Localization and Mapping in a complex environment: a constraint programming approach

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L. Jaulin
ENSTA-Brest, LabSTICC.

http://www.ensta-bretagne.fr/jaulin/

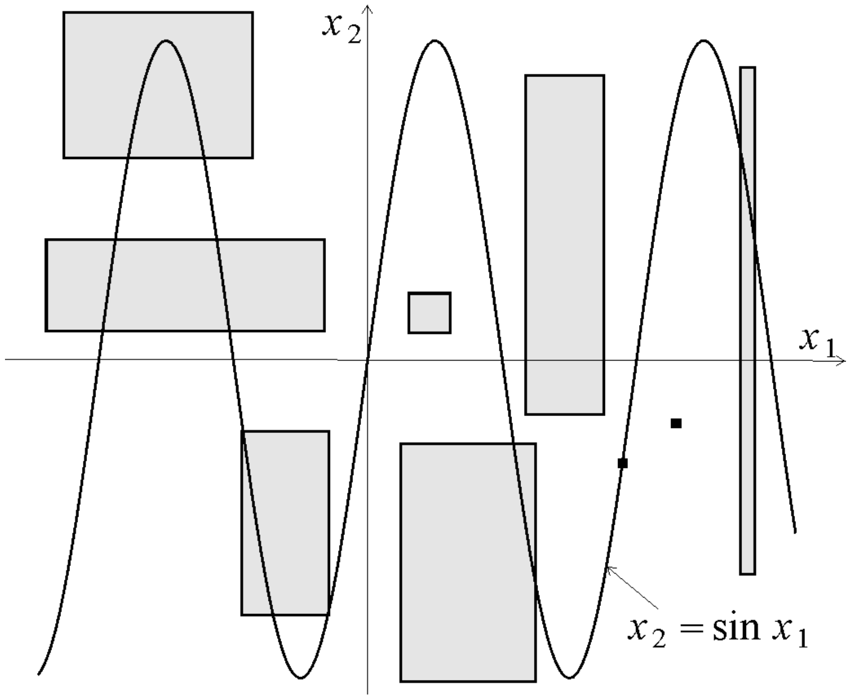
1 Contractors

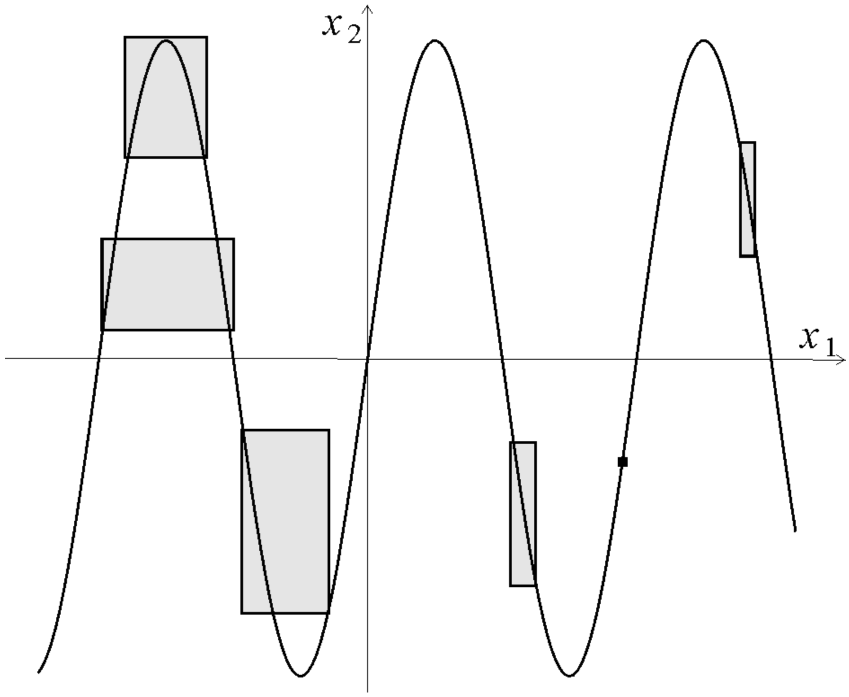
The operator $\mathcal{C} : \mathbb{IR}^n \rightarrow \mathbb{IR}^n$ is a *contractor* for the equation $f(\mathbf{x}) = 0$, if

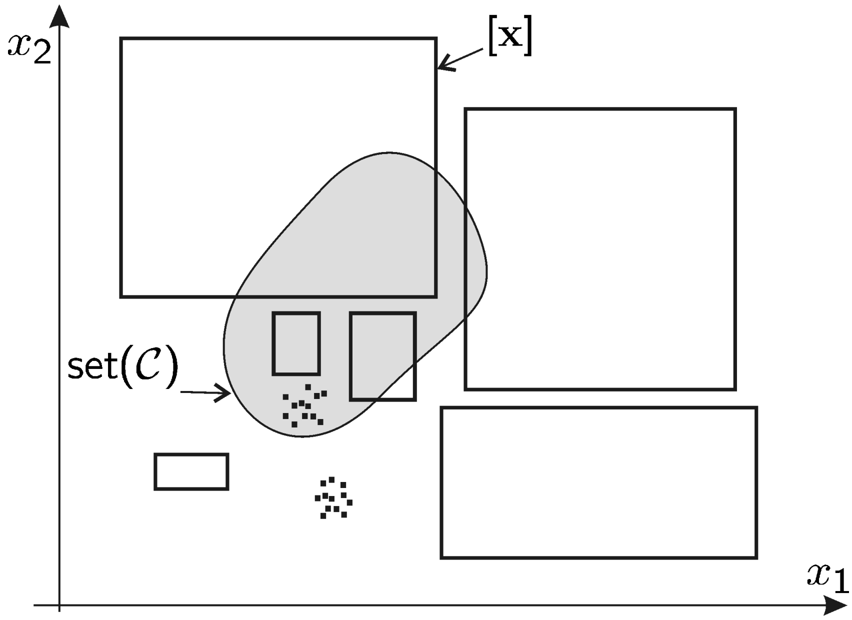
$$\begin{cases} \mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathbf{x} \in [\mathbf{x}] \text{ and } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathcal{C}([\mathbf{x}]) & \text{(consistence)} \end{cases}$$

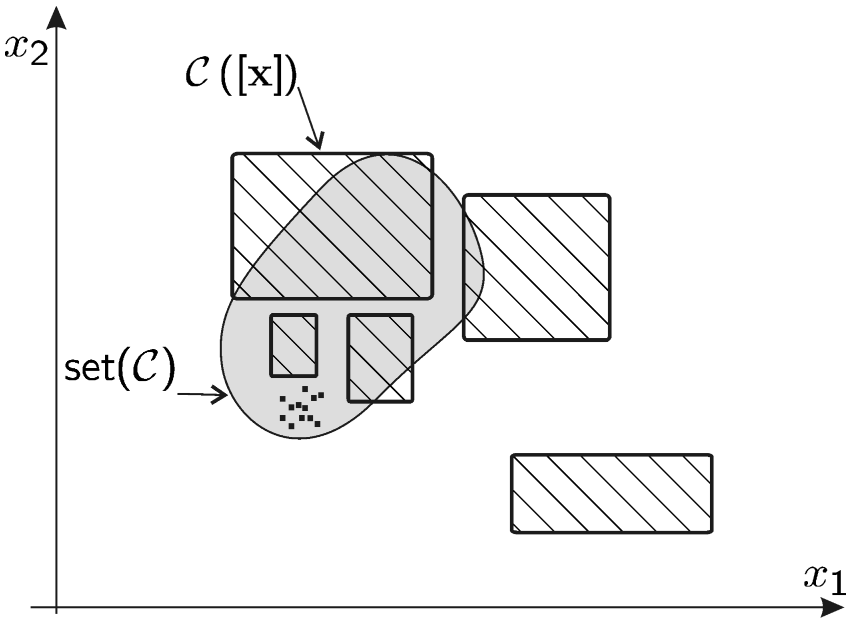
Example. Consider the primitive equation:

$$x_2 = \sin x_1.$$


x_2
x_1
$x_2 = \sin x_1$


x_2
x_1


x_2
$[\mathbf{x}]$
set($\mathcal{C}$)
x_1



x_2
$\mathcal{C}([\mathbf{x}])$
set$(\mathcal{C})$
x_1

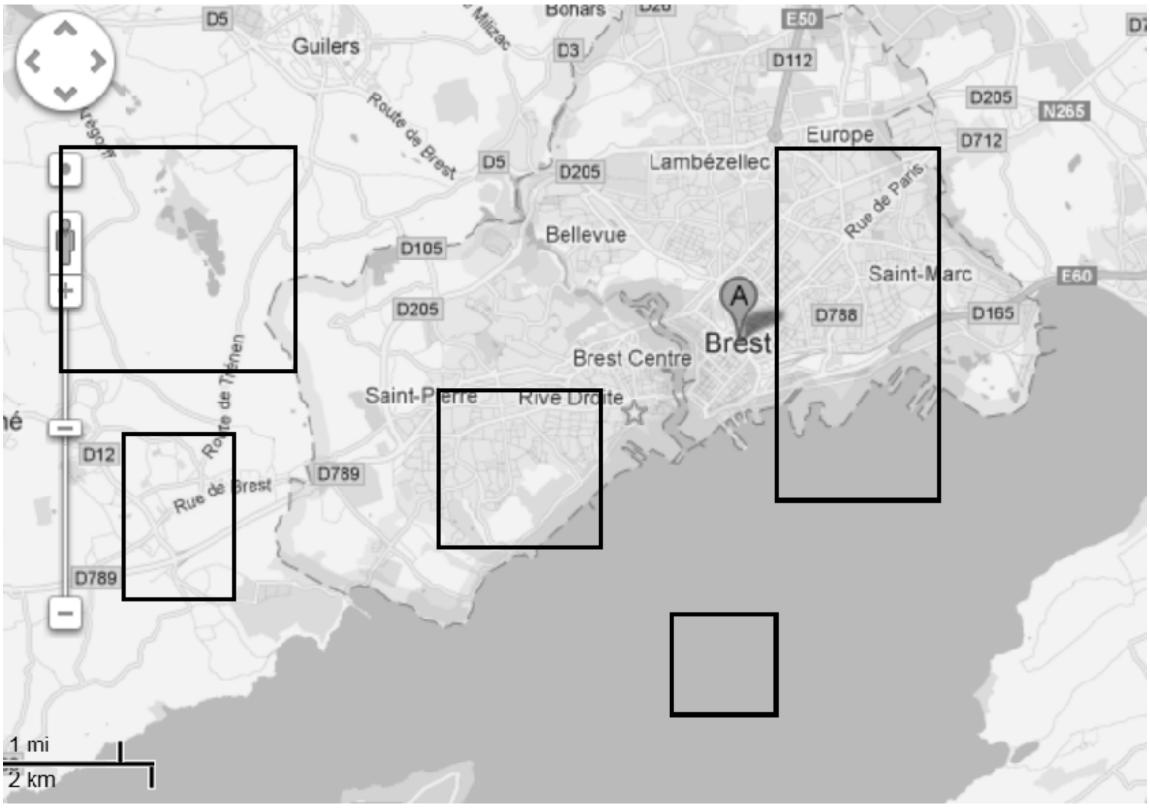
$\mathcal{C}$ is *monotonic* if	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}([\mathbf{x}]) \subset \mathcal{C}([\mathbf{y}])$
$\mathcal{C}$ is *minimal* if	$\mathcal{C}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathsf{set}\,(\mathcal{C})]$
$\mathcal{C}$ is *idempotent* if	$\mathcal{C}\,(\mathcal{C}([\mathbf{x}])) = \mathcal{C}([\mathbf{x}])$
$\mathcal{C}$ is *continuous* if	$\mathcal{C}\,(\mathcal{C}^{\infty}([\mathbf{x}])) = \mathcal{C}^{\infty}([\mathbf{x}])$.

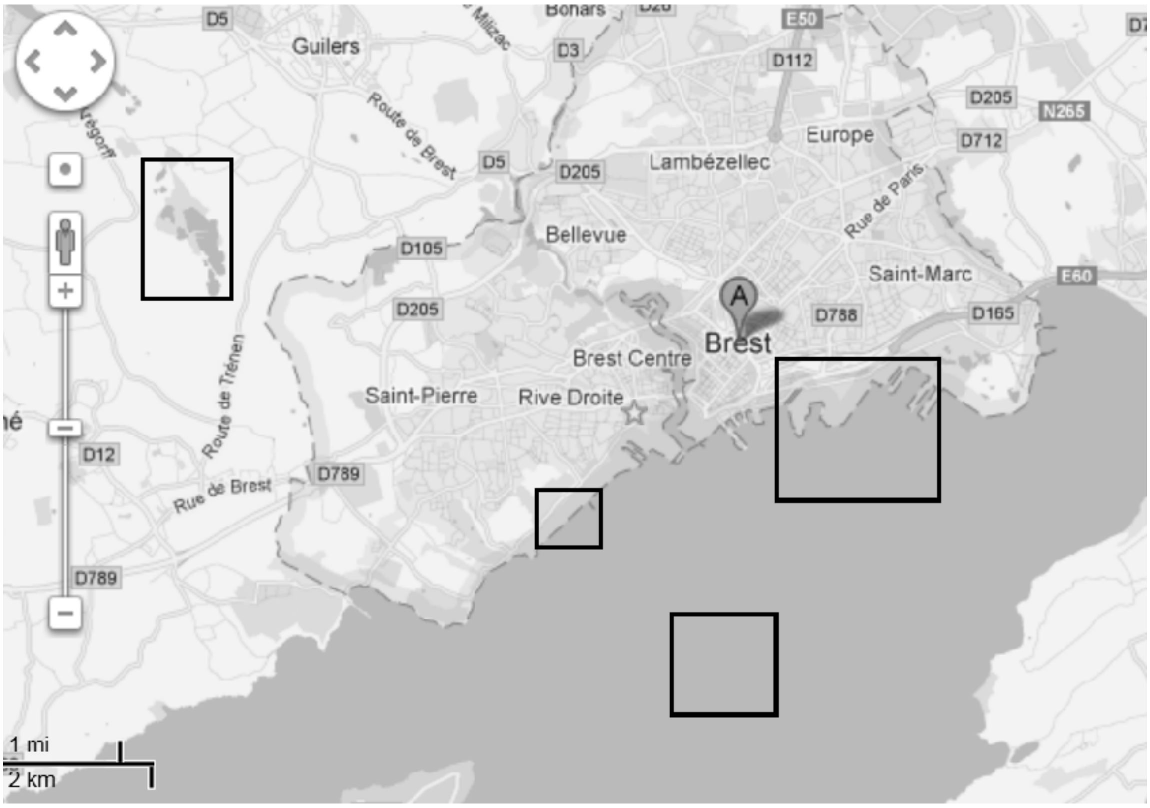
Contractor algebra

intersection	$(\mathcal{C}_1 \cap \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1([\mathbf{x}]) \cap \mathcal{C}_2([\mathbf{x}])$
union	$(\mathcal{C}_1 \cup \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} [\mathcal{C}_1([\mathbf{x}]) \cup \mathcal{C}_2([\mathbf{x}])]$
composition	$(\mathcal{C}_1 \circ \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1(\mathcal{C}_2([\mathbf{x}]))$
reiteration	$\mathcal{C}^\infty \overset{\text{def}}{=} \mathcal{C} \circ \mathcal{C} \circ \mathcal{C} \circ \dots$

Contractor on images

The robot with coordinates (x_1, x_2) is in the water.


