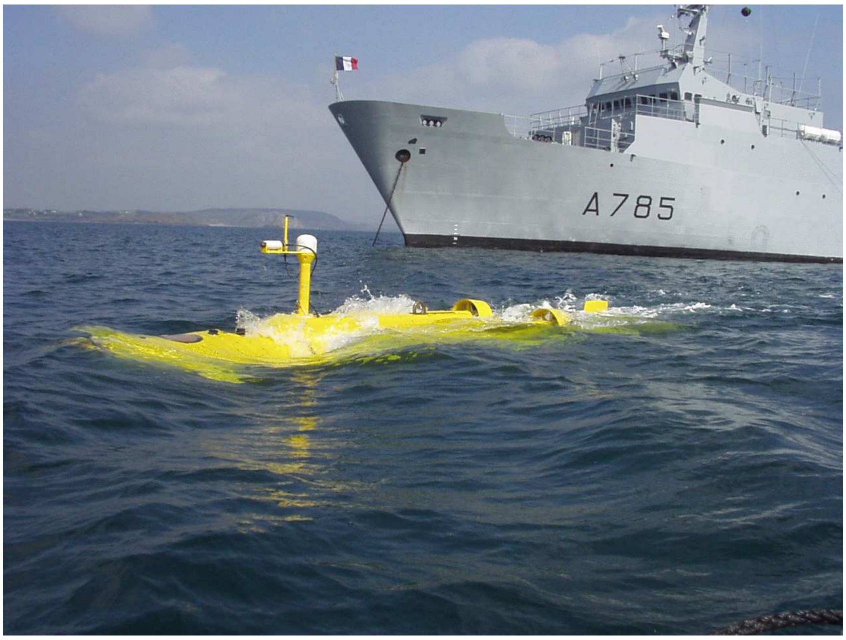




4 SLAM



Redermor, GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)


