ENSTA
BRETAGNE

Combining probabilistic and set membership filters

Interval Extended Kalman Filter

Localisation with Interval Filtering

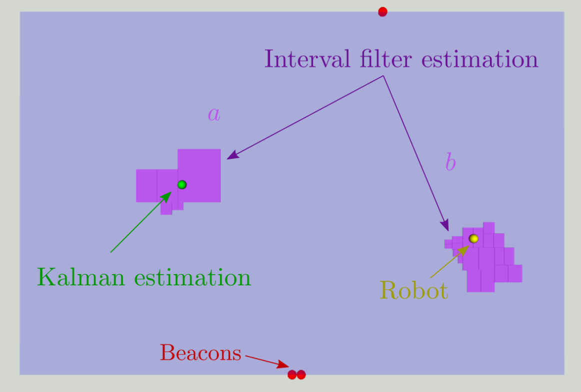
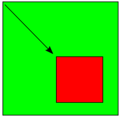
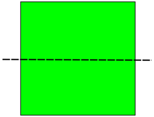


Figure: Interval filter in a static 2D scenario

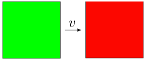
- Contraction



- Subpaving



- Dead reckoning



Contraction to a connected set

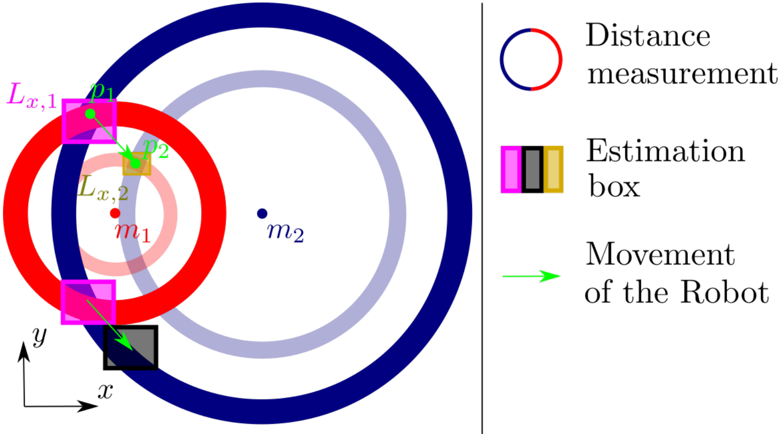


Figure: Situation where the initial set is not connected

Limits of the extended Kalman filter

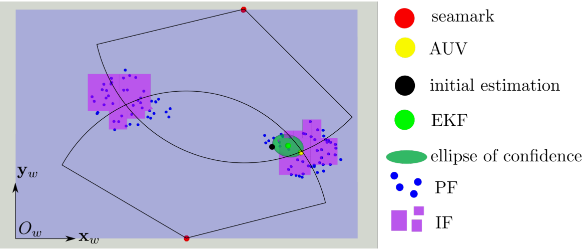


Figure: Interval filter in a static 2D scenario

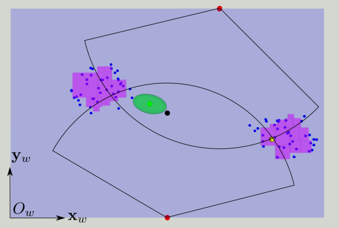


Figure: Interval filter in a static 2D scenario

Extended Kalman filter - 1

Discrete nonlinear state space system:

$$\begin{cases} \boldsymbol{x}_{k+1} &= \boldsymbol{f}_k\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) \\ \boldsymbol{y}_k &= \boldsymbol{g}_k\left(\boldsymbol{x}_k\right) \end{cases} \tag{1}$$

Lineralization at $\bar{\boldsymbol{x}}_k$:

$$\begin{cases} \boldsymbol{x}_{k+1} &\simeq \boldsymbol{f}_k\left(\bar{\boldsymbol{x}}_k, \boldsymbol{u}_k\right) + \frac{\partial \boldsymbol{f}_k(\bar{\boldsymbol{x}}_k, \boldsymbol{u}_k)}{\partial \boldsymbol{x}} \cdot \left(\boldsymbol{x}_k - \bar{\boldsymbol{x}}_k\right) \\ \boldsymbol{y}_k &\simeq \boldsymbol{g}_k\left(\bar{\boldsymbol{x}}_k\right) + \frac{d\boldsymbol{g}_k(\bar{\boldsymbol{x}}_k)}{d\boldsymbol{x}} \cdot \left(\boldsymbol{x}_k - \bar{\boldsymbol{x}}_k\right) \end{cases} \tag{2}$$

