Distributed localization of a group of underwater robots

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Presentation available at http://youtu.be/qDtnTzzY9ms

1 Interval trajectories

A trajectory is a function $\mathbf{f} : \mathbb{R} \to \mathbb{R}^n$. For instance

$$\mathbf{f}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

is a trajectory.

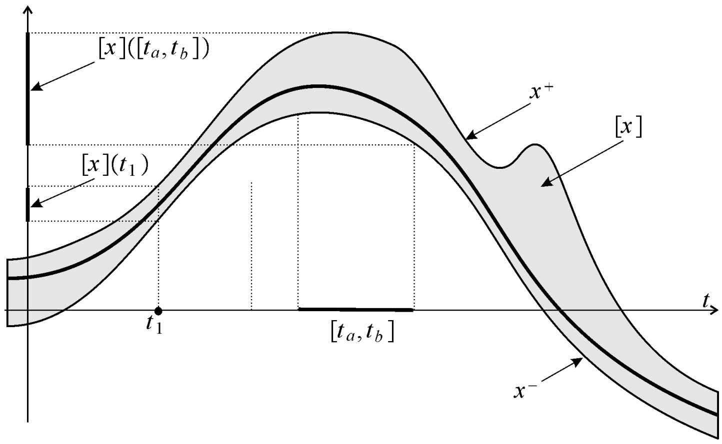
Order relation

$$\mathbf{f} \leq \mathbf{g} \Leftrightarrow \forall t, \forall i, f_i\left(t\right) \leq g_i\left(t\right).$$

We have

$$\begin{aligned}
\mathbf{h} &= \mathbf{f} \wedge \mathbf{g} \Leftrightarrow \forall t, \forall i, h_i(t) = \min\left(f_i(t), g_i(t)\right), \\
\mathbf{h} &= \mathbf{f} \vee \mathbf{g} \Leftrightarrow \forall t, \forall i, h_i(t) = \max\left(f_i(t), g_i(t)\right).
\end{aligned}$$

The set of trajectories is a lattice. Interval of trajectories (tubes) can be defined.



2 Tube arithmetics

If $[x]$ and $[y]$ are two scalar tubes, we have

$$[z] = [x] + [y] \Rightarrow [z](t) = [x](t) + [y](t) \quad \text{(sum)}$$
$$[z] = \text{shift}_a([x]) \Rightarrow [z](t) = [x](t+a) \quad \text{(shift)}$$
$$[z] = [x] \circ [y] \Rightarrow [z](t) = [x]([y](t)) \quad \text{(compositi}$$
$$[z] = \int [x] \Rightarrow [z](t) = \left[\int_0^t x^-(\tau)\,d\tau, \int_0^t x^+(\tau)\,d\tau\right] \quad \text{(integral)}$$

3 Tube contractors

Example 1. Consider $x(t) \in [x](t)$ with the constraint

$$\forall t,\ x(t) = x(t+1)$$

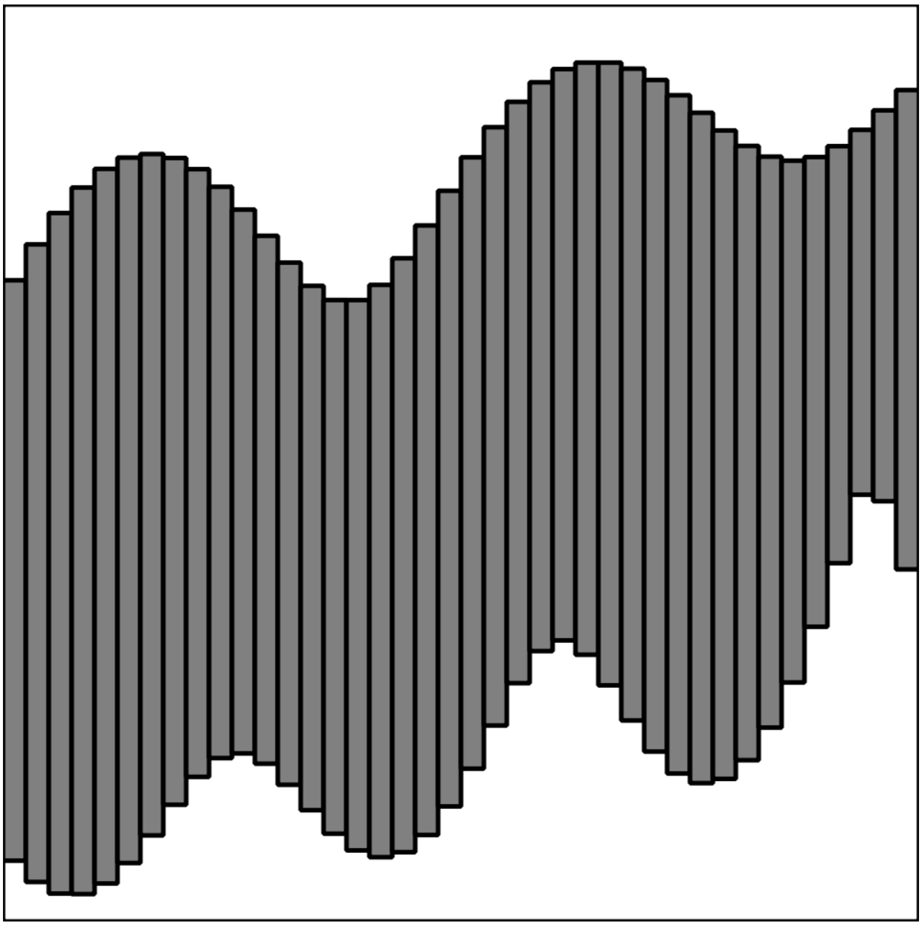
Contract the tube $[x](t)$.

