Localization of an AUV using interval analysis

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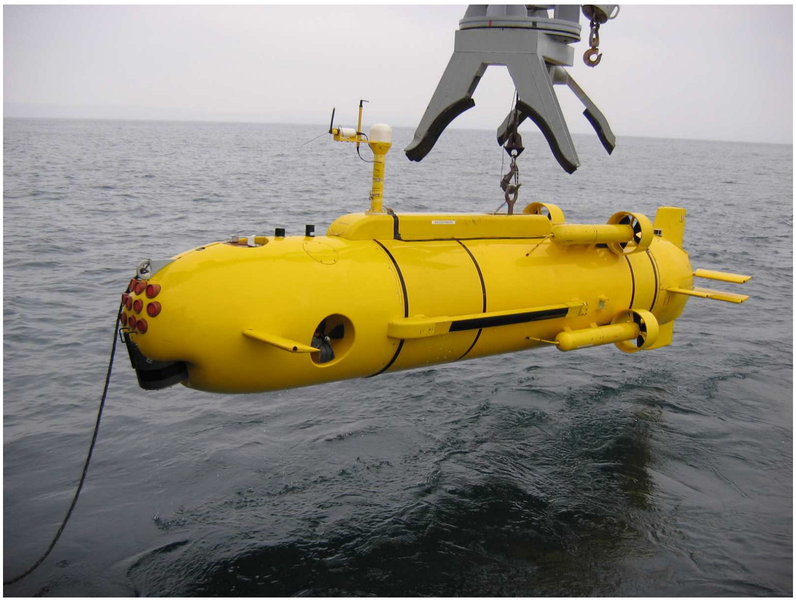
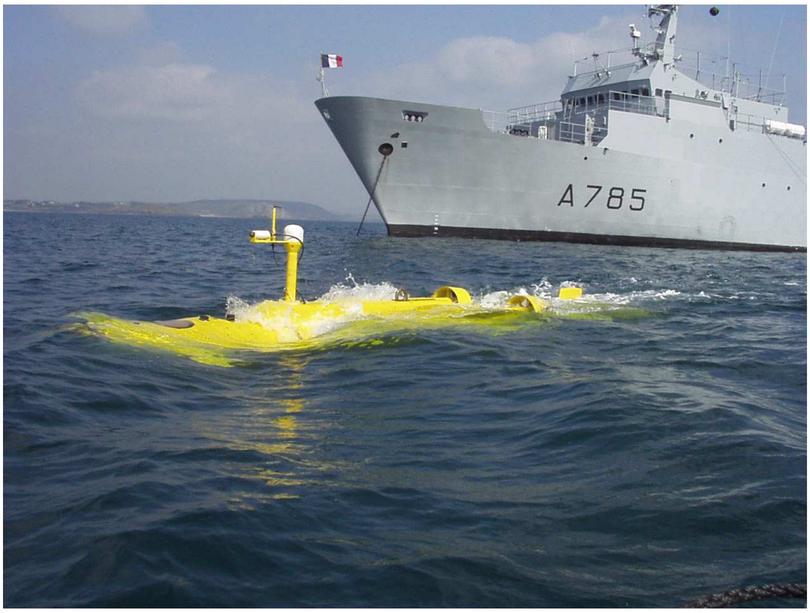
February 12, 2007.

1 The Redermor



The *Redermor*, GESMA



The *Redermor* at the surface

Show simulation

2 SLAM

Localization

When the mark locations are known, the localization problem is a state estimation problem.

The model of the system is

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x}) \end{cases}$$

where $\mathbf{x} = (x, y, z, \phi, \theta, \psi, v)$.

SLAM (simultaneous localization and mapping)

The mark locations are unknown.

Determine the location of the robot as well as the location of the marks.

Why choosing an interval constraint approach ?

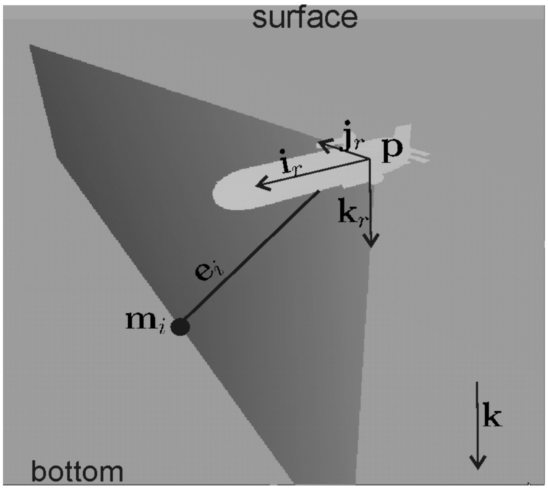
1) A reliable method is needed.
2) The model is nonlinear.
3) The pdf of the noises are unknown.
4) Reliable error bounds are provided by the sensors.
5) A huge number of redundant data are available.