A785

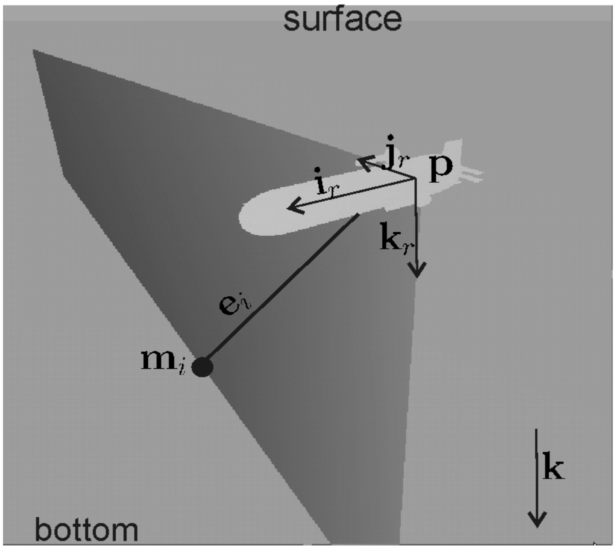
Montrer la simulation

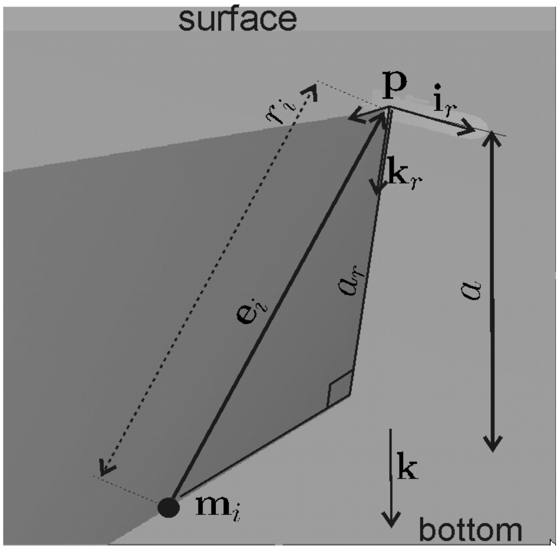
4.1 Sensors

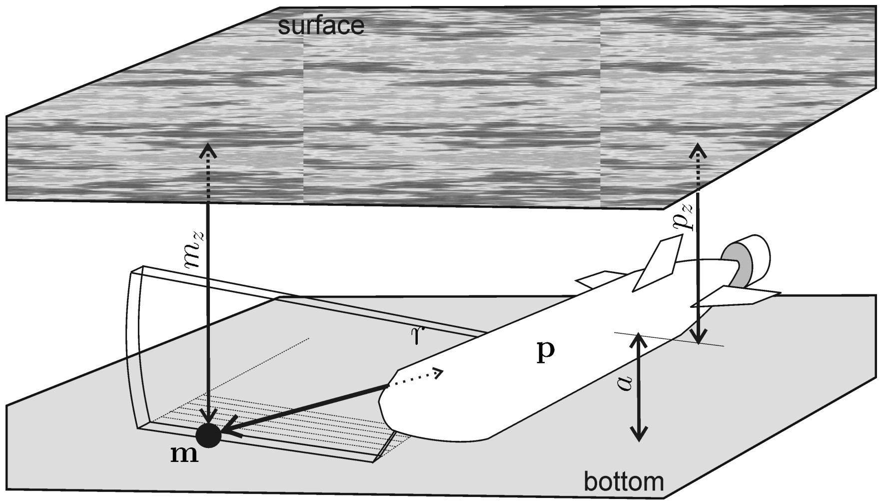
GPS (Global positioning system), only at the surface.

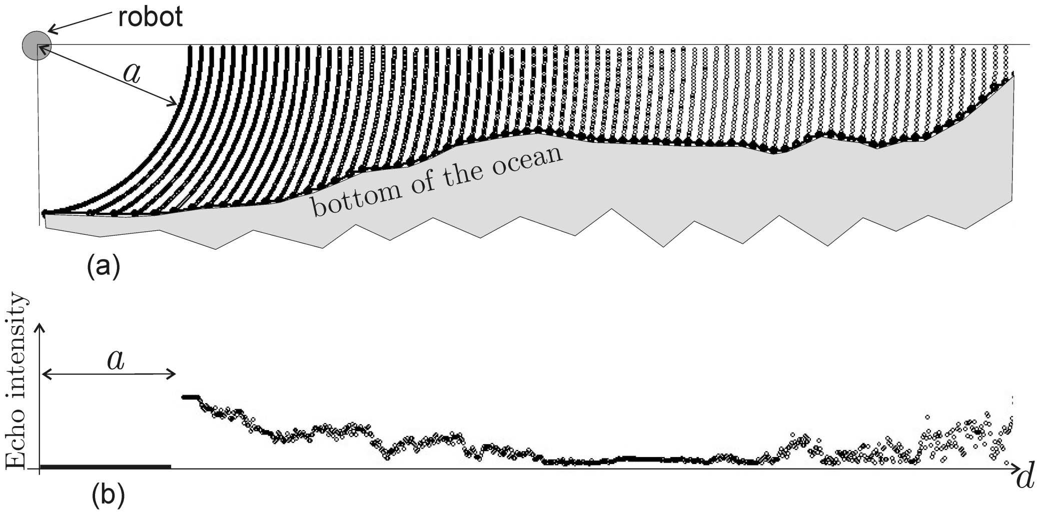
$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m$$

Sonar (KLEIN 5400 side scan sonar).