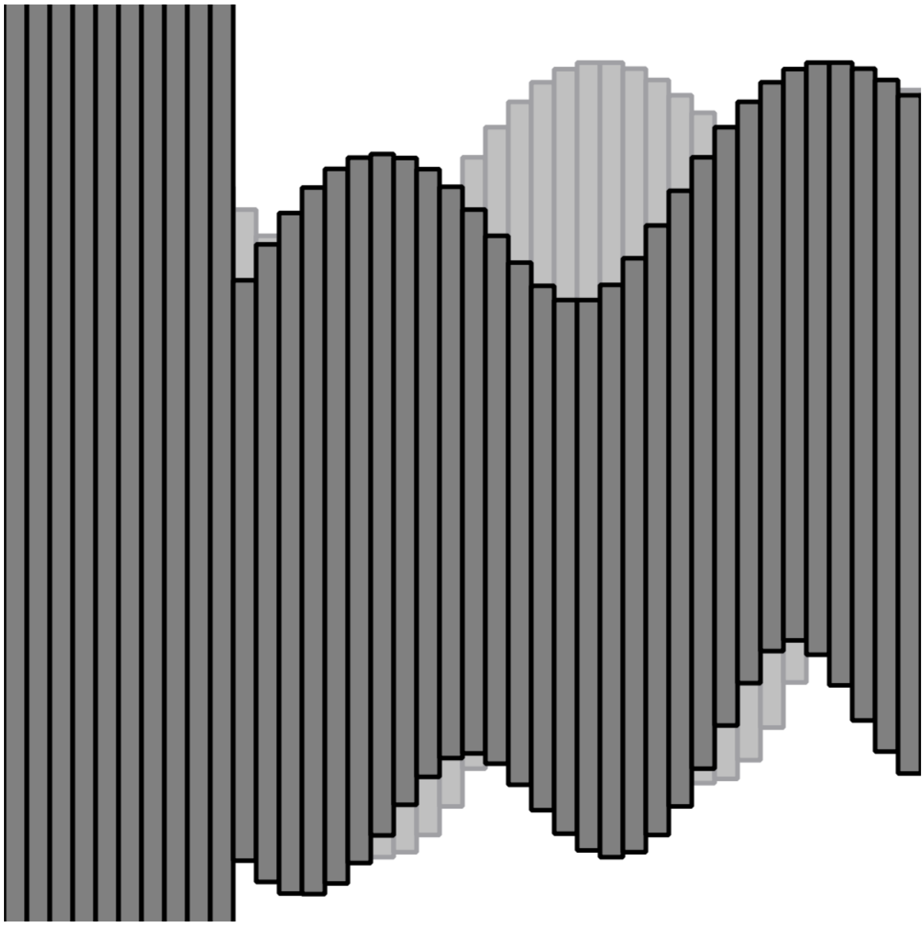
We first decompose into primitive trajectory constraints