with $\boldsymbol{A}_k = \frac{\partial \boldsymbol{f}_k(\bar{\boldsymbol{x}}_k, \boldsymbol{u}_k)}{\partial \boldsymbol{x}}$ and $\boldsymbol{C}_k = \frac{d\boldsymbol{g}_k(\bar{\boldsymbol{x}}_k)}{d\boldsymbol{x}}$

$$\begin{cases} \boldsymbol{x}_{k+1} &\simeq \boldsymbol{A}_k \cdot \boldsymbol{x}_k + \underbrace{\left(\boldsymbol{f}_k\left(\bar{\boldsymbol{x}}_k, \boldsymbol{u}_k\right) - \boldsymbol{A}_k \cdot \bar{\boldsymbol{x}}_k\right)}_{\boldsymbol{v}_k} \\ \underbrace{\left(\boldsymbol{y}_k - \boldsymbol{g}_k\left(\bar{\boldsymbol{x}}_k\right) + \boldsymbol{C}_k \cdot \bar{\boldsymbol{x}}_k\right)}_{\boldsymbol{z}_k} &\simeq \boldsymbol{C}_k \cdot \boldsymbol{x}_k \end{cases} \tag{3}$$

Extended Kalman filter - 2

with $\bar{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k$:

$$
\begin{aligned}
\mathbf{C}_k &= \frac{\partial \mathbf{g}_k(\mathbf{x}_k)}{\partial \mathbf{x}} && \text{(observation matrix)}\\
\tilde{\mathbf{z}}_k &= \boldsymbol{z}_k - \boldsymbol{C}_k \cdot \hat{\boldsymbol{x}}_k = \mathbf{y}_k - \mathbf{g}_k(\hat{\mathbf{x}}_k) && \text{(innovation)}\\
\mathbf{S}_k &= \mathbf{C}_k \Gamma_k \mathbf{C}_k^T + \Gamma_{\beta_k} && \text{(innovation covariance)}\\
\mathbf{K}_k &= \Gamma_k \mathbf{C}_k^T \mathbf{S}_k^{-1} && \text{(Kalman gain)}\\
\mathbf{A}_k &= \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}(\mathbf{x}_k + \mathbf{K}_k \mathbf{z}_k, \mathbf{u}_k) && \text{(evolution matrix)}\\
\hat{\mathbf{x}}_{k+1} &= \boldsymbol{A}_k \cdot (\hat{\mathbf{x}}_k + \mathbf{K}_k \mathbf{z}_k) + \boldsymbol{v}_k = \mathbf{f}_k(\hat{\mathbf{x}}_k + \mathbf{K}_k \mathbf{z}_k, \mathbf{u}_k) && \text{(predicted estimation)}\\
\Gamma_{k+1} &= \mathbf{A}_k(\mathbf{I} - \mathbf{K}_k \mathbf{C}_k)\Gamma_k \mathbf{A}_k^T + \Gamma_{\alpha_k} && \text{(predicted covariance)}
\end{aligned} \tag{4}
$$

Interval extended Kalman filter

with $\bar{\boldsymbol{x}}_k$ from interval filtering:

$$
\begin{aligned}
\mathbf{C}_k &= \frac{\partial \mathbf{g}_k(\overline{\mathbf{x}}_k)}{\partial \mathbf{x}} && \text{(observation matrix)} \\
\mathbf{z}_k &= \mathbf{y}_k - \mathbf{g}_k(\hat{\mathbf{x}}_k) + \mathbf{C}_k \cdot (\overline{\mathbf{x}}_k - \hat{\mathbf{x}}_k) && \text{(innovation)} \\
\mathbf{S}_k &= \mathbf{C}_k \Gamma_k \mathbf{C}_k^T + \Gamma_{\beta_k} && \text{(covariance of the innovation)} \\
\mathbf{K}_k &= \Gamma_k \mathbf{C}_k^T \mathbf{S}_k^{-1} && \text{(Kalman gain)} \\
\mathbf{A}_k &= \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}(\overline{\mathbf{x}}_k + \mathbf{K}_k \mathbf{z}_k, \mathbf{u}_k) && \text{(evolution matrix)} \\
\hat{\mathbf{x}}_{k+1} &= \mathbf{f}_k(\hat{\mathbf{x}}_k + \mathbf{K}_k \mathbf{z}_k, \mathbf{u}_k) + \mathbf{A}_k(\hat{\mathbf{x}}_k - \overline{\mathbf{x}}_k) && \text{(predicted estimation)} \\
\Gamma_{k+1} &= \mathbf{A}_k(\mathbf{I} - \mathbf{K}_k \mathbf{C}_k)\Gamma_k \mathbf{A}_k^T + \Gamma_{\alpha_k} && \text{(predicted covariance)}
\end{aligned}
\tag{5}
$$

