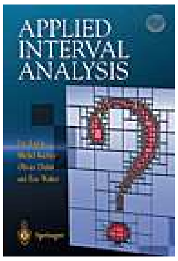
Interval constraint propagation; applications to control, estimation and robotics

Brest, UBO, Nov 13, 2009



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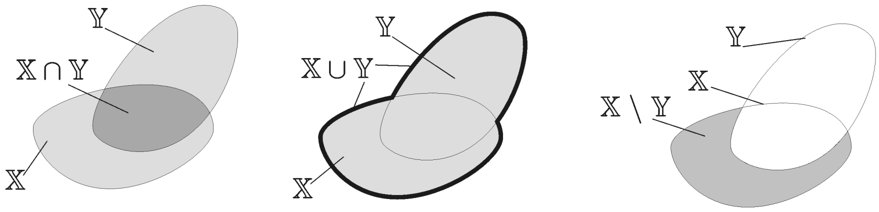
1 Set computation

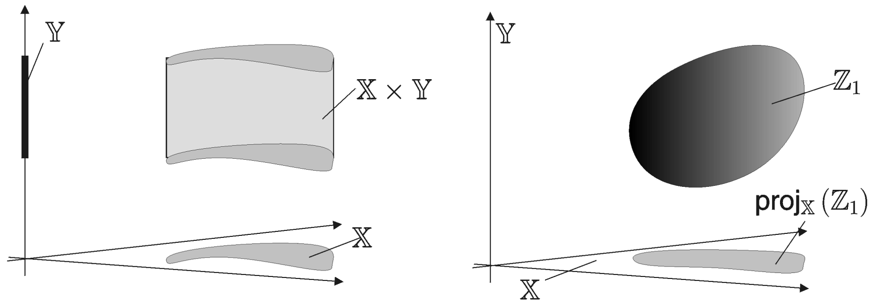
1.1 Basic notions on set theory

We define

$$
\begin{aligned}
\mathbb{X} \cap \mathbb{Y} &\stackrel{\text{def}}{=} \{x \mid x \in \mathbb{X} \text{ and } x \in \mathbb{Y}\} \\
\mathbb{X} \cup \mathbb{Y} &\stackrel{\text{def}}{=} \{x \mid x \in \mathbb{X} \text{ or } x \in \mathbb{Y}\} \\
\mathbb{X} \setminus \mathbb{Y} &\stackrel{\text{def}}{=} \{x \mid x \in \mathbb{X} \text{ and } x \notin \mathbb{Y}\} \\
\mathbb{X} \times \mathbb{Y} &\stackrel{\text{def}}{=} \{(x, y) \mid x \in \mathbb{X} \text{ and } y \in \mathbb{Y}\} \\
\text{proj}_{\mathbb{X}}(\mathbb{Z}) &\stackrel{\text{def}}{=} \{x \in \mathbb{X} \mid \exists y \in \mathbb{Y},\ (x, y) \in \mathbb{Z}\}.
\end{aligned}
$$



$\mathbb{Y}$
$\mathbb{X} \cap \mathbb{Y}$
$\mathbb{X}$
$\mathbb{Y}$
$\mathbb{X} \cup \mathbb{Y}$
$\mathbb{X}$
$\mathbb{Y}$
$\mathbb{X}$
$\mathbb{X} \setminus \mathbb{Y}$



$\mathbb{Y}$
$\mathbb{X} \times \mathbb{Y}$
$\mathbb{X}$
$\mathbb{Y}$
$\mathbb{Z}_1$
$\mathsf{proj}_{\mathbb{X}}(\mathbb{Z}_1)$
$\mathbb{X}$

Exercise: If $\mathbb{X} = \{a, b, c, d\}$ and $\mathbb{Y} = \{b, c, x, y\}$, then

$$\begin{aligned} \mathbb{X} \cap \mathbb{Y} &= ? \\ \mathbb{X} \cup \mathbb{Y} &= ? \\ \mathbb{X} \setminus \mathbb{Y} &= ? \\ \mathbb{X} \times \mathbb{Y} &= ? \end{aligned}$$