Guilers
Route de Brest
Lambézellec
Europe
Bellevue
Saint-Marc
Rue de Paris
Brest
Brest Centre
Saint-Pierre
Rive Droite
Route de Trénen
Rue de Brest
D5
D3
E50
D112
D205
N265
D712
D105
E60
D788
D165
D12
D789
1 mi
2 km



Guilers
Bohars
Route de Brest
D5
D3
D26
E50
D112
D205
N265
Europe
D712
Lambézellec
D205
Bellevue
Rue de Paris
D105
Saint-Marc
E60
D205
Brest
D788
D165
Brest Centre
Saint-Pierre
Rive Droite
Route de Trénen
D12
D789
Rue de Brest
D789
1 mi
2 km

Building contractors for equations

Consider the primitive equation

$$x_1 + x_2 = x_3$$

with $x_1 \in [x_1]$, $x_2 \in [x_2]$, $x_3 \in [x_3]$.

We have

$$
\begin{array}{llll}
x_3 = x_1 + x_2 \Rightarrow & x_3 \in & [x_3] \cap ([x_1] + [x_2]) & // \text{ forward} \\
x_1 = x_3 - x_2 \Rightarrow & x_1 \in & [x_1] \cap ([x_3] - [x_2]) & // \text{ backward} \\
x_2 = x_3 - x_1 \Rightarrow & x_2 \in & [x_2] \cap ([x_3] - [x_1]) & // \text{ backward}
\end{array}
$$

The contractor associated with $x_1 + x_2 = x_3$ is thus

$$\mathcal{C}\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} = \begin{pmatrix} [x_1] \cap ([x_3] - [x_2]) \\ [x_2] \cap ([x_3] - [x_1]) \\ [x_3] \cap ([x_1] + [x_2]) \end{pmatrix}$$

2 Solver

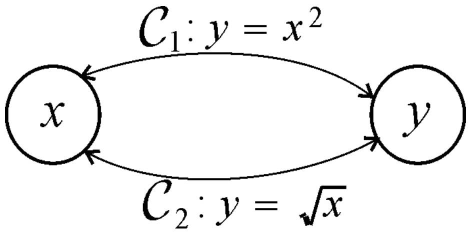
Example 1. Solve the system

$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

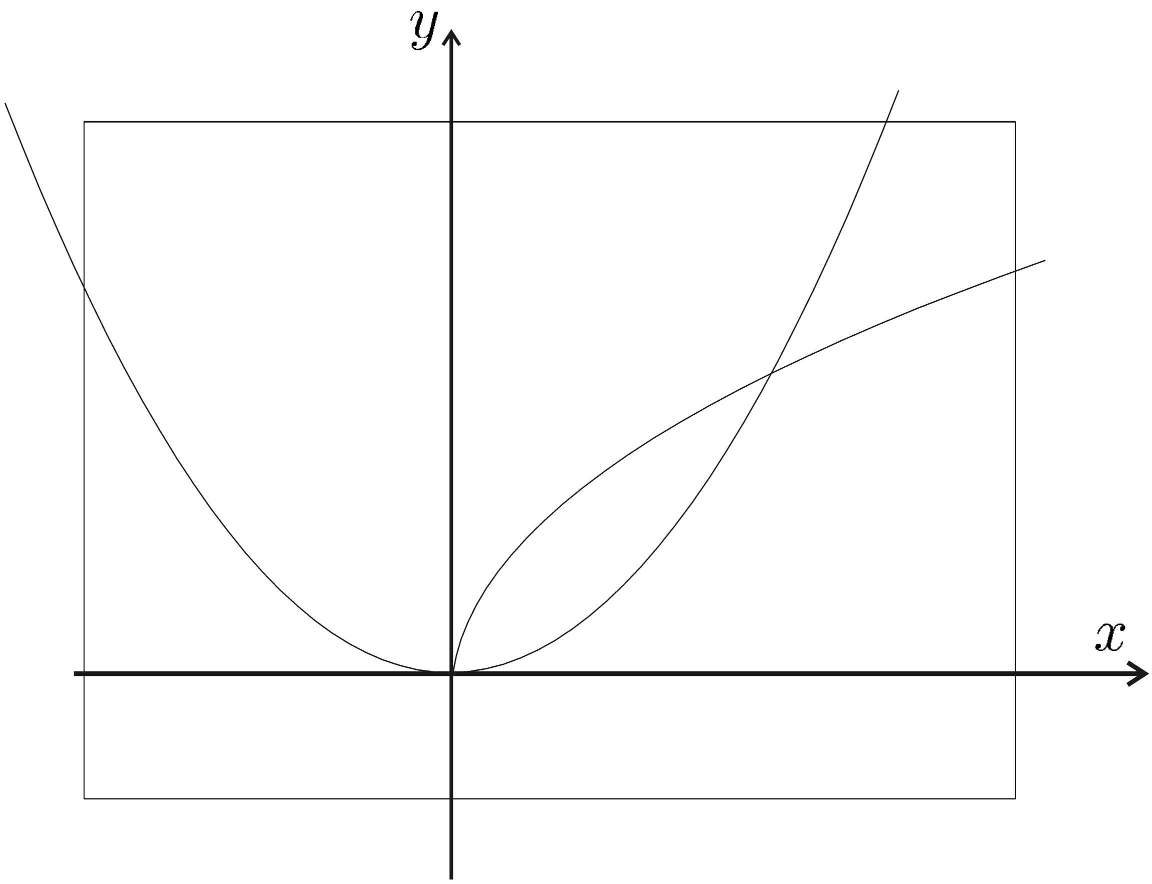
We build two contractors

$$\mathcal{C}_1 : \left\{ \begin{array}{l} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{array} \right. \quad \text{associated to } y = x^2$$

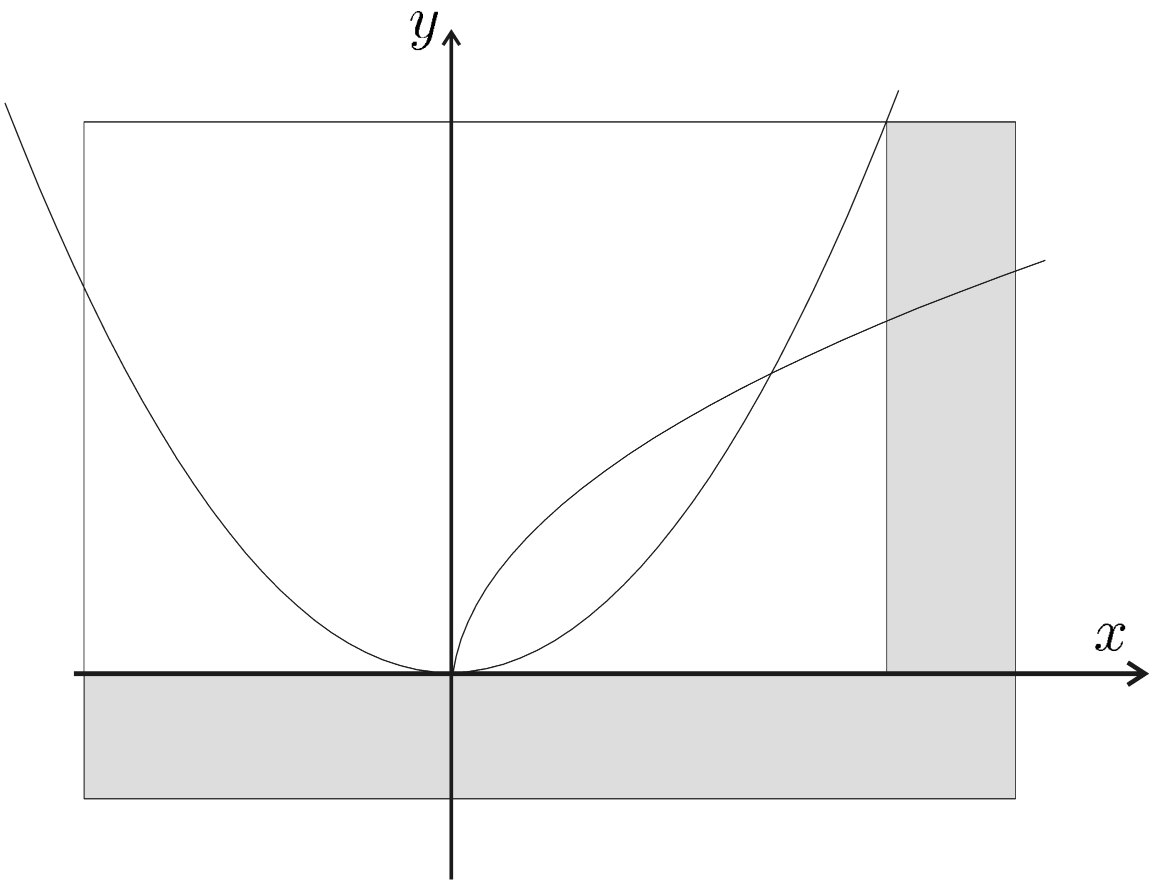
$$\mathcal{C}_2 : \left\{ \begin{array}{l} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{array} \right. \quad \text{associated to } y = \sqrt{x}$$