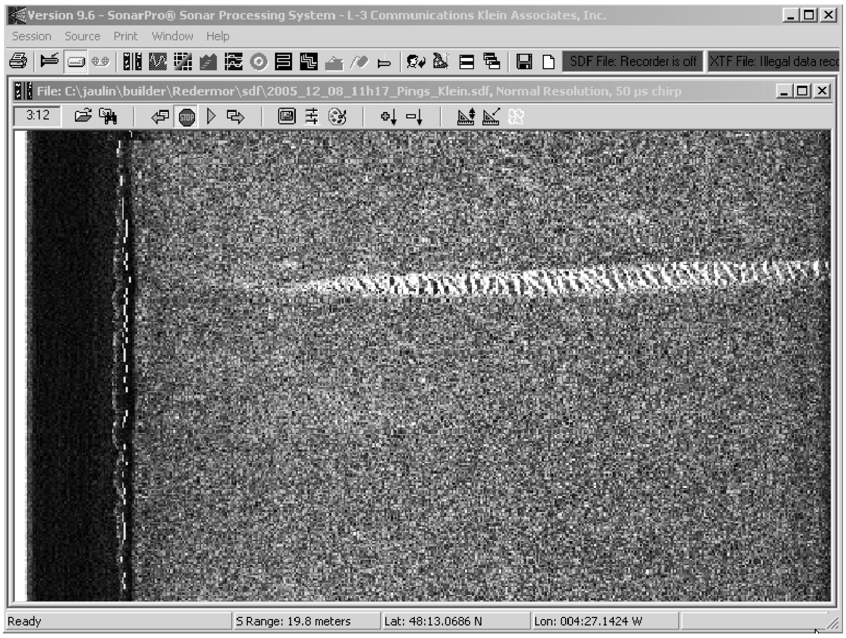




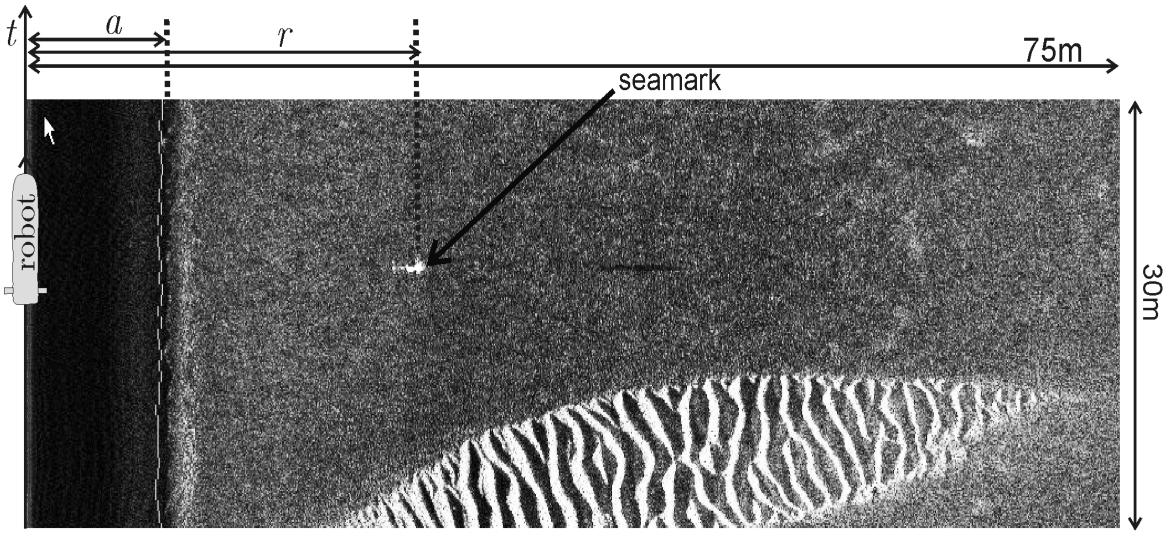

surface
m_z
p_z
r
$\mathbf{p}$
a
$\mathbf{m}$
bottom



robot
a
bottom of the ocean
(a)
Echo intensity
a
d
(b)



Screenshot of SonarPro



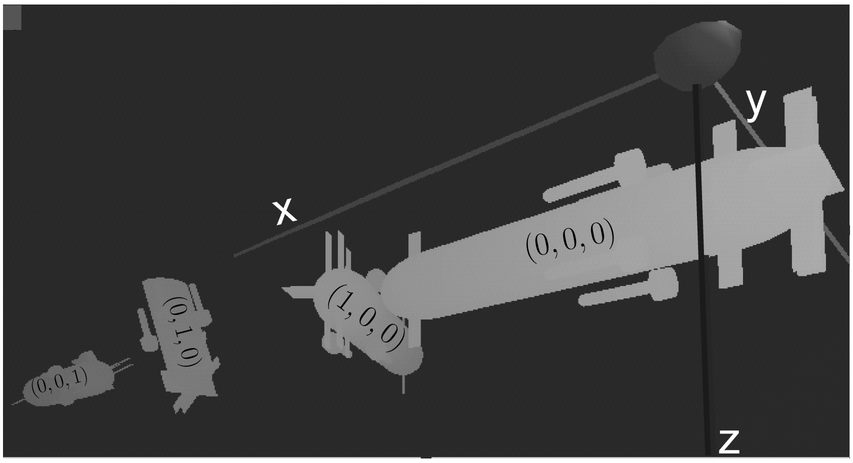
Mine detecttion with SonarPro

Loch-Doppler returns the speed robot $\mathbf{v}_r$.

