Toward dynamic epistemic robotics
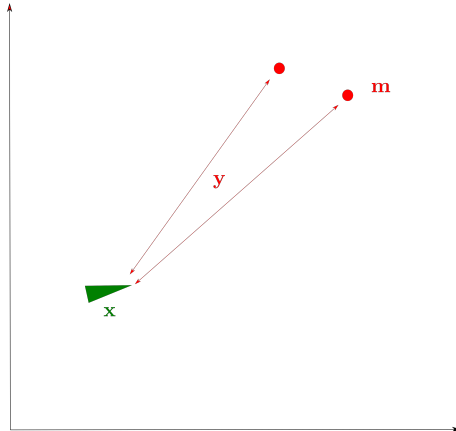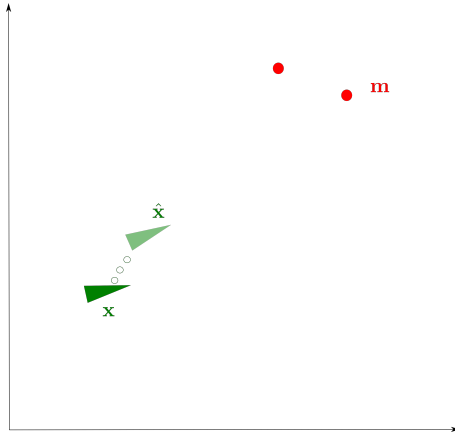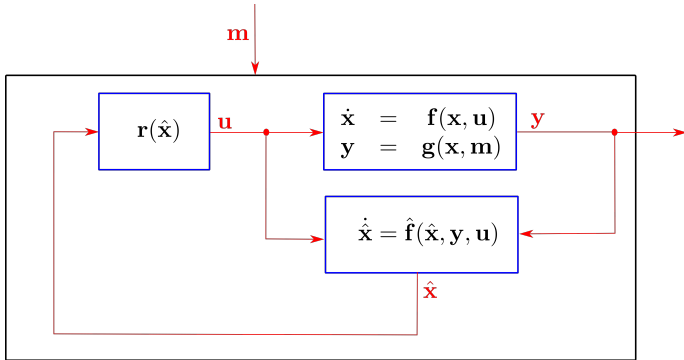
Epistemic observer

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
\mathbf{y} &= \mathbf{g}(\mathbf{x}, \mathbf{m})
\end{aligned}
$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$
$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{m})$$
$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{f}}(\hat{\mathbf{x}}, \mathbf{y}, \mathbf{u})$$
$$\mathbf{u} = \mathbf{r}(\hat{\mathbf{x}})$$

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{r}(\hat{\mathbf{x}})) = \mathbf{f}(\mathbf{x}, \hat{\mathbf{x}})$$
$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{f}}(\hat{\mathbf{x}}, \mathbf{g}(\mathbf{x}, \mathbf{m}), \mathbf{r}(\hat{\mathbf{x}})) = \hat{\mathbf{f}}(\mathbf{x}, \hat{\mathbf{x}}, \mathbf{m})$$
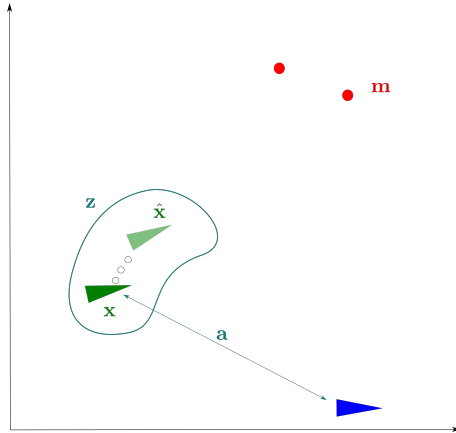
If we set $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$, we get

$$\dot{\mathbf{z}} = \varphi(\mathbf{z}, \mathbf{m})$$

Assume that we can observe the motion of the robot

$$
\begin{aligned}
\dot{\mathbf{z}} &= \varphi(\mathbf{z}, \mathbf{m}) \\
\mathbf{a} &= \psi(\mathbf{x})
\end{aligned}
$$

where $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$.

We can build an observer for $\mathbf{z}$:

$$
\begin{aligned}
\dot{\mathbf{z}} &= \varphi(\mathbf{z}, \mathbf{m}) \\
\mathbf{a} &= \psi(\mathbf{x}) \\
\dot{\hat{\mathbf{z}}} &= \hat{\varphi}(\mathbf{a}, \hat{\mathbf{z}})
\end{aligned}
$$
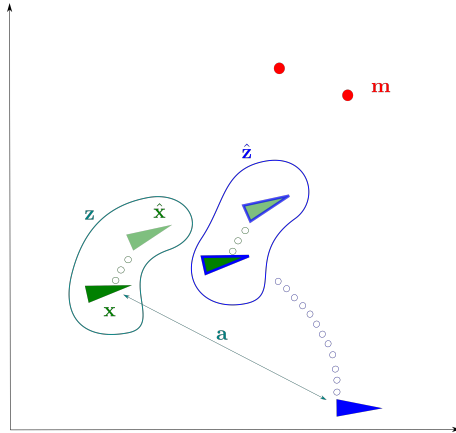
Cyber attack

Use **m** as an input to control **x**.