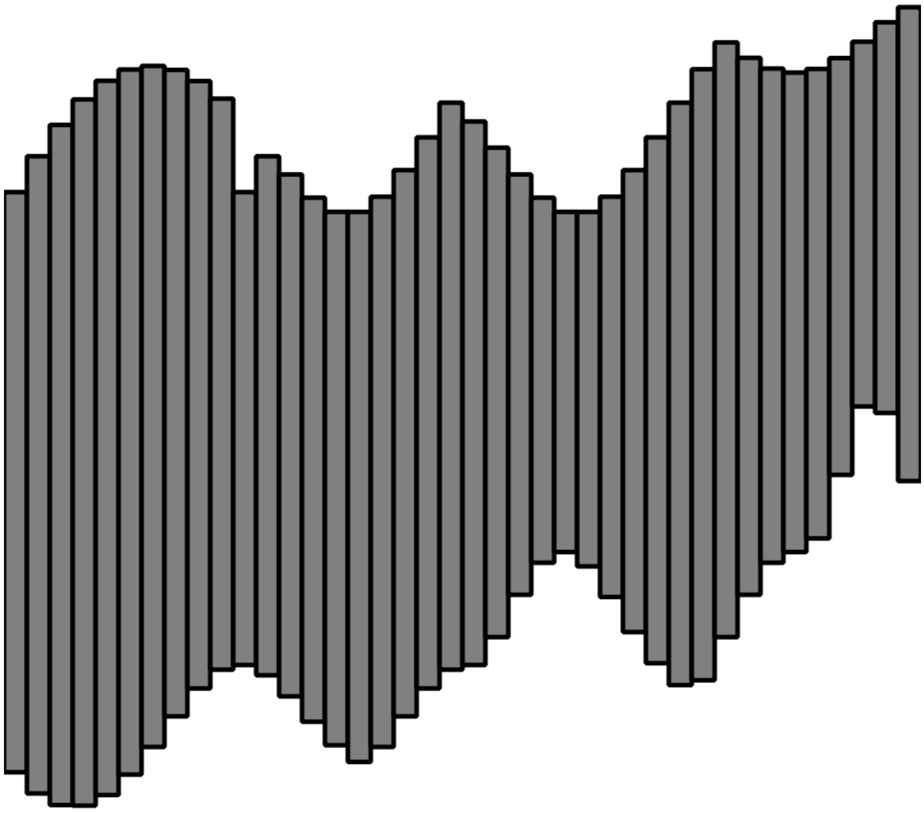
$$\begin{aligned} x(t) &= a(t+1) \\ x(t) &= a(t) . \end{aligned}$$