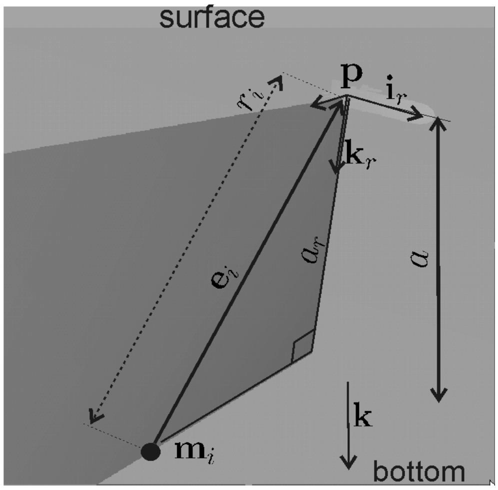
3 Sensors

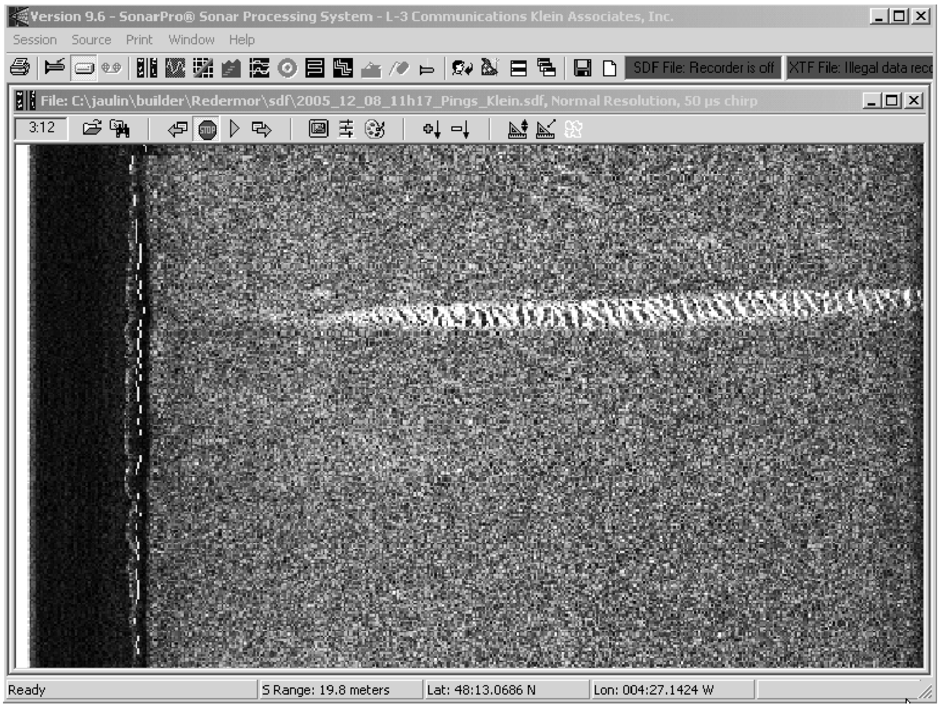
A GPS (Global positioning system), at the surface only.

$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m$$

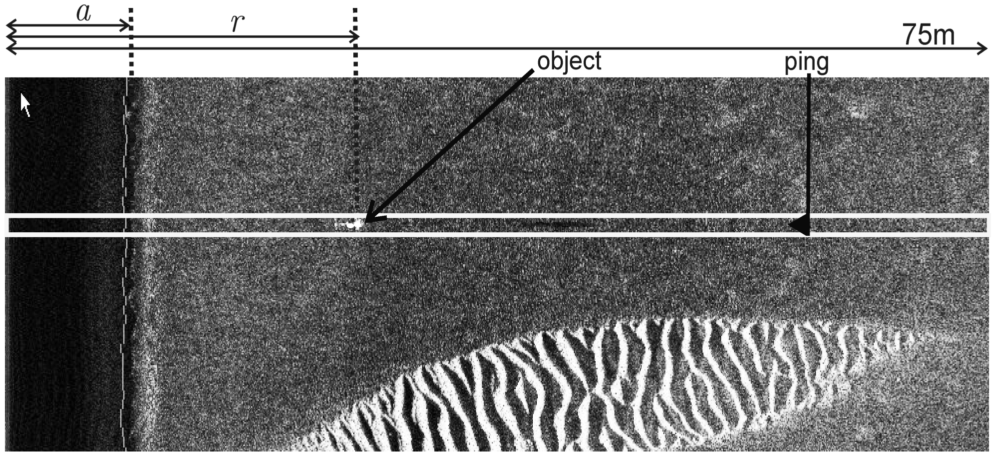
A sonar (KLEIN 5400 side scan sonar). Gives the distance r between the robot to the detected object.