Exercise: If $\mathbb{X} = \{a, b, c, d\}$ and $\mathbb{Y} = \{b, c, x, y\}$, then

$$\begin{aligned}
\mathbb{X} \cap \mathbb{Y} &= \{b, c\} \\
\mathbb{X} \cup \mathbb{Y} &= \{a, b, c, d, x, y\} \\
\mathbb{X} \setminus \mathbb{Y} &= \{a, d\} \\
\mathbb{X} \times \mathbb{Y} &= \{(a, b), (a, c), (a, x), (a, y), \\
&\quad \ldots, (d, b), (d, c), (d, x), (d, y)\}
\end{aligned}$$

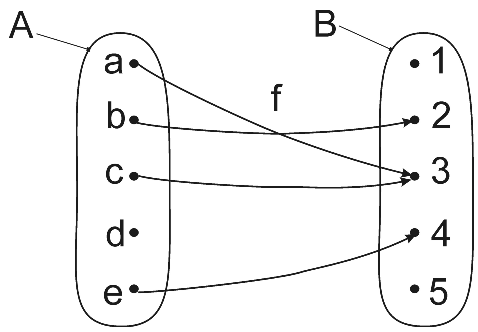
The *direct image* of $\mathbb{X}$ by f is

$$f(\mathbb{X}) \triangleq \{f(x) \mid x \in \mathbb{X}\}.$$

The *reciprocal image* of $\mathbb{Y}$ by f is

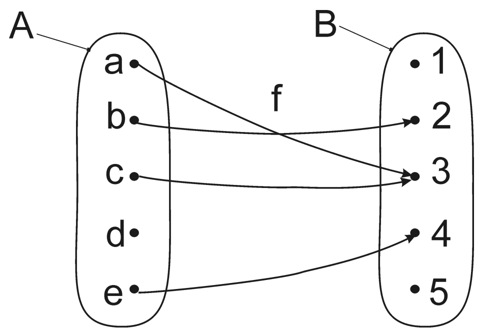
$$f^{-1}(\mathbb{Y}) \triangleq \{x \in \mathbb{X} \mid f(x) \in \mathbb{Y}\}.$$

Exercise: If f is defined as follows



$$\begin{aligned} f(A) &= ?. \\ f^{-1}(B) &= ?. \\ f^{-1}(f(A)) &= ? \\ f^{-1}(f(\{b,c\})) &= ?. \end{aligned}$$

Exercise: If f is defined as follows



$$
\begin{aligned}
f(A) &= \{2,3,4\} = \mathsf{Im}(f). \\
f^{-1}(B) &= \{a,b,c,e\} = \mathsf{dom}(f). \\
f^{-1}(f(A)) &= \{a,b,c,e\} \subset A \\
f^{-1}(f(\{b,c\})) &= \{a,b,c\}.
\end{aligned}
$$

Exercise: If $f(x) = x^2$, then

$$\begin{aligned} f([2,3]) &= ? \\ f^{-1}([4,9]) &= ?. \end{aligned}$$

Exercise: If $f(x) = x^2$, then

$$\begin{aligned} f([2,3]) &= [4,9] \\ f^{-1}([4,9]) &= [-3,-2] \cup [2,3]. \end{aligned}$$

This is consistent with the property

$$f\left(f^{-1}(\mathbb{Y})\right) \subset \mathbb{Y}.$$

1.2 Interval arithmetic

If $\diamond \in \{+, -, ., /, \mathsf{max}, \mathsf{min}\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \,.$$

For instance,

$$\begin{array}{rl} [-1, 3] + [2, 5] & = [?, ?], \\ [-1, 3].[2, 5] & = [?, ?], \\ [-1, 3]/[2, 5] & = [?, ?], \\ [-1, 3] \vee [2, 5] & = [?, ?]. \end{array}$$

If $\diamond \in \{+, -, ., /, \text{max}, \text{min}\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \,.$$

For instance,

$$\begin{aligned} [-1,3] + [2,5] &= [1,8], \\ [-1,3].[2,5] &= [-5,15], \\ [-1,3]/[2,5] &= [-\tfrac{1}{2}, \tfrac{3}{2}], \\ [-1,3] \vee [2,5] &= [2,5]. \end{aligned}$$

$$
\begin{aligned}
[x^-, x^+] + [y^-, y^+] = \quad & [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] = \quad & [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
& x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+], \\
[x^-, x^+] \vee [y^-, y^+] = \quad & [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}
$$

If $f \in \{\text{cos, sin,sqr, sqrt, log, exp, } \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] .$$