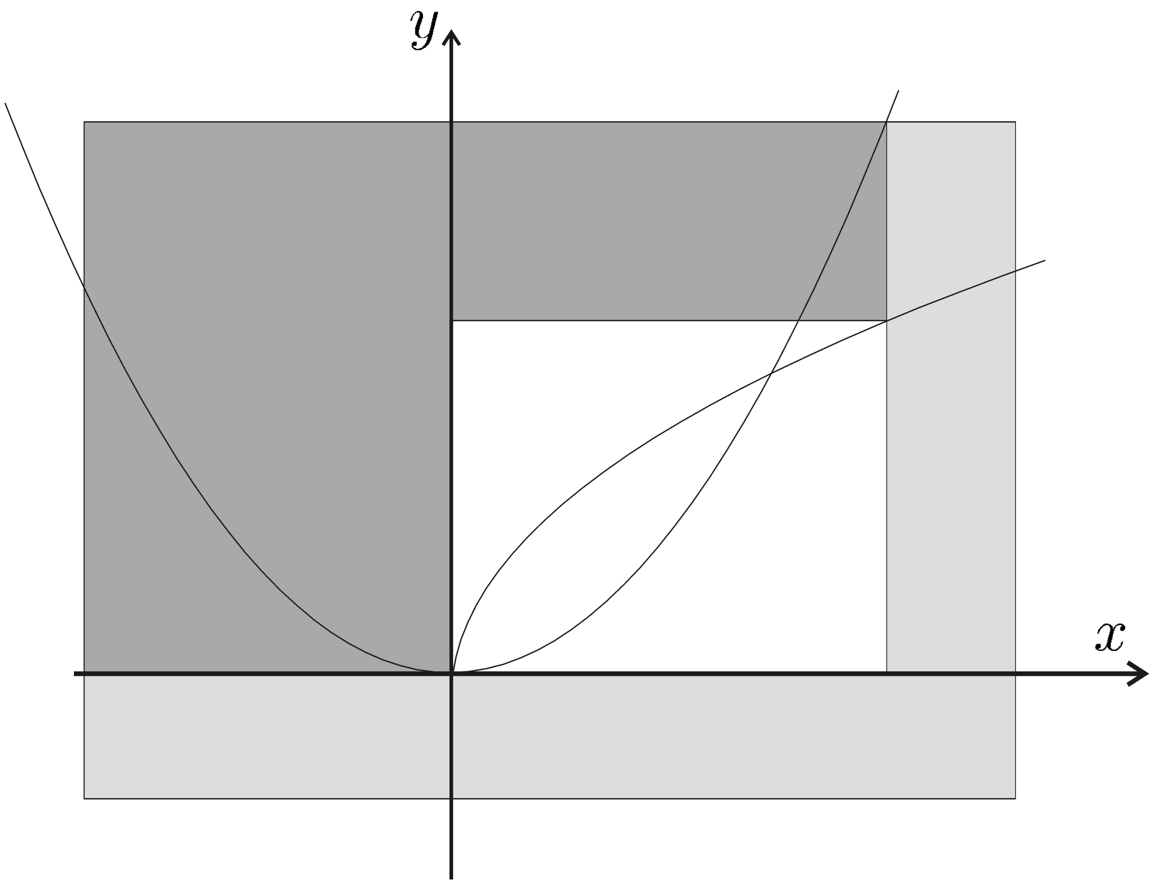
Contractor graph

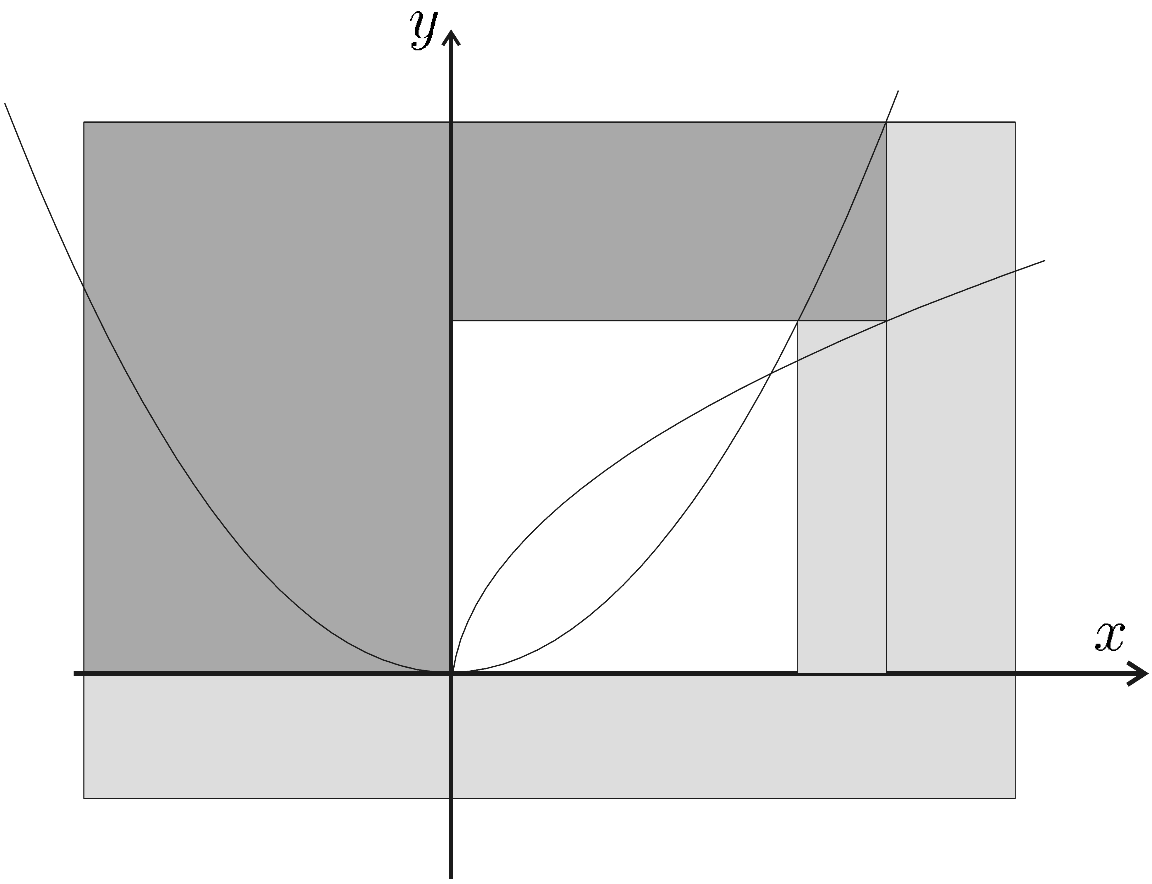


y
x

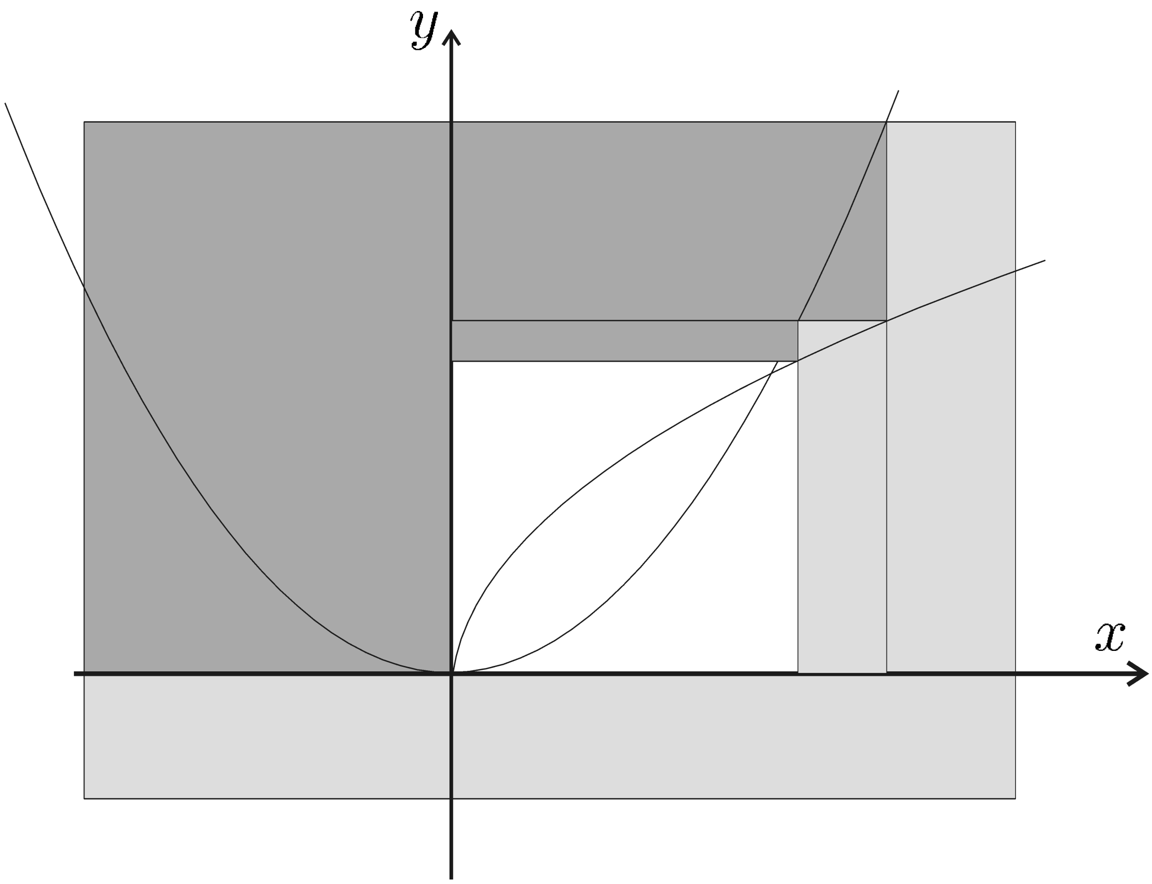


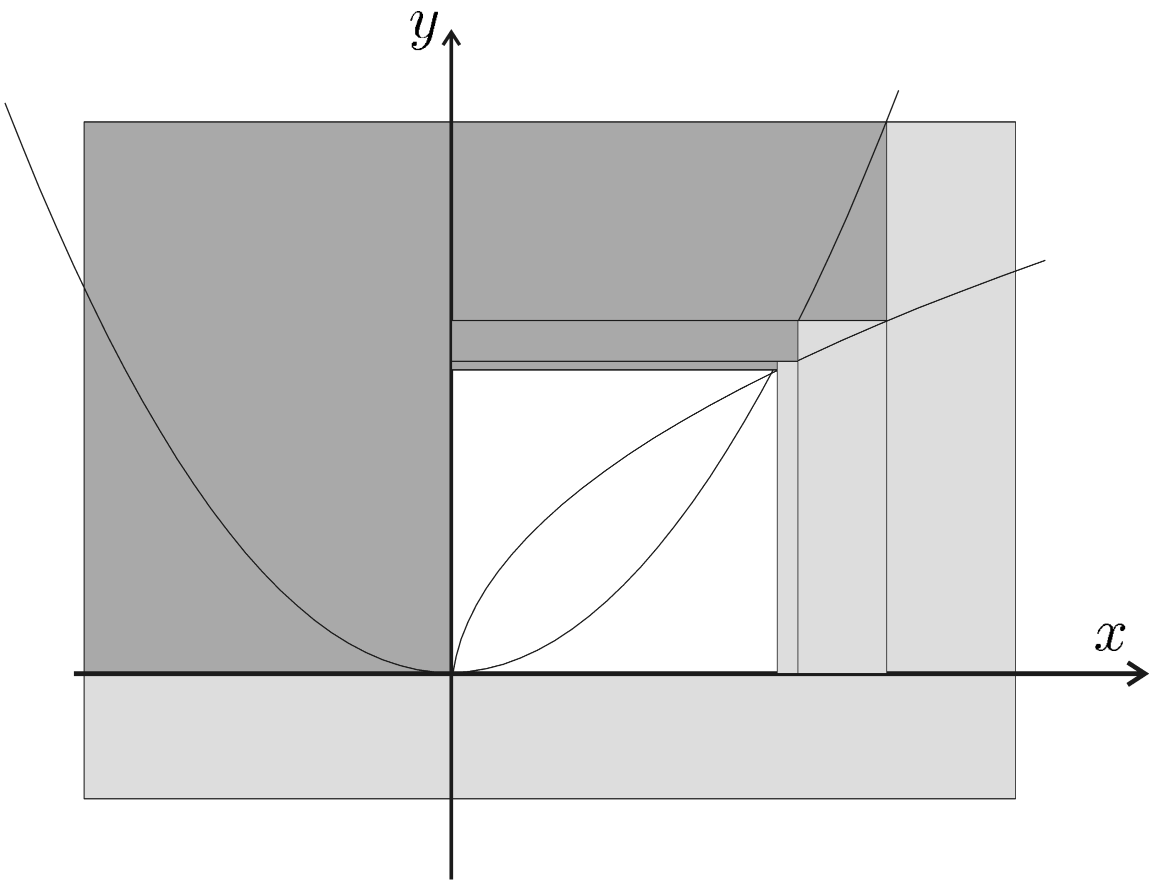
y
x


y
x

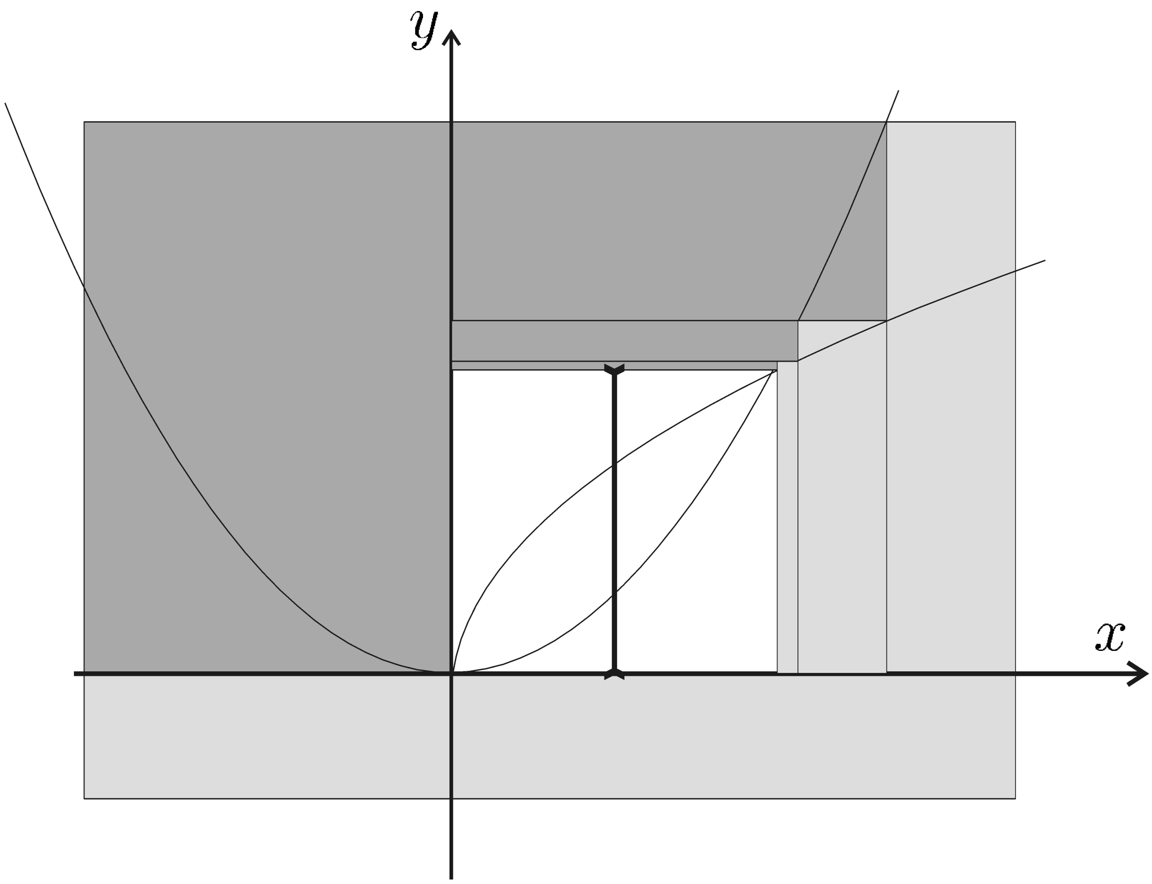


y
x

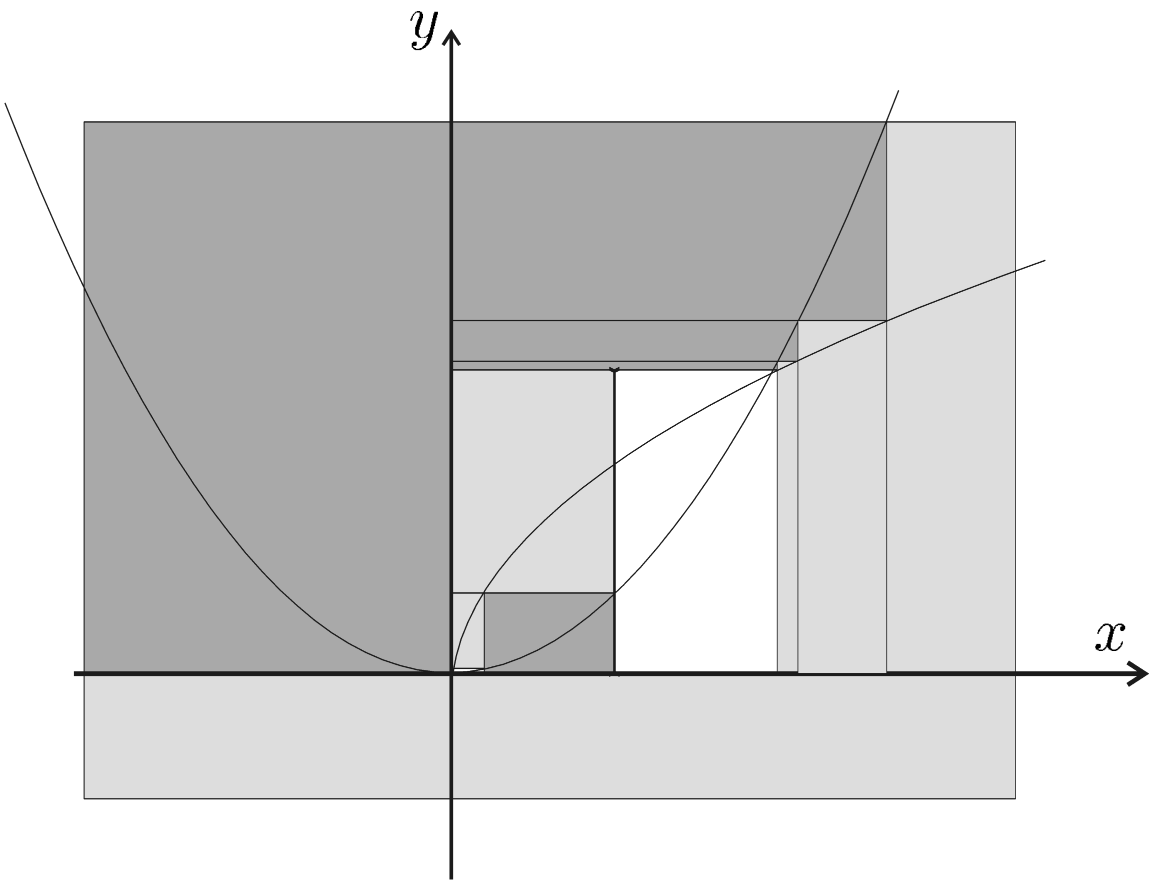

y
x



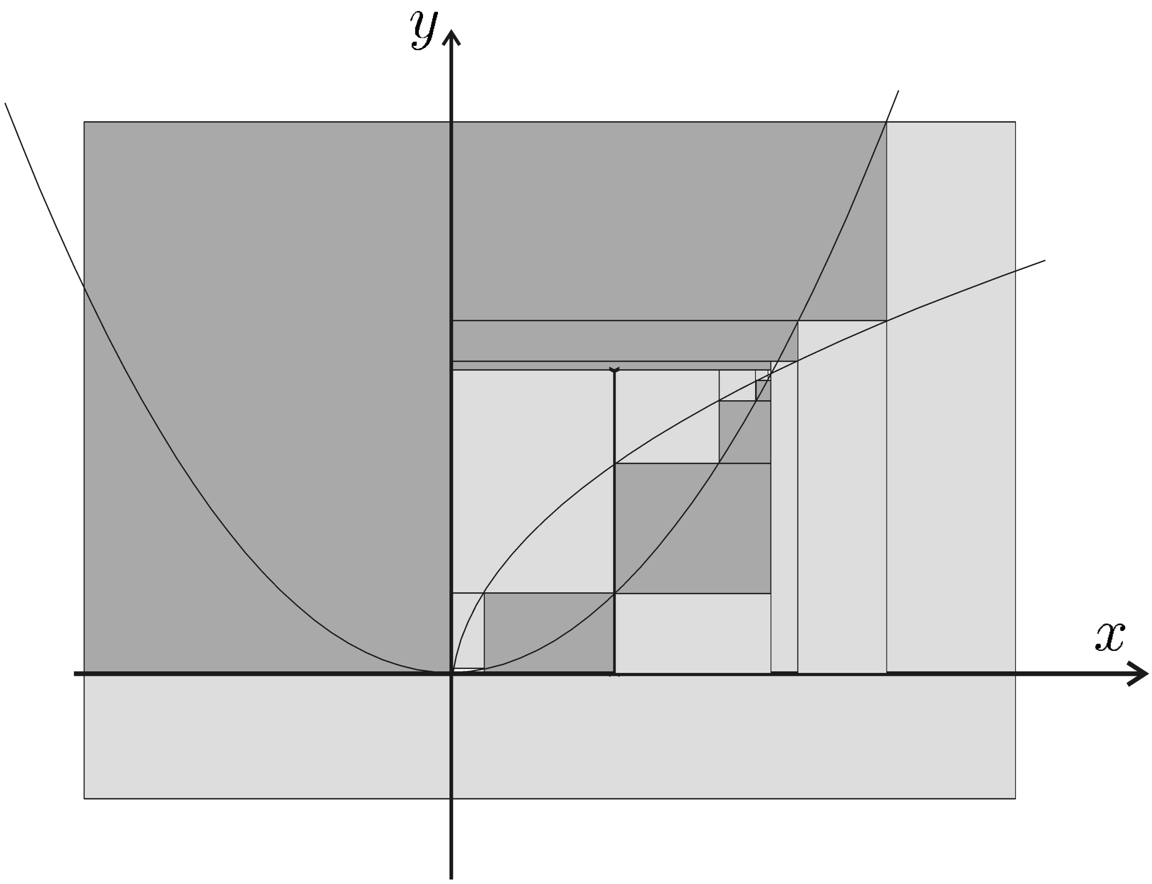
y
x



y
x



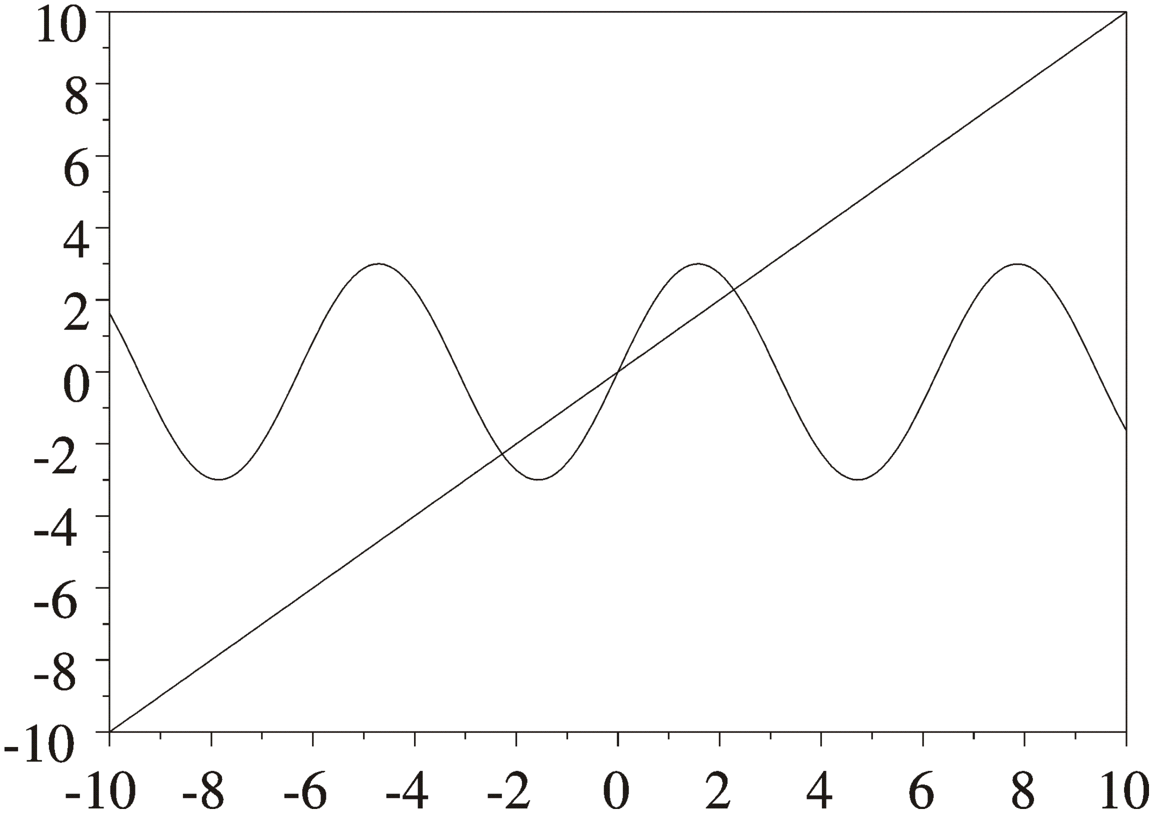
y
x

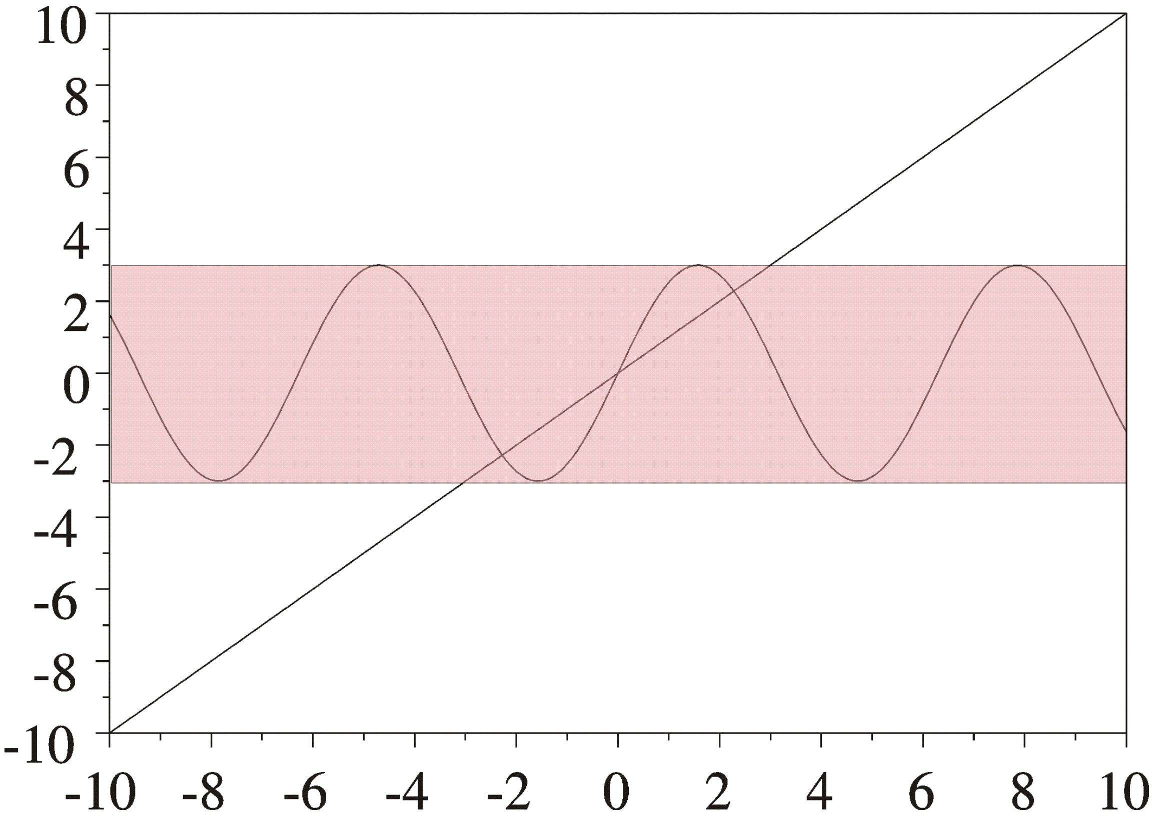


y
x