Screenshot of SonarPro



Detection of a mine using SonarPro

A Loch-Doppler. Returns the speed of the robot $\mathbf{v}_r$ and the altitude a of the robot $\pm$ 10cm.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ, the pitch θ and the head ψ.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$

4 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines have been detected by the sonar:

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

5 Constraints satisfaction problem

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t).\mathbf{R}_{\theta}(t).\mathbf{R}_{\varphi}(t),$$

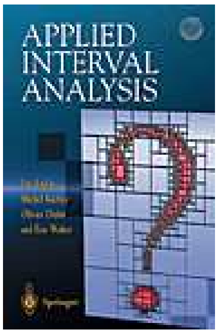
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t)$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^{\mathsf{T}}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5].$$

6 Interval Constraint Propagation



6.1 Interval arithmetic

If $\diamond \in \{+, -, ., /\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \, .$$

For instance,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3].[2, 5] &= [-5, 15], \\ [-1, 3]/[2, 5] &= [-\tfrac{1}{2}, \tfrac{3}{2}]. \end{aligned}$$

If $f \in \{\mathsf{cos},\ \mathsf{sin}, \mathsf{sqr},\ \mathsf{sqrt},\ \mathsf{log},\ \mathsf{exp},\ \dots\}$

$$f\left([x]\right) = \left[\{f(x) \mid x \in [x]\}\right].$$

For instance,

$$\begin{aligned} \mathsf{sin}\left([0,\pi]\right) &= [0,1], \\ \mathsf{sqr}\left([-1,3]\right) &= [-1,3]^2 = [0,9], \\ \mathsf{sqrt}\left([-10,4]\right) &= \sqrt{[-10,4]} = [0,2]. \end{aligned}$$

6.2 Constraint contraction

Let x, y, z be 3 variables such that

$$\left\{\begin{array}{c} x \in [-\infty, 5] \\ y \in [-\infty, 4] \\ z \in [6, \infty] \\ z = x + y \end{array}\right. \Leftrightarrow \left\{\begin{array}{c} x \in [2, 5] \\ y \in [1, 4] \\ z \in [6, 9] \\ z = x + y \end{array}\right.$$

The values < 2 for x, < 1 for y and > 9 for z are said to be inconsistent.

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$,

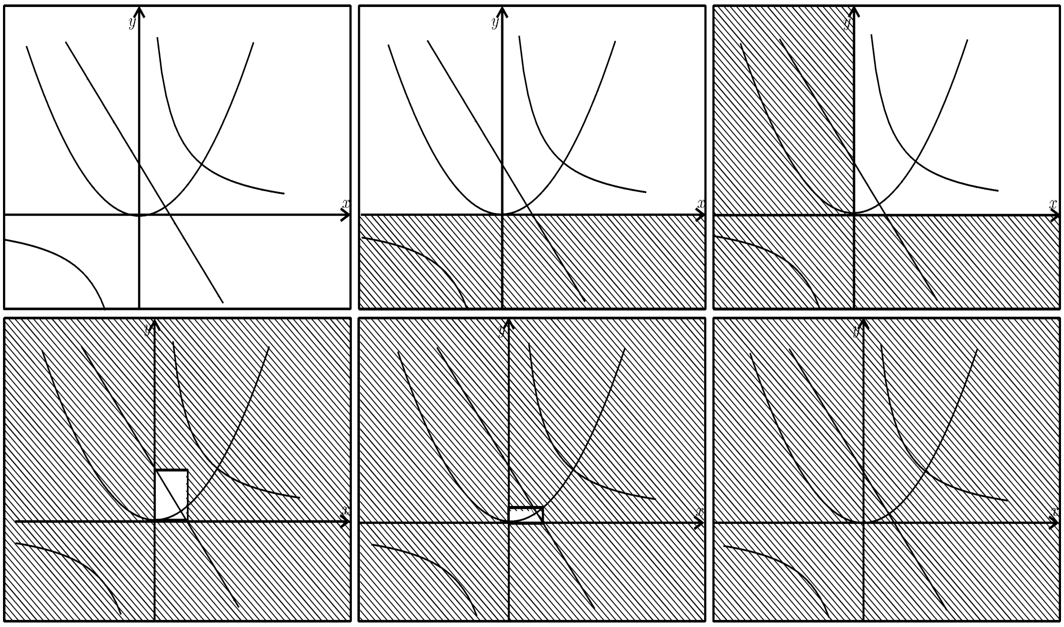
$$\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}$$

