GUARANTEED LOCALIZATION OF AN UNDERWATER ROBOT USING BATHYMETRY DATA AND INTERVAL ANALYSIS

Guaranteed localization of an underwater robot using bathymetry data and interval analysis

- The problem
- Interval analysis
- Contractors
- Application to our problem
- Conclusion

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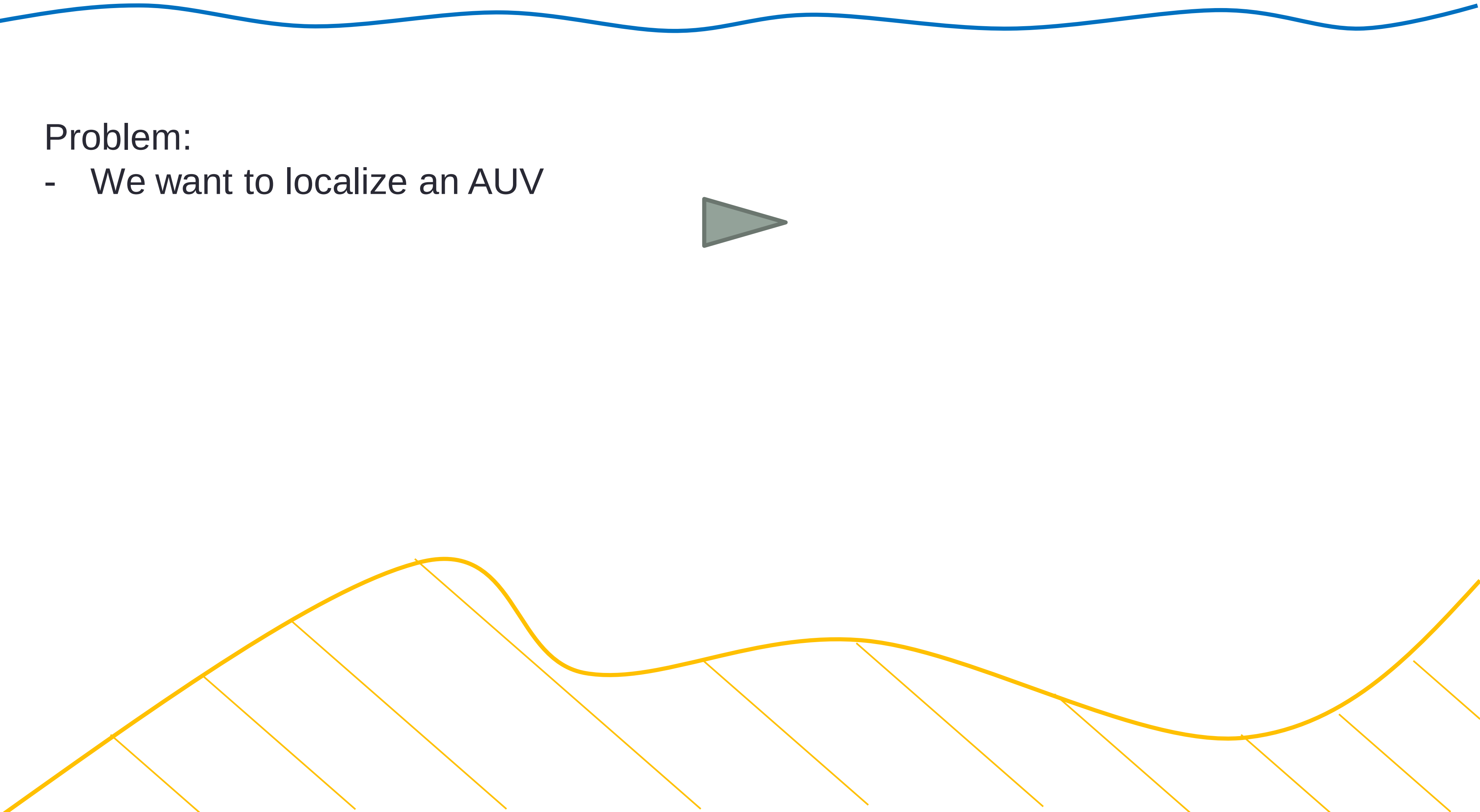
Problem:

- We want to localize an AUV



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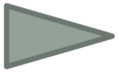
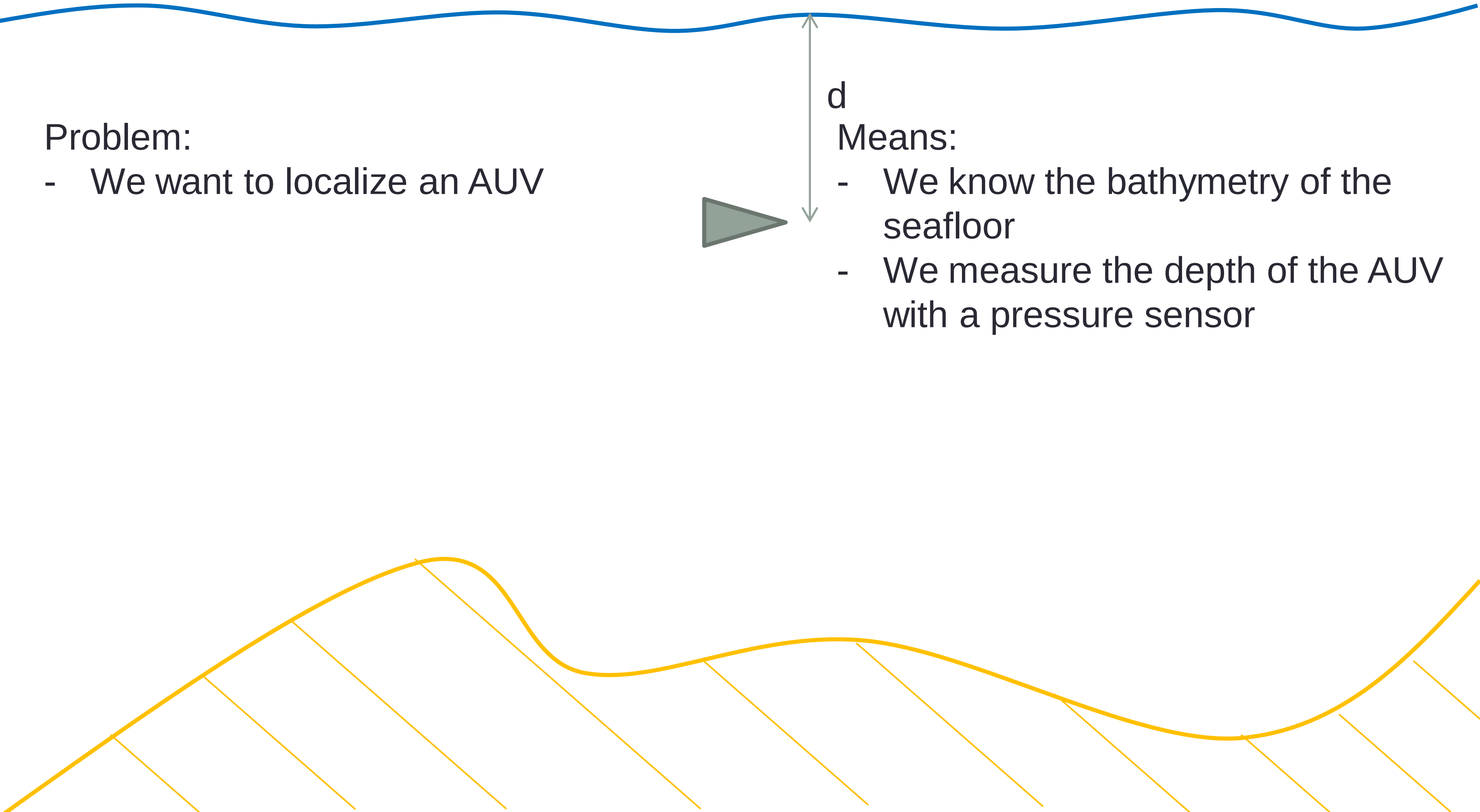


Means:

- We know the bathymetry of the seafloor



Problem:

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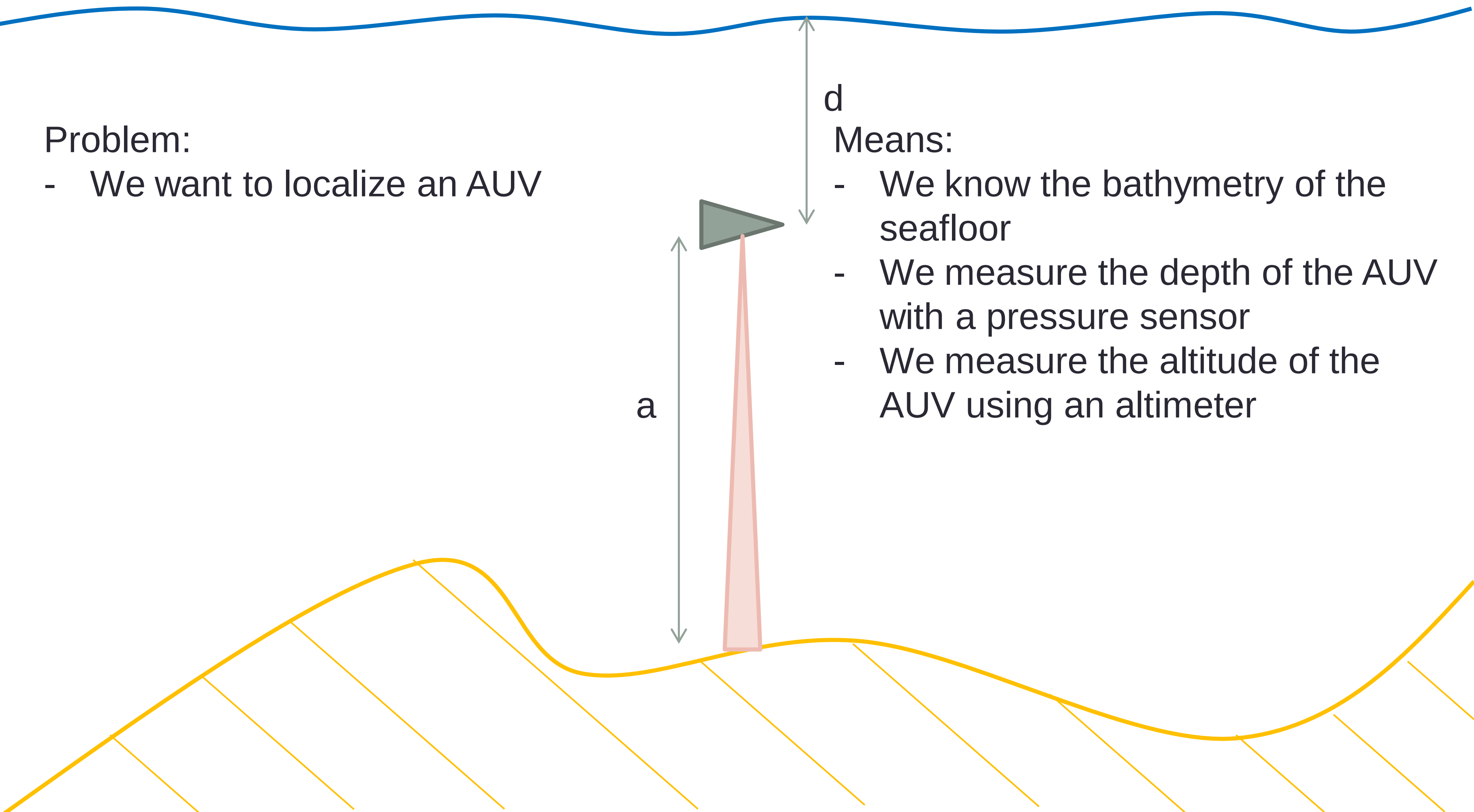
Means:

- We know the bathymetry of the seafloor
- We measure the depth of the AUV with a pressure sensor



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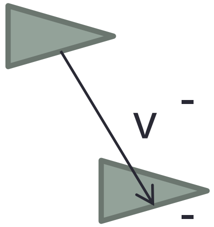
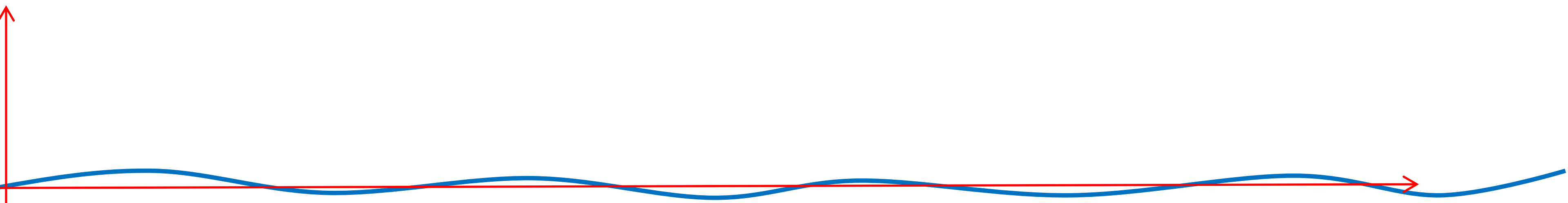


Means:

- We know the bathymetry of the seafloor
- We measure the depth of the AUV with a pressure sensor
- We measure the altitude of the AUV using an altimeter
- We measure the speed of the AUV with an IMU/DVL coupling



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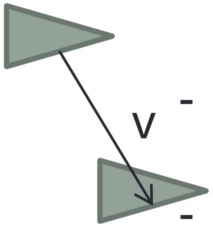

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In our presentation, z axis points towards the top, and 0 is at sea level

State equations for the AUV

Evolution model:

$$\dot{p} = v$$

p is the position of the AUV

v is its speed

Observation model:

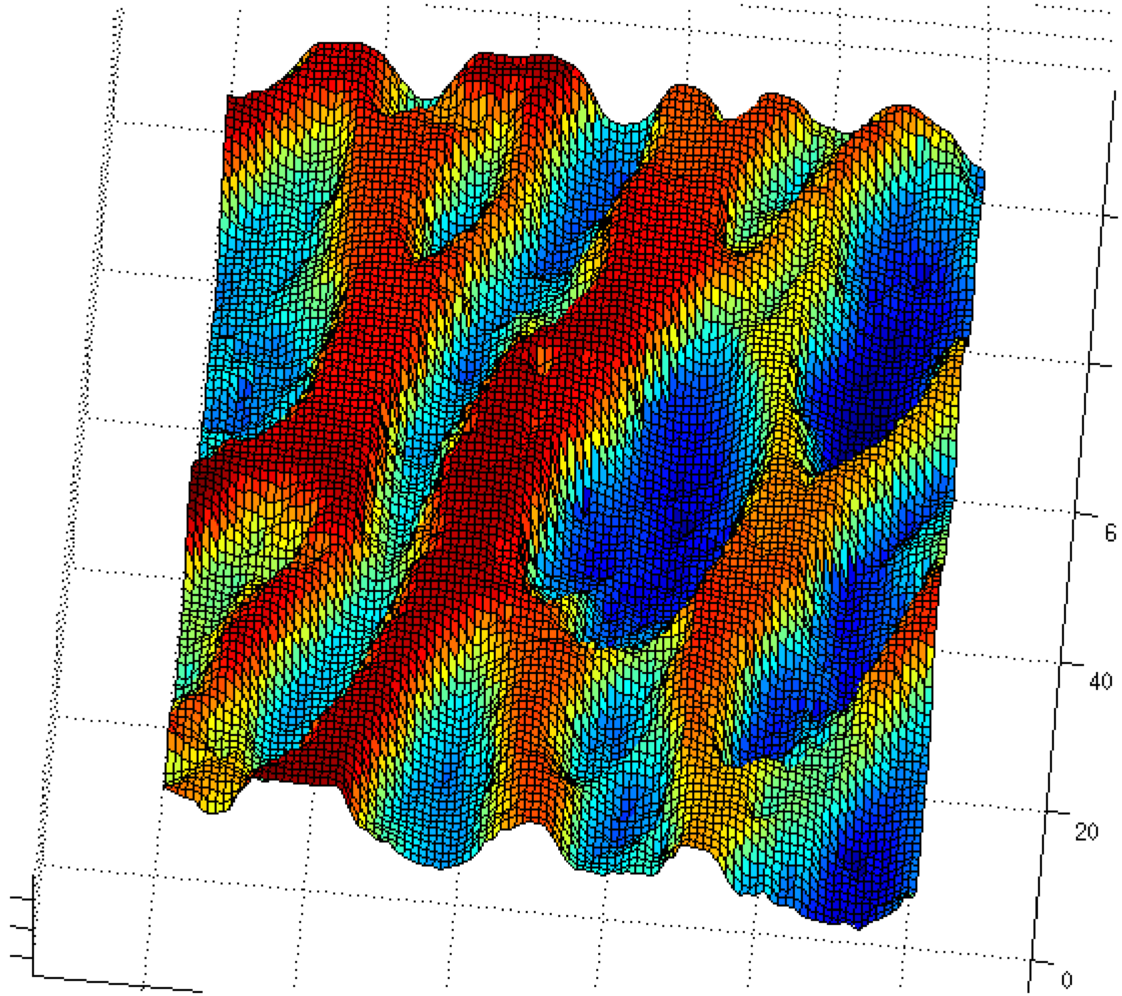
$$z = d$$

d is its depth as measured by the pressure sensor

$$z = a + \boldsymbol{M}(x, y)$$

- Problem: we don’t have an expression for the function

$\boldsymbol{M}(x, y)$



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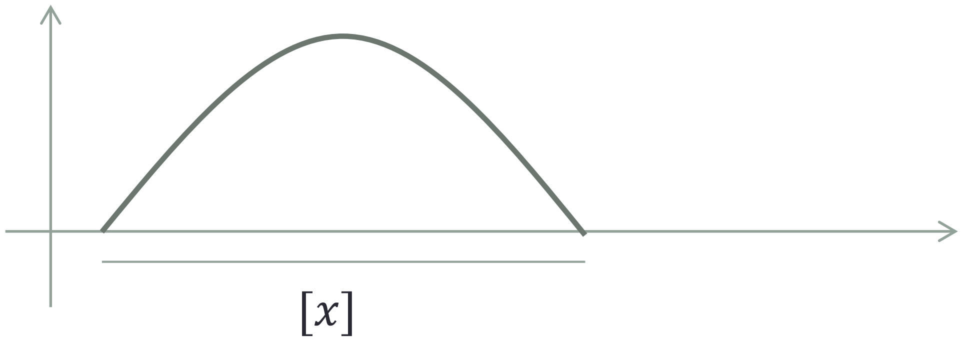
- An interval $[x] = [\underline{x}, \overline{x}] \in IR^n$ is a closed, connected subset of R^n
- Arithmetic operations such as +,-,/ are defined on intervals
- Elementary functions such as sin, tan, exp can be extended to intervals
- We use intervals to treat uncertainties in a non-probabilistic way