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] . \tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

Inertial central (Octans III from IXSEA).

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



Six mines have been detected.

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

4.2 Constraints

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t)\mathbf{R}_{\theta}(t)\mathbf{R}_{\varphi}(t),$$

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_{r}(t),$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^{\mathsf{T}}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5]$$

```
//---------------------------------------------------
Constants
N = 59996; // Number of time steps
Variables
  R[N-1][3][3], // rotation matrices
  p[N][3], // positions
  v[N-1][3], // speed vectors
  phi[N-1],theta[N-1],psi[N-1]; // Euler angles
  px[N],py[N]; // for display only
//---------------------------------------------------
```

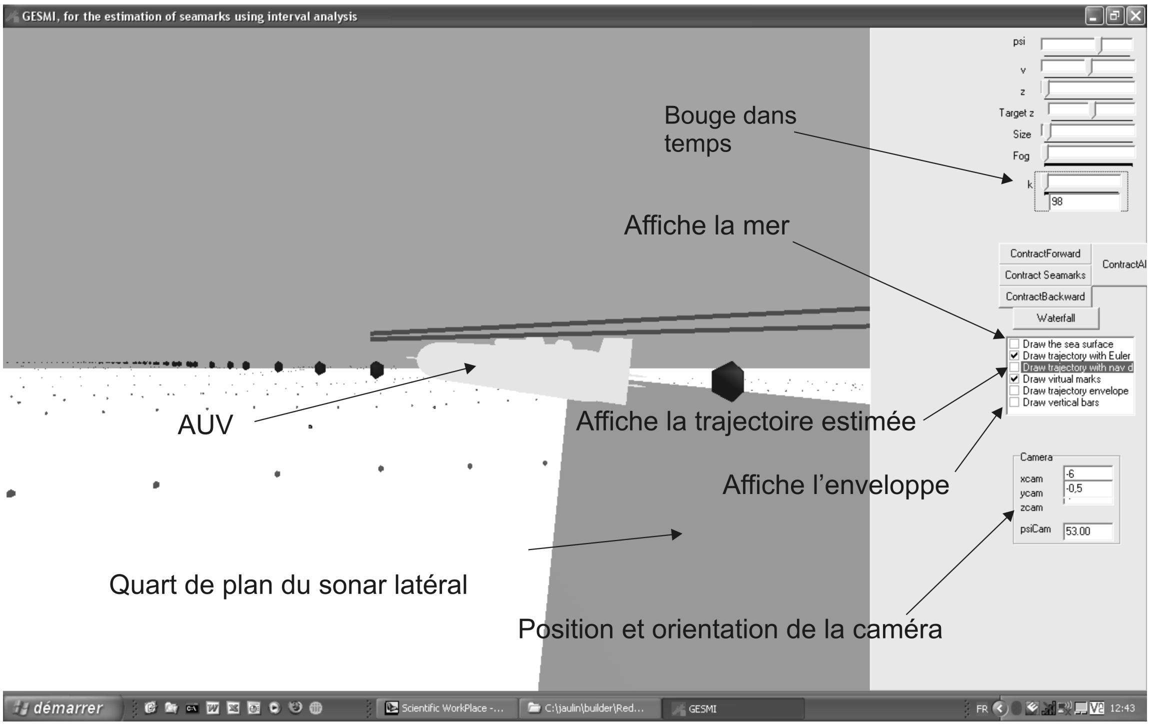
```
function R[3][3]=euler(phi,theta,psi)
  cphi = cos(phi);
  sphi = sin(phi);
  ctheta = cos(theta);
  stheta = sin(theta);
  cpsi = cos(psi);
  spsi = sin(psi);
  R[1][1]=ctheta*cpsi;
  R[1][2]=-cphi*spsi+stheta*cpsi*sphi;
  R[1][3]=spsi*sphi+stheta*cpsi*cphi;
  R[2][1]=ctheta*spsi;
  R[2][2]=cpsi*cphi+stheta*spsi*sphi;
  R[2][3]=-cpsi*sphi+stheta*cphi*spsi;
  R[3][1]=-stheta;
  R[3][2]=ctheta*sphi;
  R[3][3]=ctheta*cphi;
end
```

```
contractor-list rotation
  for k=1:N-1;
    R[k]=euler(phi[k],theta[k],psi[k]);
  end
end
//---------------------------------------------------------
contractor-list statequ
  for k=1:N-1;
    p[k+1]=p[k]+0.1*R[k]*v[k];
  end
end
//---------------------------------------------------------
contractor init
  inter k=1:N-1;
    rotation(k)
  end
end
```