Test-cases

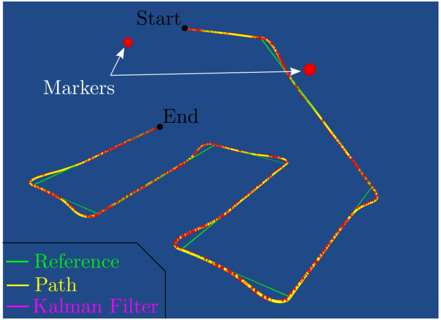


Figure: Precise localisation with the Extended Kalman Filter

Figure: Photo of the Riptide AUV

Control strategy

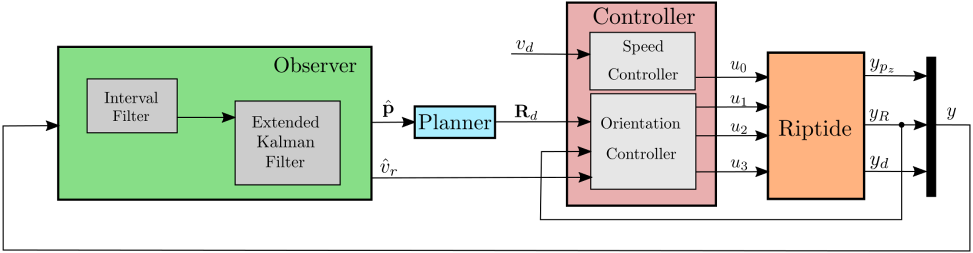


Figure: Block diagram fo the system

Modelling of the system

The skew-symmetric matrix:

$$\mathbf{w}\wedge = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix} \quad (6)$$

Differential equations of the system:

$$\begin{cases} \dot{\mathbf{p}} & = \mathbf{R} \cdot (v_r \; 0 \; 0)^T \\ \dot{\mathbf{R}} & = \mathbf{R} \cdot (\mathbf{w}_r \wedge) \\ \dot{v}_r & = q_1 \cdot u_0^2 - q_2 \cdot v_r \cdot |v_r| \\ \mathbf{w}_r & = v_r \cdot \mathbf{B} \cdot (u_1 \; u_2 \; u_3)^T \end{cases} \quad (7)$$

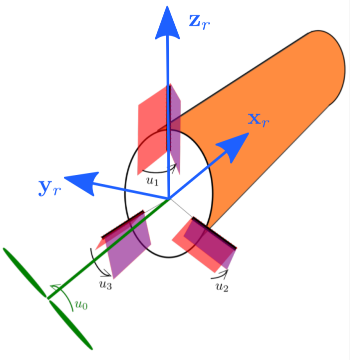


Figure: System's inputs

Controller design

Speed controller:

$$u_0 = \sqrt{\frac{q_2}{q_1}} v_d \tag{8}$$

Orientation error:

$$\begin{cases} \mathbf{e}_w & = \mathbf{R}^T \cdot \wedge^{-1} \log(\mathbf{R}_d \cdot \mathbf{R}^T) \\ \dot{\mathbf{e}}_w & = -\mathbf{w}_r = -\mathbf{K} \cdot \mathbf{e}_w \end{cases} \tag{9}$$

Orientation controller:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \frac{1}{v_r} \mathbf{B}^{-1} \mathbf{w}_r = \frac{1}{v_r} \mathbf{B}^{-1} \cdot \mathbf{K} \cdot \mathbf{e}_w \tag{10}$$