- Intervals enables us to easily approximate sets in a guaranteed manner

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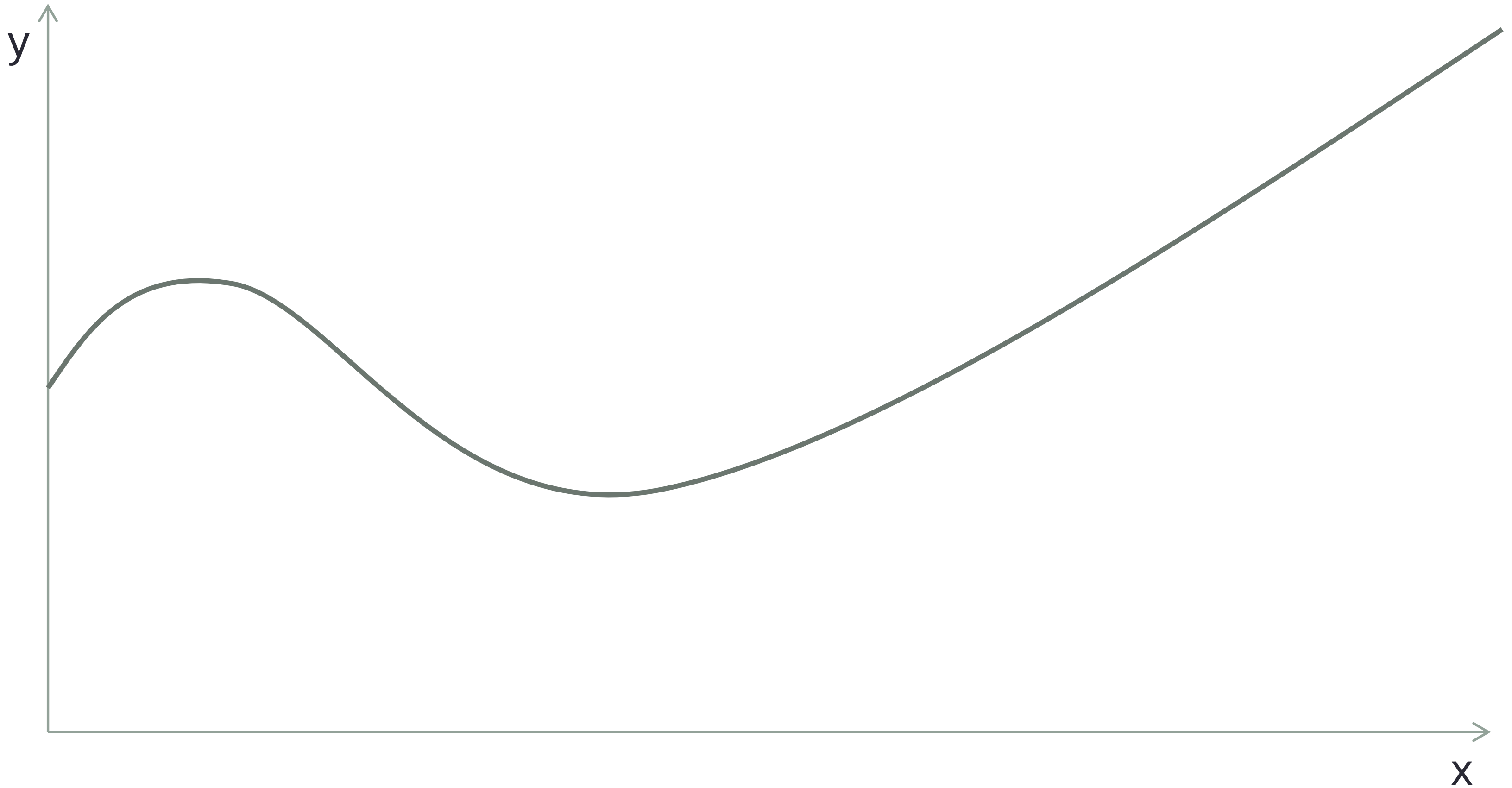
- The epigraph **E** of a function *f(x)*:$R^n \to R$ is defined as:

$$E = \{x \in R^n, y \in R | y - f(x) \leq 0\}$$

- Example:



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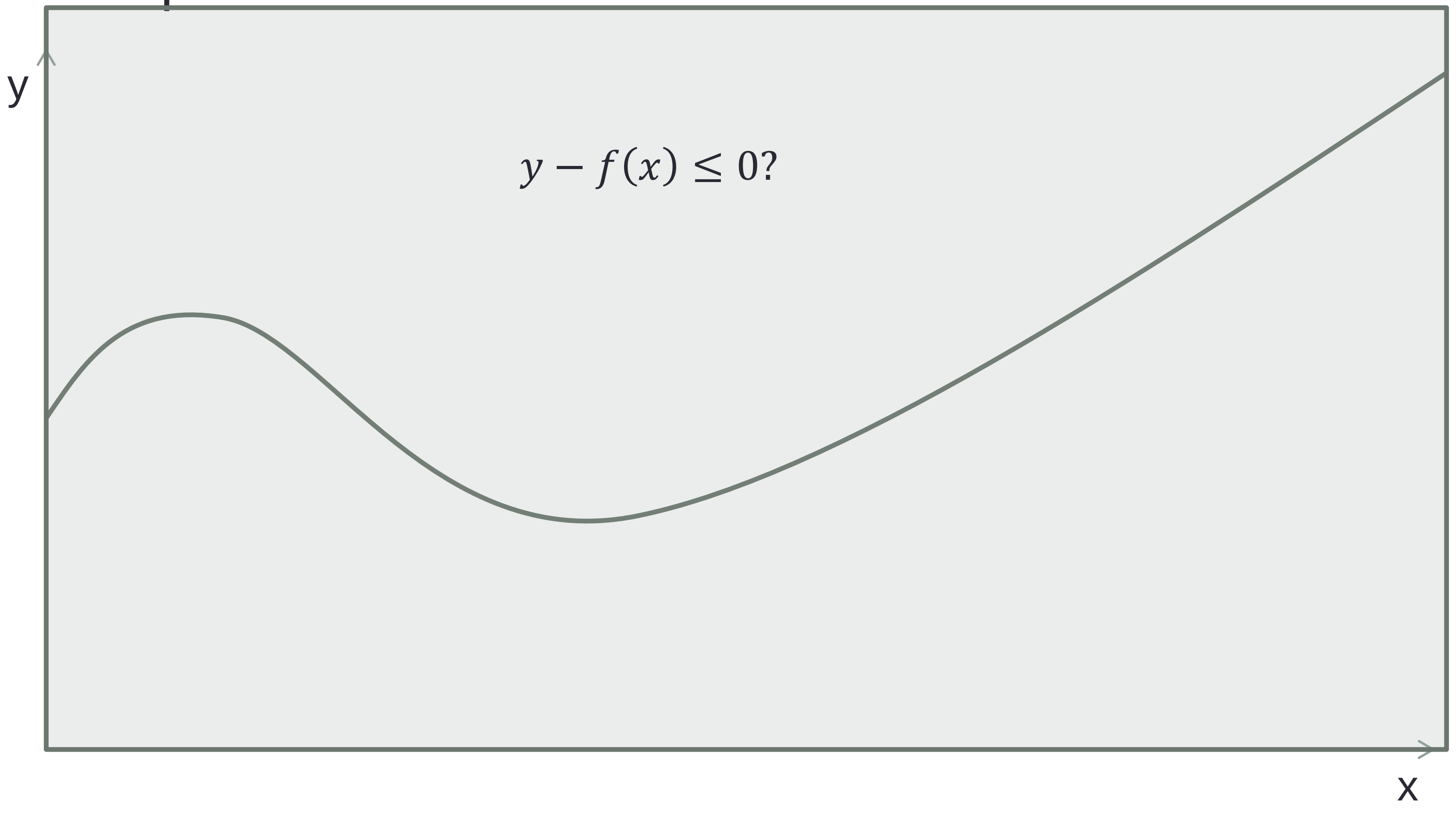

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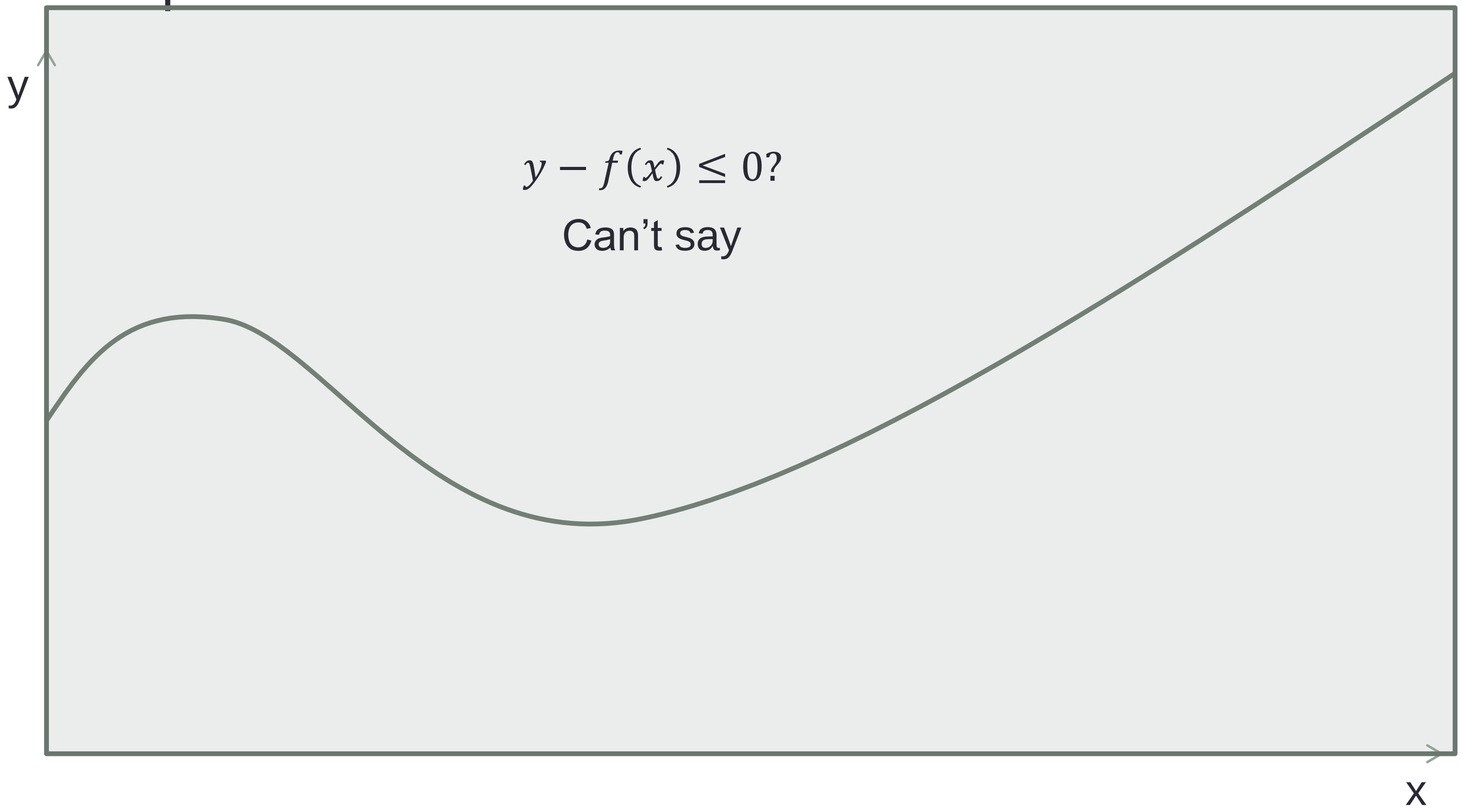

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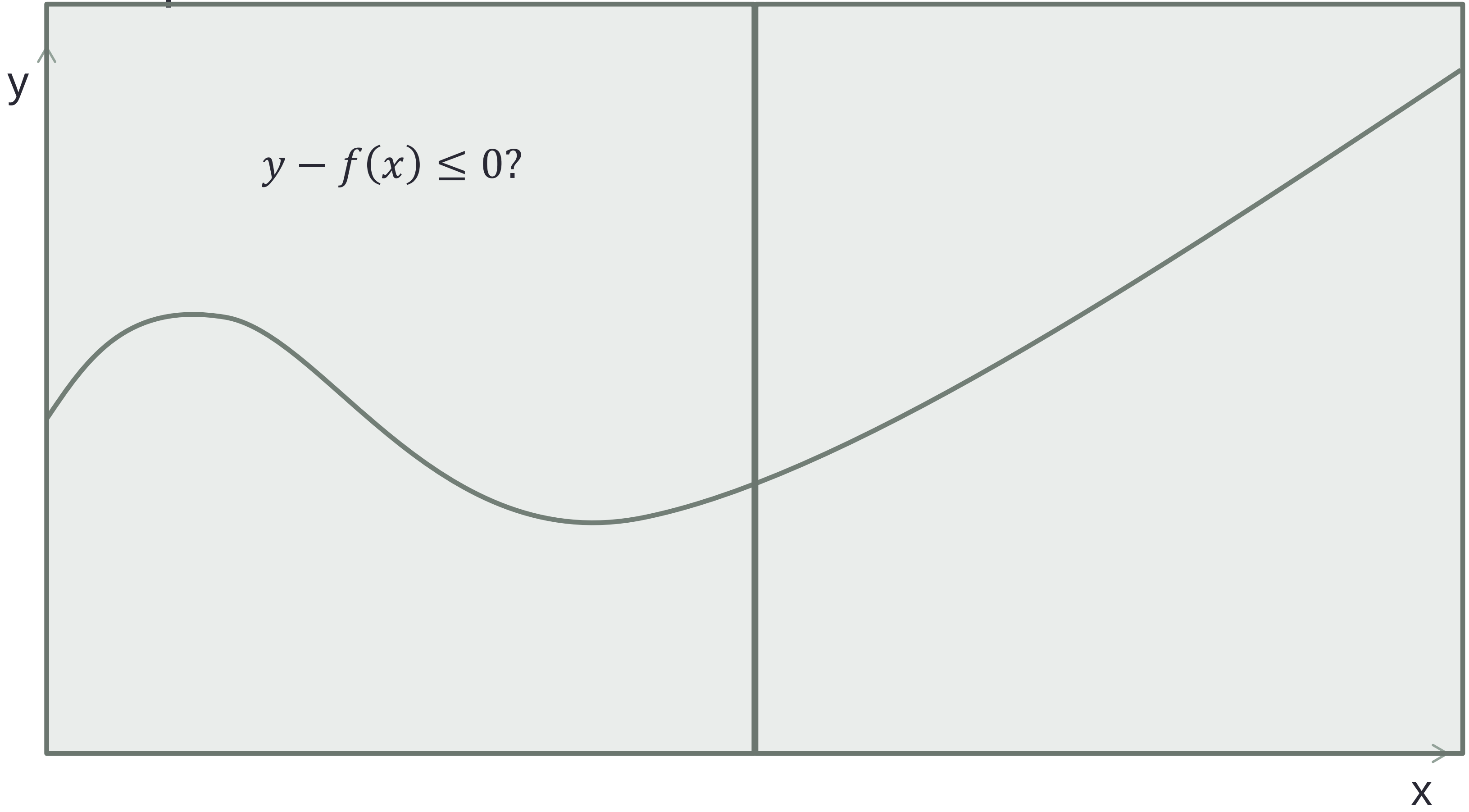

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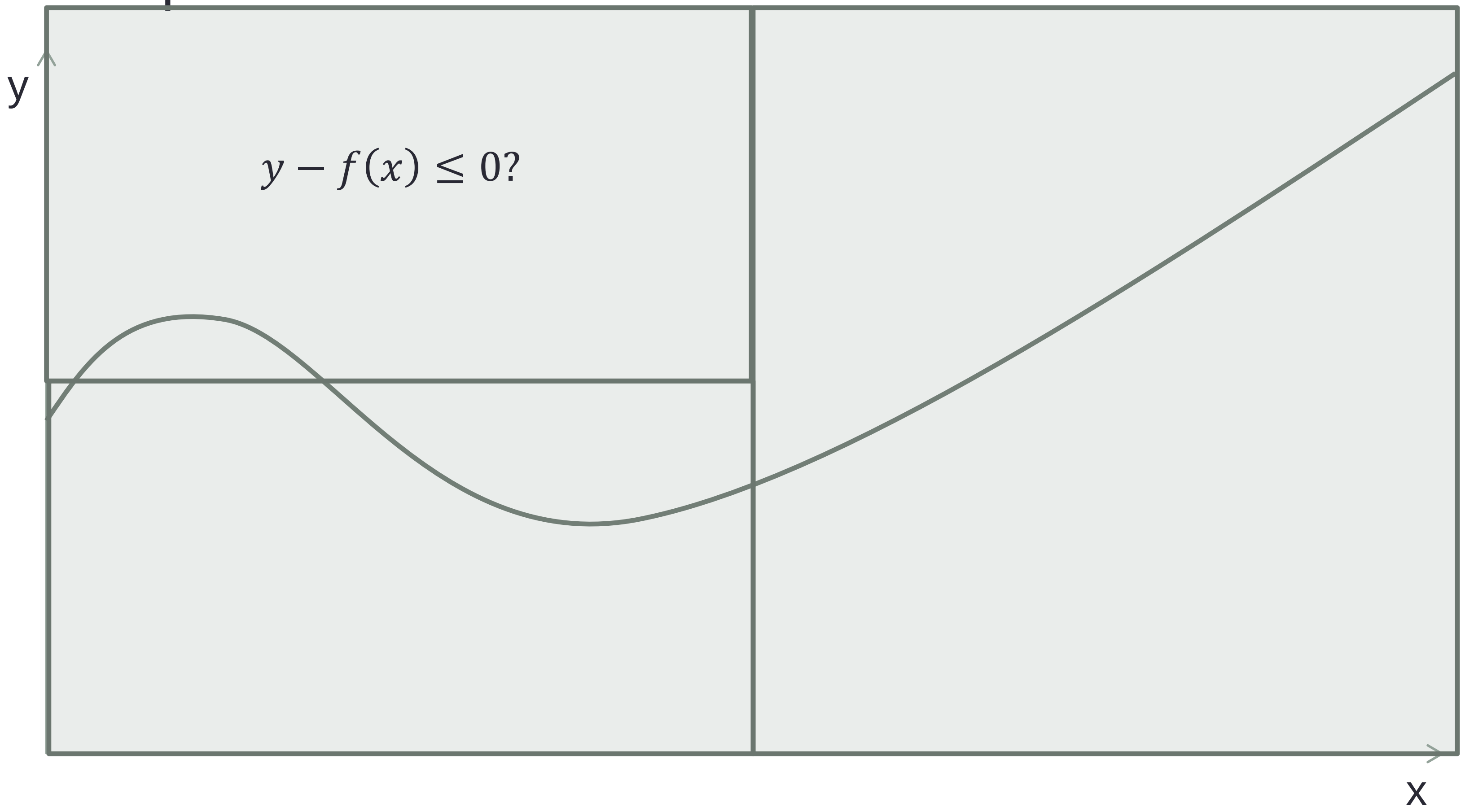