For instance,

$$\begin{aligned}
\sin([0,\pi]) &= ?, \\
\text{sqr}([-1,3]) &= [-1,3]^2 = ?, \\
\text{abs}([-7,1]) &= ?, \\
\text{sqrt}([-10,4]) &= \sqrt{[-10,4]} = ?, \\
\log([-2,-1]) &= ?.
\end{aligned}$$

If $f \in \{\cos, \sin, \mathrm{sqr}, \mathrm{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

For instance,

$$\begin{aligned}
\sin([0,\pi]) &= [0,1], \\
\mathrm{sqr}([-1,3]) &= [-1,3]^2 = [0,9], \\
\mathrm{abs}([-7,1]) &= [0,7], \\
\mathrm{sqrt}([-10,4]) &= \sqrt{[-10,4]} = [0,2], \\
\log([-2,-1]) &= \emptyset.
\end{aligned}$$

1.3 Boxes

A *box*, or *interval vector* $[\mathbf{x}]$ of $\mathbb{R}^n$ is

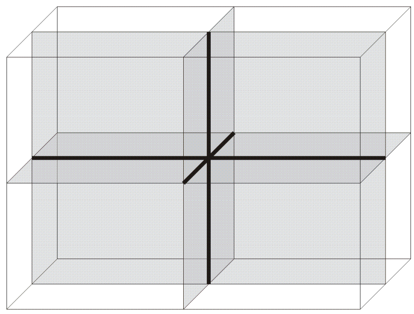
$$[\mathbf{x}] = [x_1^-, x_1^+] \times \cdots \times [x_n^-, x_n^+] = [x_1] \times \cdots \times [x_n].$$

The set of all boxes of $\mathbb{R}^n$ will be denoted by $\mathbb{IR}^n$.

The *width* $w([\mathbf{x}])$ of a box $[\mathbf{x}]$ is the length of its largest side. For instance

$$w([1,2]\times[-1,3]) = 4$$

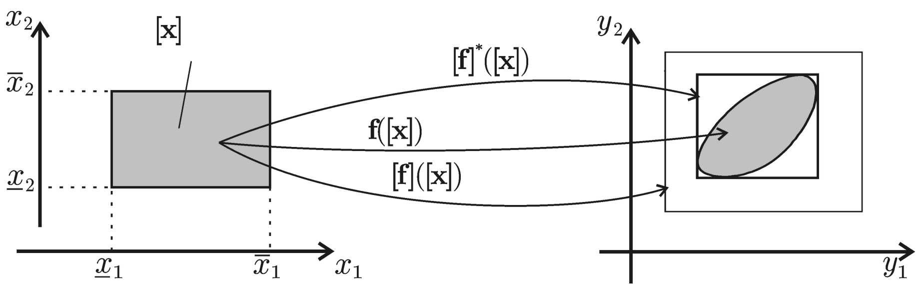
The *principal plane* of $[\mathbf{x}]$ is the symmetric plane $[\mathbf{x}]$ perpendicular to its largest side.



1.4 Inclusion function

The interval function $[\mathbf{f}]$ from $\mathbb{IR}^n$ to $\mathbb{IR}^m$, is an *inclusion function* of $\mathbf{f}$ if

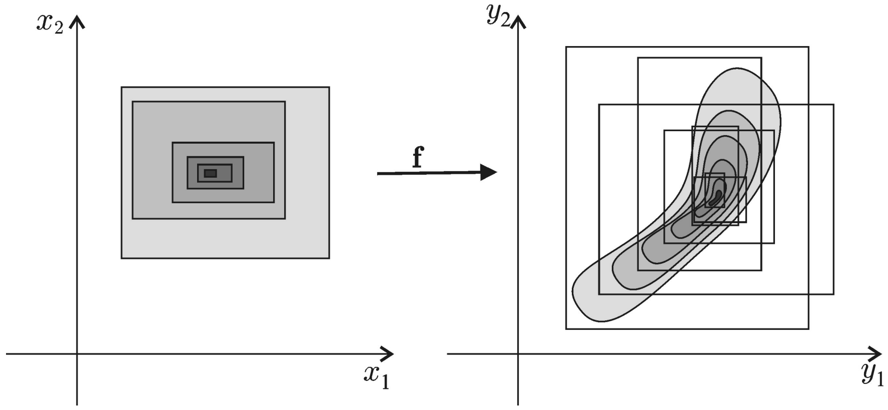
$$\forall\,[\mathbf{x}] \in \mathbb{IR}^n, \quad \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]\,([\mathbf{x}]).$$