6.3 Constraint propagation

Consider the three constraints

$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$

To each variable we assign the domain $[-\infty, \infty]$.
Constraint propagation amounts to contract all constraints until equilibrium.


y
x

$$
\begin{aligned}
(C_1) \Rightarrow \quad & y \in \quad [-\infty,\infty]^2 \;=\; [0,\infty] \\
(C_2) \Rightarrow \quad & x \in \quad 1/[0,\infty] \;= [0,\infty] \\
(C_3) \Rightarrow \quad & y \in \quad [0,\infty] \;\cap\; ((-2)\,.[0,\infty] + 1) \\
& \qquad\;\; = [0,\infty] \;\cap\; ([-\infty,1]) = [0,1] \\
& x \in \quad [0,\infty] \;\cap\; (-[0,1]/2 + 1/2) = [0,\tfrac{1}{2}] \\
(C_1) \Rightarrow \quad & y \in \quad [0,1] \cap [0,1/2]^2 = [0,1/4] \\
(C_2) \Rightarrow \quad & x \in \quad [0,1/2] \;\cap\; 1/[0,1/4] = \emptyset \\
& y \in \quad [0,1/4] \cap 1/\emptyset = \emptyset
\end{aligned}
$$

6.4 Decomposition

For more complex constraints, we have to perform a decomposition.

$$x + \mathsf{sin}(y) - xz \leq \mathsf{0},$$
$$x \in [-1, 1], y \in [-1, 1], z \in [-1, 1]$$

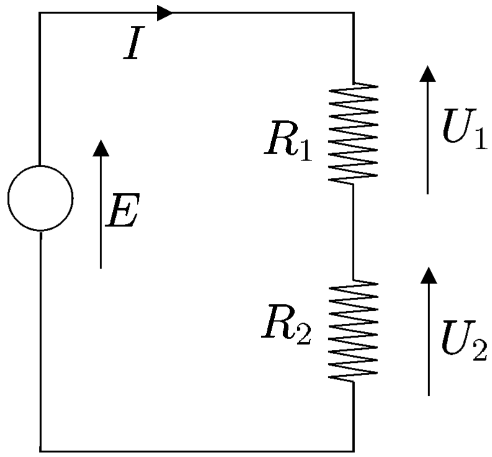
can be decomposed into

$$\begin{cases} a = \mathsf{sin}(y) & x \in [-1, 1] \quad a \in [-\infty, \infty] \\ b = x + a & y \in [-1, 1] \quad b \in [-\infty, \infty] \\ c = xz & z \in [-1, 1] \quad c \in [-\infty, \infty] \\ b - c = d & \qquad\qquad\quad\; d \in [-\infty, \mathsf{0}] \end{cases}$$

Matrix constraints should also be decomposed

$$\begin{aligned}\mathbf{A}.\mathbf{x} &= \mathbf{b} \Leftrightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ &\Leftrightarrow \left\{ \begin{array}{rcl} a_{11}x_1 + a_{12}x_2 &=& b_1 \\ a_{21}x_1 + a_{22}x_2 &=& b_2 \end{array} \right.\end{aligned}$$

6.5 Estimation problem



Constraints

$$
\begin{aligned}
P &= EI;\ E = (R_1 + R_2)\, I; \\
U_1 &= R_1 I;\ U_2 = R_2 I;\ E = U_1 + U_2.
\end{aligned}
$$