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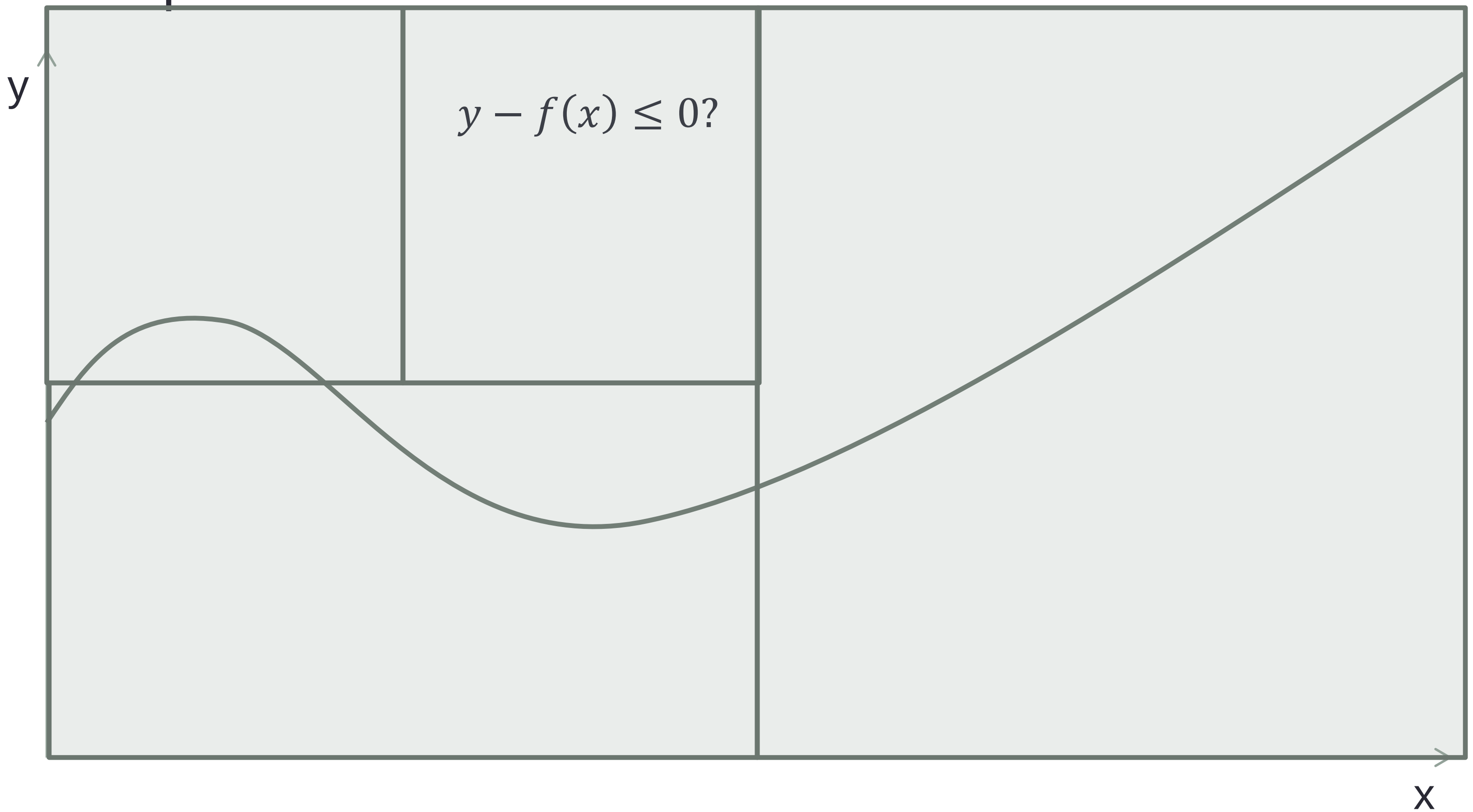

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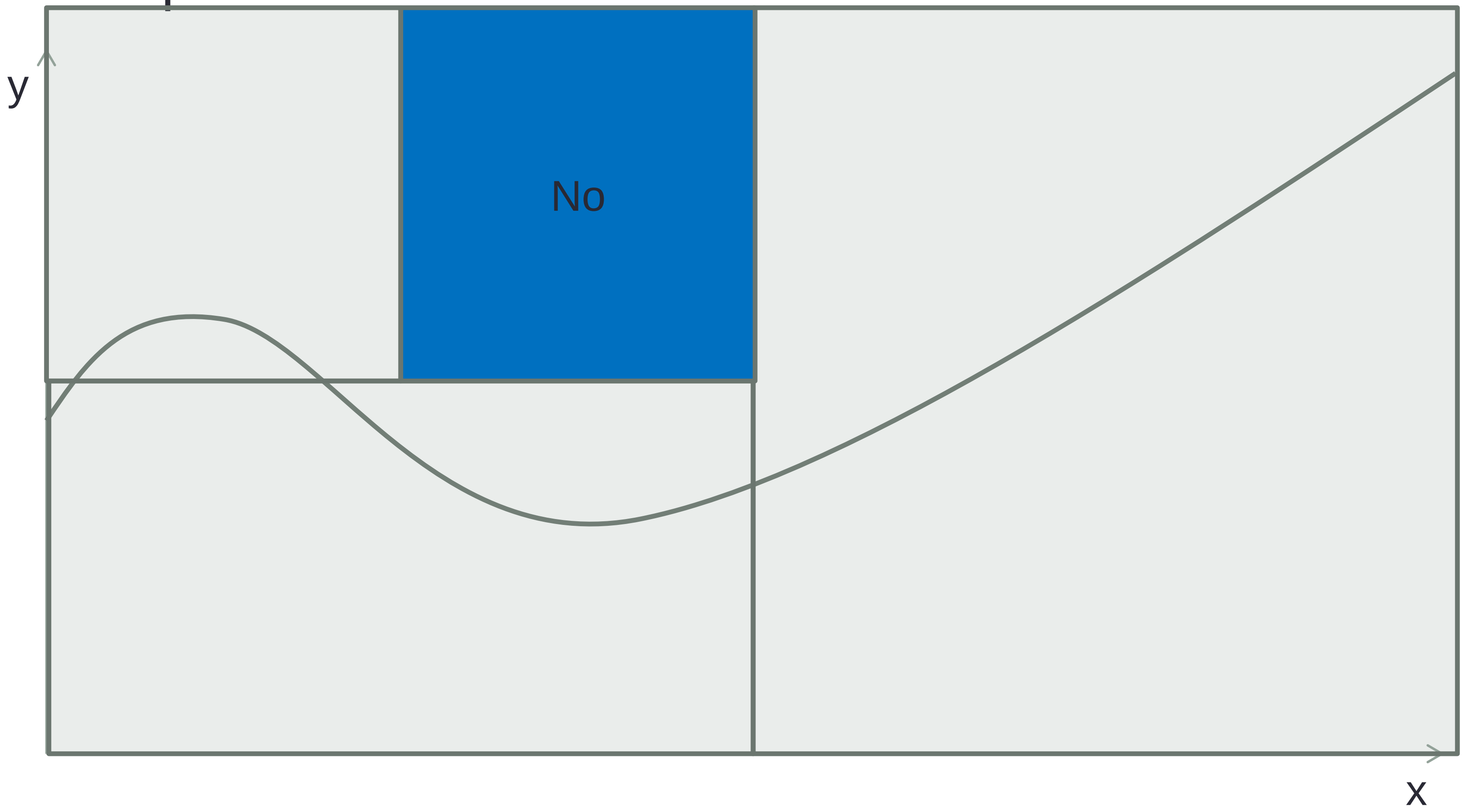

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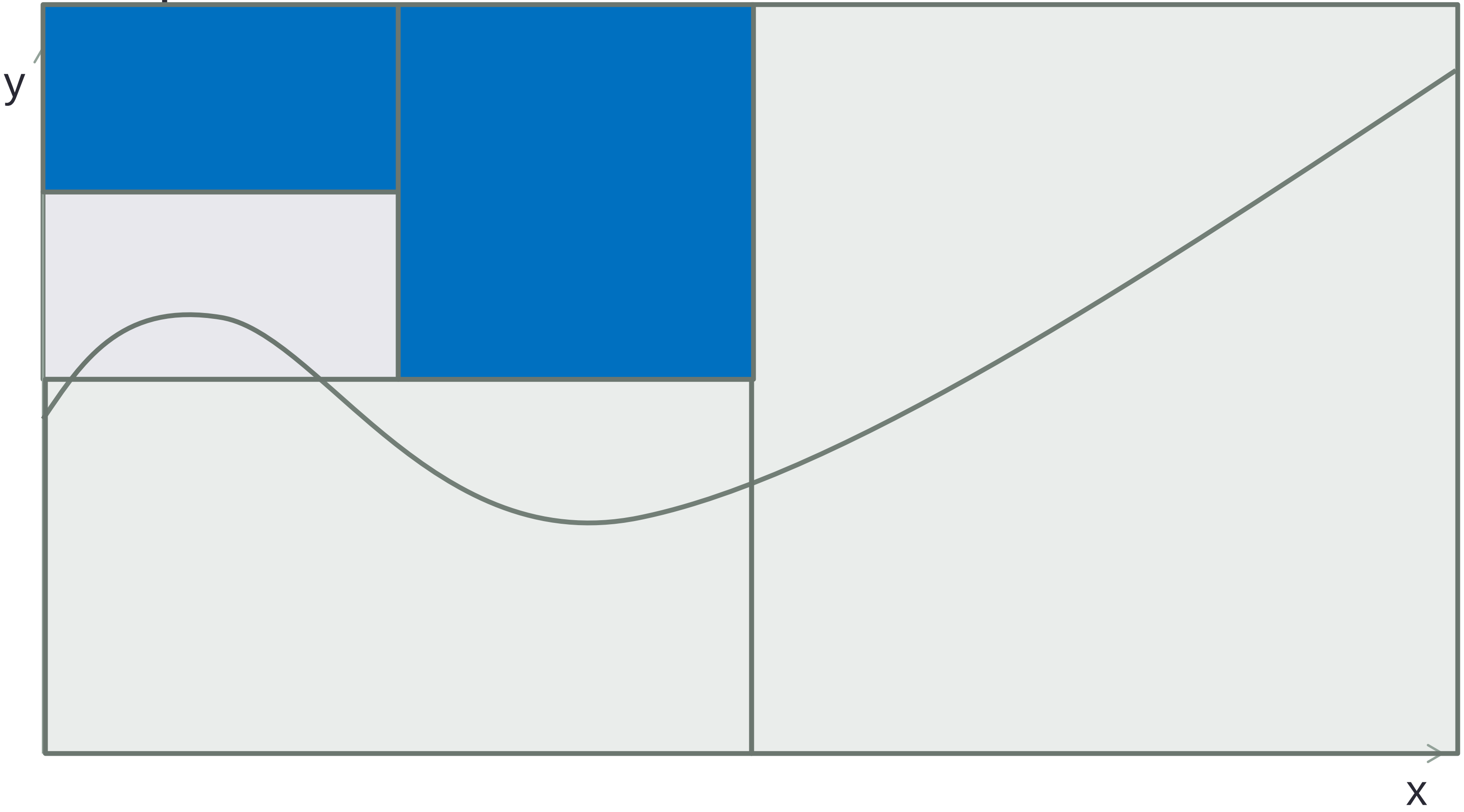

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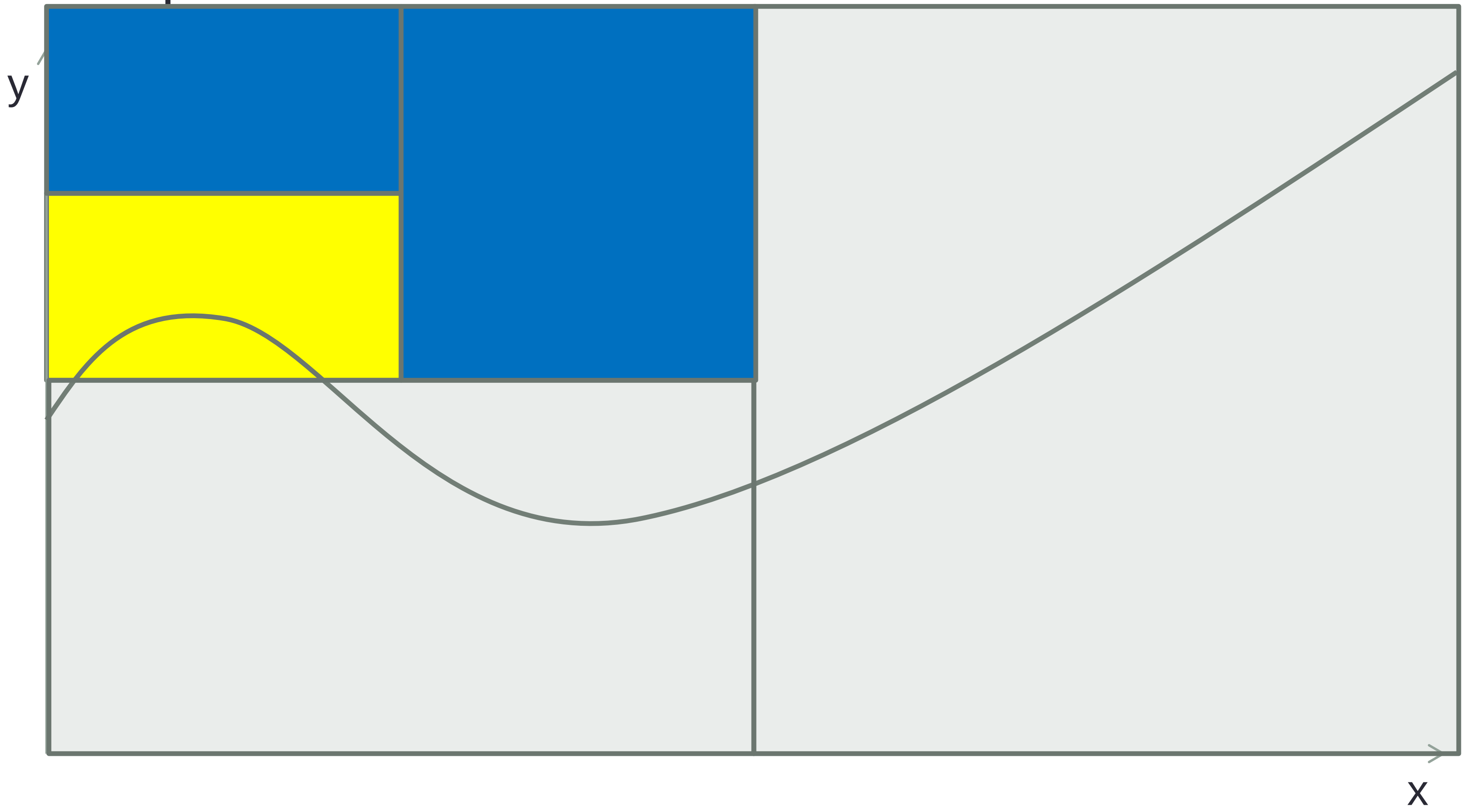

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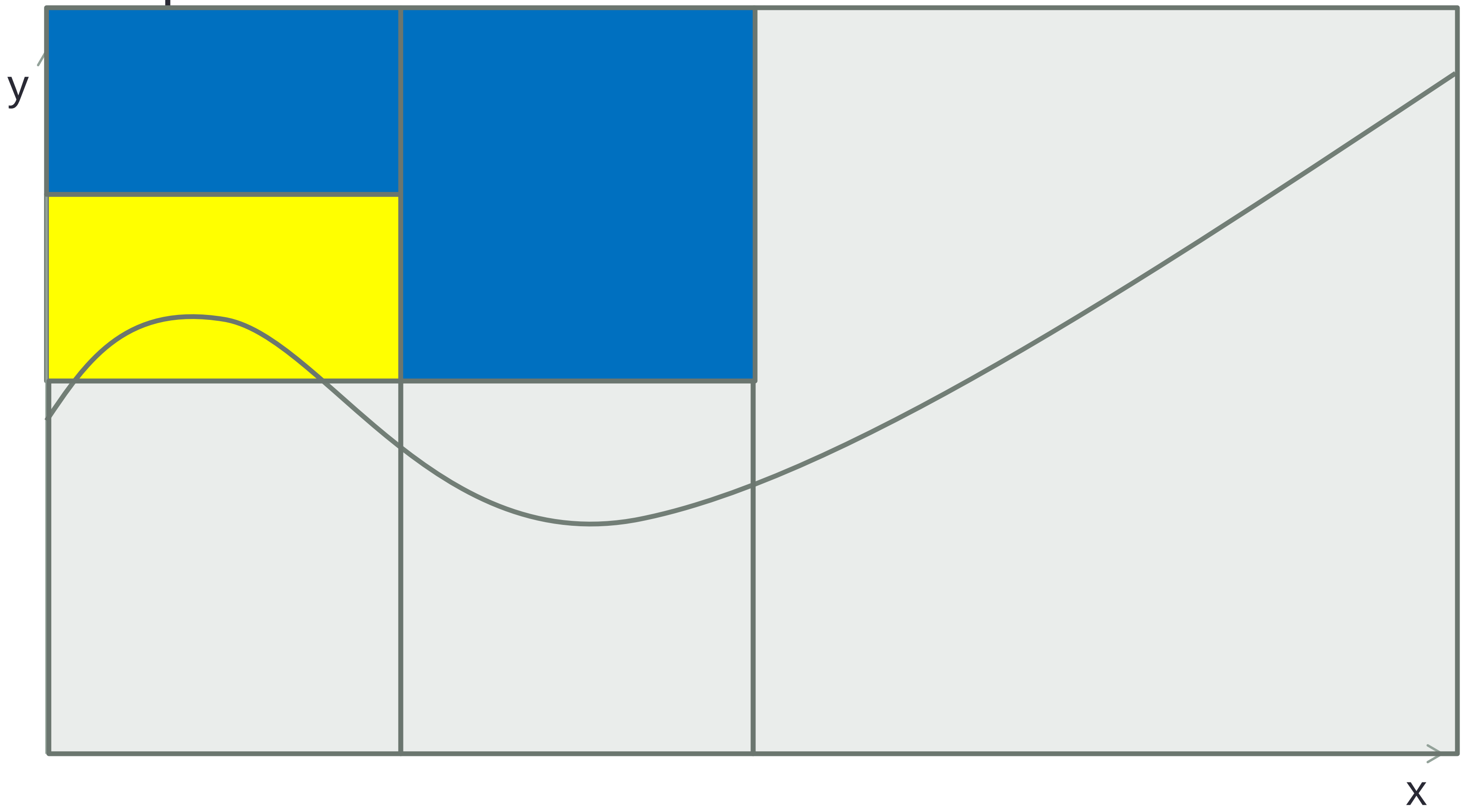