The vector **m** may correspond to the position of a landmark (e.g., a satellite).

We want to build a controller:

$$\mathbf{m} = \rho(\hat{\mathbf{z}}, t)$$

Distributed knowledge

A thinks that B thinks that A is here

B thinks that A thinks it is here

A thinks it is here

A is here

B thinks it is here

B is here

$\mathbf{x}_A$

$\mathbf{x}_B$
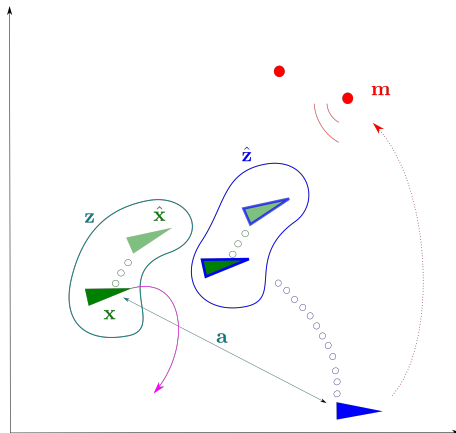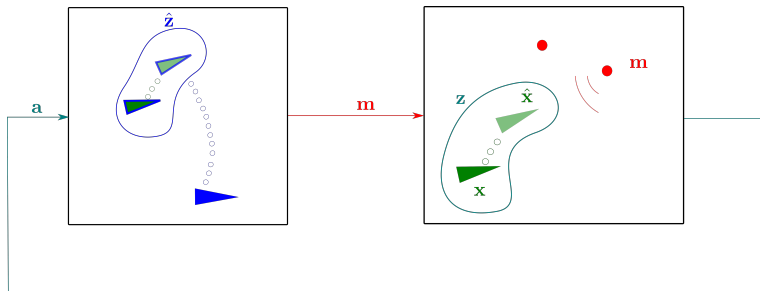
B thinks that A is here
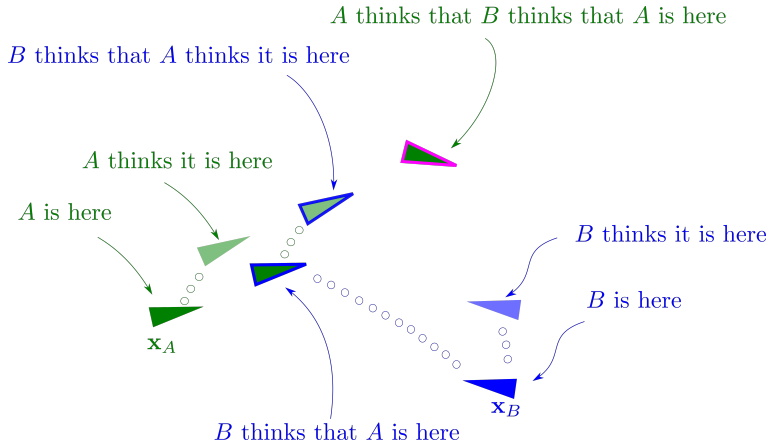
Communication

A communication is a measure of the knowledge.

A thinks that B thinks that A is here or here

B thinks that A thinks it is here
or here

A thinks it is here

A is here

$\mathbf{x}_A$

B is here
and everybody knows it

$\mathbf{x}_B$

B thinks that A is here or here

A thinks that *B* thinks that *A* is here or here

B thinks that *A* thinks it is here
or here

A thinks it is here

A is here

$\mathbf{x}_A$

I see you

B is here
and everybody knows it

$\mathbf{x}_B$

B thinks that *A* is here or here

A thinks that B thinks that A is here or here

B thinks that A thinks it is here

A thinks it is here

A is here

B is here
and everybody knows it

$\mathbf{x}_A$

I see you

B thinks that A is here

$\mathbf{x}_B$

A thinks that B thinks that A is here or here

B thinks that A thinks it is here

A thinks it is here

A is here

B is here
and everybody knows it

$\mathbf{x}_A$

B thinks that A is here

OK

$\mathbf{x}_B$

A thinks that B thinks that A is here

B thinks that A thinks it is here

A thinks it is here

A is here

$\mathbf{x}_A$

B is here
and everybody knows it

OK

$\mathbf{x}_B$

B thinks that A is here