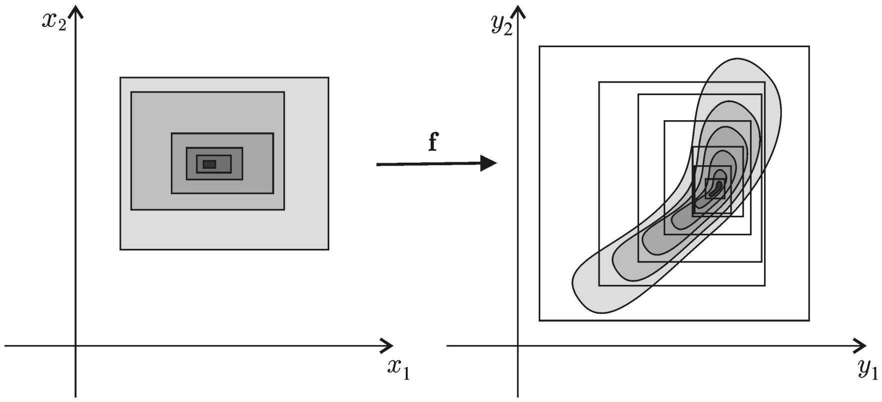
Inclusion functions $[\mathbf{f}]$ and $[\mathbf{f}]^*$; here, $[\mathbf{f}]^*$ is minimal.

The inclusion function $[\mathbf{f}]$ is

monotonic	if	$([\mathbf{x}] \subset [\mathbf{y}]) \Rightarrow ([\mathbf{f}]([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{y}]))$
minimal	if	$\forall [\mathbf{x}] \in \mathbb{IR}^n,\ [\mathbf{f}]([\mathbf{x}]) = [\mathbf{f}([\mathbf{x}])]$
thin	if	$w([\mathbf{x}]) = 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}]) = 0$
convergent	if	$w([\mathbf{x}]) \to 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}]) \to 0.$



Convergent but non-monotonic inclusion function



Convergent and monotonic inclusion function

The natural inclusion function for $f(x) = x^2 + 2x + 4$ is

$$[f]([x]) = [x]^2 + 2[x] + 4.$$

If $[x] = [-3, 4]$, we have

$$\begin{aligned} [f]([-3,4]) &= [-3,4]^2 + 2[-3,4] + 4 \\ &= [0,16] + [-6,8] + 4 \\ &= [-2,28]. \end{aligned}$$

Note that $f([-3, 4]) = [3, 28] \subset [f]([-3, 4]) = [-2, 28]$.

A minimal inclusion function for

$$\mathbf{f}: \begin{array}{ccc} \mathbb{R}^2 & \rightarrow & \mathbb{R}^3 \\ (x_1, x_2) & \mapsto & \left(x_1 x_2, x_1^2, x_1 - x_2\right). \end{array}$$

is

$$[\mathbf{f}]: \begin{array}{ccc} \mathbb{IR}^2 & \rightarrow & \mathbb{IR}^3 \\ ([x_1], [x_2]) & \rightarrow & \left([x_1] * [x_2], [x_1]^2, [x_1] - [x_2]\right). \end{array}$$

If $\mathbf{f}$ is given by the algorithm

Algorithm $\mathbf{f}$(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: $\mathbf{y} = (y_1, y_2)$)
$z := x_1$;
for $k :=$ 0 to 100
$\qquad z := x_2(z + kx_3)$;
next;
$y_1 := z$;
$y_2 :=$ sin(zx_1);

Its natural inclusion function is

```
Algorithm [f](in: [x], out: [y])
[z] := [x1];
for k := 0 to 100
    [z] := [x2] * ([z] + k * [x3]);
next;
[y1] := [z] ;
[y2] := sin([z] * [x1]);
```

Here, $[\mathbf{f}]$ is a convergent, thin and monotonic inclusion function for $\mathbf{f}$.

1.5 Subpavings

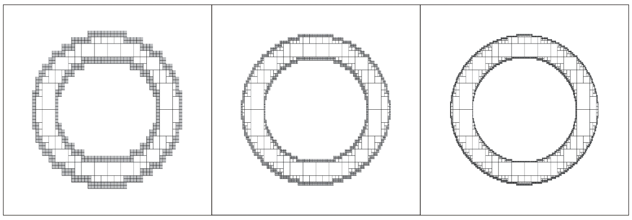
A subpaving of $\mathbb{R}^n$ is a set of non-overlapping boxes of $\mathbb{R}^n$.