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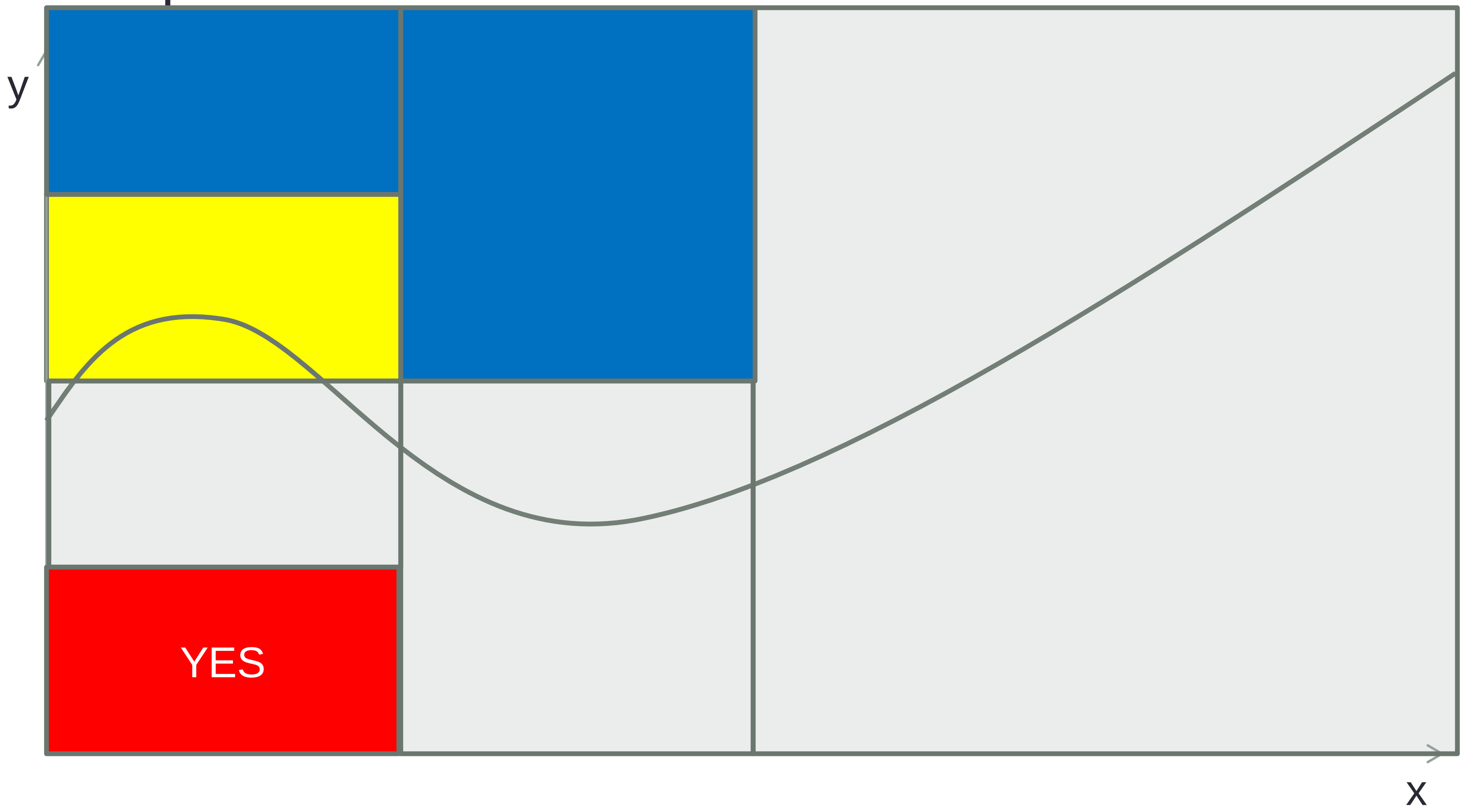
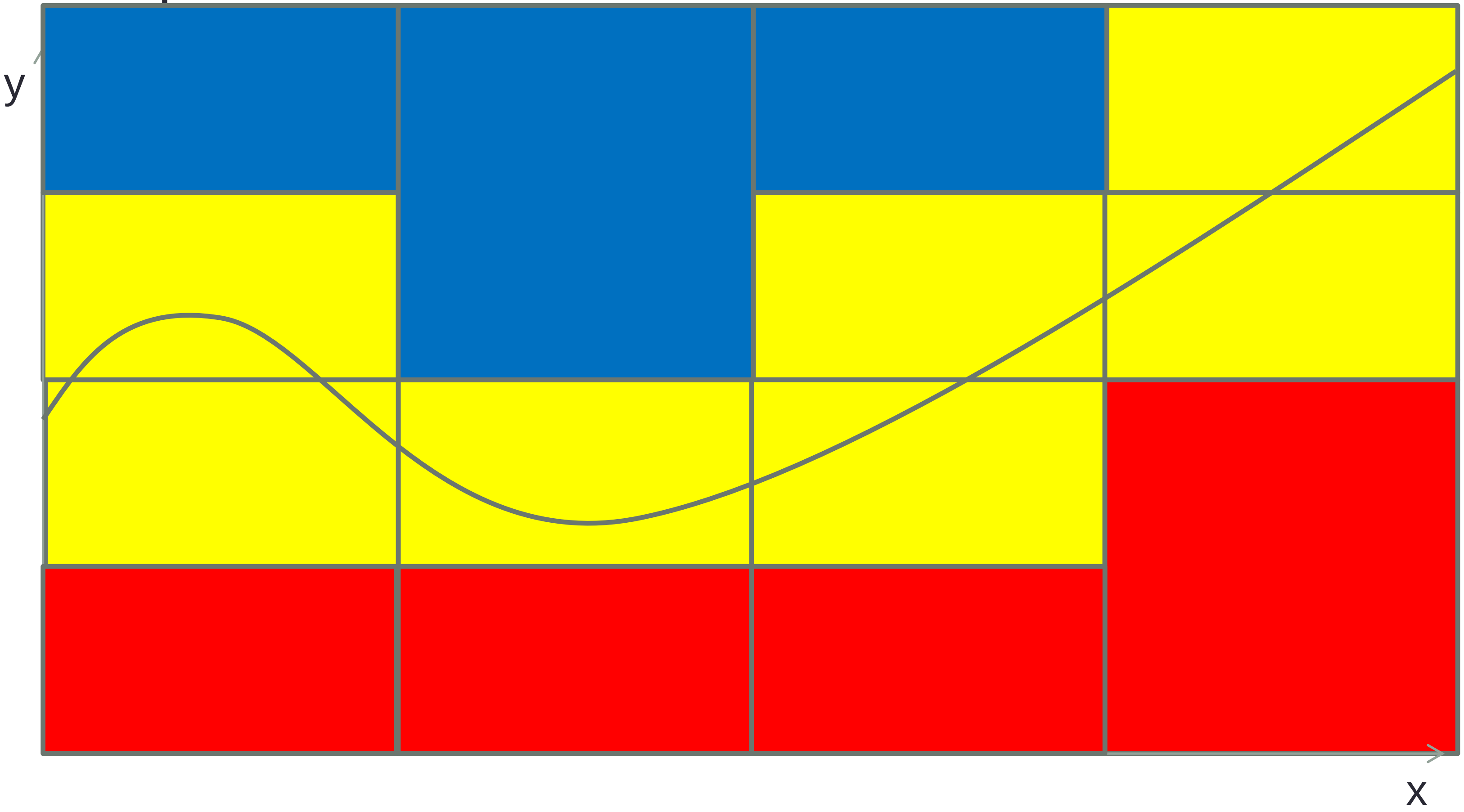
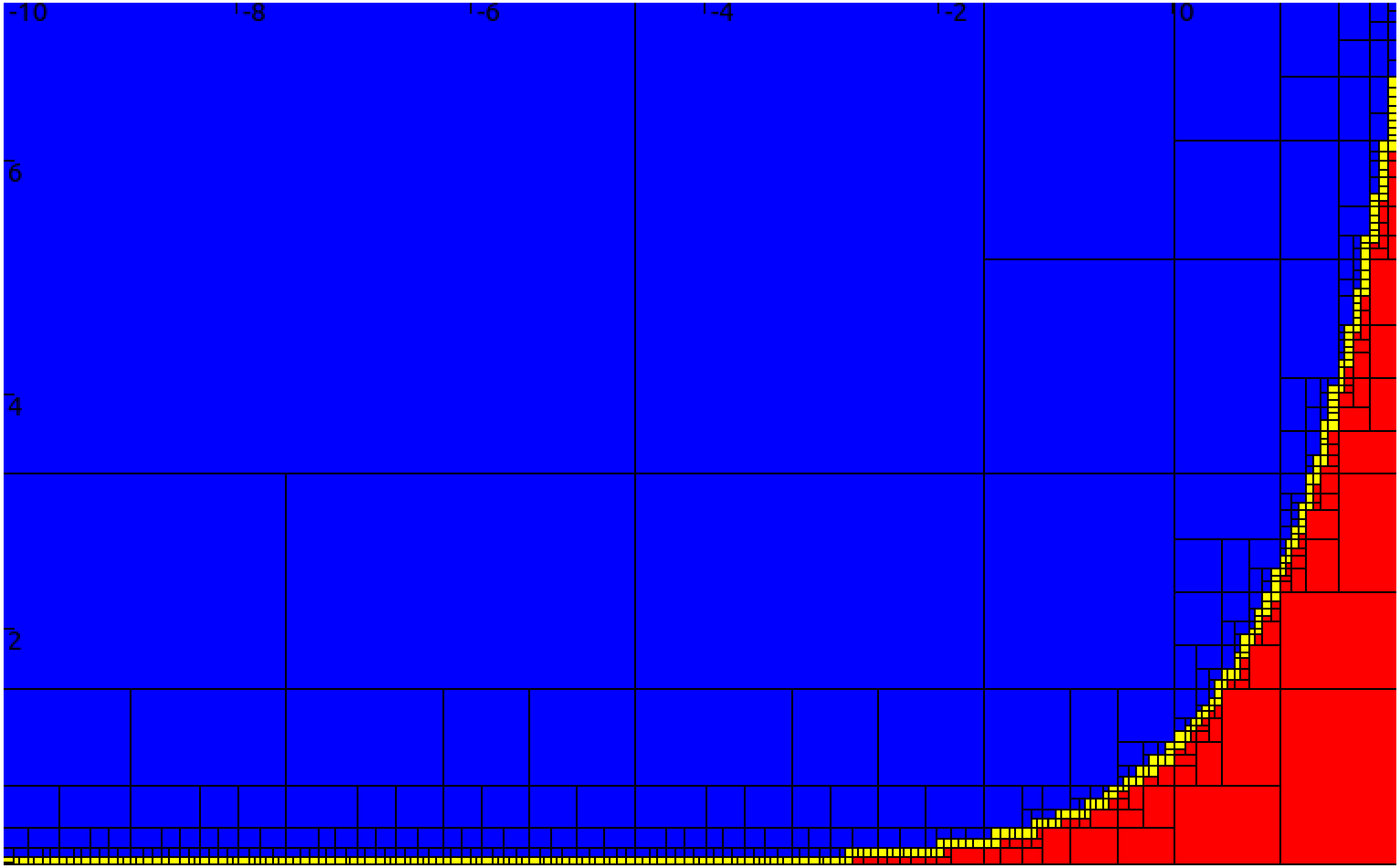


- Example:

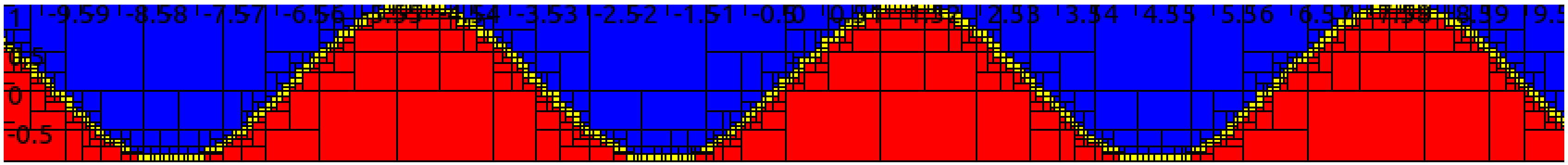
Etc…

- Example:





Epigraph of $y = e^x$



Epigraph of $y = \cos x$

Epigraph of $y = \tan x$

- The epigraph is stored in a binary-tree structure
  - This enables a very quick access to the boxes stored (O(log(n)))
- This is the representation we use to store and manipulate the bathymetry data

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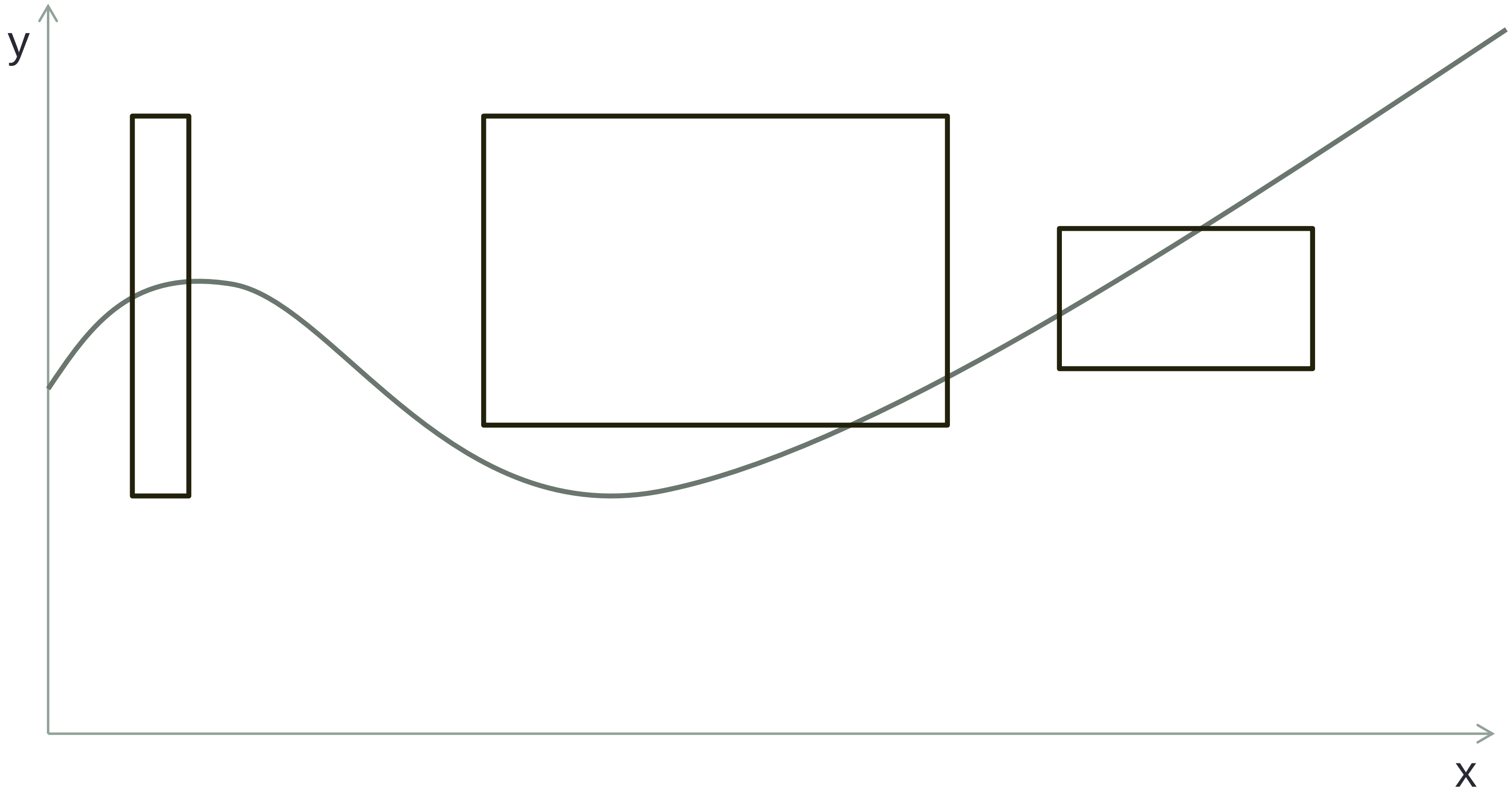
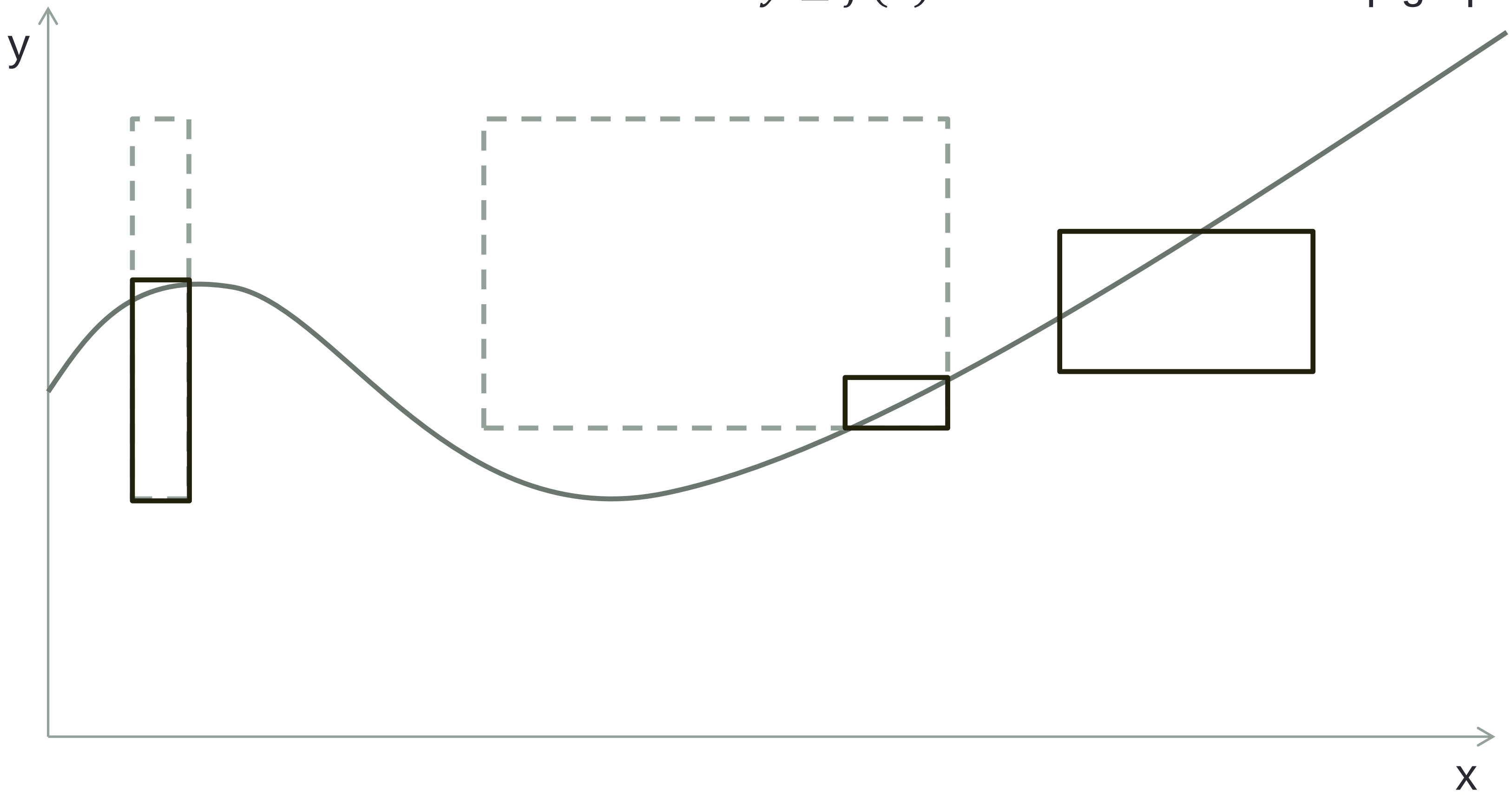
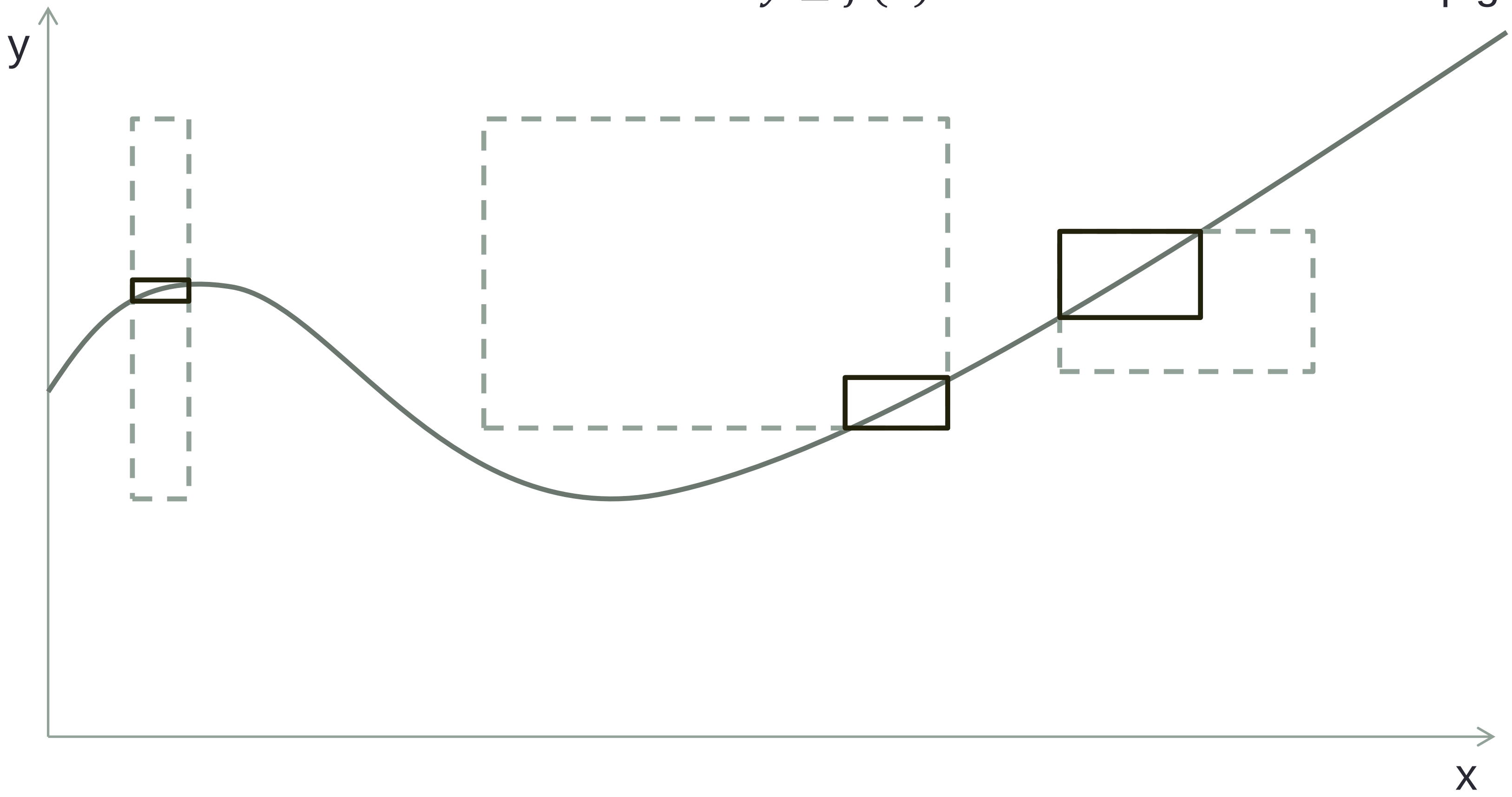
- A contractor *C* associated to a set X is an operator which associates to a box $[x] \in IR^n$ an other box $C([x]) \in IR^n$ such that the following properties are satisfied:
  - $C([x]) \subset [x]$ (contractance)
  - $C([x]) \cap [x] = [x] \cap X$ (completeness)