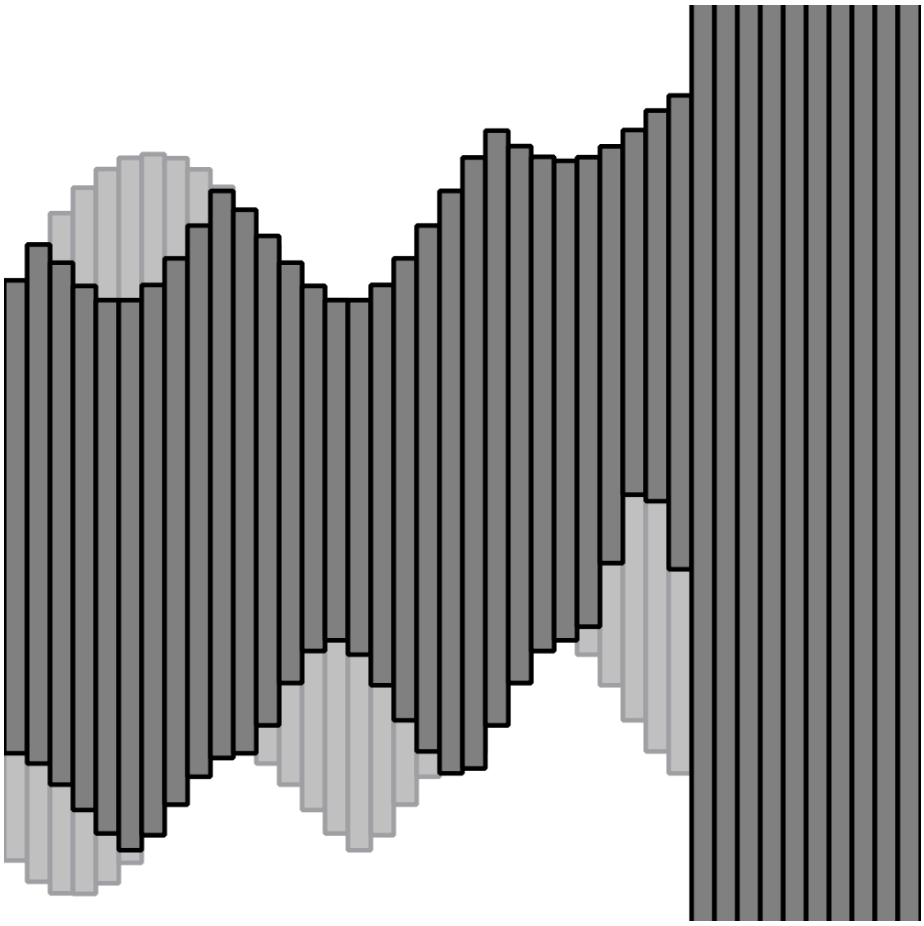
Contractors

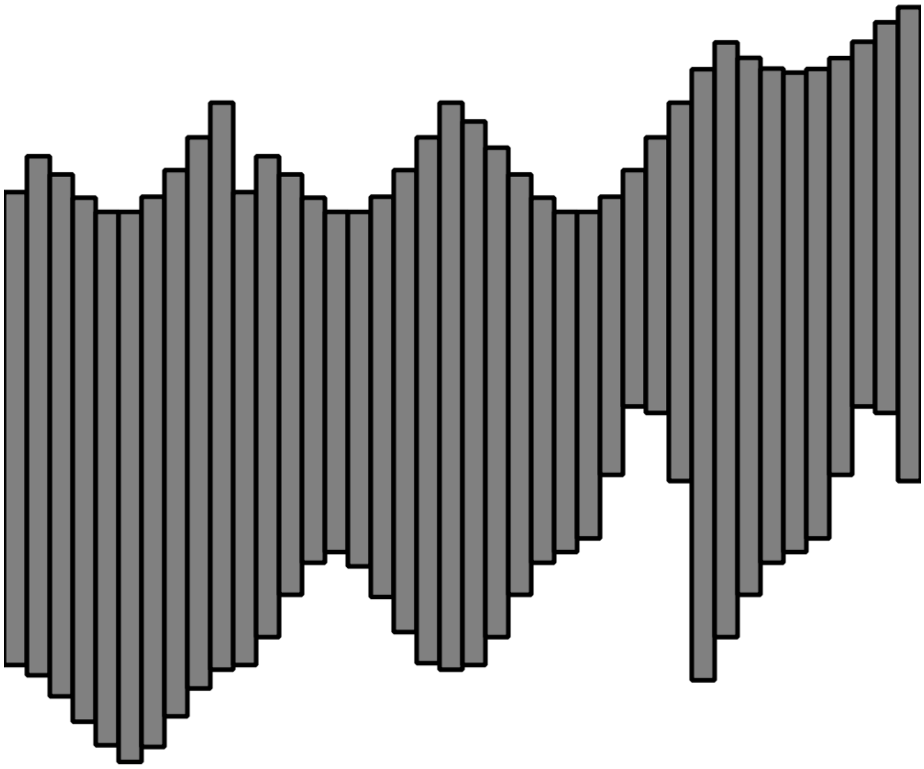
$$
\begin{aligned}
[x](t) &:= [x](t) \cap [a](t+1) \\
[a](t) &:= [a](t) \cap [x](t-1) \\
[x](t) &:= [x](t) \cap [a](t) \\
[a](t) &:= [a](t) \cap [x](t)
\end{aligned}
$$