Compact sets $\mathbb{X}$ can be bracketed between inner and outer subpavings:

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Example.

$$\mathbb{X} = \{(x_1, x_2) \bigm| x_1^2 + x_2^2 \in [1, 2]\}.$$



Set operations such as $\mathbb{Z} := \mathbb{X} + \mathbb{Y}$, $\mathbb{X} := \mathbf{f}^{-1}(\mathbb{Y})$, $\mathbb{Z} := \mathbb{X} \cap \mathbb{Y} \ldots$ can be approximated by subpaving operations.

1.6 Set inversion

Let $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ and let $\mathbb{Y}$ be a subset of $\mathbb{R}^m$. Set inversion is the characterization of

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}).$$

We shall use the following tests.

$$\begin{array}{lll} \text{(i)} & [\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y} & \Rightarrow \quad [\mathbf{x}] \subset \mathbb{X} \\ \text{(ii)} & [\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset & \Rightarrow \quad [\mathbf{x}] \cap \mathbb{X} = \emptyset. \end{array}$$

Boxes for which these tests failed, will be bisected, except if they are too small.

Stack-queue

A *queue* is a list on which two operations are allowed :

- add an element at the end (*push*)
- remove the first element (*pull*).

A *stack* is a list on which two operations are allowed :

- add an element at the beginning of the list (*stack*)
- remove the first element (*pop*).

Example: Let $\mathcal{L}$ be an empty queue.

k	operation	result
0		$\mathcal{L} = \emptyset$
1	push $(\mathcal{L}, a)$	$\mathcal{L} = \{a\}$
2	push $(\mathcal{L}, b)$	$\mathcal{L} = \{a, b\}$
3	$x :=$ pull $(\mathcal{L})$	$x = a, \mathcal{L} = \{b\}$
4	$x :=$ pull $(\mathcal{L})$	$x = b, \mathcal{L} = \emptyset.$

If $\mathcal{L}$ is a stack, the table becomes

k	operation	result
0		$\mathcal{L} = \emptyset$
1	stack $(\mathcal{L}, a)$	$\mathcal{L} = \{a\}$
2	stack $(\mathcal{L}, b)$	$\mathcal{L} = \{a, b\}$
3	$x :=$ pop $(\mathcal{L})$	$x = b, \mathcal{L} = \{a\}$
4	$x :=$ pop $(\mathcal{L})$	$x = a, \mathcal{L} = \emptyset.$

.

Algorithm Sivia(in: $[\mathbf{x}](0), \mathbf{f}, \mathbb{Y}$)

1 $\mathcal{L} := \{[\mathbf{x}](0)\}$;
2 pull $[\mathbf{x}]$ from $\mathcal{L}$;
3 if $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y}$, draw($[\mathbf{x}]$, 'red');
4 elseif $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset$, draw($[\mathbf{x}]$, 'blue');
5 elseif $w([\mathbf{x}]) < \varepsilon$, {draw ($[\mathbf{x}]$, 'yellow')};
6 else bisect $[\mathbf{x}]$ and push into $\mathcal{L}$;
7 if $\mathcal{L} \neq \emptyset$, go to 2

If $\Delta\mathbb{X}$ denotes the union of yellow boxes and if $\mathbb{X}^-$ is the union of red boxes then :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^- \cup \Delta\mathbb{X}.$$