y
x

$y \leq f(x)$ ie: the box is in the epigraph



$y \geq f(x)$ ie: the box is not in the epigraph



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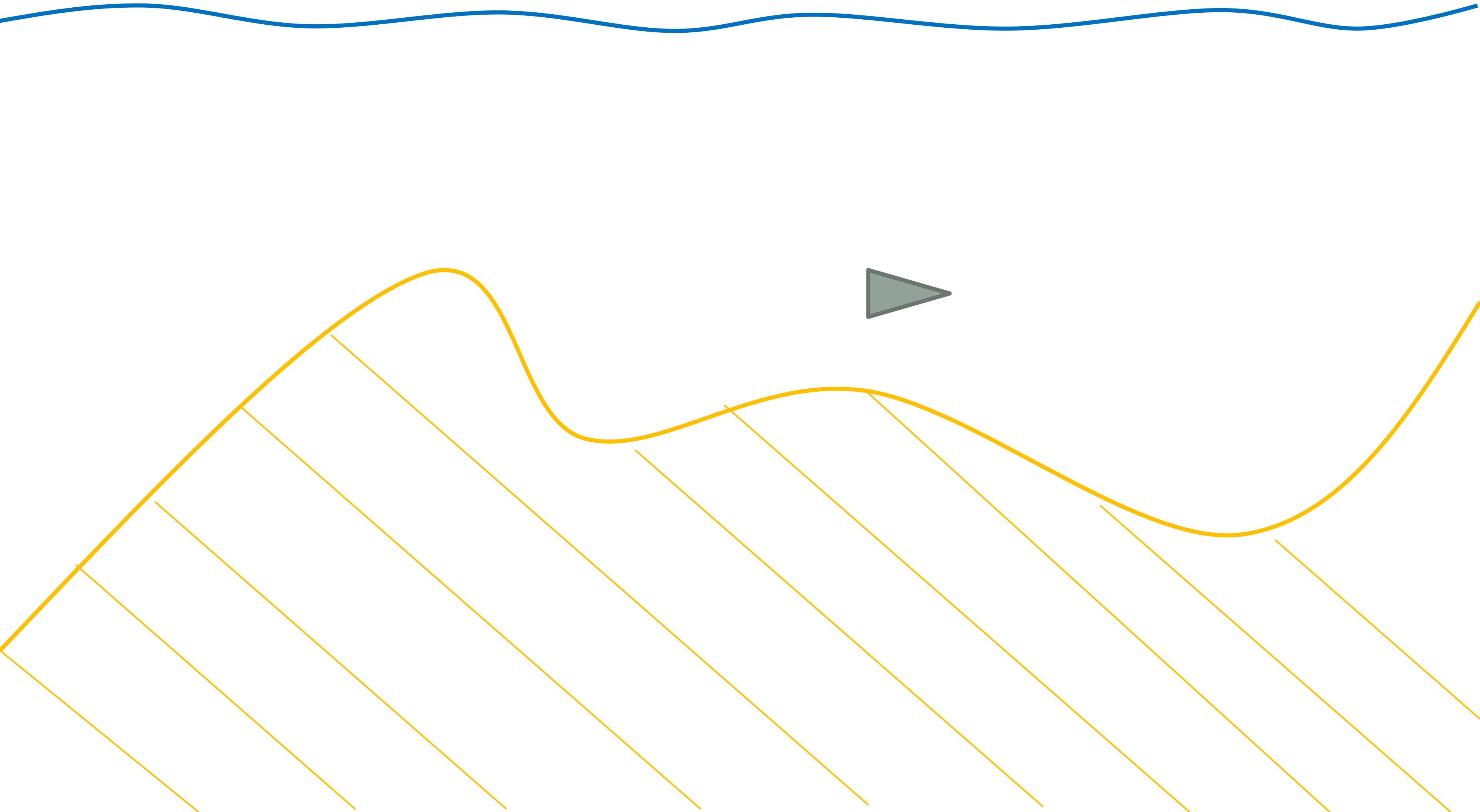
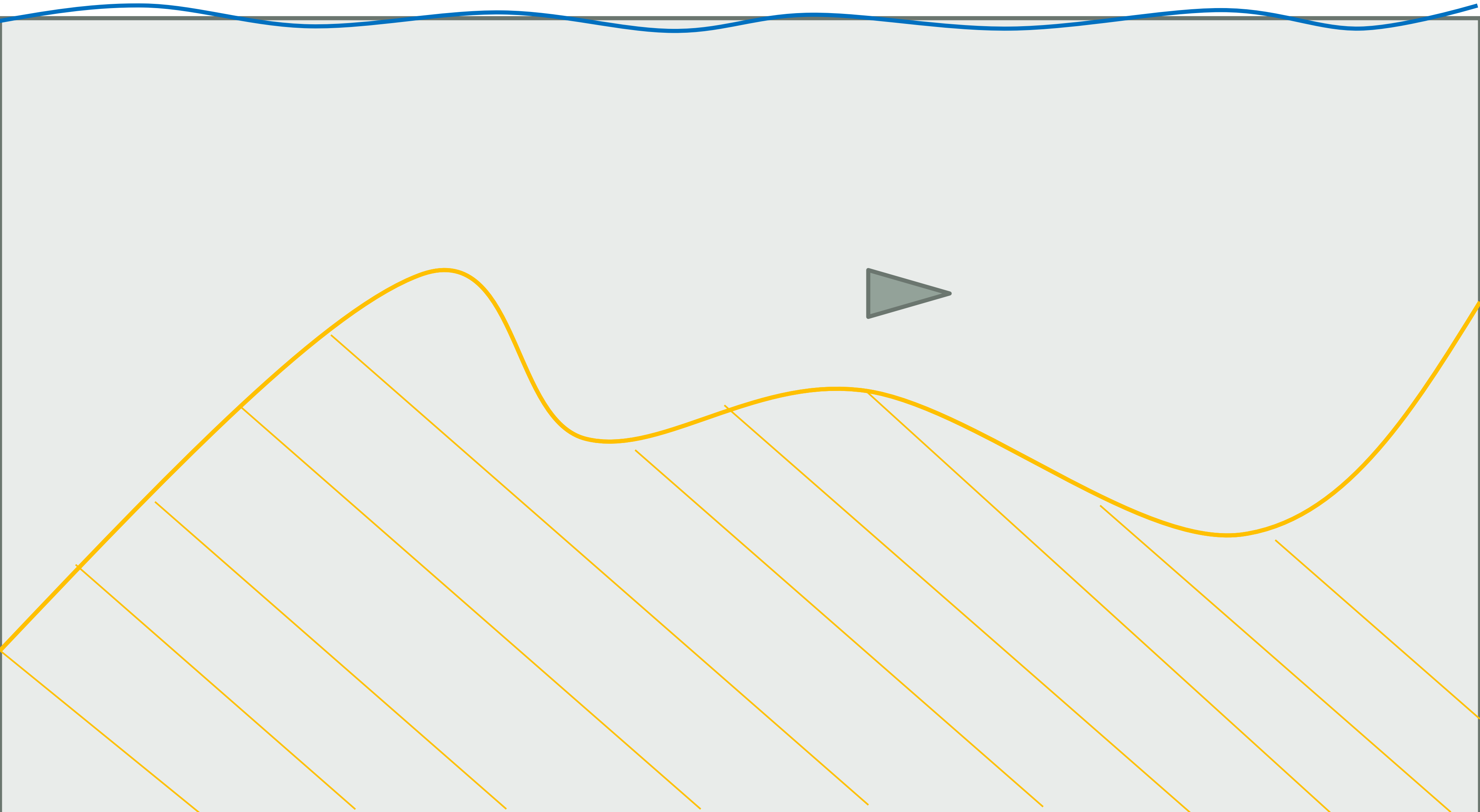
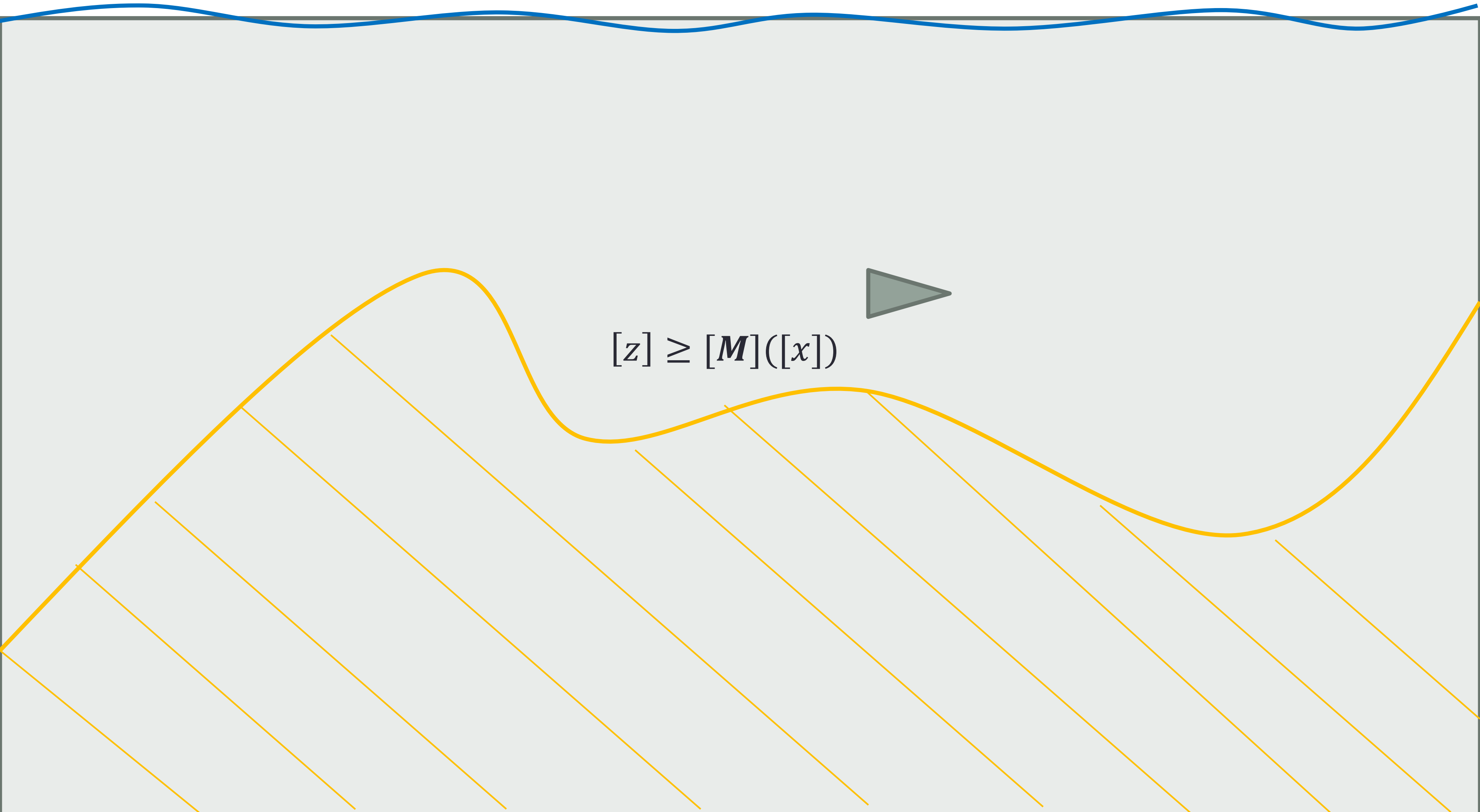
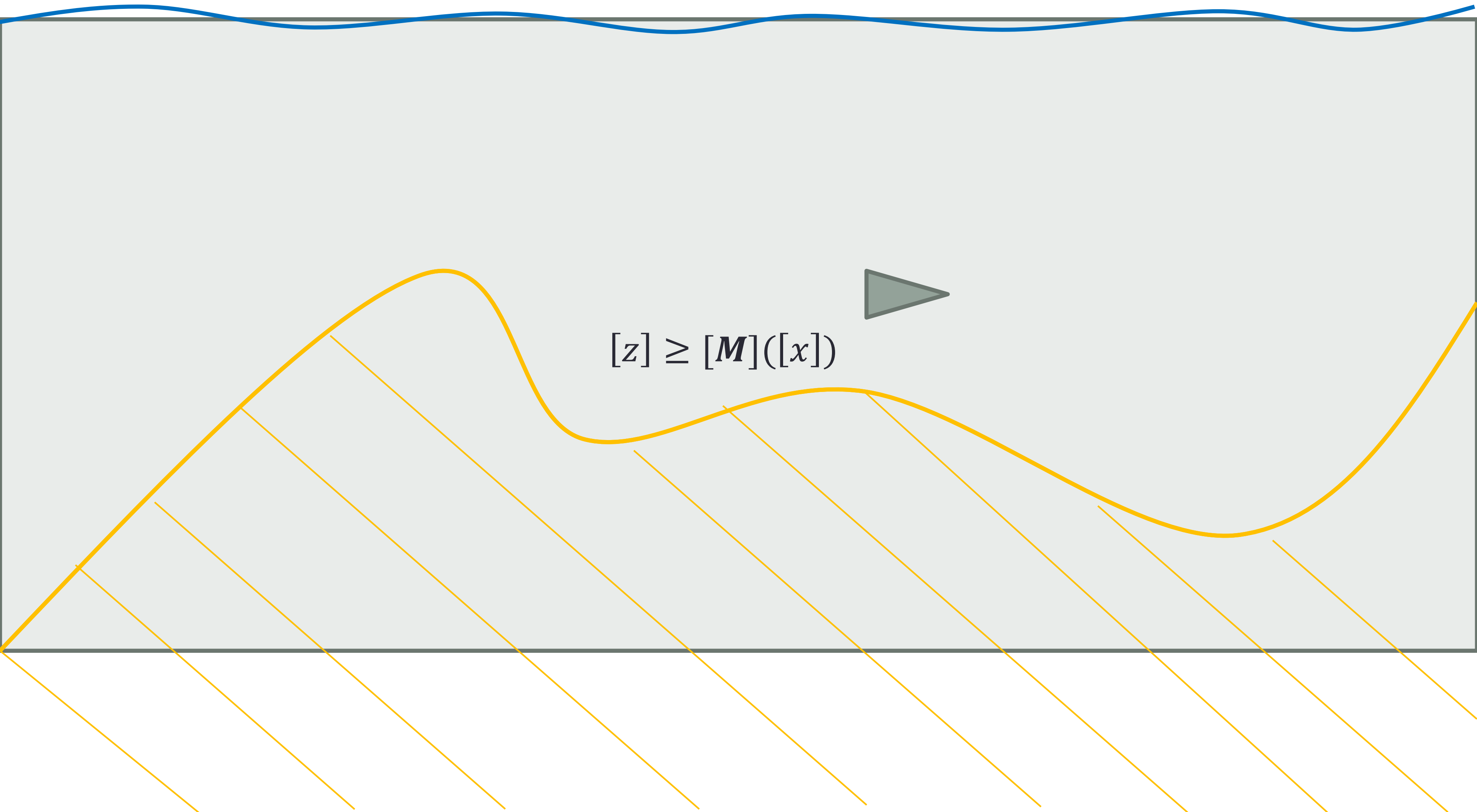