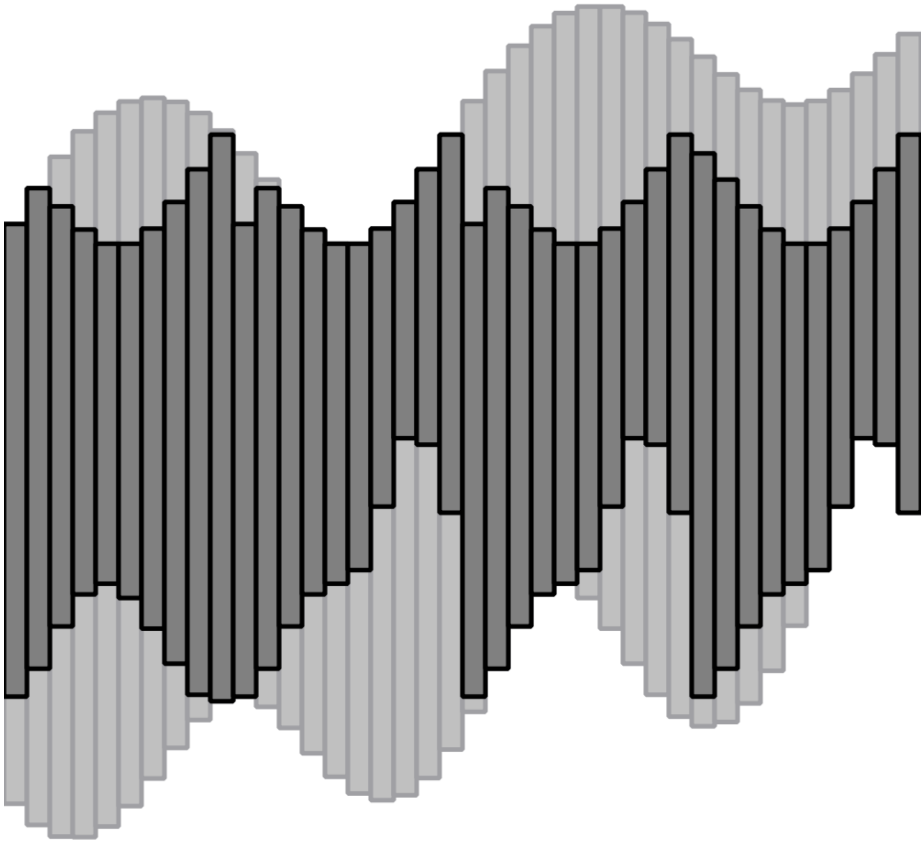


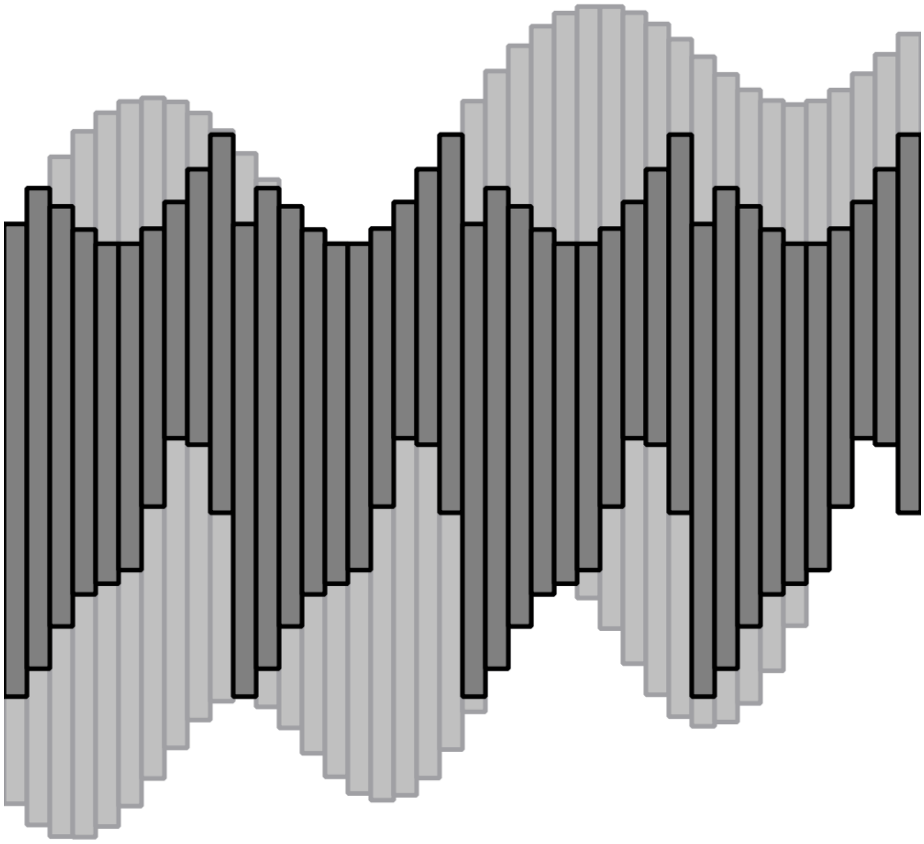












Example 2. Consider for instance the differential constraint

$$\begin{array}{rcl} \dot{x}(t) & = & x(t+1)\cdot u(t), \\ x(t) & \in & [x](t), \dot{x}(t) \in [\dot{x}](t), u(t) \in [u](t) \end{array}$$

We decompose as follows

$$\left\{ \begin{array}{rcl} x(t) & = & x(0) + \int_0^t y(\tau)\, d\tau \\ y(t) & = & a(t)\cdot u(t). \\ a(t) & = & x(t+1) \end{array} \right.$$

Possible contractors are

$$\left\{\begin{array}{lcl}
[x](t) & = & [x](t)\cap\left([x](0)+\int_0^t [y](\tau)\,d\tau\right)\\
[y](t) & = & [y](t)\cap[a](t)\cdot[u](t)\\
[u](t) & = & [u](t)\cap\frac{[y](t)}{[a](t)}\\
[a](t) & = & [a](t)\cap\frac{[y](t)}{[u](t)}\\
[a](t) & = & [a](t)\cap[x](t+1)\\
[x](t) & = & [x](t)\cap[a](t-1)
\end{array}\right.$$

4 Time-space estimation

Classical state estimation

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) & t \in \mathbb{R} \\ \mathbf{0} & = & \mathbf{g}(\mathbf{x}(t), t) & t \in \mathbb{T} \subset \mathbb{R}. \end{cases}$$

Space constraint $\mathbf{g}(\mathbf{x}(t), t) = 0$.