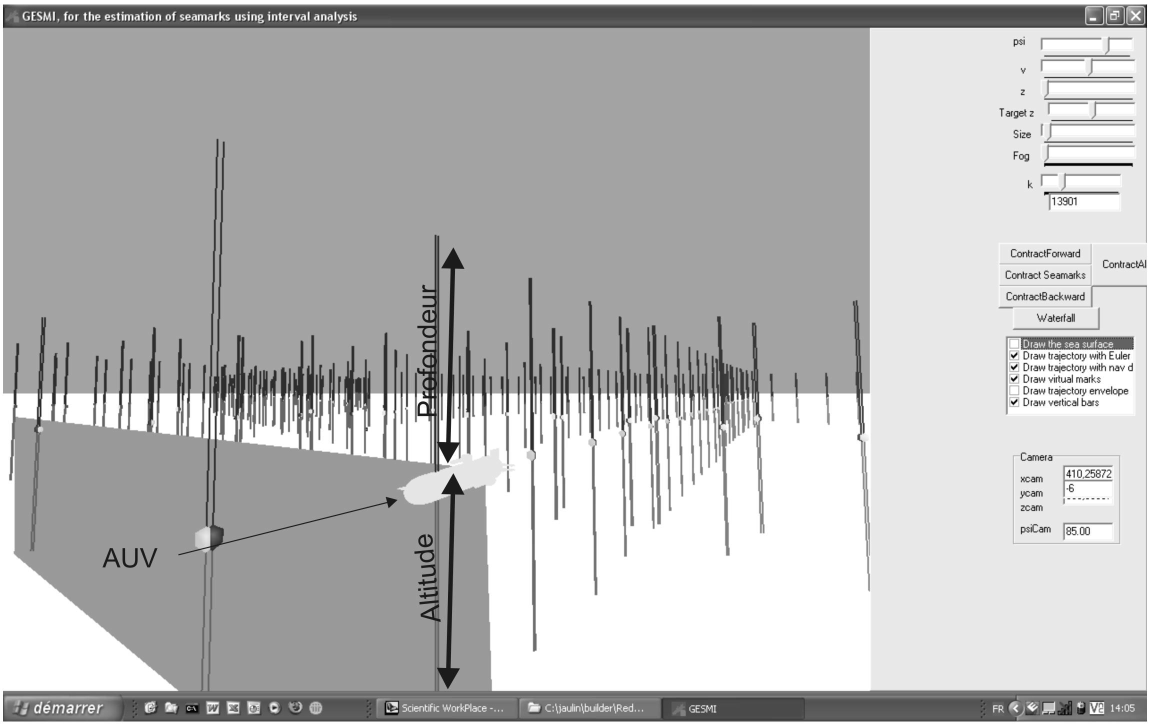
```
contractor fwd
  inter k=1:N-1;
    statequ(k)
  end
end
//---------------------------------------------------
contractor bwd
  inter k=1:N-1;
    statequ(N-k)
  end
end
```