$$[z] \geq [\boldsymbol{M}]([x])$$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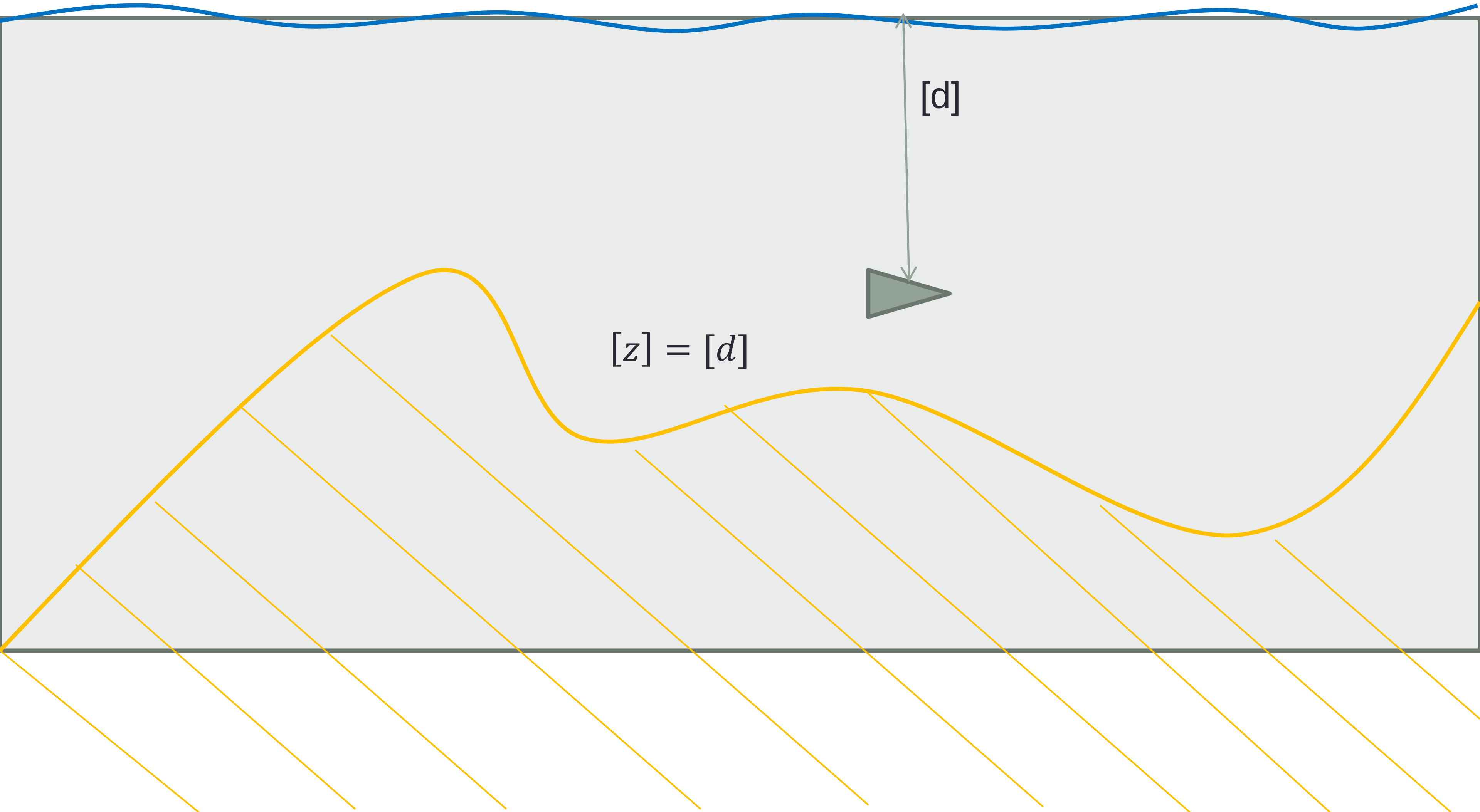
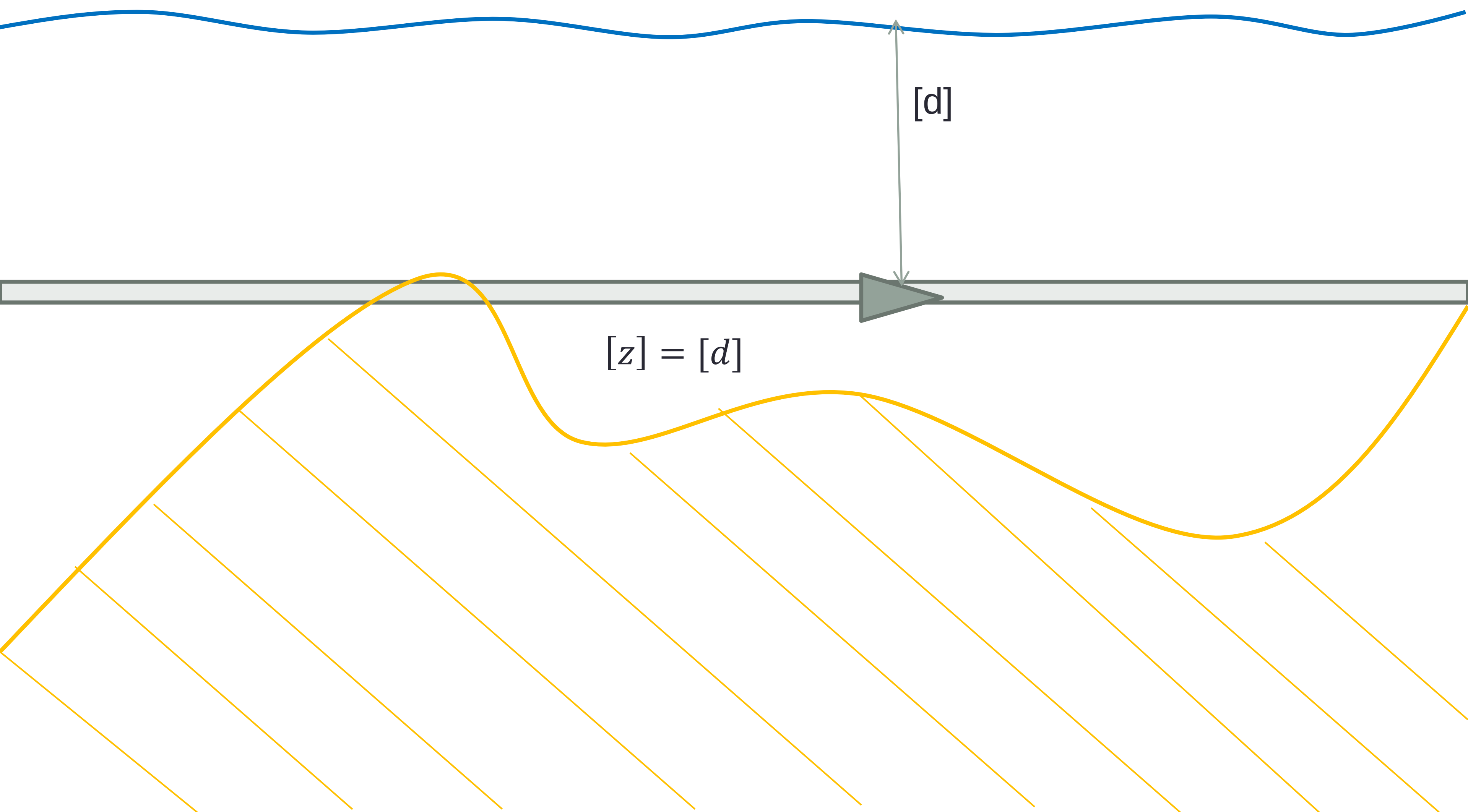

[d]
$[z] = [d]$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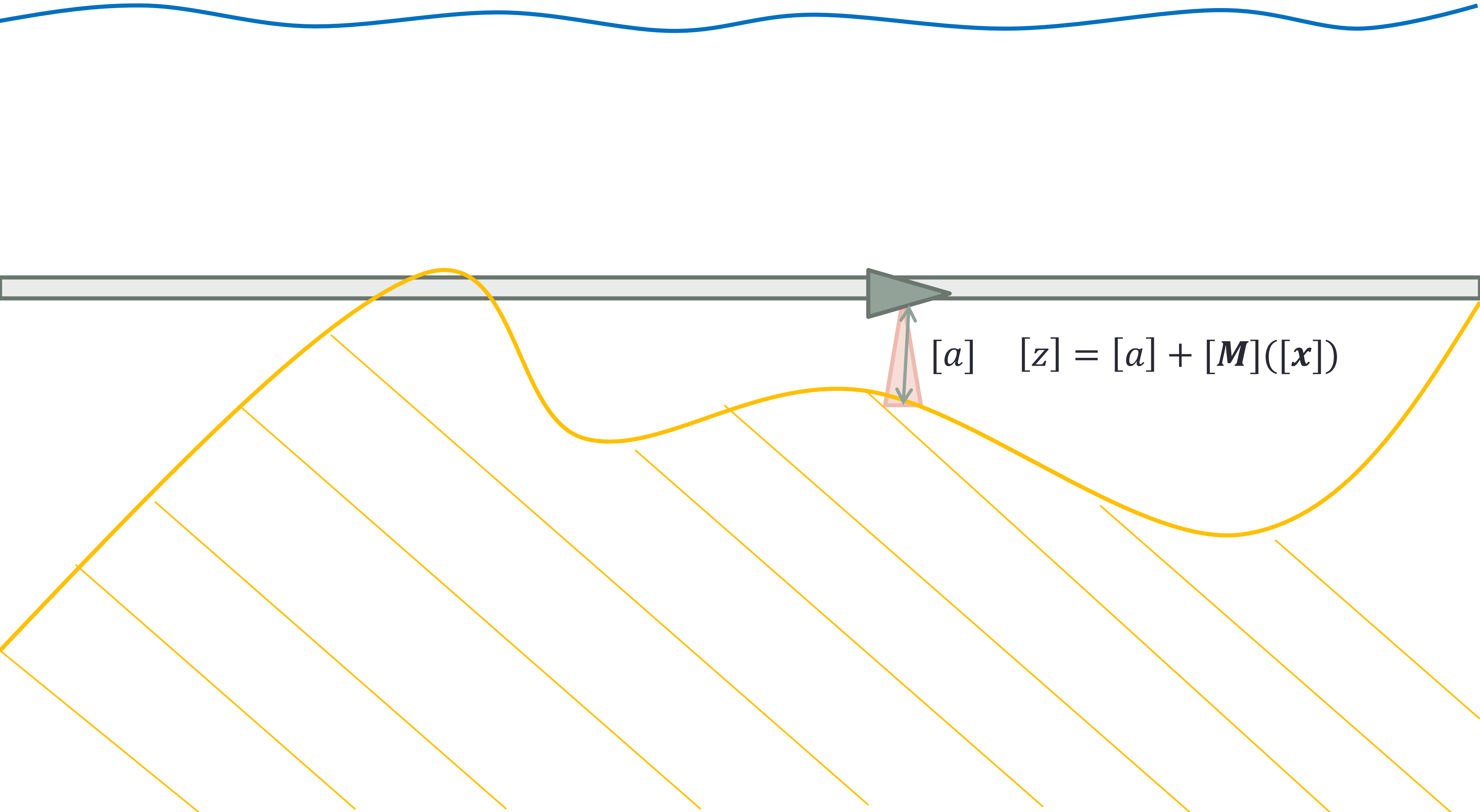
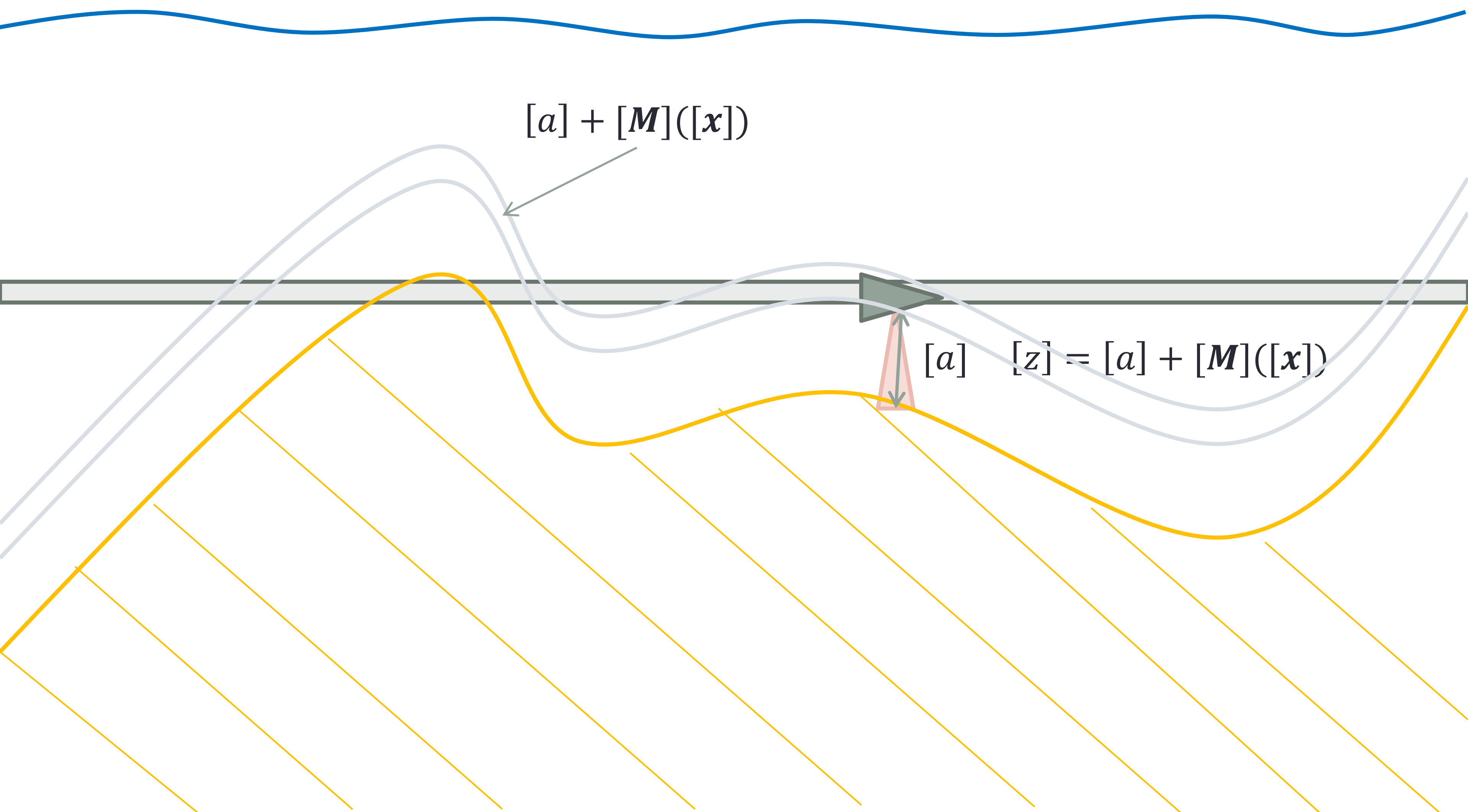