Example.

$$\begin{cases} \dot{x}_1 = x_3 \cos x_4 \\ \dot{x}_2 = x_3 \cos x_4 \\ \dot{x}_3 = u_1 \\ \dot{x}_4 = u_2 \\ (x_1(5) - 1)^2 + (x_2(5) - 2)^2 - 4 = 0 \\ (x_1(7) - 1)^2 + (x_2(7) - 2)^2 - 9 = 0 \end{cases}$$

With time-space constraints

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) & t \in \mathbb{R} \\ \mathbf{0} & = & \mathbf{g}(\mathbf{x}(t), \mathbf{x}(t'), t, t') & (t, t') \in \mathbb{T} \subset \mathbb{R} \times \mathbb{R}. \end{cases}$$

Example. An ultrasonic underwater robot with state

$$\mathbf{x} = (x_1, x_2, \dots) = (x, y, \theta, v, \dots)$$

At time t the robot emits an onmidirectional sound. At time t' it receives it

$$\left(x_1 - x_1^{'}\right)^2 + \left(x_2 - x_2^{'}\right)^2 - c\left(t - t'\right)^2 = 0.$$

5 Mass spring problem

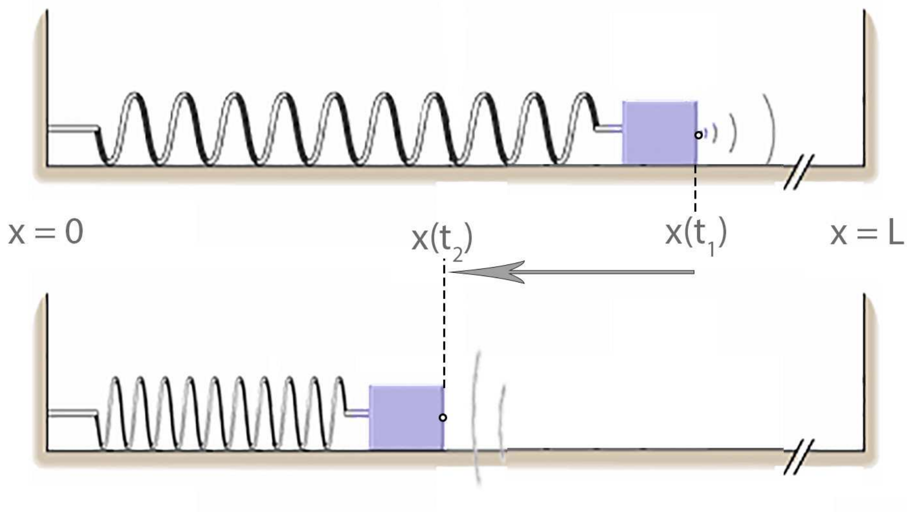
The mass spring satisfies

$$\ddot{x} + \dot{x} + x - x^3 = 0$$

i.e.

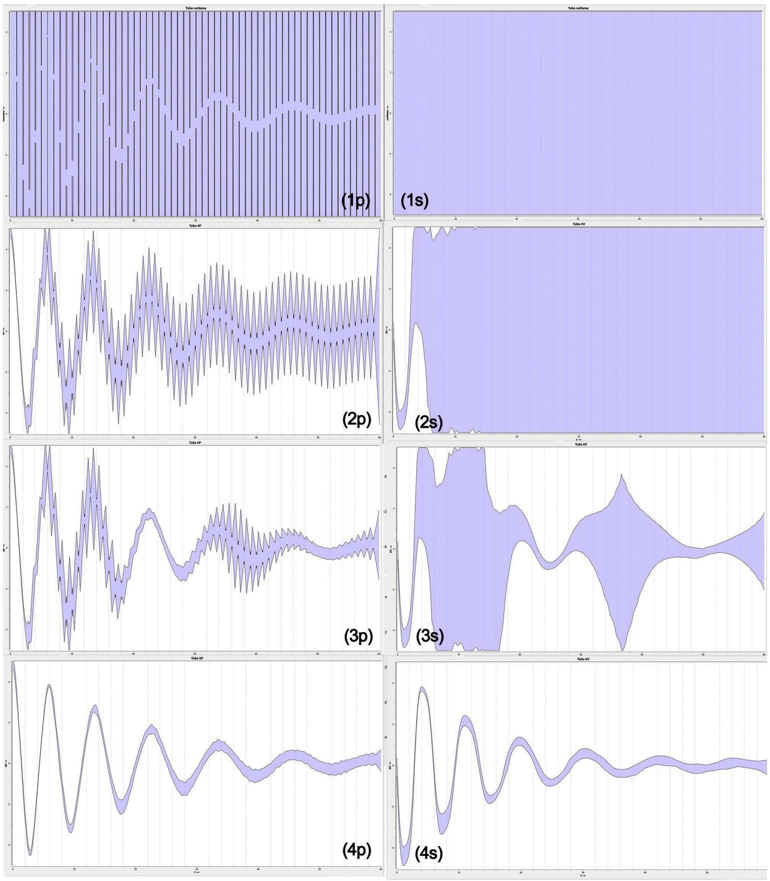
$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_2 - x_1 + x_1^3 \end{cases}$$