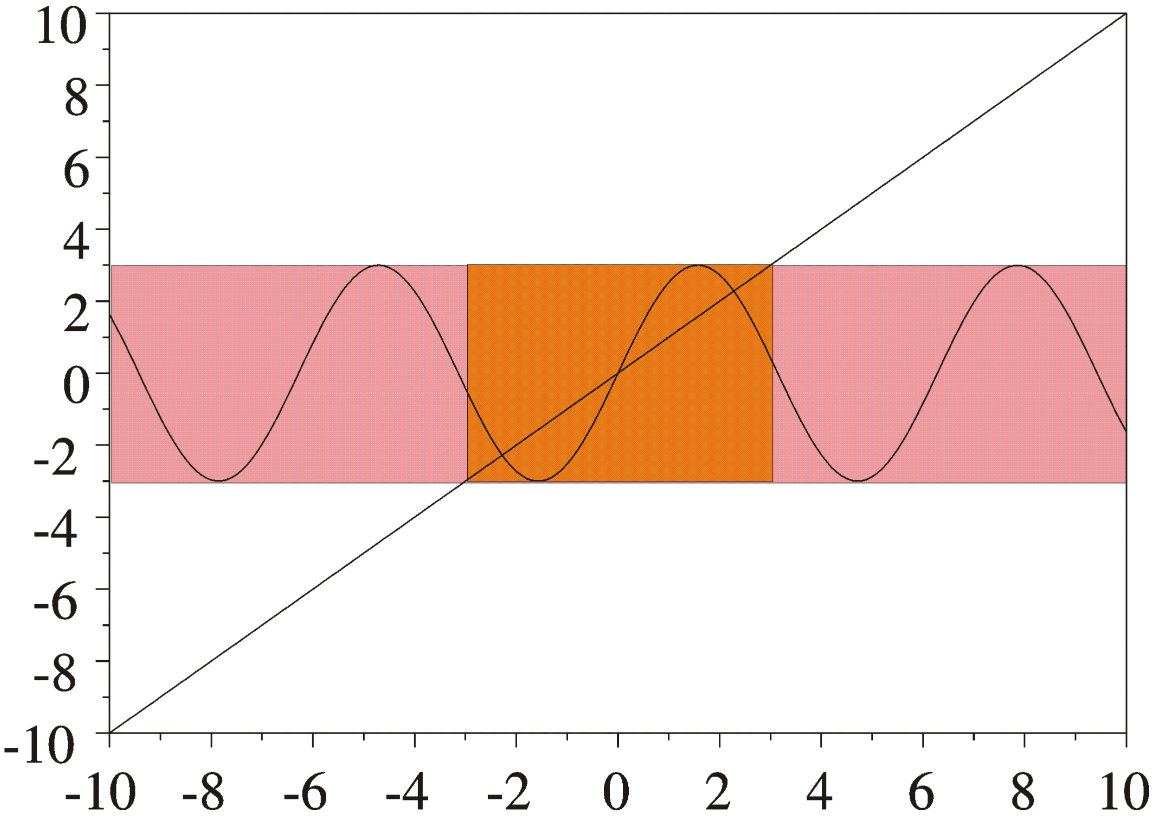
Example 2. Consider the system

$$\begin{cases} y & = & 3\sin(x) \\ y & = & x \end{cases} \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 0 2 4 6 8 10



10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

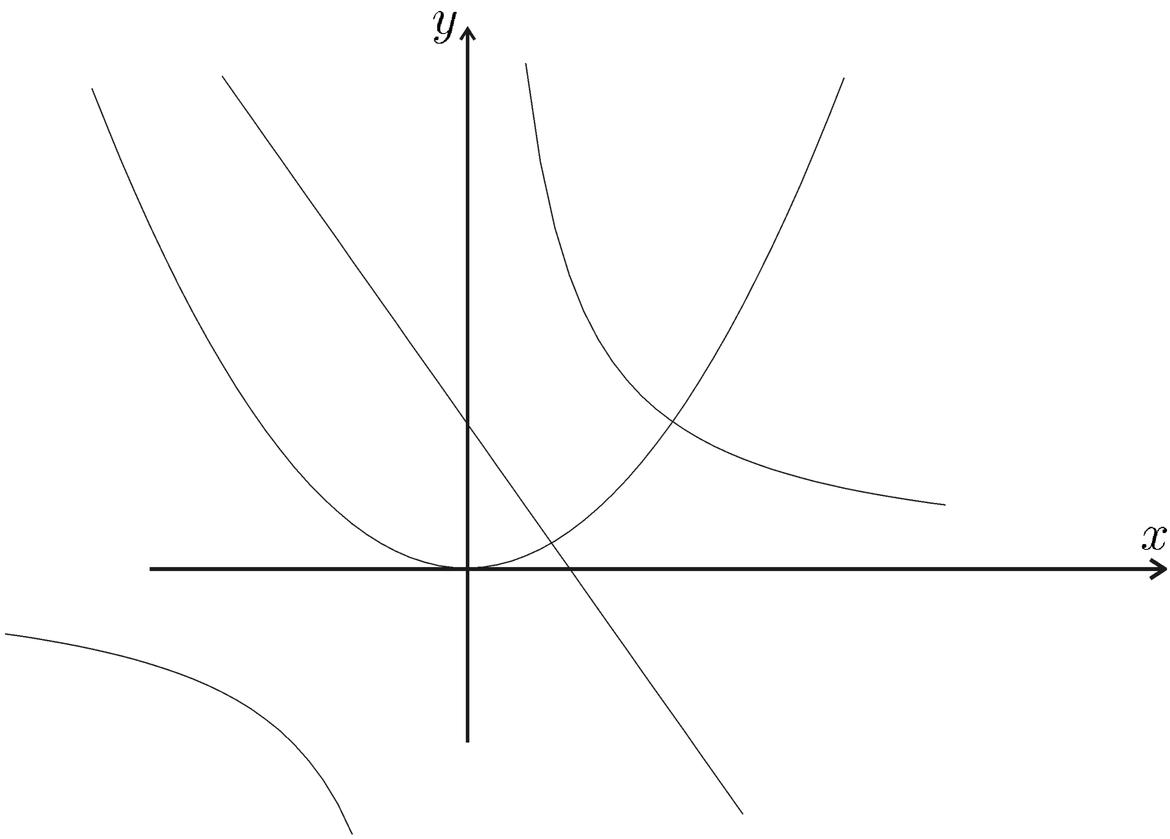


We converge the largest box $[\mathbf{x}]$ such that

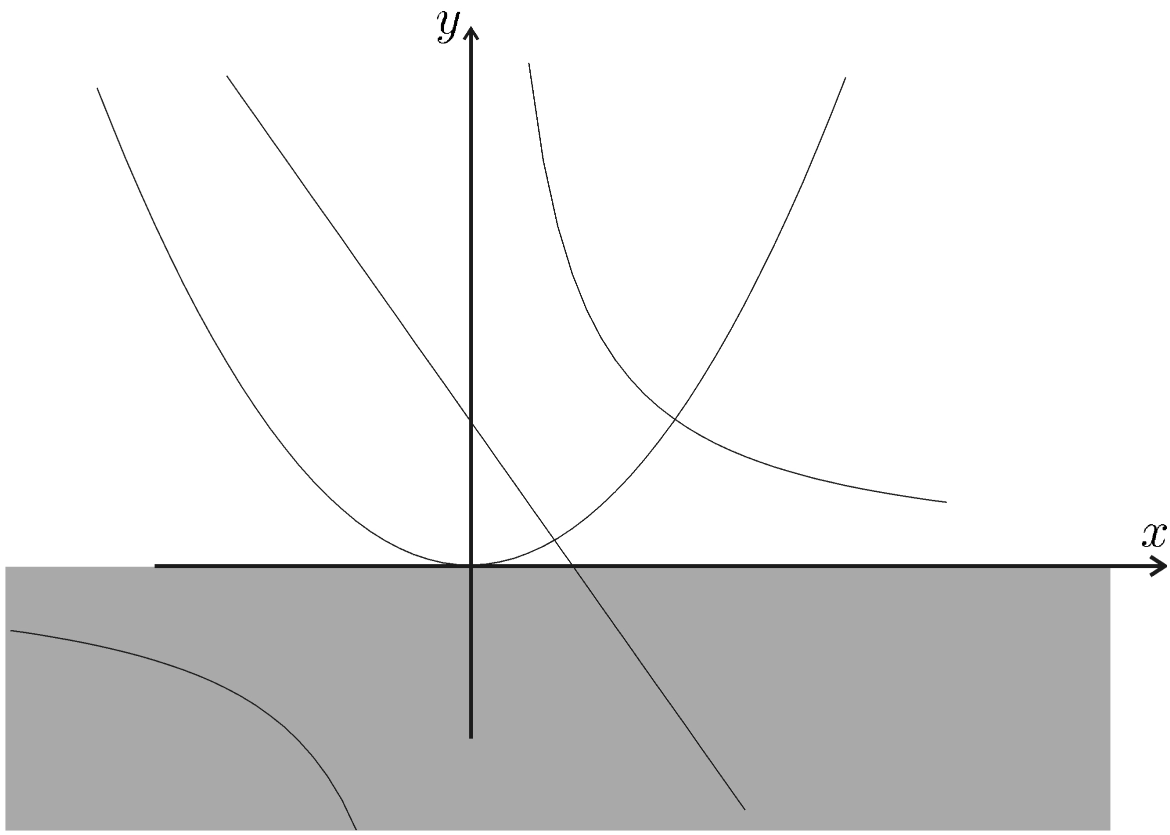
$$\mathcal{C}_1([\mathbf{x}]) = \mathcal{C}_2([\mathbf{x}]) = [\mathbf{x}].$$