$[a]$
$[z] = [a] + [\boldsymbol{M}]([\boldsymbol{x}])$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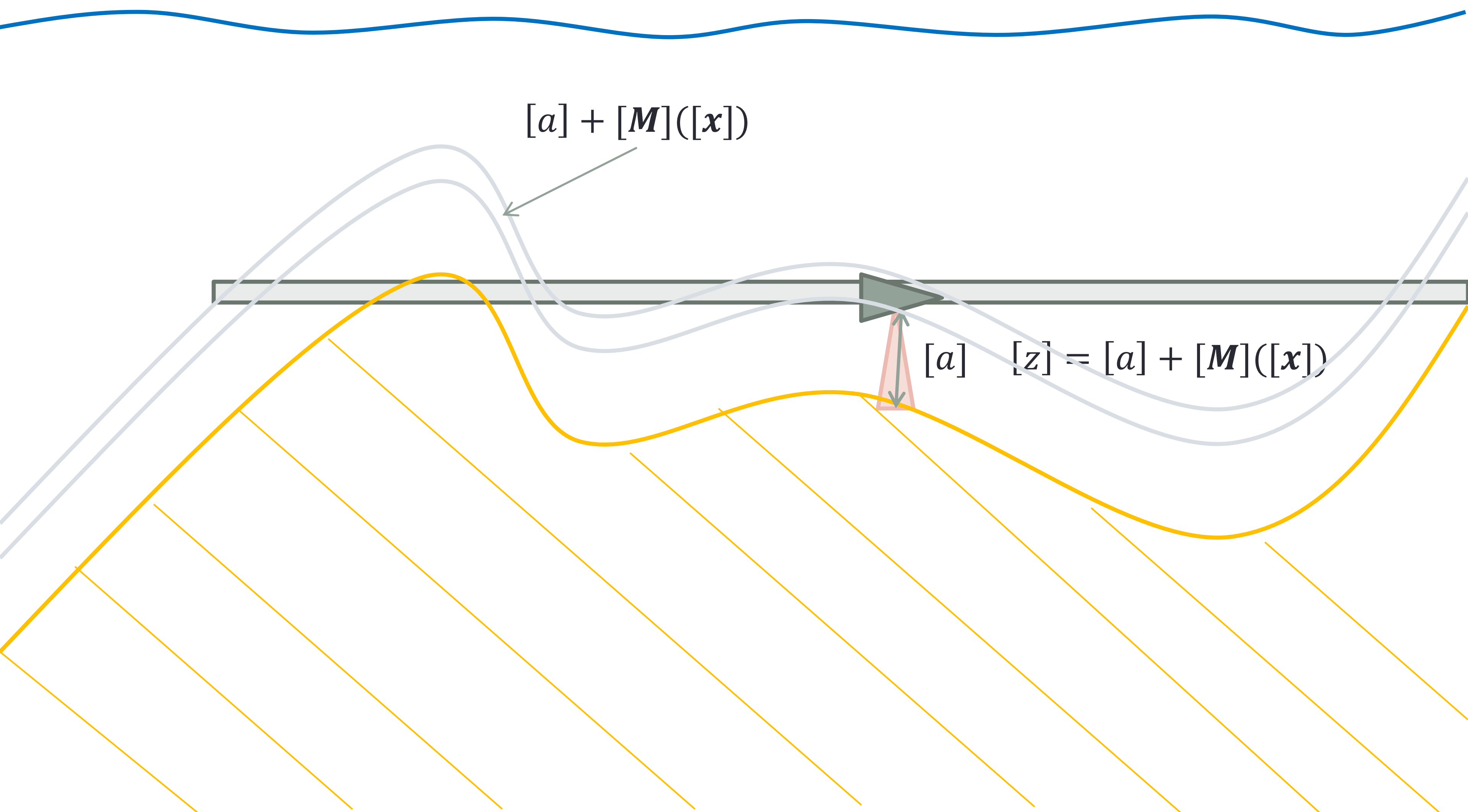
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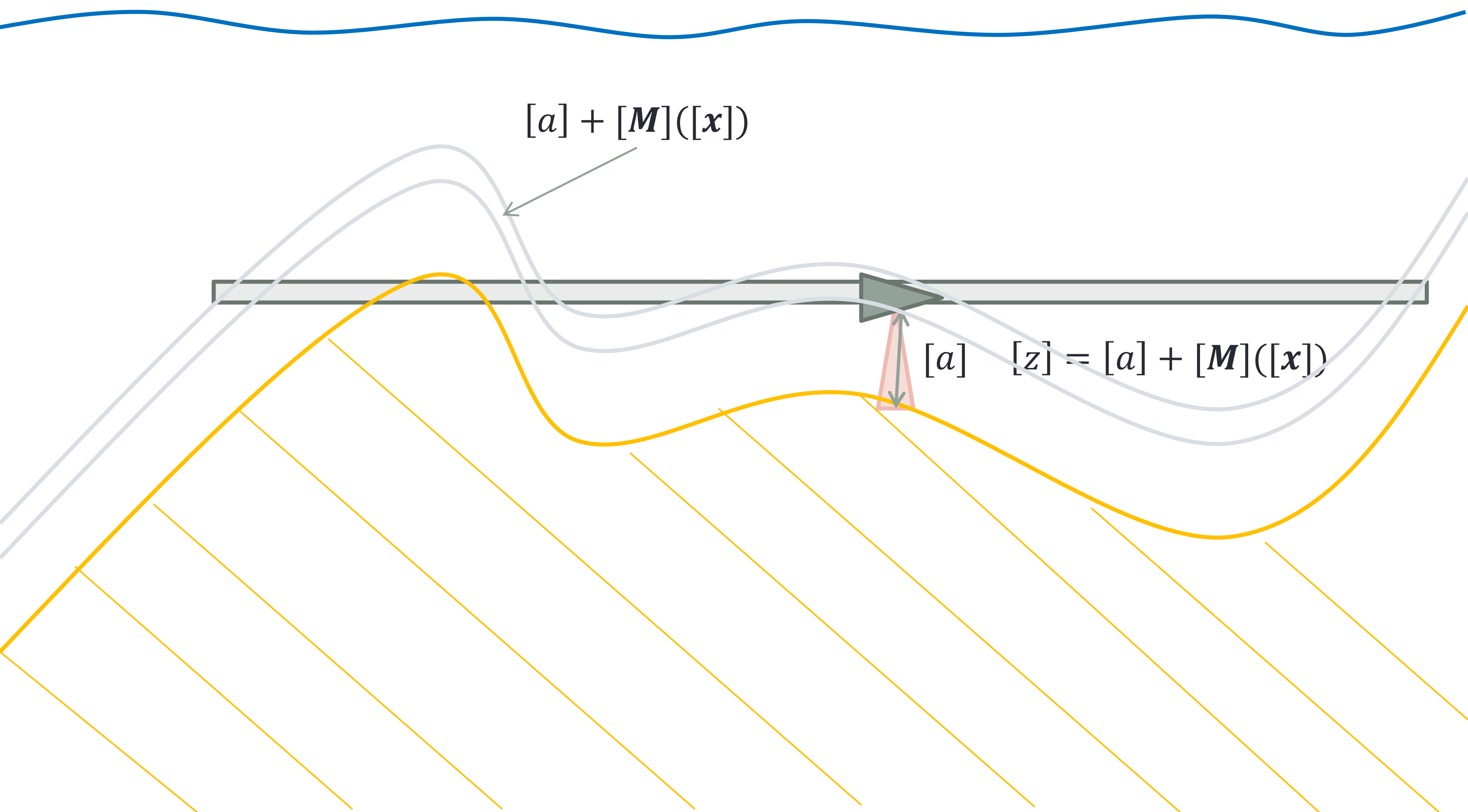
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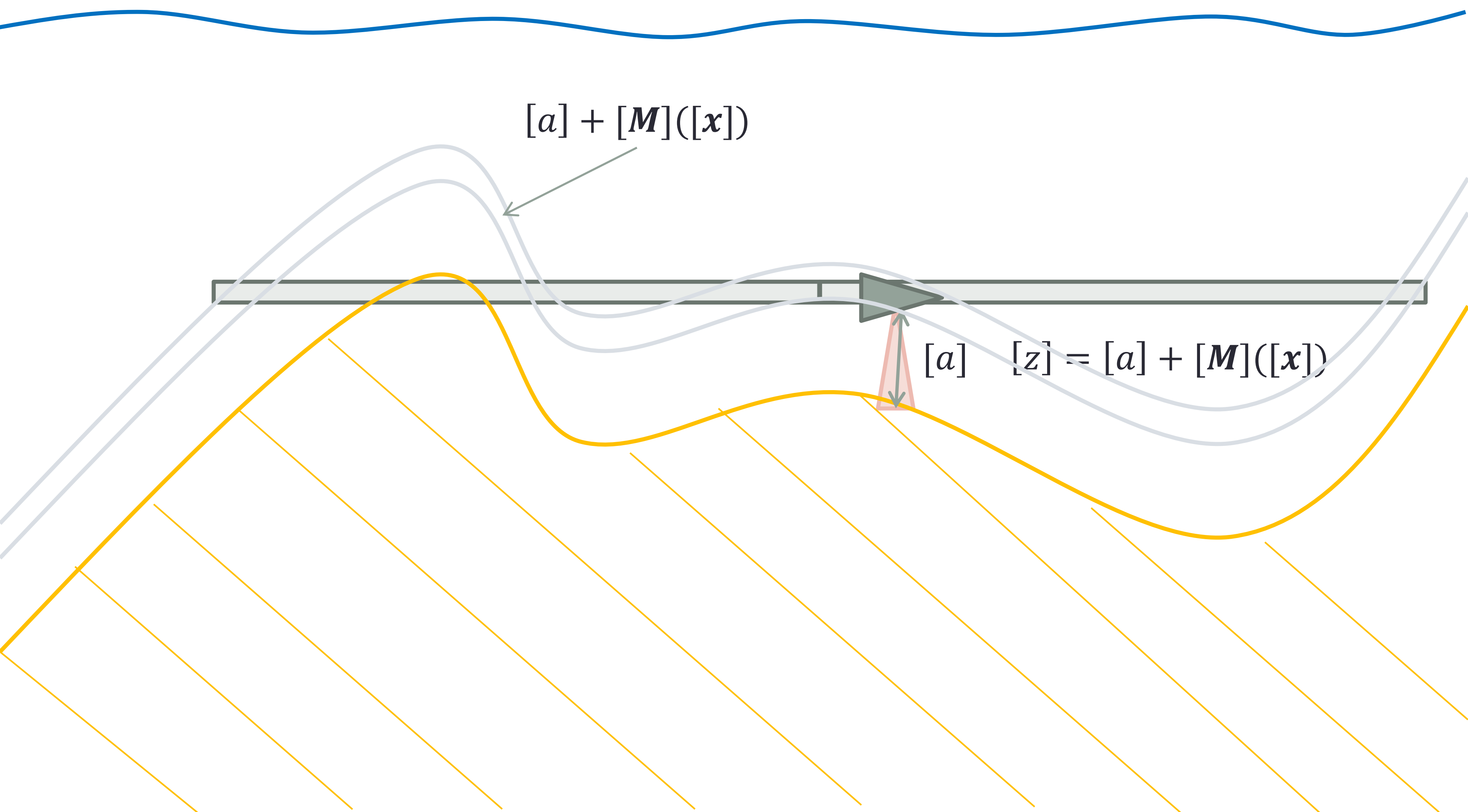
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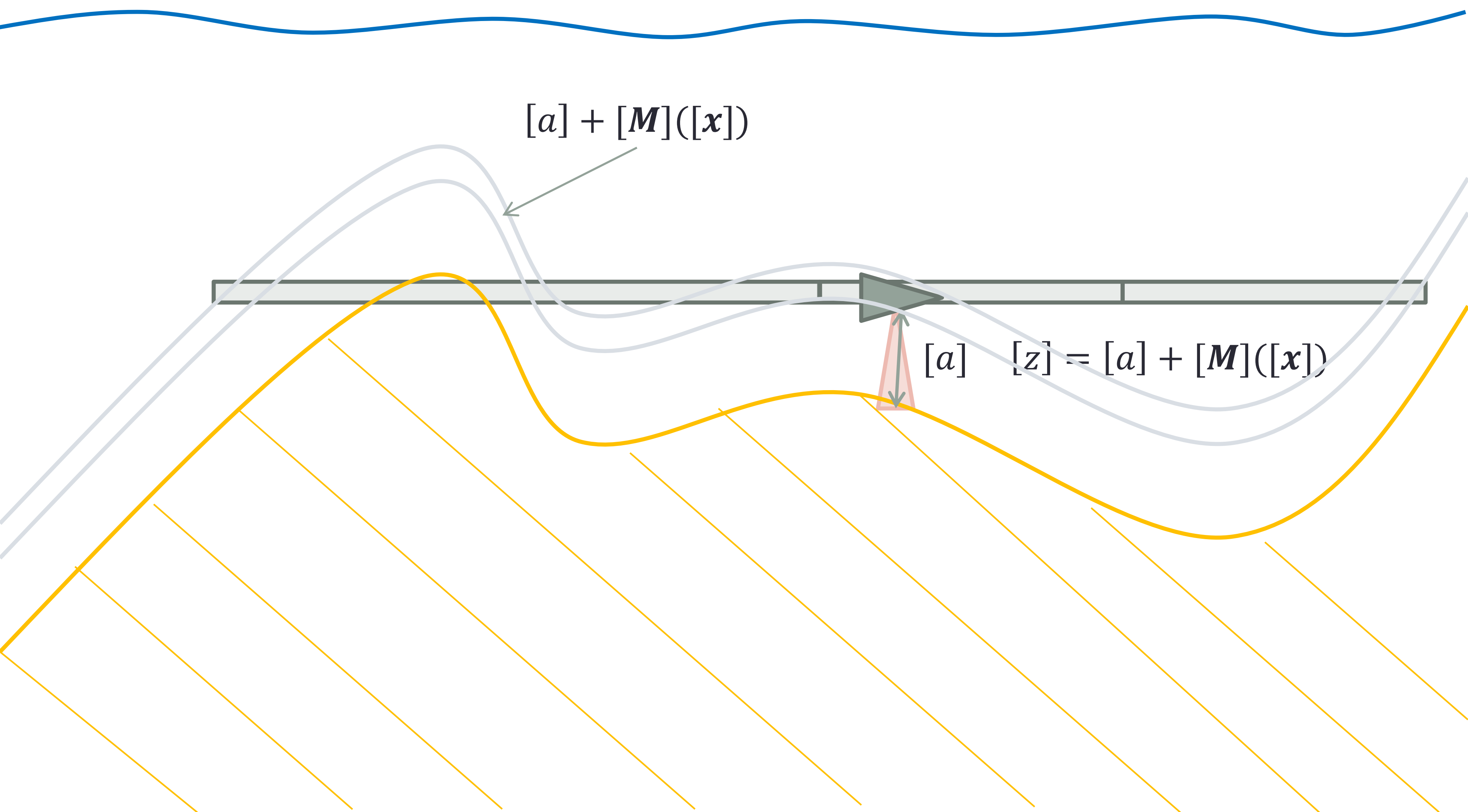
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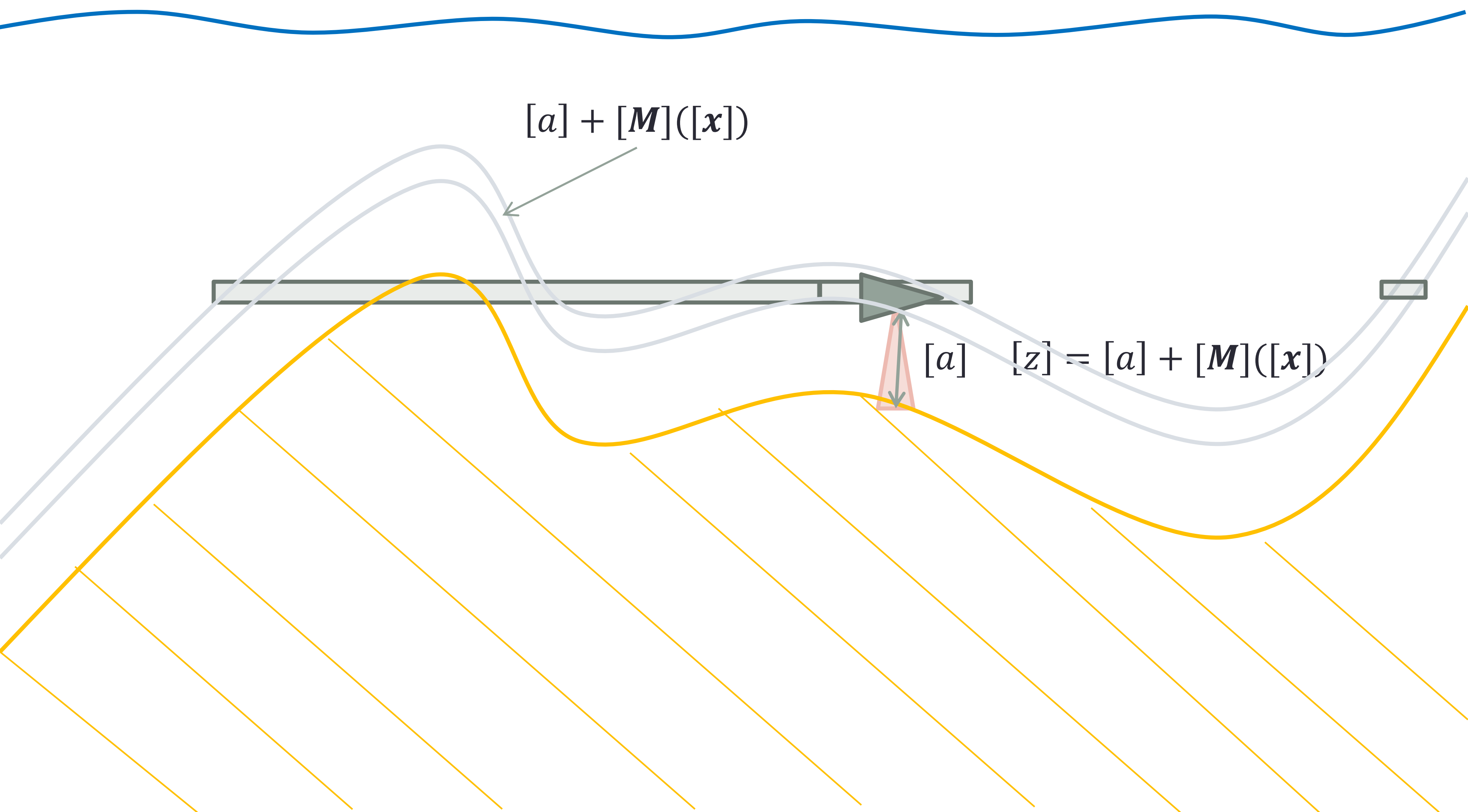
$[a] + [\boldsymbol{M}]([\boldsymbol{x}])$
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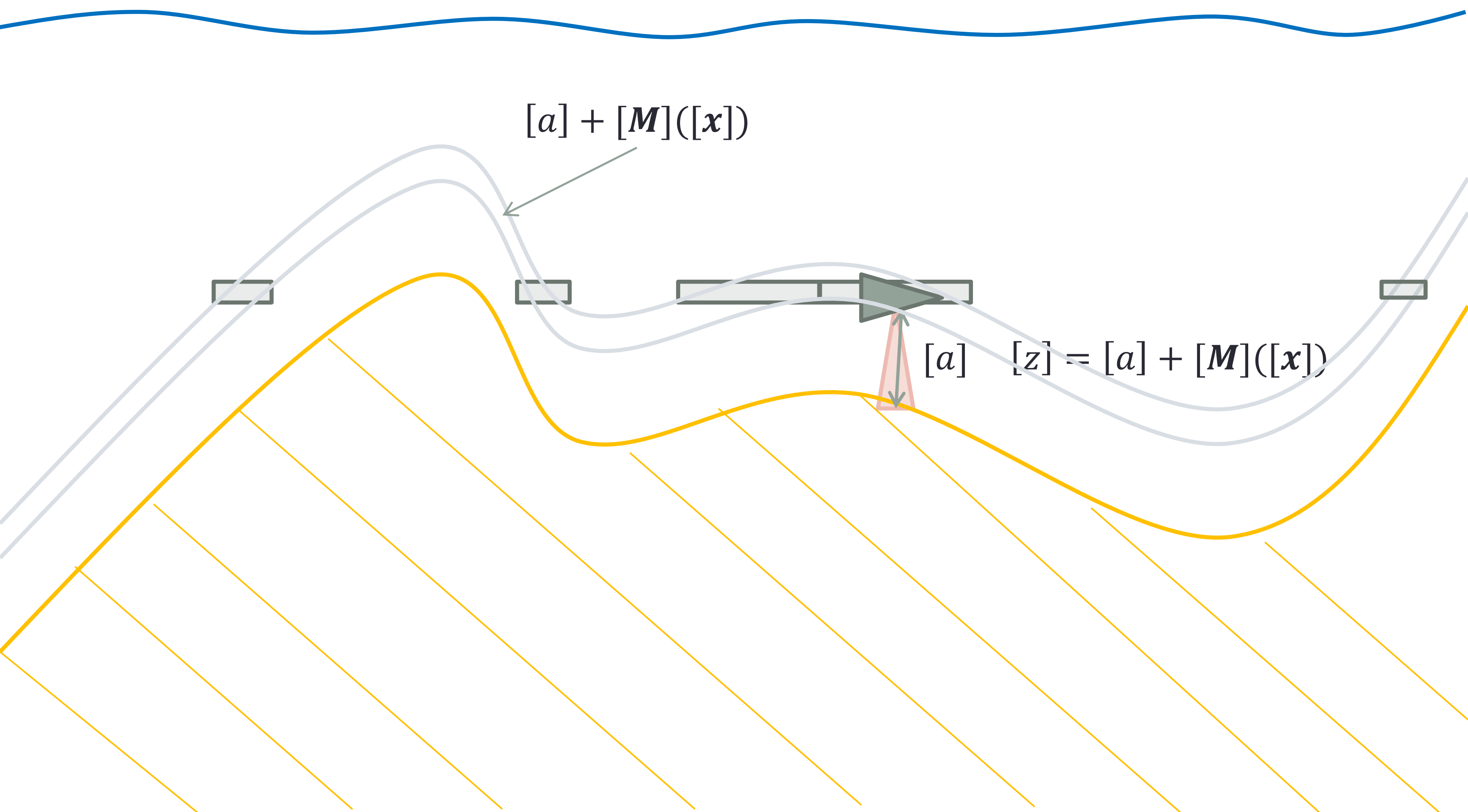
$[a] + [\boldsymbol{M}]([\boldsymbol{x}])$
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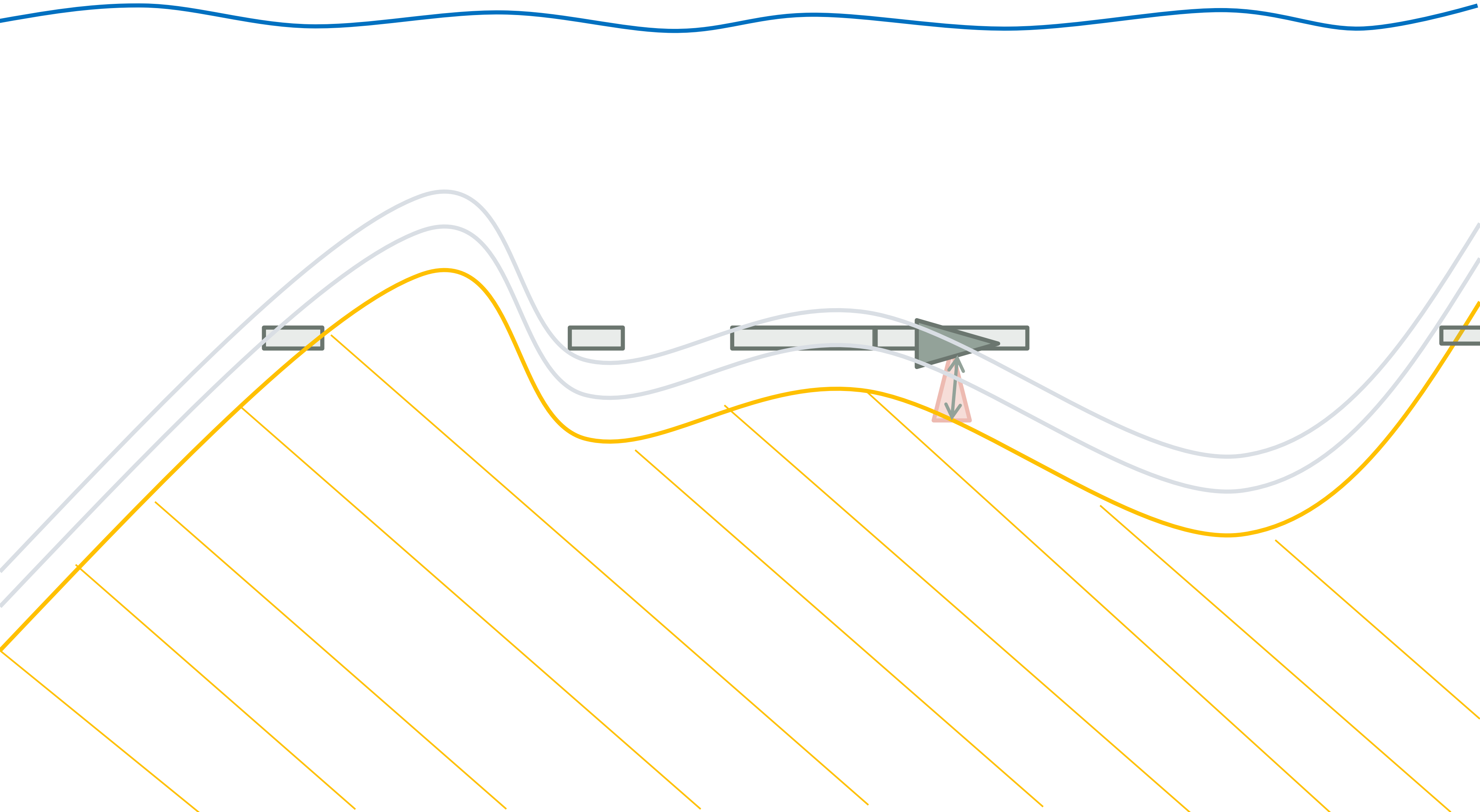