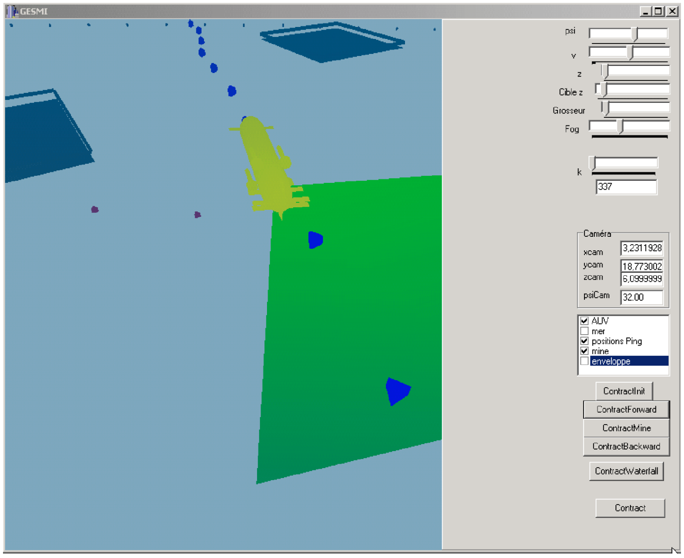
Initial domains

$$
\begin{array}{cc}
R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\
E \in [23, 26]\text{V}, & I \in [4, 8]\text{A}, \\
U_1 \in [10, 11]\text{V}, & U_2 \in [14, 17]\text{V}, \\
P \in [124, 130]\text{W}, &
\end{array}
$$

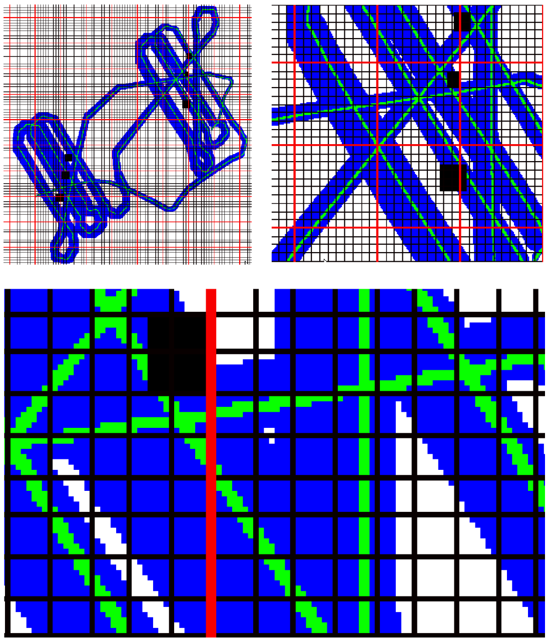
We get the contracted domains

$$
\begin{array}{cc}
R_1 \in [\mathbf{1.84}, \mathbf{2.31}]\,\Omega, & R_2 \in [\mathbf{2.58}, \mathbf{3.35}]\Omega, \\
E \in [\mathbf{24}, 26]\mathrm{V}, & I \in [\mathbf{4.769}, \mathbf{5.417}]\,\mathrm{A}, \\
U_1 \in [10, 11]\mathrm{V}, & U_2 \in [14, \mathbf{16}]\mathrm{V}, \\
P \in [124, 130]\mathrm{W}, &
\end{array}
$$

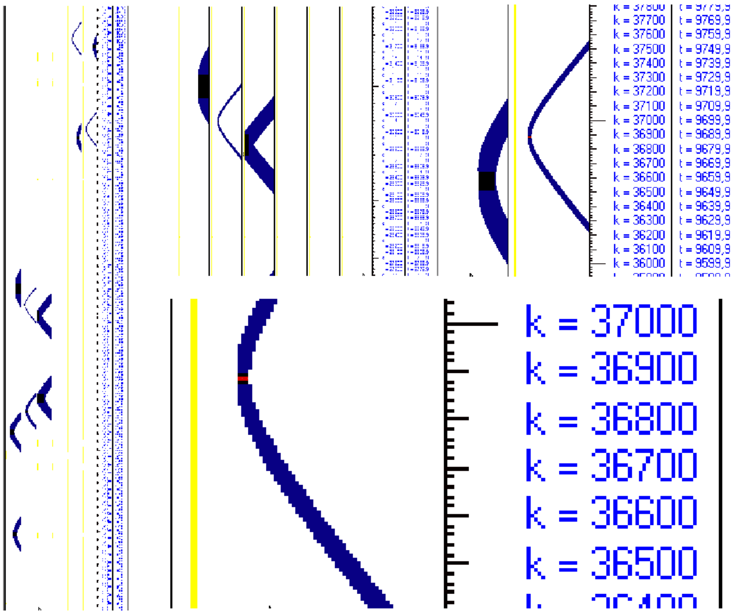
7 GESMI



GESMI (Guaranteed Estimation of Sea Mines with Intervals)



Trajectory reconstructed by GESMI



Waterfall reconstructed by GESMI