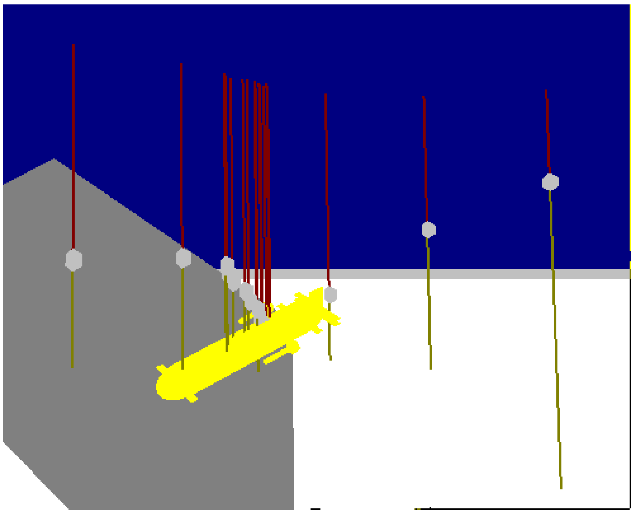
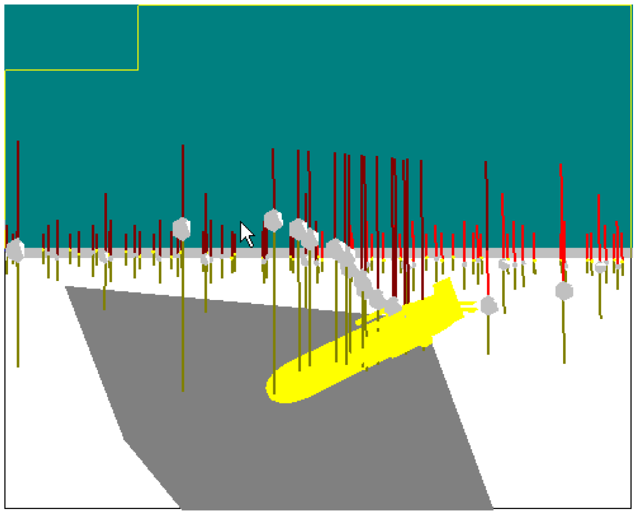
```
main
  p[1] :=read("gps_init.dat");
  v :=read("Quimper_v.dat");
  phi :=read("Quimper_phi.dat");
  theta :=read("Quimper_theta.dat");
  psi :=read("Quimper_psi.dat");
  init;
  fwd;
  bwd;
  column(p,px,1);
  column(p,py,2);
  print("--- Robot positions: ---");
  newplot("gesmi.dat");
  plot(px,py,color(rgb(1,1,1),rgb(0,0,0)));
end
```