The initial state is unknown.



x = 0
x(t2)
x(t1)
x = L

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_2 - x_1 + x_1^3 \\ L - x_1(t_1) + L - x_1(t_2) = c(t_2 - t_1) \,. \end{cases}$$


(1p)
(1s)
(2p)
(2s)
(3p)
(3s)
(4p)
(4s)

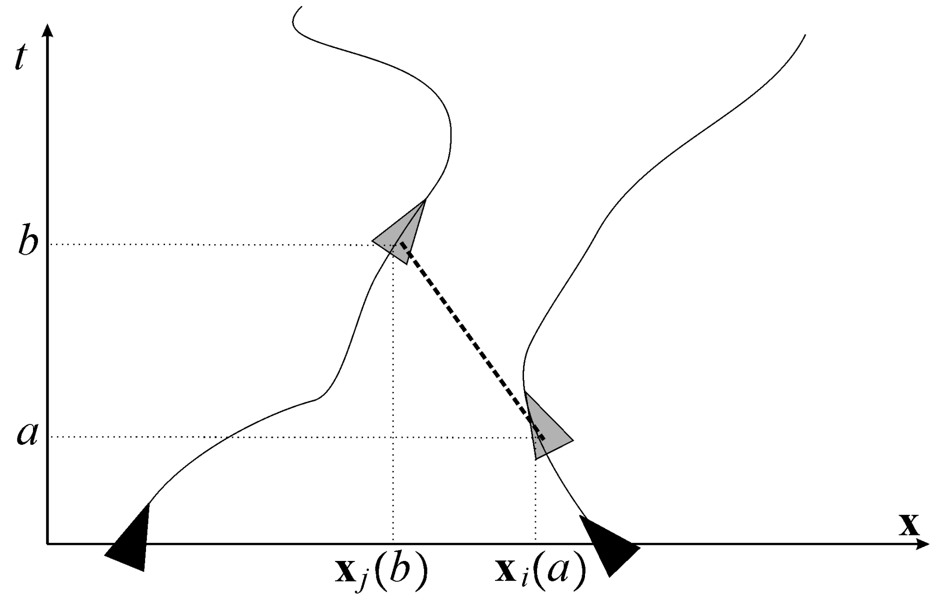
6 Swarm localization

Consider n robots $\mathcal{R}_1, \dots, \mathcal{R}_n$ described by

$$\dot{\mathbf{x}}_i = \mathbf{f}\left(\mathbf{x}_i, \mathbf{u}_i\right), \mathbf{u}_i \in \left[\mathbf{u}_i\right].$$

Omnidirectional sounds are emitted and received.

A *ping* is a 4-uple (a, b, i, j) where a is the emission time, b is the reception time, i is the emitting robot and j the receiver.



t
b
a
x
$\mathbf{x}_j(b)$
$\mathbf{x}_i(a)$

With the time space constraint

$$\dot{\mathbf{x}}_i = \mathbf{f}\left(\mathbf{x}_i, \mathbf{u}_i\right), \mathbf{u}_i \in \left[\mathbf{u}_i\right].$$
$$g\left(\mathbf{x}_{i(k)}\left(a\left(k\right)\right), \mathbf{x}_{j(k)}\left(b\left(k\right)\right), a\left(k\right), b\left(k\right)\right) = 0$$

where

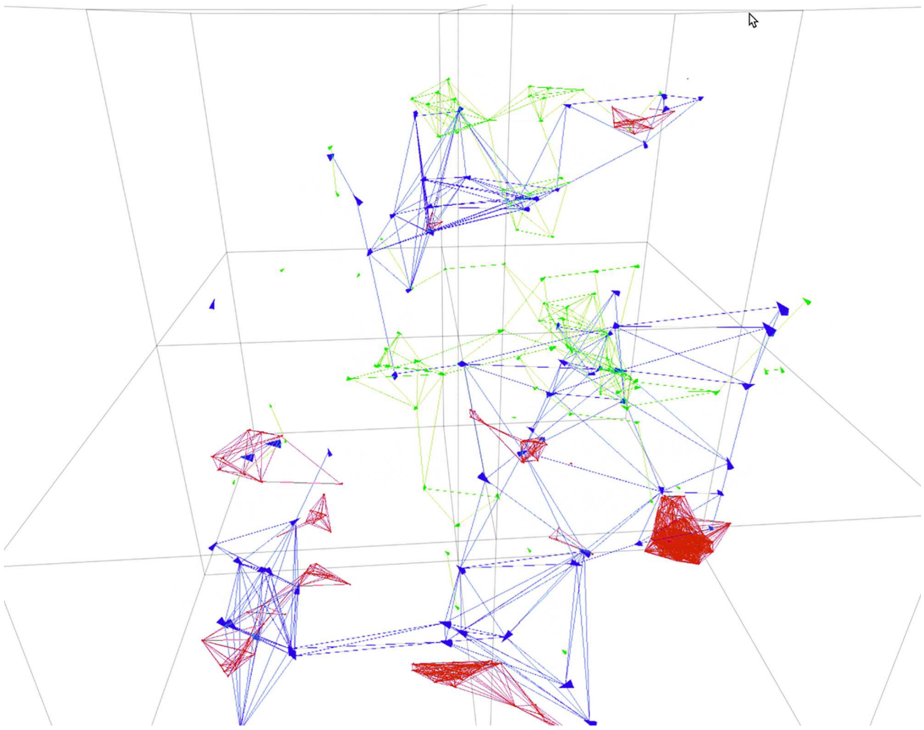
$$g\left(\mathbf{x}_i, \mathbf{x}_j, a, b\right) = \|x_1 - x_2\| - c\left(b - a\right).$$