Example 3. Consider the following problem

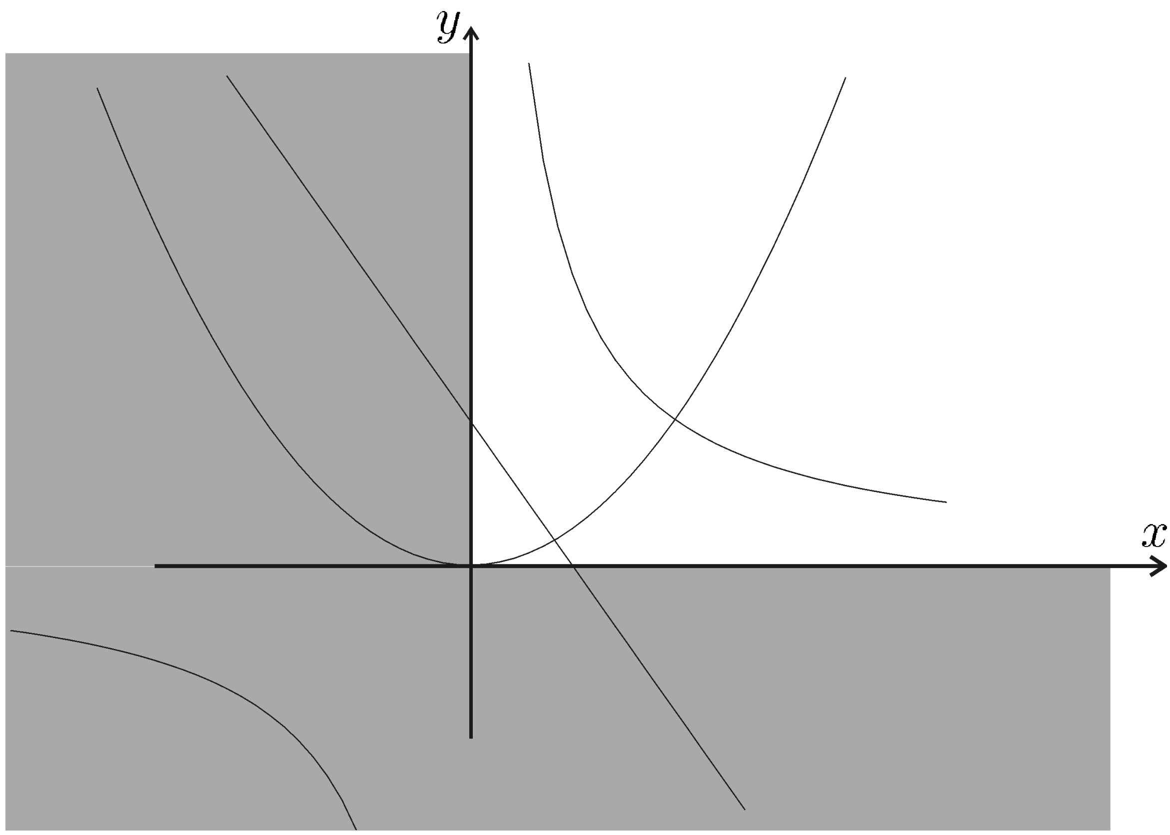
$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$