1.7 Image evaluation

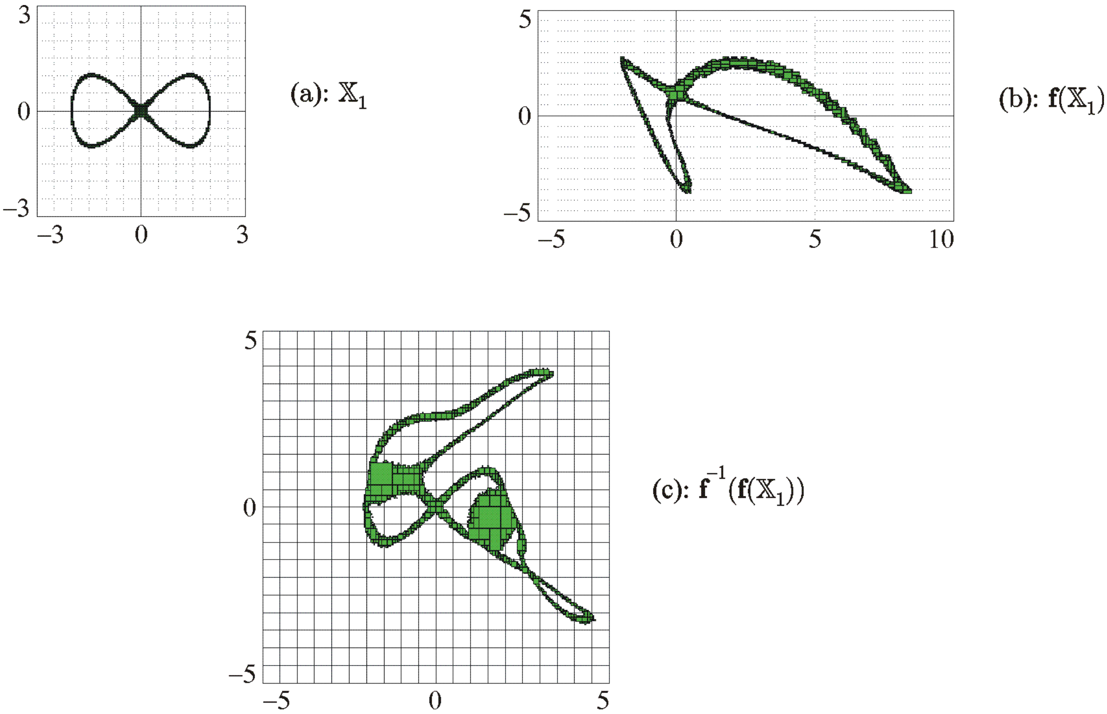
Define

$$\mathbf{f}(x_1, x_2) = \begin{pmatrix} (x_1 - 1)^2 - 1 + x_2 \\ -x_1^2 + (x_2 - 1)^2 \end{pmatrix},$$

and

$$\mathbb{X}_1 = \left\{ (x_1, x_2) \in \mathbb{R}^2 \,\middle|\, x_1^4 - x_1^2 + 4x_2^2 \in [-0.1, 0.1] \right\}.$$

We shall compute $\mathbb{X}_1$, $\mathbf{f}(\mathbb{X}_1)$ and $\mathbf{f}^{-1} \circ \mathbf{f}(\mathbb{X}_1)$.



3
0
−3
−3
0
3
(a): $\mathbb{X}_1$
5
0
−5
−5
0
5
10
(b): $\mathbf{f}(\mathbb{X}_1)$
5
0
−5
−5
0
5
(c): $\mathbf{f}^{-1}(\mathbf{f}(\mathbb{X}_1))$

2 Applications of set computation

2.1 Bounded-error estimation

Model : $\phi(\mathbf{p}, t) = p_1 e^{-p_2 t}$.

Prior feasible box for the parameters : $[\mathbf{p}] \subset \mathbb{R}^2$

Measurement times : $t_1, t_2, \ldots, t_m$

Data bars : $[y_1^-, y_1^+], [y_2^-, y_2^+], \ldots, [y_m^-, y_m^+]$

$\mathbb{S} = \{\mathbf{p} \in [\mathbf{p}], \phi(\mathbf{p}, t_1) \in [y_1^-, y_1^+], \ldots, \phi(\mathbf{p}, t_m) \in [y_m^-, y_m^+]$

If

$$\boldsymbol{\phi}(\mathbf{p}) = \begin{pmatrix} \phi(\mathbf{p}, t_1) \\ \\ \phi(\mathbf{p}, t_m) \end{pmatrix}$$