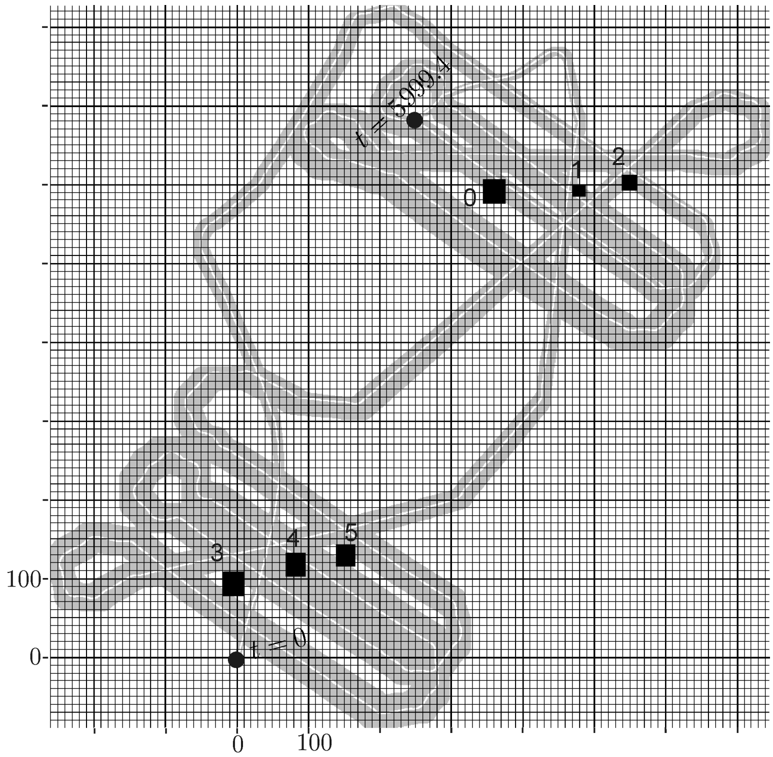
4.3 GESMI


GESMI, for the estimation of seamarks using interval analysis
psi
v
z
Target z
Size
Fog
k
98
ContractForward
Contract Seamarks
ContractBackward
ContractAl
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
ycam
zcam
psiCam
-6
-0,5
53.00
Bouge dans temps
Affiche la mer
AUV
Affiche la trajectoire estimée
Affiche l'enveloppe
Quart de plan du sonar latéral
Position et orientation de la caméra
démarrer
GESMI
12:43

```
% This file has been generated by a generator
% and will be used by GESMI to solve a SLAM problem
% Note : every line starting by a '%' is considered as a comment by GESMI.
% ----------------------------------------------
%
% This file contains
% 1) Some basic information about the scope of the sonar and the sampling time
% 2) The prior domains for the seamarks
% 3) The coordinates for some virtual marks
% 4) sensor data (angles, speeds, depth, altitude, position) with bounds
% 5) ping table : at the end of this file
%
% ----------------------------------------------
% maximal distance of the lateral sonar and error error interval [min,max]
  75.0    1.0
% Sampling time
0.1
% ----------------------------------------------
% Domain for seamarks detected using the software SonarPro
% ----------------------------------------------
%  Initial domains for the mines
% xmin    xmax    ymin     ymax    zmin    zmax
-10000   10000  -10000   10000     0      100
-10000   10000  -10000   10000     0      100
-10000   10000  -10000   10000     0      100
-10000   10000  -10000   10000     0      100
-10000   10000  -10000   10000     0      100
-10000   10000  -10000   10000     0      100
$
%----------------------------------------------
% Virtual marks (only for graphism, not used for computation)
%----------------------------------------------
% x              y               z     colorRed   ColorGreen   ColorBlue  LocalFrame
594.0533723    374.7293035      19       0.1         0          0          0       % seamark 0
599.6093723    484.5528643      19       0.1        0.1         0          0       % seamark 1
601.4613723    557.3573086      19       0.1         0         0.1         0       % seamark 2
94.01337232    -2.868189501     19        0         0.1         0          0       % seamark 3
119.9413723    84.75155672      20        0          0         0.1         0       % seamark 4
127.3493723    156.3280366      20        0          0          0          0       % seamark 5
0             0           0      1      0          0     1    % origin of the local frame
689.96        264.10      0      0      0          1     1    % final GPS
$
%----------------------------------------------
%----------------------------------------------
%t      phi        dphi         theta      dtheta          psi        dpsi          vx        dvx
0    -0.011505   0.0001745329252   0.033556   0.0001745329252   0.415613   0.0052679
0.1    -0.012272   0.0001745329252   0.034994   0.0001745329252   0.416284   0.00526
0.2    -0.012847   0.0001745329252   0.037583   0.0001745329252   0.417051   0.00526
0.3    -0.013422   0.0001745329252   0.040938   0.0001745329252   0.418202   0.00526
0.4    -0.014093   0.0001745329252   0.044773   0.0001745329252   0.419831   0.00526
0.5    -0.014285   0.0001745329252   0.0488   0.0001745329252   0.422228   0.0052679
0.6    -0.013998   0.0001745329252   0.052539   0.0001745329252   0.425488   0.00526
0.7    -0.012943   0.0001745329252   0.055703   0.0001745329252   0.42961   0.005267
0.8    -0.011121   0.0001745329252   0.0581   0.0001745329252   0.434308   0.0052679
0.9    -0.008725   0.0001745329252   0.059825   0.0001745329252   0.439773   0.00526
1    -0.005944   0.0001745329252   0.060496   0.0001745329252   0.445334   0.0052679
1.1    -0.003547   0.0001745329252   0.0604   0.0001745329252   0.450511   0.0052679
1.2    -0.001917   0.0001745329252   0.059729   0.0001745329252   0.455017   0.00526
1.3    -0.001438   0.0001745329252   0.058867   0.0001745329252   0.458564   0.00526
1.4    -0.002013   0.0001745329252   0.058387   0.0001745329252   0.461057   0.00526
1.5    -0.003356   0.0001745329252   0.058483   0.0001745329252   0.462303   0.00526
1.6    -0.005561   0.0001745329252   0.059346   0.0001745329252   0.462591   0.00526
1.7    -0.007958   0.0001745329252   0.060976   0.0001745329252   0.462112   0.00526
```


GESMI, for the estimation of seamarks using interval analysis
Profondeur
Altitude
AUV
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
ContractAll
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw trajectory with nav d
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
410,25872
ycam
-6
zcam
psiCam
85.00
démarrer
Scientific WorkPlace -...
C:\jaulin\builder\Red...
GESMI
FR
14:05