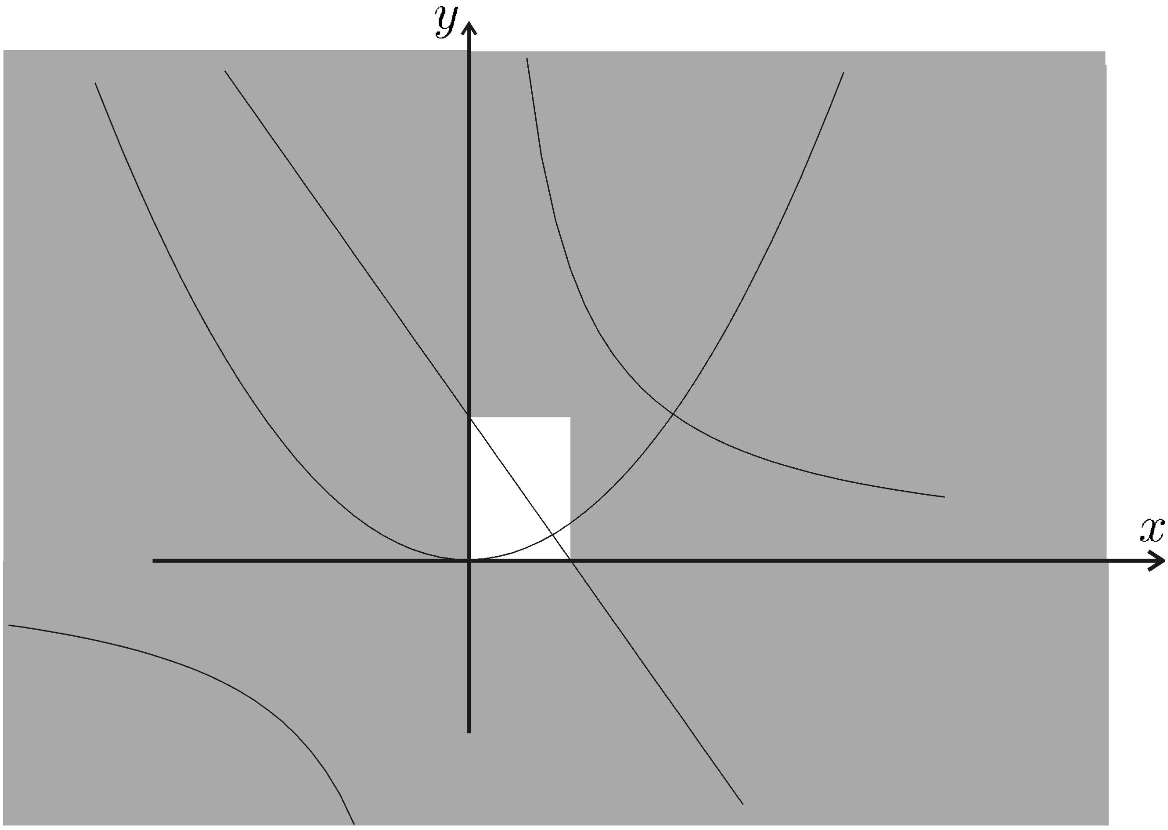
y
x

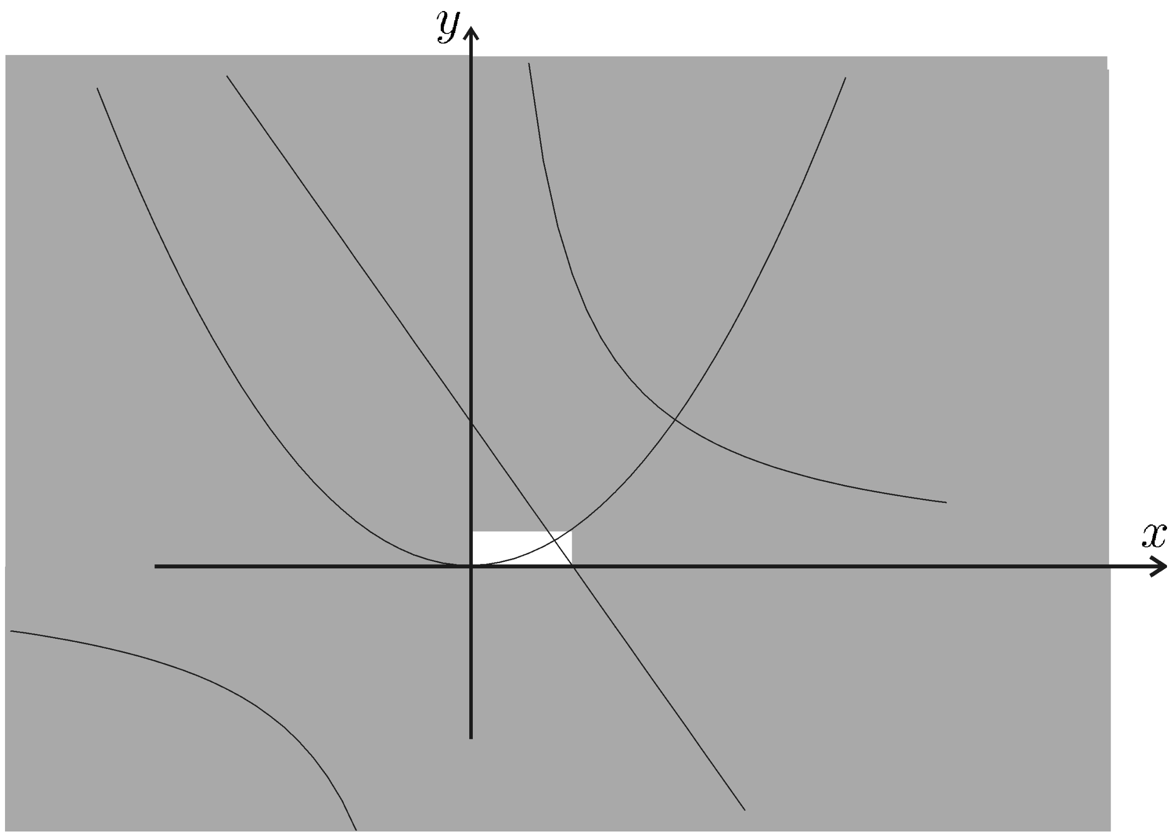


y
x

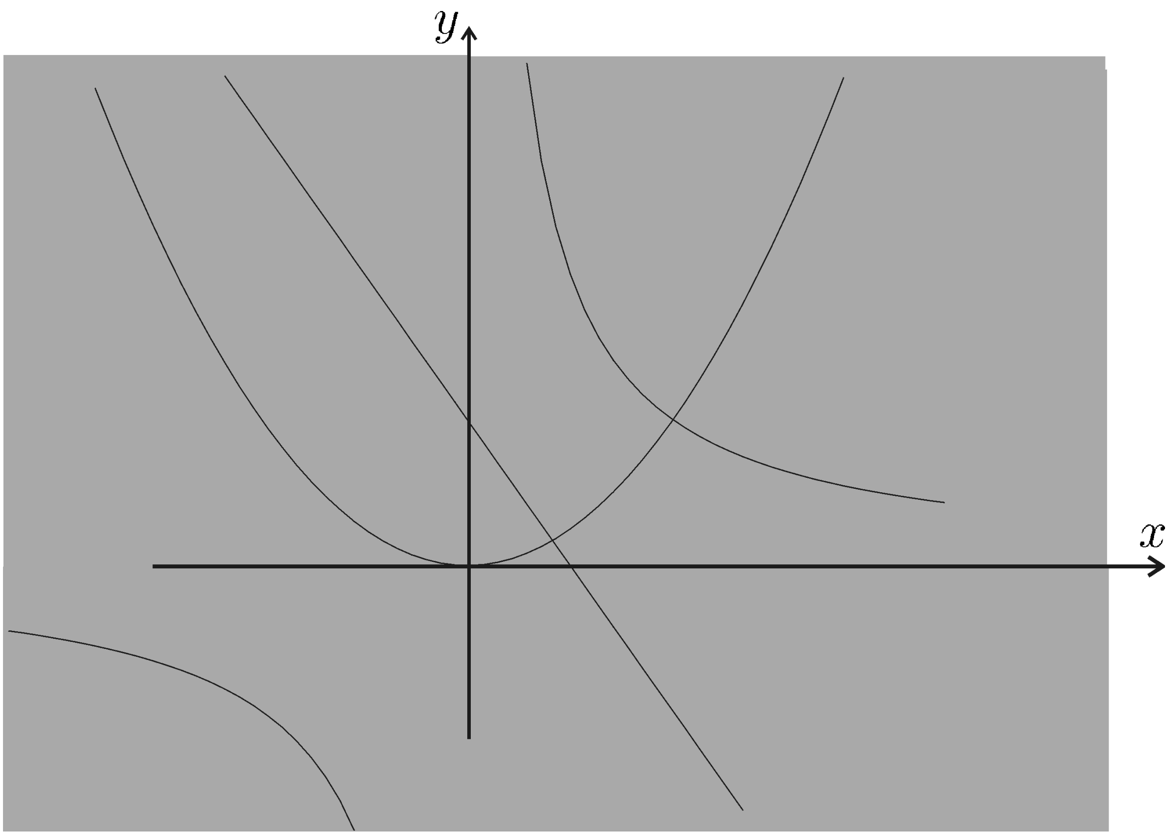


y
x


y
x

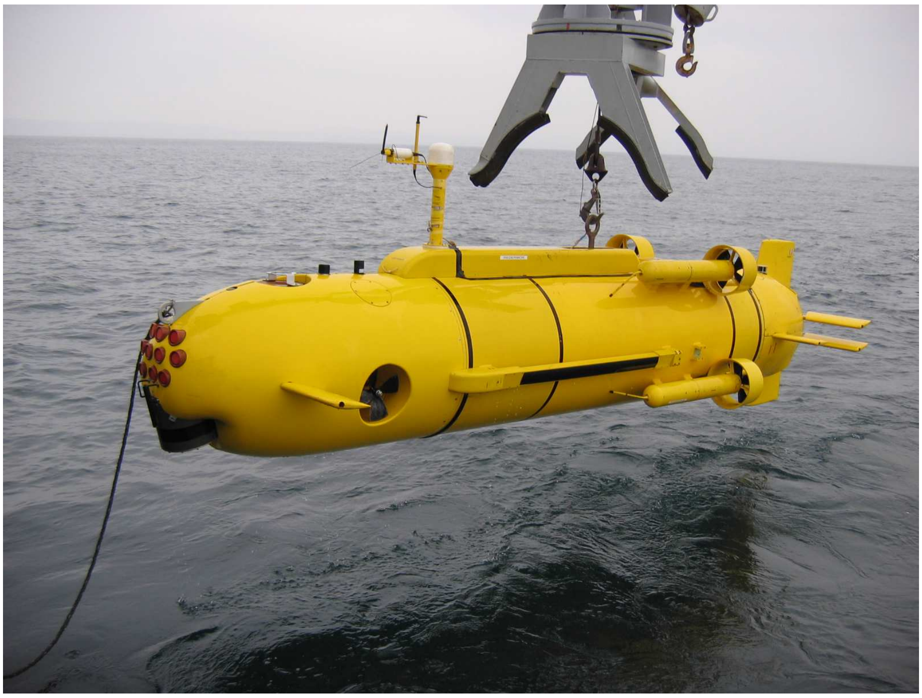


y
x

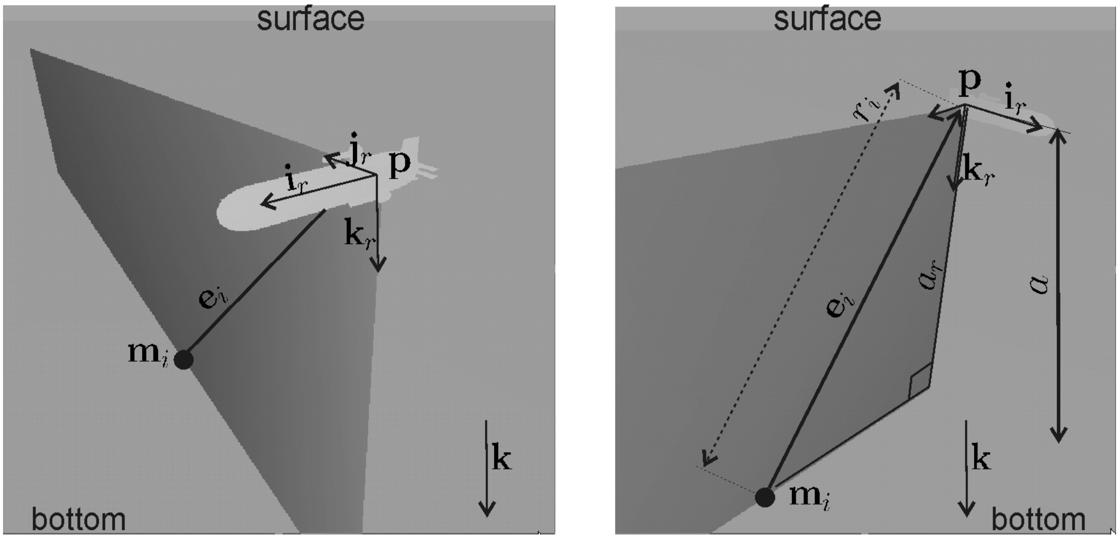

y
x

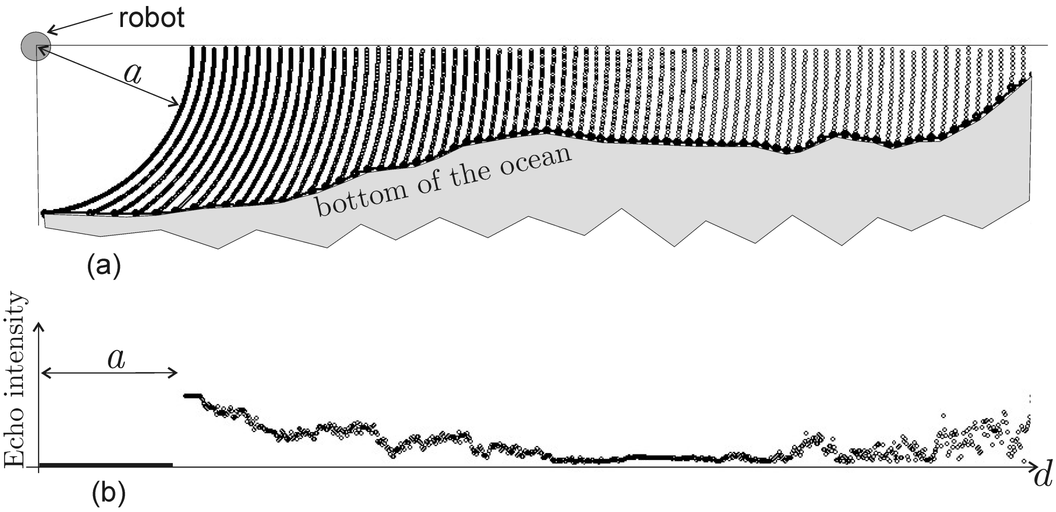
$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty, \infty]^2 \; = \; [0, \infty] \\
(C_2) \Rightarrow & x \in & 1/[0, \infty] \; = [0, \infty] \\
(C_3) \Rightarrow & y \in & [0, \infty] \; \cap \; ((-2)\,.[0, \infty] + 1) \\
 & & = [0, \infty] \; \cap \; ([-\infty, 1]) = [0, 1] \\
 & x \in & [0, \infty] \; \cap \; (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0, 1] \cap [0, 1/2]^2 = [0, 1/4] \\
(C_2) \Rightarrow & x \in & [0, 1/2] \; \cap \; 1/[0, 1/4] = \emptyset \\
 & y \in & [0, 1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

3 SLAM with point marks

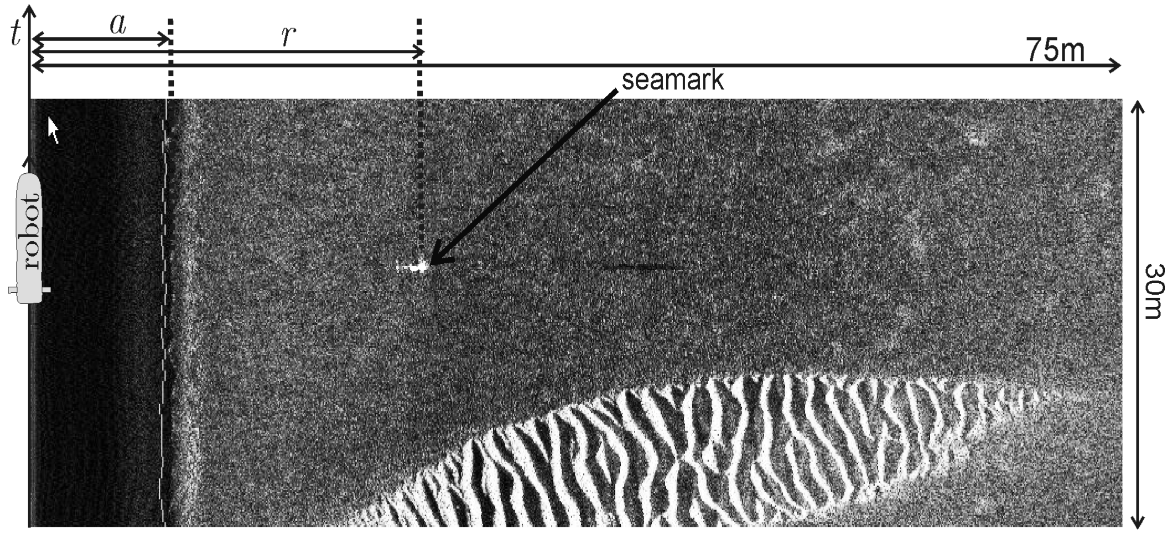


Show the video


surface
$\mathbf{j}_r$
$\mathbf{i}_r$
$\mathbf{p}$
$\mathbf{k}_r$
$\mathbf{e}_i$
$\mathbf{m}_i$
$\mathbf{k}$
bottom
surface
$\mathbf{p}$
$\mathbf{i}_r$
r_i
$\mathbf{k}_r$
$\mathbf{e}_i$
a_r
a
$\mathbf{k}$
$\mathbf{m}_i$
bottom



robot
a
bottom of the ocean
(a)
Echo intensity
a
d
(b)



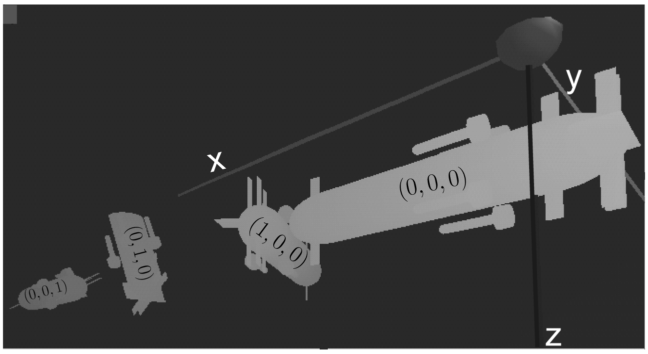
Mine detection with SonarPro

Loch-Doppler returns the speed robot $\mathbf{v}_r$.

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] \, . \tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

Inertial central (Octans III from IXSEA).

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



Six mines have been detected.

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

3.1 Constraints

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \cdot \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) \cdot \frac{\pi}{180}\right) & 0 \end{pmatrix} \cdot \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

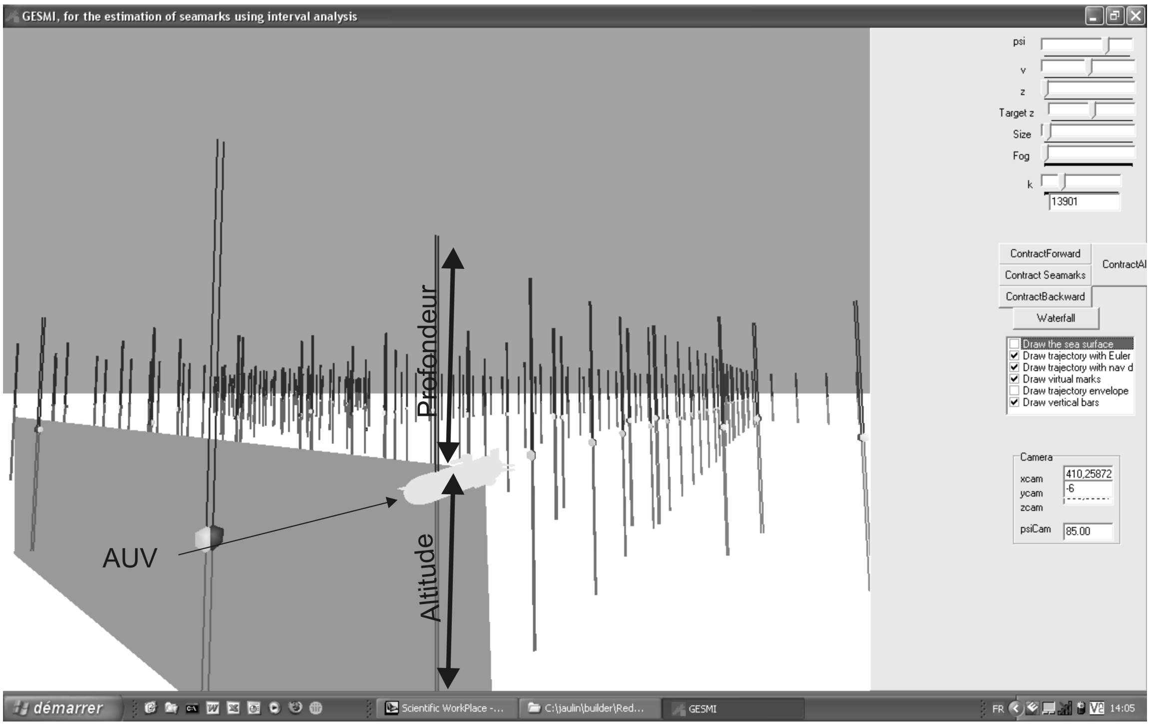
$$\mathbf{R}(t) = \mathbf{R}_\psi(t) \cdot \mathbf{R}_\theta(t) \cdot \mathbf{R}_\varphi(t),$$

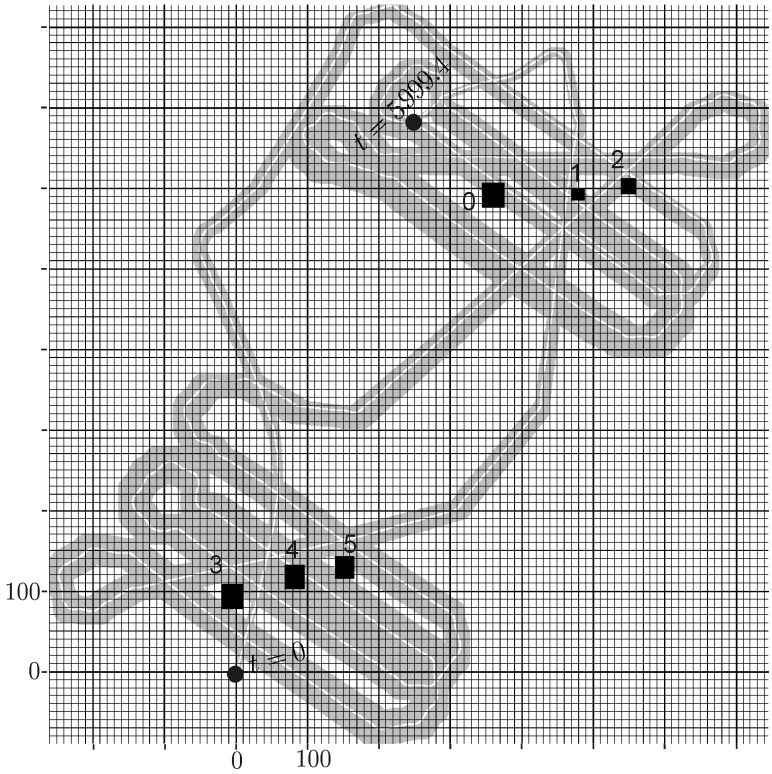
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t) \cdot \mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^\mathsf{T}(\tau(i)) \cdot (\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2}.$$

3.2 GESMI


GESMI, for the estimation of seamarks using interval analysis
Profondeur
Altitude
AUV
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
ycam
zcam
psiCam
410,25872
-6
85.00
démarrer
Scientific WorkPlace -...
GESMI
FR
14:05



$t = 5999.4$
0
1
2
3
4
5
100
0
$t = 0$
0
100

4 Vaimos

Vaimos (IFREMER and ENSTA)

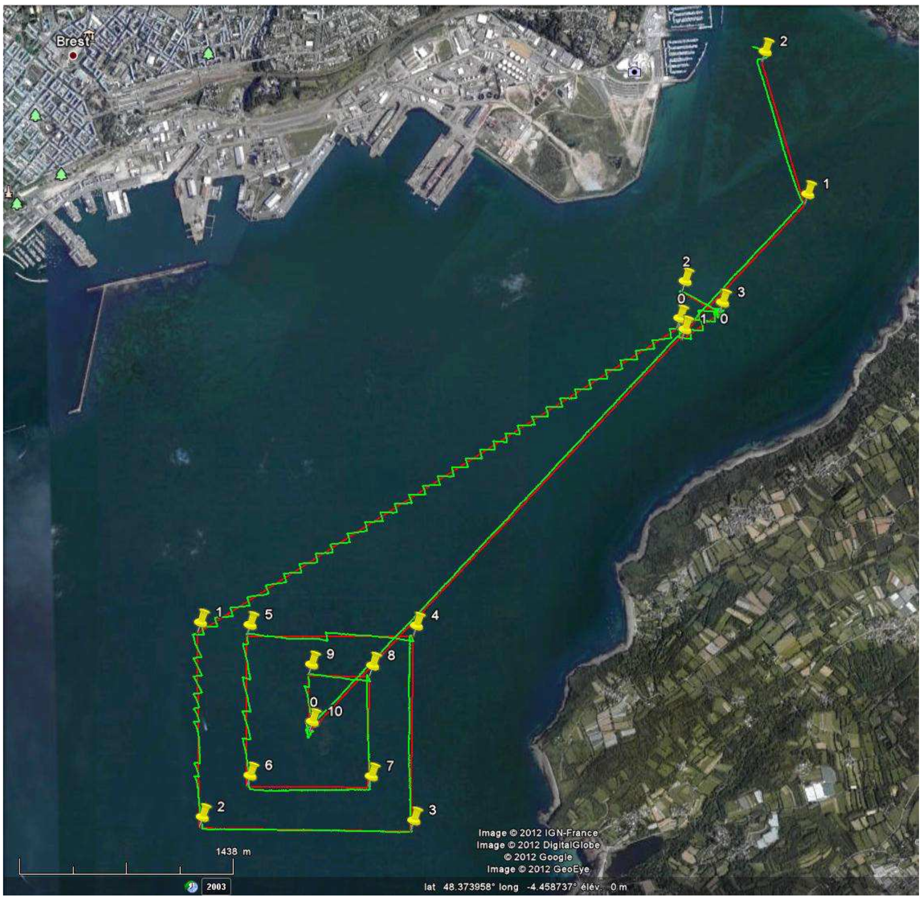
The robot satisfies a state equation

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{u}\right).$$

With the controller $\mathbf{u} = \mathbf{g}\left(\mathbf{x}\right)$, the robot satisfies an equation of the form

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}\right).$$

Brest


Brest
Image © 2012 IGN-France
Image © 2012 DigitalGlobe
© 2012 Google
Image © 2012 GeoEye
1438 m
2003
lat 48.373958° long -4.458737° élév. 0 m

Brest-Douarnenez. January 17, 2012, 8am


Brest
PRO
BR D98 505
YAMAHA

miniji
F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Le Conquet
Plougastel-Daoulas
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
lat 48.230580° long -4.564526° élév 6 m
Altitude 53.47 km
©2010 Google
Start
vaimos.txt -...
Logs
sftp://root...
Command Pr...
Command Pr...
data
Interface po...
VCtrl (Runni...
VAIMOS aut...
Google Earth
FR
98%
01:42

Middle of Atlantic ocean

350 km made by Vaimos in 53h, September 6-9, 2012.