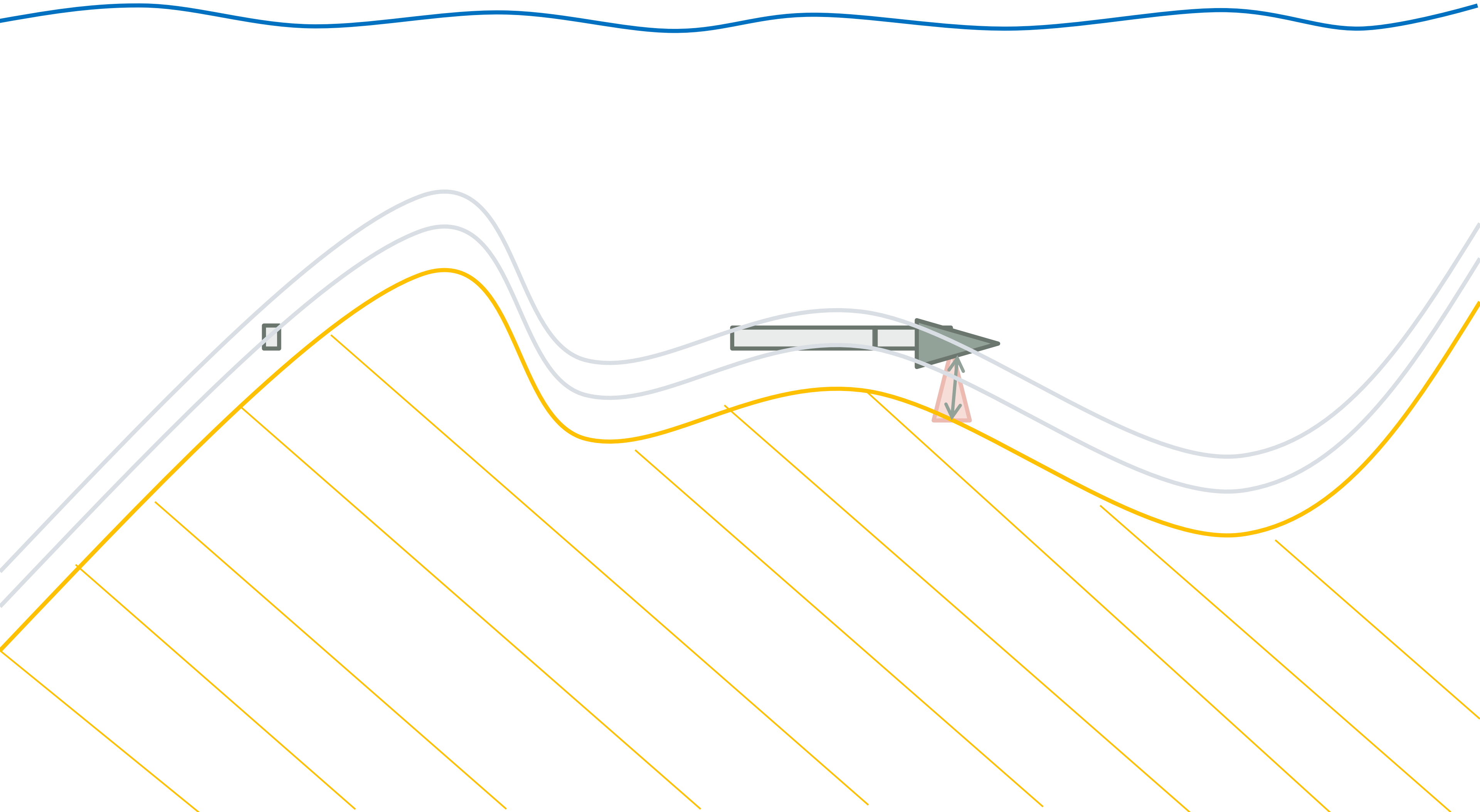

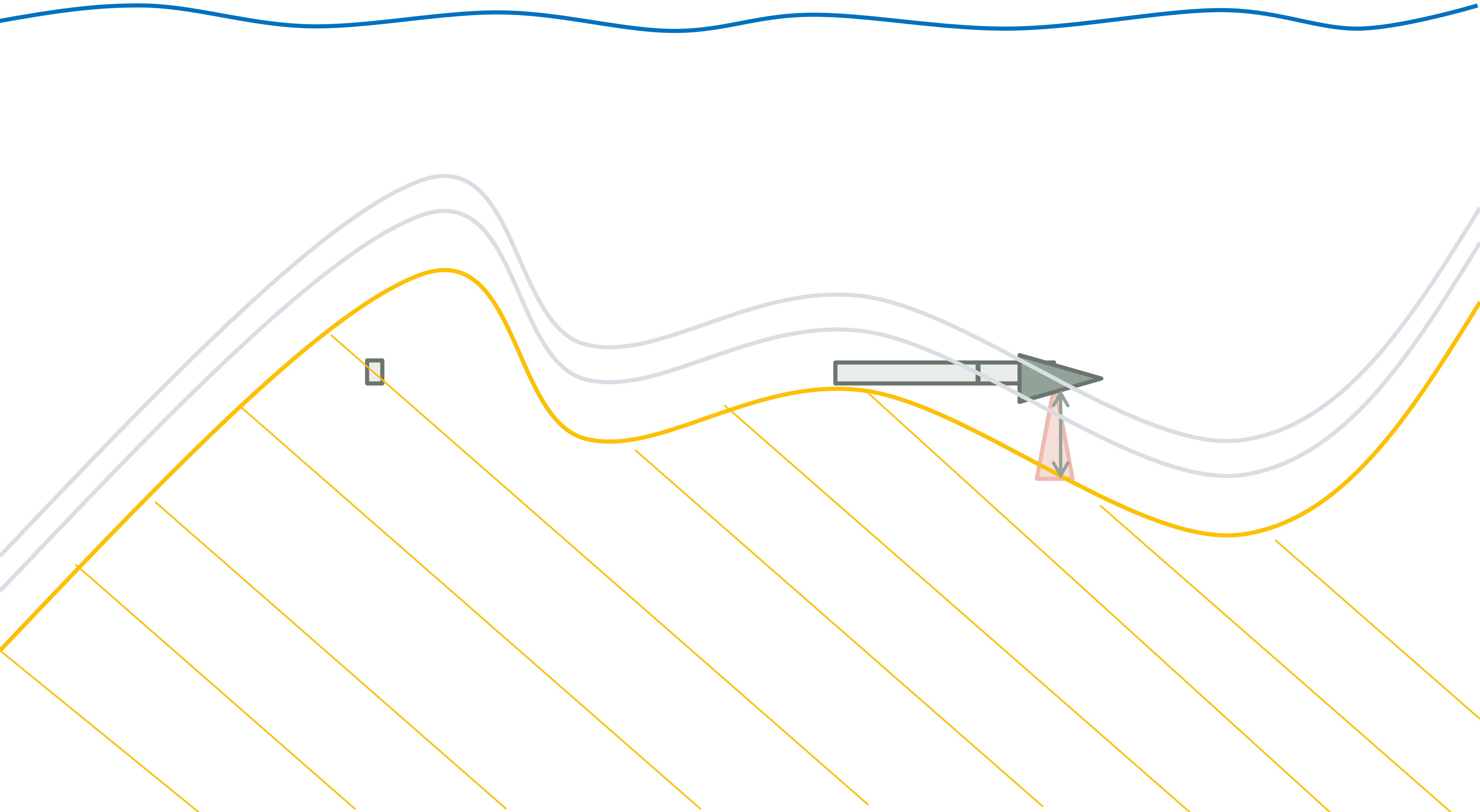

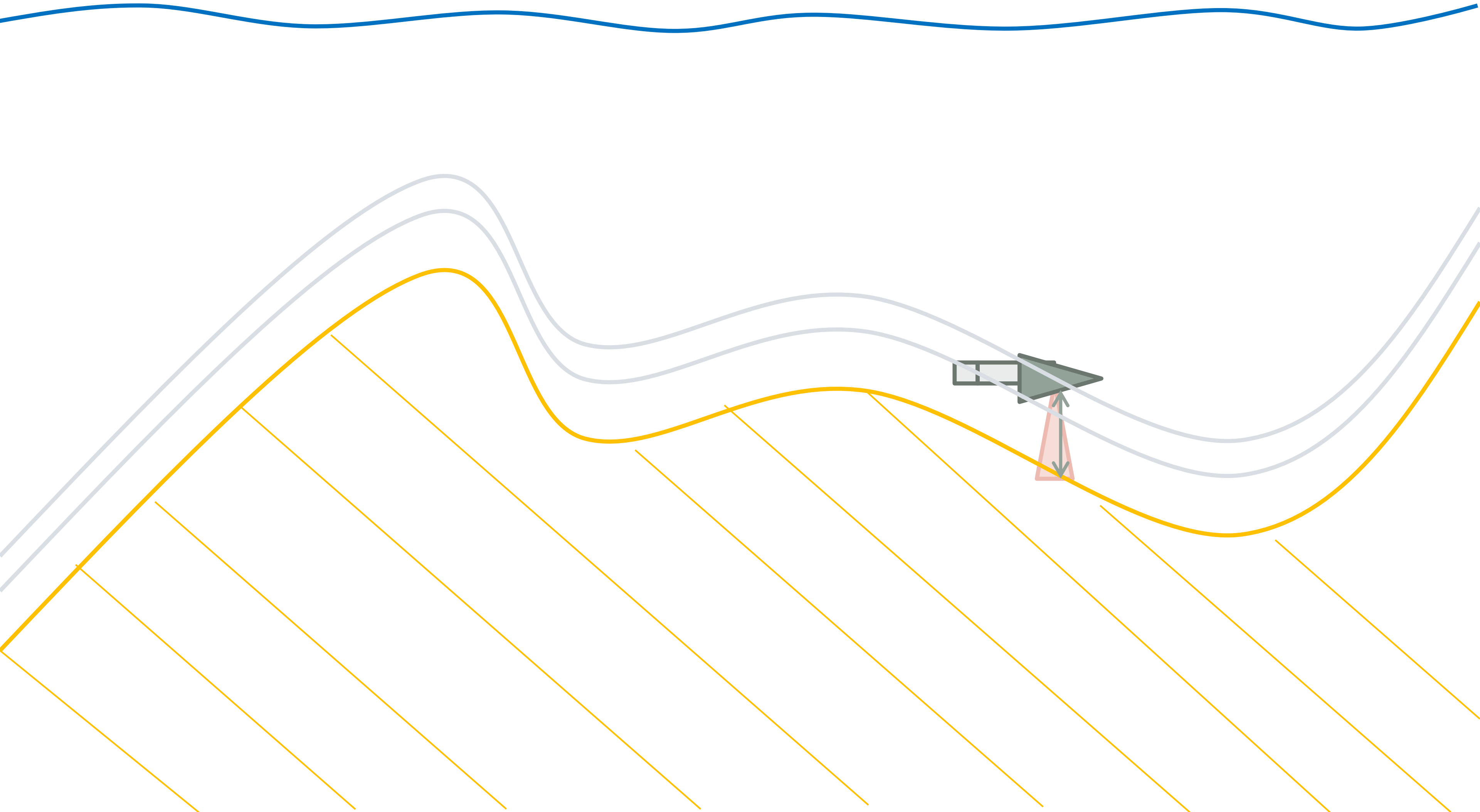

- After some time, if
  - The motion sensors are precise enough
  - The topology of the seafloor is rich enough
- We should converge to the correct position

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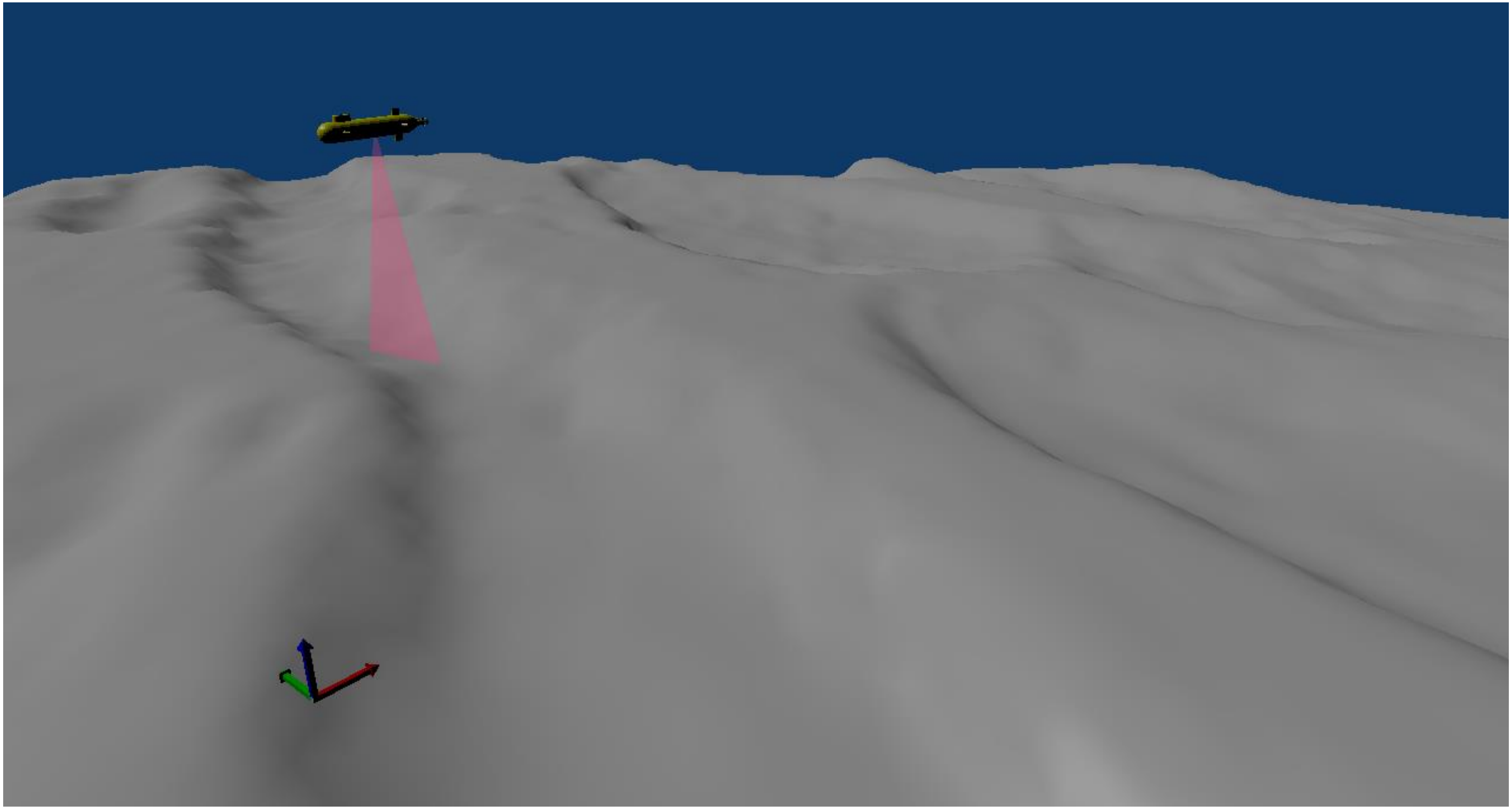
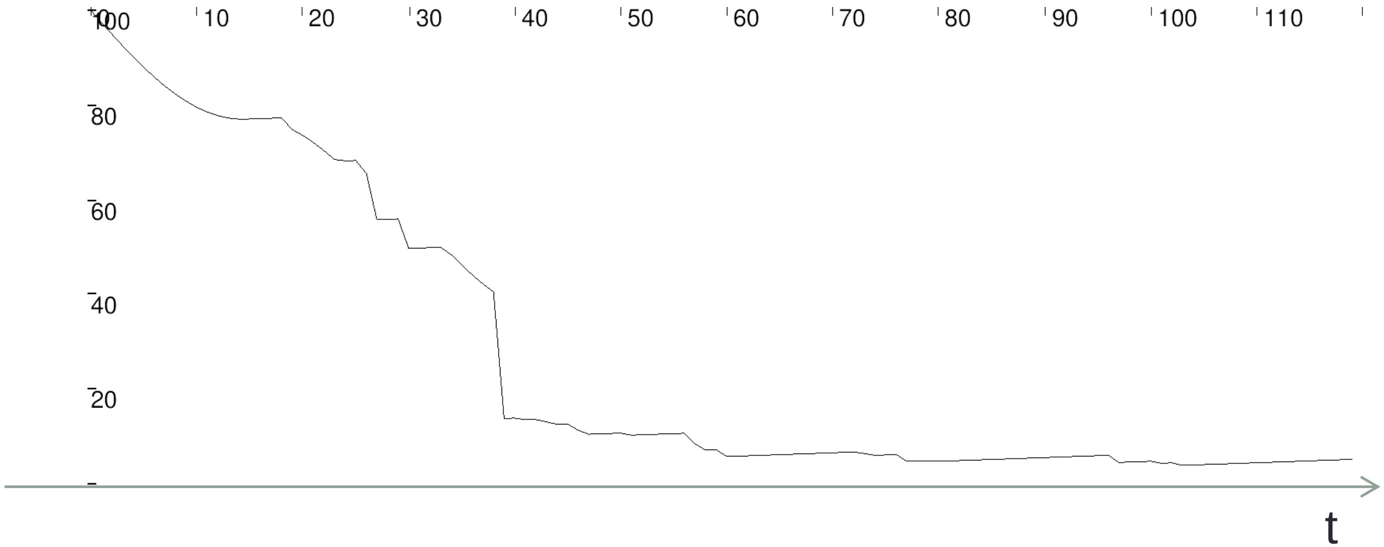
- We simulated a navigation in a 3D environment
- The AUV follows an helicoïdal trajectory with a radius R=15m at a speed V=1,57m/s
- The speed is measured with a confidence of +-4cm/s
- The depth is measured with a confidence of +-10cm
- The altitude is measured with a confidence of +-10cm
- The initial position of the AUV is totally unknown
- The map dimension is 100m X 100m



- Cf: video

Width of the uncertainty corridors

Uncertainty (m) along the x axis



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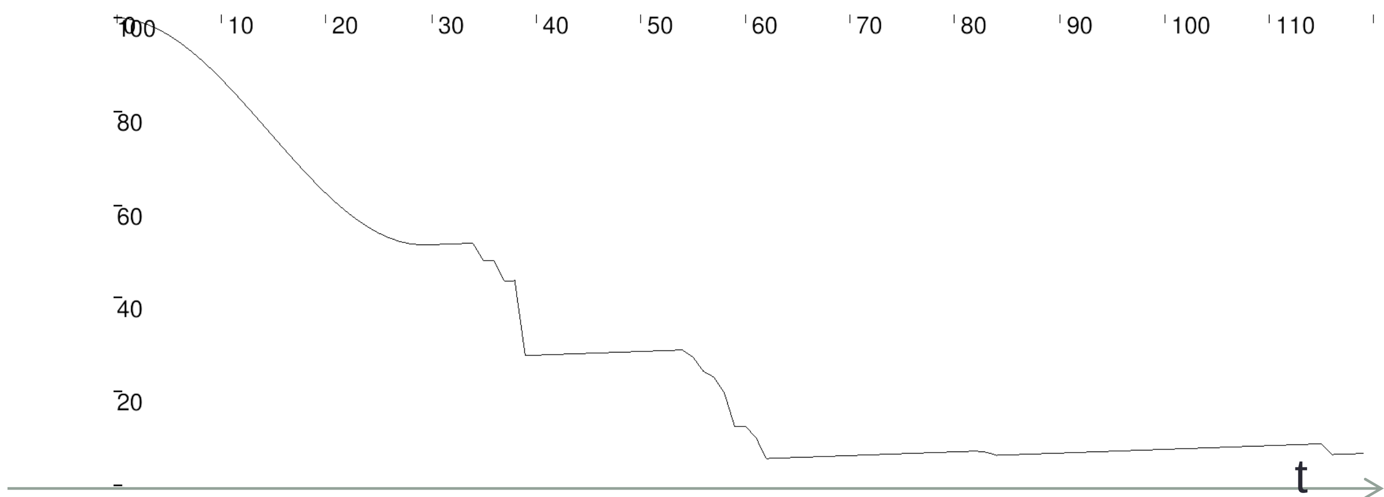

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- We presented a bathymetry-based localization method for an AUV using interval analysis
- The algorithm is:
  - Fast
  - Guaranteed
- However, storing the epigraph of the map requires a lot of memory, and might not be suited for very large maps (>1GB)
- Propagating information back in time could greatly enhance the contractions
- Can we chose a trajectory that will help us converge quickly?

- Questions?