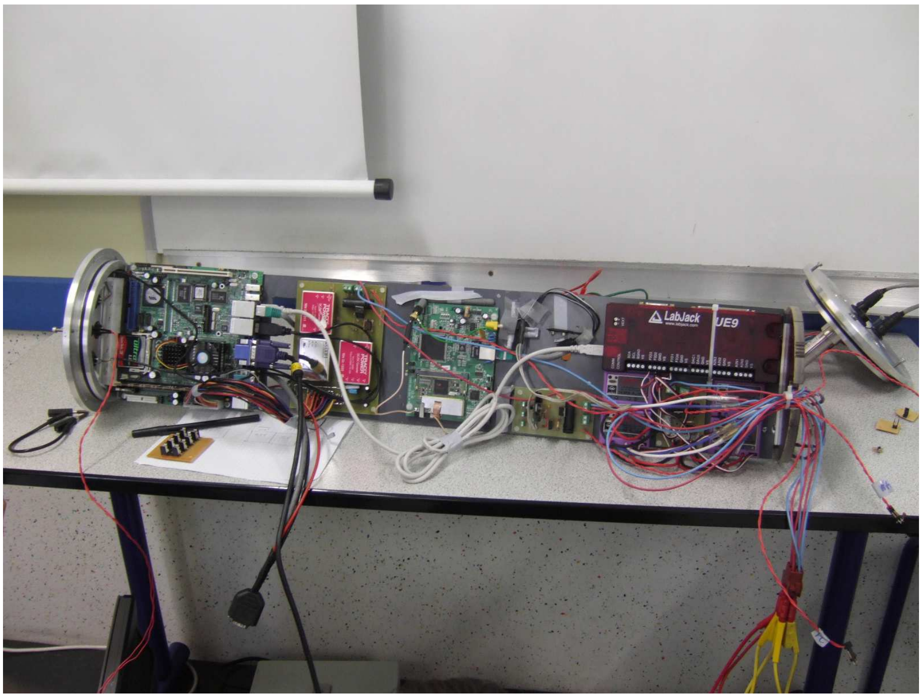



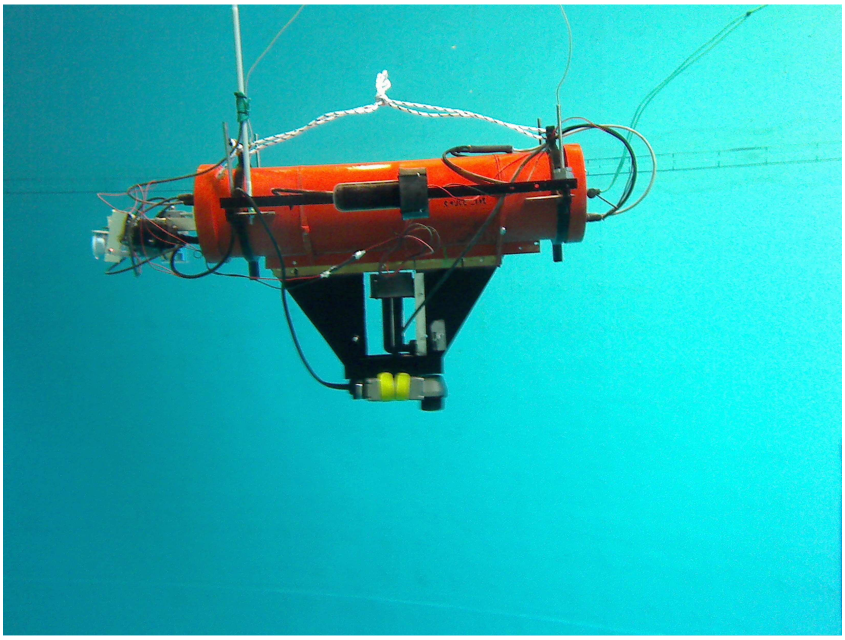

t = 5999.4
0
1
2
3
4
5
t = 0
100
0
0
100

5 Robustness

Montrer le logiciel de démo de Jan Sliwka

Portsmouth, July 12-15, 2007.


LabJack
UE9



Robot Sauc'isse dans une piscine