5 Range-only SLAM

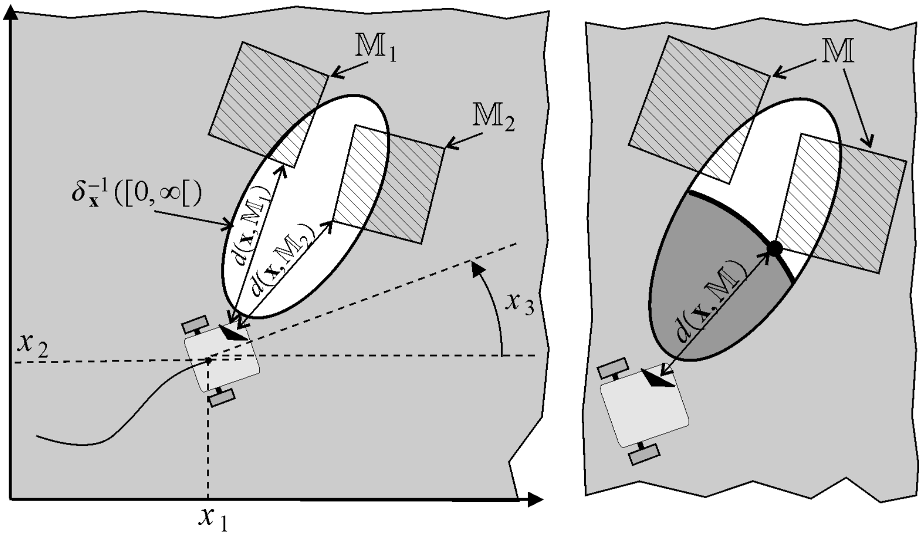
$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) & \text{(evolution equation)} \\ z(t) &= d(\mathbf{x}(t), \mathbb{M}) & \text{(map equation)} \end{cases}$$

where $t \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, $\mathbb{M} \in \mathcal{C}(\mathbb{R}^q)$ is the occupancy map.

Unknown: the map $\mathbb{M}$ and the trajectory $\mathbf{x}$.

Assumption. d corresponds to a *rangefinder*, i.e.,

$$\begin{cases} d(\mathbf{x}, \mathbb{M}_1 \cup \mathbb{M}_2) = \min\{d(\mathbf{x}, \mathbb{M}_1), d(\mathbf{x}, \mathbb{M}_2)\} \\ d(\mathbf{x}, \emptyset) = +\infty. \end{cases}$$



Impact, covering and dug zones

Define the function $\delta_{\mathbf{x}} : \mathbb{R}^q \longrightarrow \mathbb{R}$ as

$$\delta_{\mathbf{x}}(\mathbf{a}) = d(\mathbf{x}, \{\mathbf{a}\}).$$

For given $\mathbf{x}$ and z, we define

covering zone	$\delta_{\mathbf{x}}^{-1}([0,\infty[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < \infty\}$
impact zone	$\delta_{\mathbf{x}}^{-1}(\{z\}) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) = z\}$
dug zone	$\delta_{\mathbf{x}}^{-1}([0,z[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < z\}$

The dug zone does not intersect $\mathbb{M}$, i.e.,

$$z = d(\mathbf{x}, \mathbb{M}) \Rightarrow \delta_{\mathbf{x}}^{-1}([0, z[) \cap \mathbb{M} = \emptyset.$$

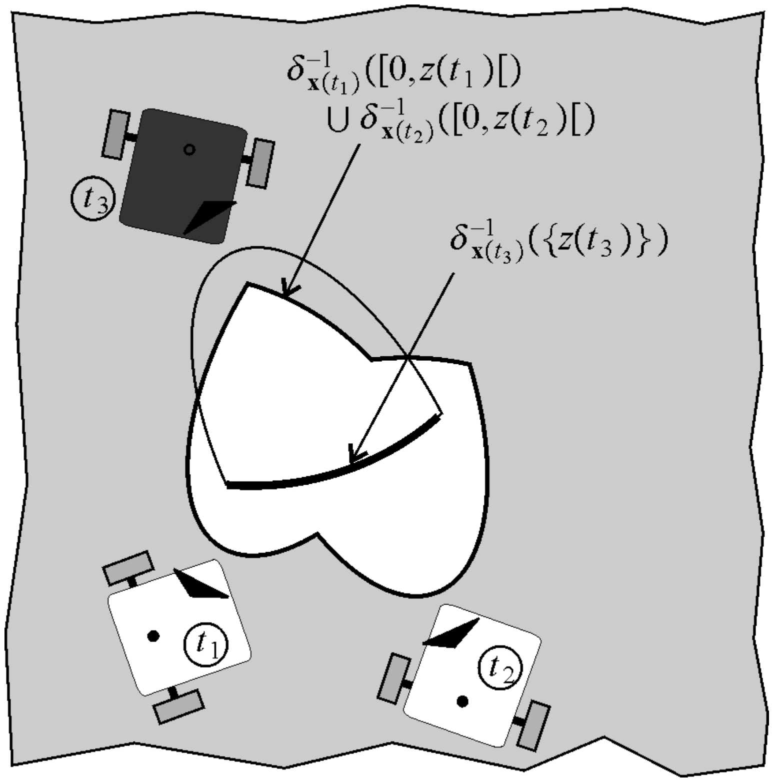
The set $\mathbb{D} = \bigcup_{t \in [t]} \delta_{\mathbf{x}(t)}^{-1}([0, z(t)[)$ is called the *dug space*.
We have

$$\mathbb{D} \cap \mathbb{M} = \emptyset.$$

.

For all $\mathbf{x}$, the impact zone intersects the map, i.e,

$$z = d\left(\mathbf{x}, \mathbb{M}\right) \Rightarrow \delta_{\mathbf{x}}^{-1}\left(\{z\}\right) \cap \mathbb{M} \neq \emptyset.$$



$\delta^{-1}_{\mathbf{x}(t_1)}([0,z(t_1)[)$
$\cup\,\delta^{-1}_{\mathbf{x}(t_2)}([0,z(t_2)[)$
$\delta^{-1}_{\mathbf{x}(t_3)}(\{z(t_3)\})$
t_3
t_1
t_2

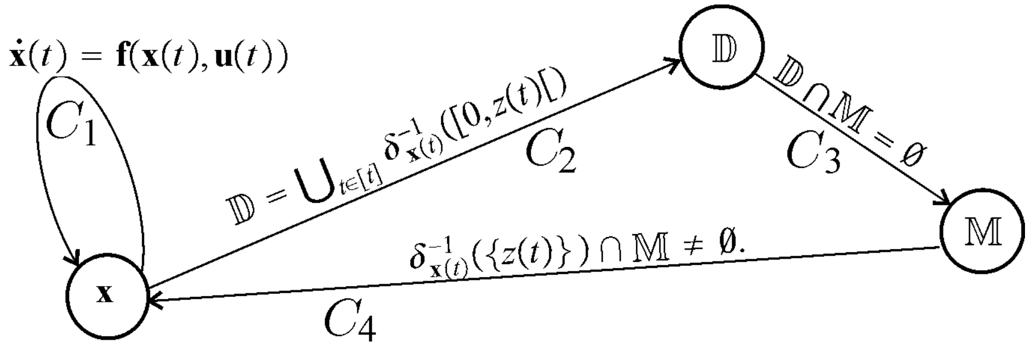
The range-only SLAM problem is a *hybrid CSP*.

Variables: $\mathbf{x}(t)$, $\mathbb{M}$ and $\mathbb{D}$.

Constraints:

$$\left.\begin{array}{ll}
(1) & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\
(2) & \mathbb{D} = \bigcup_{t \in [t]} \delta^{-1}_{\mathbf{x}(t)}([0, z(t)[) \\
(3) & \mathbb{D} \cap \mathbb{M} = \emptyset \\
(4) & \delta^{-1}_{\mathbf{x}(t)}(\{z(t)\}) \cap \mathbb{M} \neq \emptyset.
\end{array}\right\} : z(t) = d(\mathbf{x}(t), \mathbb{M})$$

Domains: $[\mathbb{M}] = [\mathbb{D}] = [\emptyset, \mathbb{R}^q]$, $[\mathbf{x}](t) = \mathbb{R}^n$ for $t > 0$ and $[\mathbf{x}](0) = \mathbf{x}(0)$.



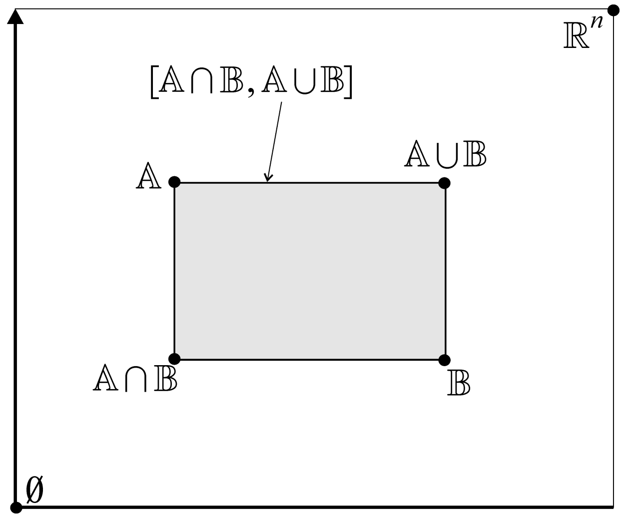
Constraint diagram of the range only SLAM problem

6 Hybrid intervals

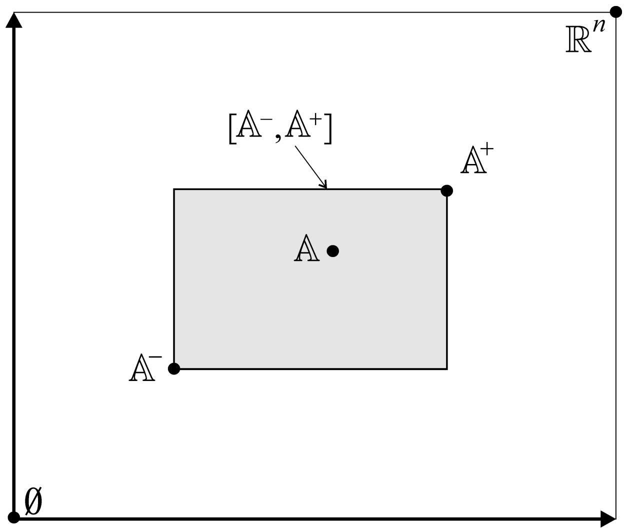
A *closed interval* (or *interval* for short) $[x]$ of a complete lattice $\mathcal{E}$ is a subset of $\mathcal{E}$ which satisfies

$$[x] = \{x \in \mathcal{E} \mid \wedge [x] \leq x \leq \vee [x]\}$$

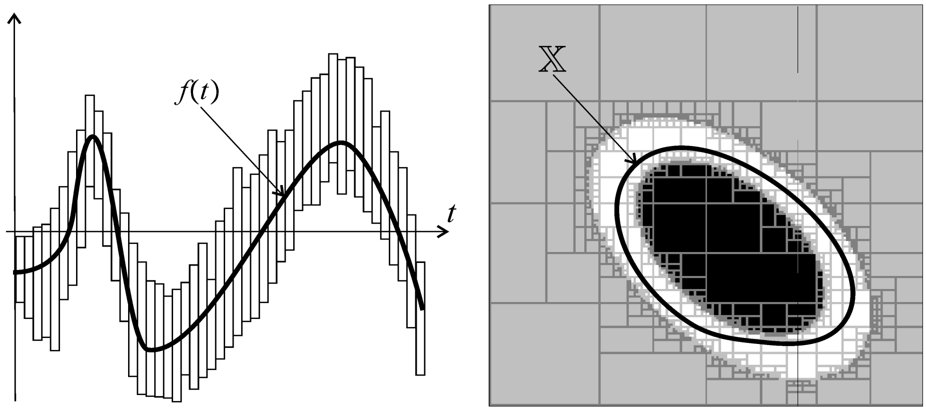
Both $\emptyset$ and $\mathcal{E}$ are intervals of $\mathcal{E}$.



Lattice $\left(\mathcal{P}\left(\mathbb{R}^n\right), \subset\right)$



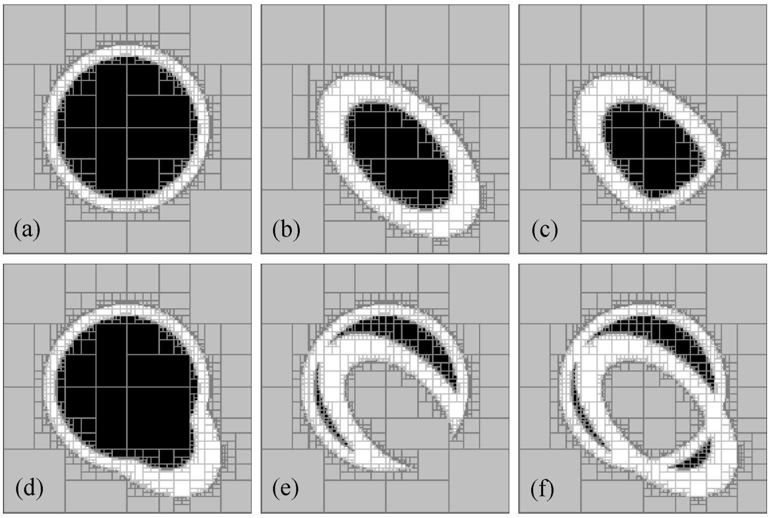
Interval in the lattice $(\mathcal{P}(\mathbb{R}^n), \subset)$



An interval function (or tube) and a set interval

Hybrid intervals. If $[x] \in \mathbb{I}\mathcal{E}_x, [y] \in \mathbb{I}\mathcal{E}_y$ then $[x] \times [y]$ is a hybrid interval.

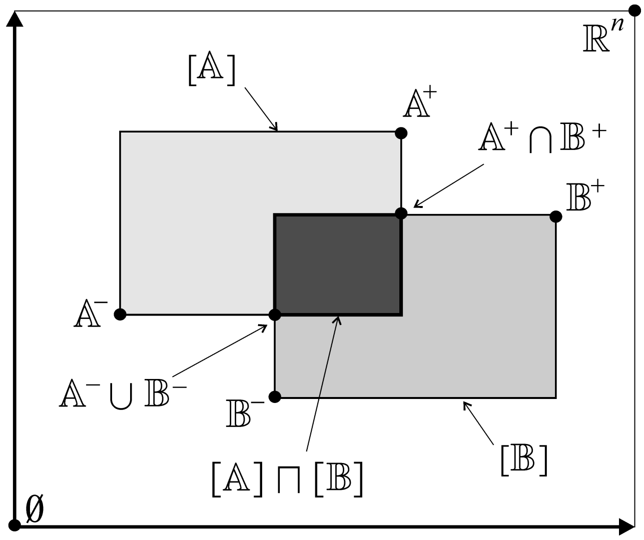
6.1 Interval arithmetic



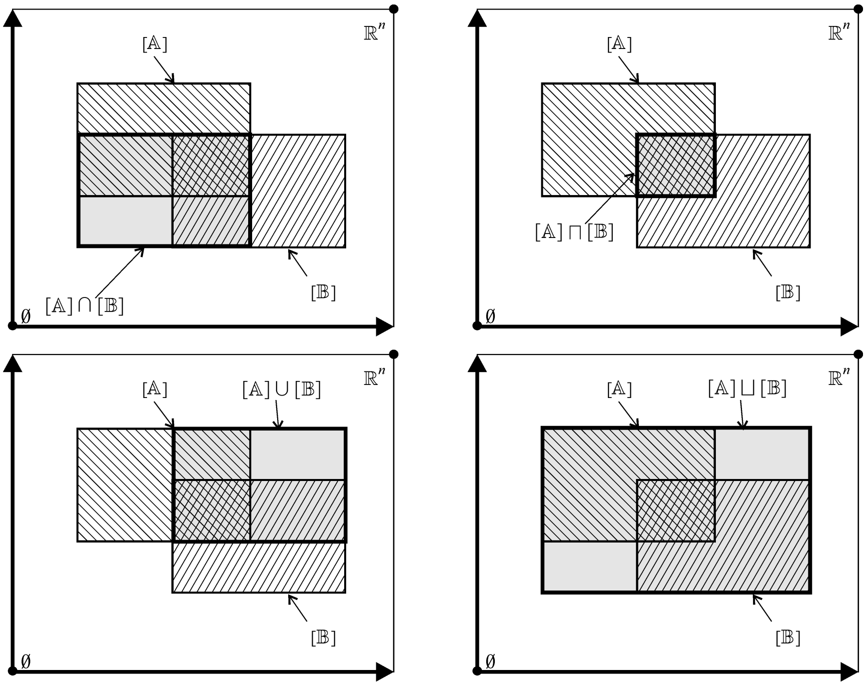
$$[\mathbb{A}],[\mathbb{B}],\ [\mathbb{A}]\cap[\mathbb{B}],\ [\mathbb{A}]\cup[\mathbb{B}],\ [\mathbb{A}]\setminus[\mathbb{B}],\ ([\mathbb{A}]\cup[\mathbb{B}])\setminus([\mathbb{A}]\cap[\mathbb{B}]).$$

Intersection.