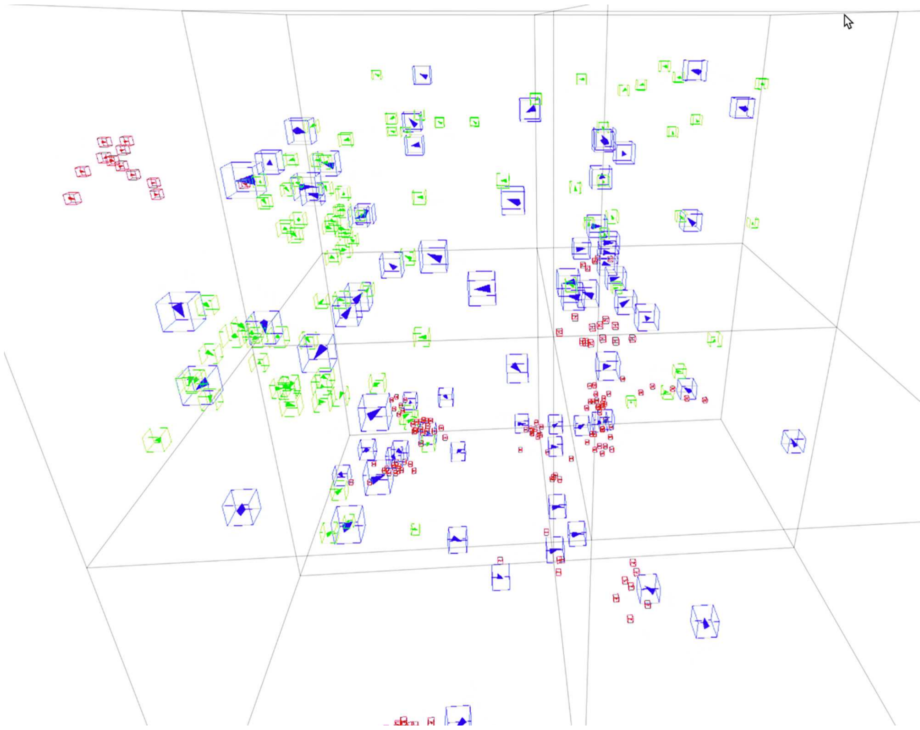
Clocks are uncertain. We only have measurements $\tilde{a}(k), \tilde{b}(k)$ of $a(k), b(k)$ thanks to clocks h_i. Thus

$$\begin{array}{l}
\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i), \mathbf{u}_i \in [\mathbf{u}_i]. \\
g\left(\mathbf{x}_{i(k)}(a(k)), \mathbf{x}_{j(k)}(b(k)), a(k), b(k)\right) = 0 \\
\tilde{a}(k) = h_{i(k)}(a(k)) \\
\tilde{b}(k) = h_{j(k)}(b(k))
\end{array}$$

The drift of the clocks is bounded

$$\begin{aligned}
&\dot{\mathbf{x}}_i = \mathbf{f}\left(\mathbf{x}_i, \mathbf{u}_i\right), \mathbf{u}_i \in \left[\mathbf{u}_i\right].\\
&g\left(\mathbf{x}_{i(k)}\left(a\left(k\right)\right), \mathbf{x}_{j(k)}\left(b\left(k\right)\right), a\left(k\right), b\left(k\right)\right) = 0\\
&\tilde{a}\left(k\right) = h_{i(k)}\left(a\left(k\right)\right)\\
&\tilde{b}\left(k\right) = h_{j(k)}\left(b\left(k\right)\right)\\
&\dot{h}_i = 1 + n_h,\ n_h \in \left[n_h\right]
\end{aligned}$$





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