Planner Design

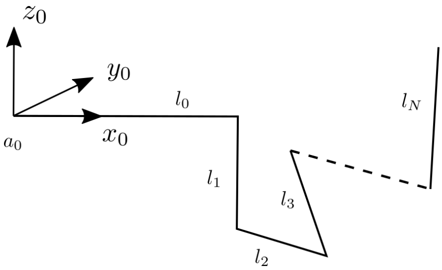


Figure: Lines to follow

$$\begin{cases} \mathbf{R}_d & = e^{\theta_e \cdot \mathbf{w}_e \wedge} \cdot \mathbf{R}_j \\ \theta_e & = \arctan(k_e \cdot e) \\ \mathbf{w}_e & = \mathbf{x}_j \wedge (\overrightarrow{MP}/MP) \\ P & = \mathbf{a}_j + < \overrightarrow{\mathbf{a}_j M} | \mathbf{x}_j > \cdot \mathbf{x}_j \end{cases} \tag{11}$$

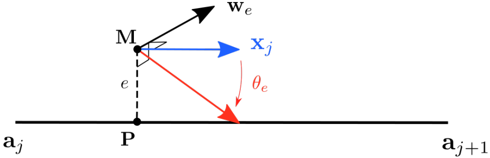


Figure: Rotation towards the line

Results in simulation - EKF vs IEKF - 1

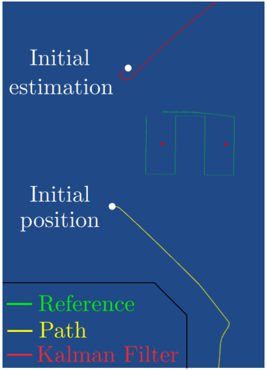


Figure: Failure of the Extended Kalman Filter

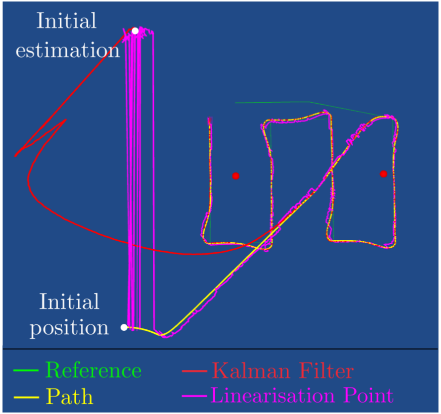


Figure: Same scenario with the IEKF

Results in simulation - EKF vs IEKF - 2

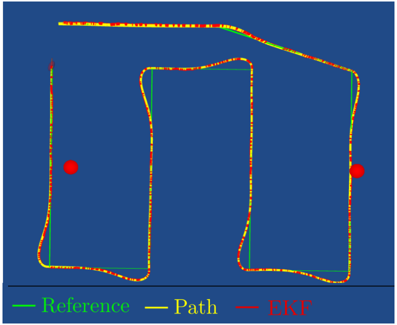


Figure: the Extended Kalman Filter with a good initial estimation

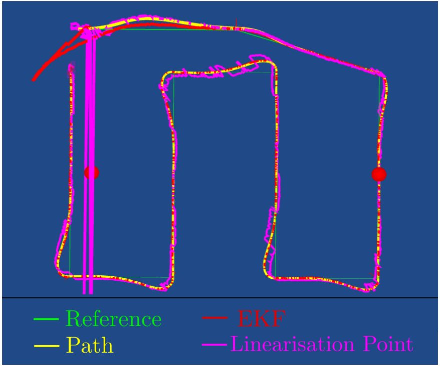


Figure: Same scenario with the IEKF

Results in simulation - IEKF with one marker

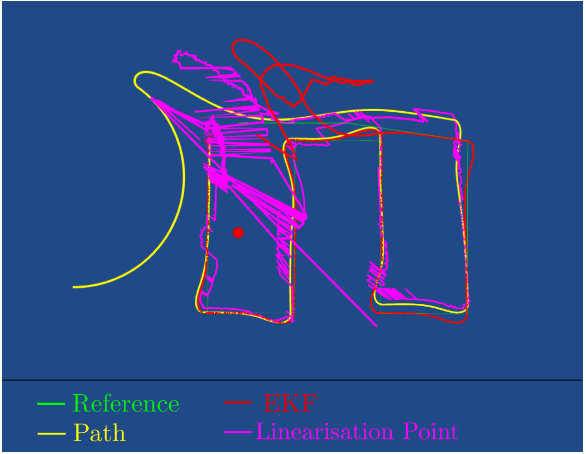


Figure: Bad initial estimation with 1 marker

Conclusion

- Collaboration of Kalman filtering and a set membership approach
- The IEKF has more integrity than the EKF in the simulated scenarios