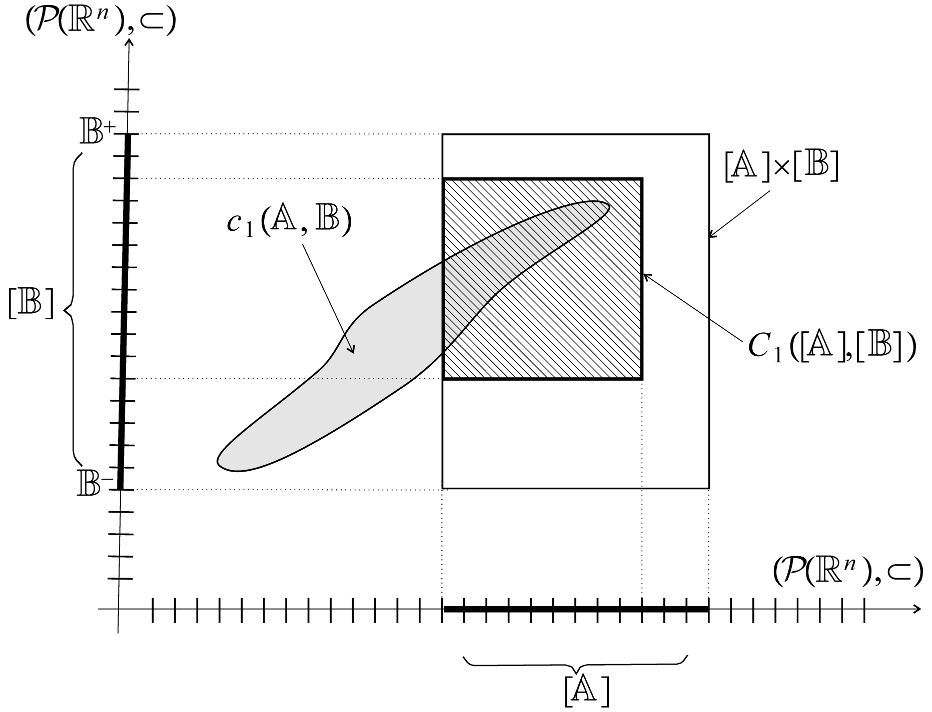
$$
\begin{aligned}
[\mathbb{A}] \sqcap [\mathbb{B}] &= \{\mathbb{X}, \mathbb{X} \in [\mathbb{A}] \text{ and } \mathbb{X} \in [\mathbb{B}]\} \\
&= \left[\mathbb{A}^- \cup \mathbb{B}^-, \mathbb{A}^+ \cap \mathbb{B}^+\right].
\end{aligned}
$$



$\mathbb{R}^n$
$[\mathbb{A}]$
$\mathbb{A}^+$
$\mathbb{A}^+ \cap \mathbb{B}^+$
$\mathbb{B}^+$
$\mathbb{A}^-$
$\mathbb{A}^- \cup \mathbb{B}^-$
$\mathbb{B}^-$
$[\mathbb{A}] \sqcap [\mathbb{B}]$
$[\mathbb{B}]$
$\emptyset$


$[\mathbb{A}]$
$\mathbb{R}^n$
$[\mathbb{A}] \cap [\mathbb{B}]$
$[\mathbb{B}]$
$\emptyset$
$[\mathbb{A}]$
$\mathbb{R}^n$
$[\mathbb{A}] \sqcap [\mathbb{B}]$
$[\mathbb{B}]$
$\emptyset$
$[\mathbb{A}]$
$[\mathbb{A}] \cup [\mathbb{B}]$
$\mathbb{R}^n$
$[\mathbb{B}]$
$\emptyset$
$[\mathbb{A}]$
$[\mathbb{A}] \sqcup [\mathbb{B}]$
$\mathbb{R}^n$
$[\mathbb{B}]$
$\emptyset$

6.2 Contractors



$(\mathcal{P}(\mathbb{R}^n),\subset)$
$\mathbb{B}^+$
$[\mathbb{A}]\times[\mathbb{B}]$
$c_1(\mathbb{A},\mathbb{B})$
$[\mathbb{B}]$
$C_1([\mathbb{A}],[\mathbb{B}])$
$\mathbb{B}^-$
$(\mathcal{P}(\mathbb{R}^n),\subset)$
$[\mathbb{A}]$

Example

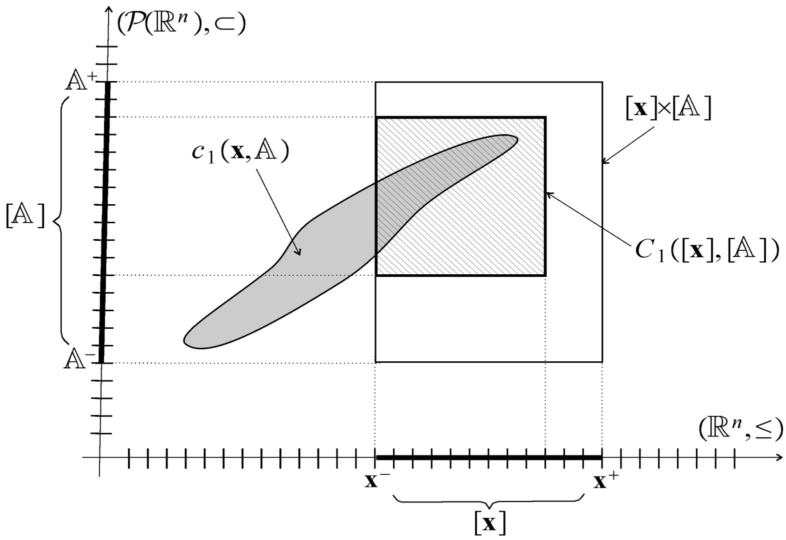
$$\left\{\begin{array}{c} \mathbb{A}\subset\mathbb{B} \\ \mathbb{A}\in[\mathbb{A}]\,,\mathbb{B}\in[\mathbb{B}]\,. \end{array}\right.$$

The optimal contractor is

$$\left\{\begin{array}{ll} \text{(i)} & [\mathbb{A}]:=[\mathbb{A}]\sqcap([\mathbb{A}]\cap[\mathbb{B}]) \\ \text{(ii)} & [\mathbb{B}]:=[\mathbb{B}]\sqcap([\mathbb{A}]\cup[\mathbb{B}]) \end{array}\right.$$

```
void Set_Contractor_Subset(paving& A,paving& B)
{ paving Z=A&B;
A=Sqcap(A,Z);
Z=B|A;
B=Sqcap(B,Z);
}
```

Hybrid contractor



Hybrid contractor $\mathcal{C}_1$

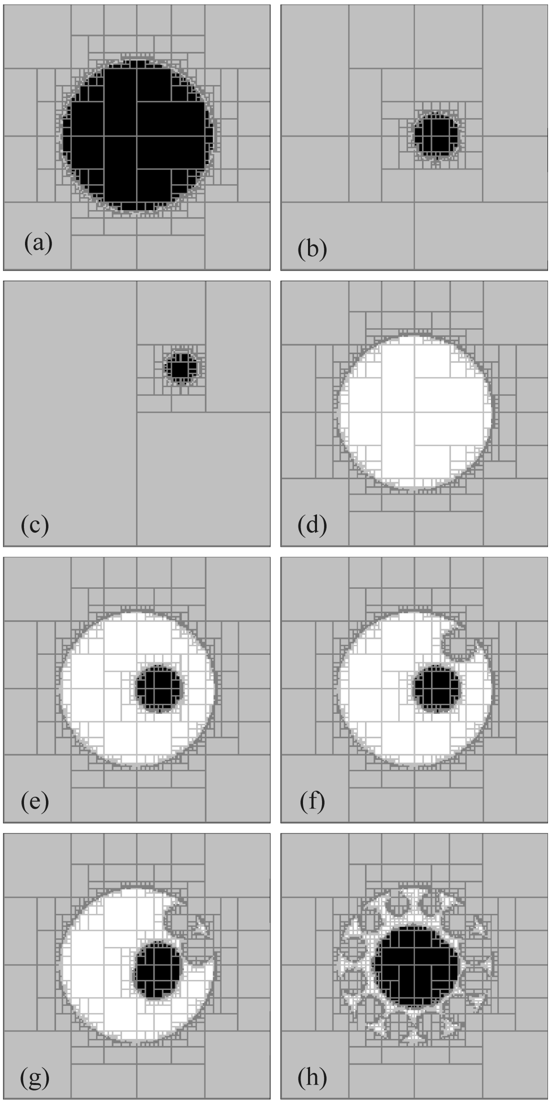
6.3 Propagation

Consider the following CSP

$$\begin{cases} \text{(i)} & \mathbb{X} \subset \mathbb{A} \\ \text{(ii)} & \mathbb{B} \subset \mathbb{X} \\ \text{(iii)} & \mathbb{X} \cap \mathbb{C} = \emptyset \\ \text{(iv)} & f(\mathbb{X}) = \mathbb{X}, \end{cases}$$

where $\mathbb{X}$ is an unknown subset of $\mathbb{R}^2$, f is a rotation with an angle of $-\frac{\pi}{6}$, and

$$\begin{cases} \mathbb{A} & = & \left\{(x_1, x_2), x_1^2 + x_2^2 \leq 3\right\} \\ \mathbb{B} & = & \left\{(x_1, x_2), (x_1 - 0.5)^2 + x_2^2 \leq 0.3\right\} \\ \mathbb{C} & = & \left\{(x_1, x_2), (x_1 - 1)^2 + (x_2 - 1)^2 \leq 0.15\right\} \end{cases}$$



(a) $[\mathbb{A}]$

(b) $[\mathbb{B}]$

(c) $[\mathbb{C}]$

(d) $\mathbb{X} \subset \mathbb{A}$

(e) $\mathbb{B} \subset \mathbb{X}$

(f) $\mathbb{X} \cap \mathbb{C} = \emptyset$

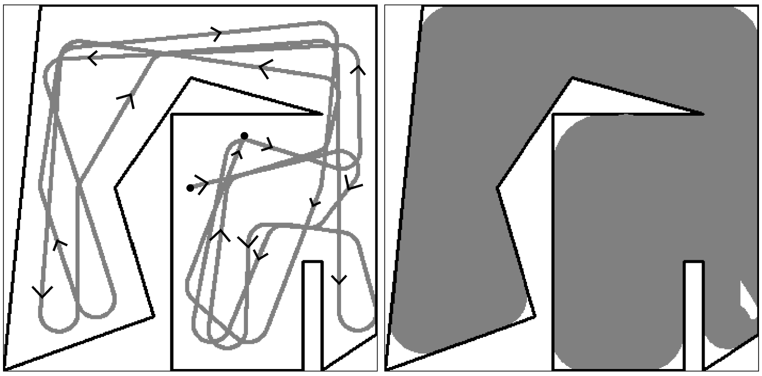
(g) $f(\mathbb{X}) = \mathbb{X}$

(h) $(f(\mathbb{X}) = \mathbb{X})^{\infty}$

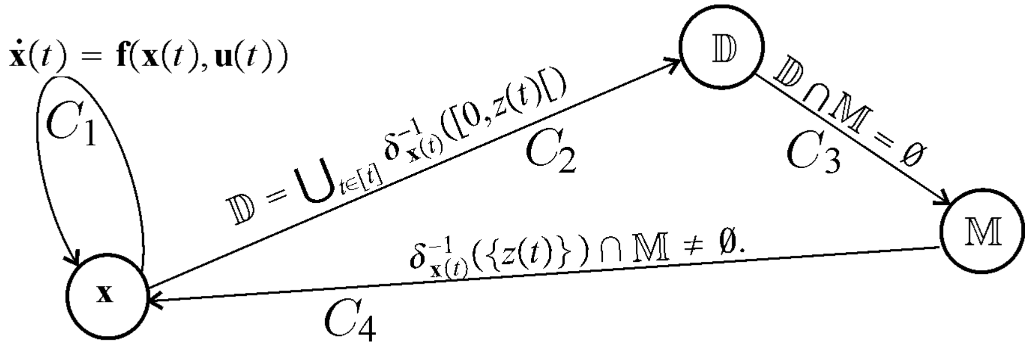
7 SLAM

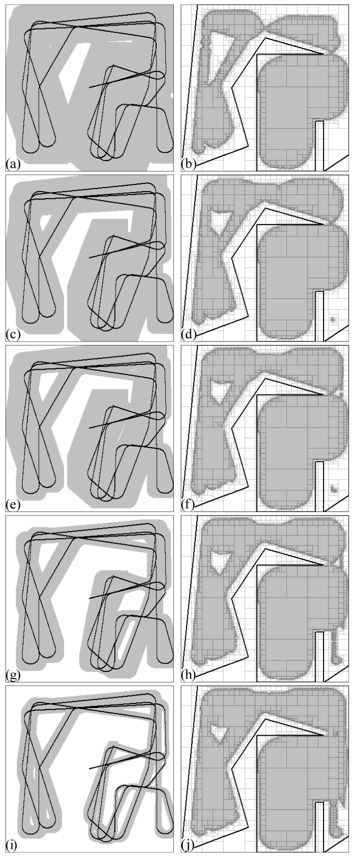
Range-only SLAM equations

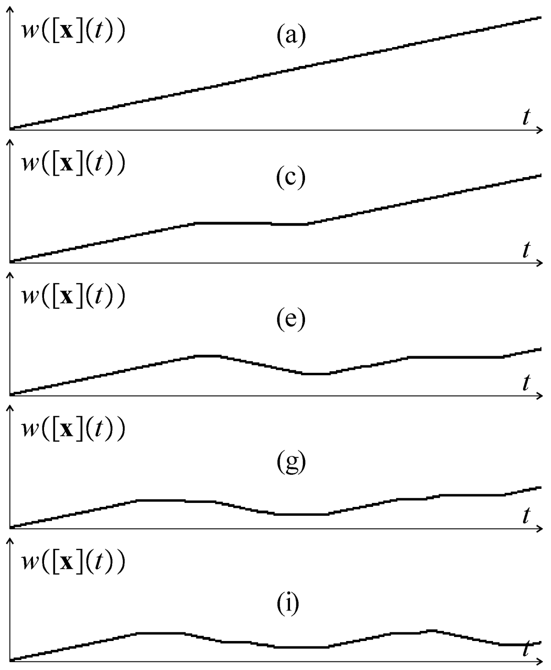
$$\left\{ \begin{array}{lll} \dot{x}_1(t) & = & u_1(t)\cos\left(u_2(t)\right) \\ \dot{x}_2(t) & = & u_1(t)\sin\left(u_2(t)\right) \\ z\left(t\right) & = & d\left(\mathbf{x}\left(t\right),\mathbb{M}\right). \end{array} \right.$$



Actual trajectory and dug space


$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$
C_1
$\mathbf{x}$
$\mathbb{D} = \bigcup_{t\in[t]} \delta^{-1}_{\mathbf{x}(t)}([0, z(t)[)$
C_2
$\mathbb{D}$
$\mathbb{D} \cap \mathbb{M} = \emptyset$
C_3
$\mathbb{M}$
$\delta^{-1}_{\mathbf{x}(t)}(\{z(t)\}) \cap \mathbb{M} \neq \emptyset.$
C_4


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)



Width of the tubes $[\mathbf{x}](t)$

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