and

$$[\mathbf{y}] = [y_1^-, y_1^+] \times \cdots \times [y_m^-, y_m^+]$$

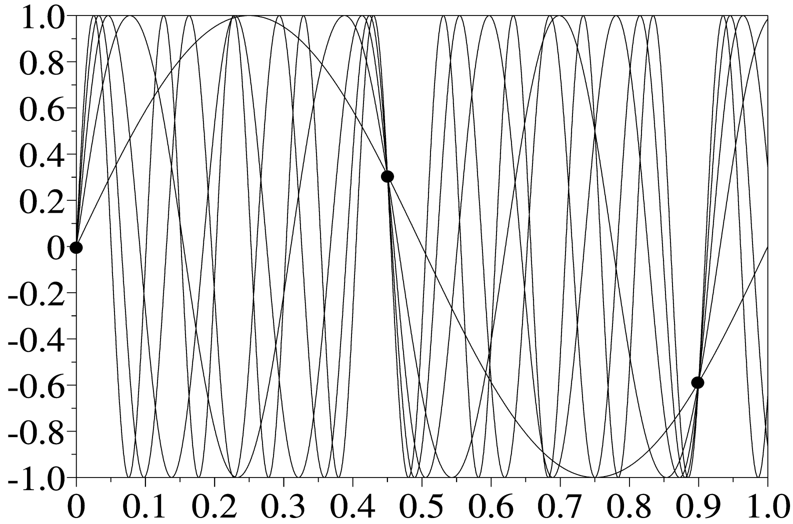
then

$$\mathbb{S} = [\mathbf{p}] \cap \boldsymbol{\phi}^{-1}([\mathbf{y}]) \,.$$

Show Setdemo (Guillaume Baffet), available at

`www.ensieta.fr/jaulin/demo.html`

If now $\phi(\mathbf{p}, t) = p_1 \sin(2\pi p_2 t)$ and $t_k = k\delta, \ldots$. $\mathbb{S}$ contains an infinite number of connected components.



2.2 Robustification against outliers

Define a *relaxing function* for the box $[\mathbf{y}] = [y_1] \times \cdots \times [y_n]$

$$\lambda(\mathbf{y}) = \pi_{[y_1]}(y_1) + \cdots + \pi_{[y_n]}(y_n)$$

where

$$\pi_{[a,b]}(x) \begin{cases} = 1 & \text{if} \quad x \in [a,b] \\ = 0 & \text{if} \quad x \notin [a,b]. \end{cases}$$

Allow up to q of the n output variables y_i to escape their prior feasible intervals. The posterior feasible set becomes

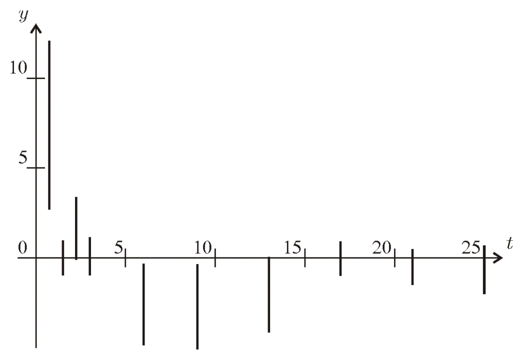
$$\hat{\mathbb{P}}_q = \{\mathbf{p} \in [\mathbf{p}] \mid \pi_{[y_1]}(\phi_1(\mathbf{p})) + \cdots + \pi_{[y_n]}(\phi_n(\mathbf{p})) \geqslant n - q\}.$$

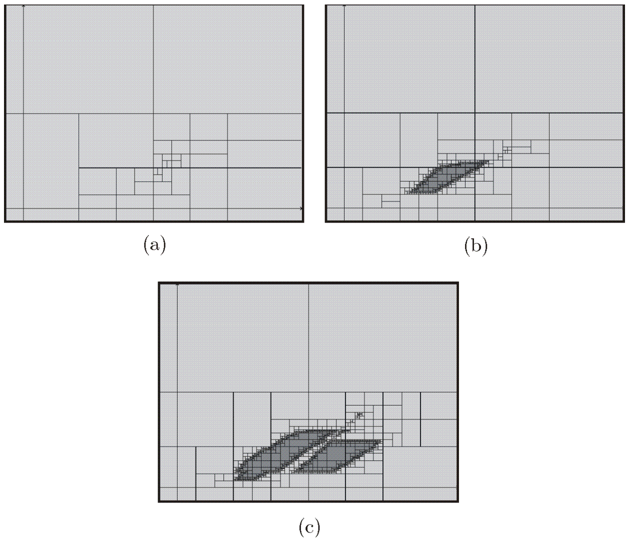
This is a set inversion problem. The set $\hat{\mathbb{P}}_q$ can thus be characterized by Sivia.

As an illustration, consider the model

$$\phi(\mathbf{p}, t) = 20\,\mathsf{exp}(-p_1 t) - 8\,\mathsf{exp}(-p_2 t)$$

with the data bars represented on the figure below





(a) no outlier assumed; (b) one outlier assumed; (c) two outliers assumed;

2.3 Sailboat

State equations

$$\begin{cases} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta - 1 \\ \dot{\theta} &= \omega \\ \dot{\delta}_s &= u_1 \\ \dot{\delta}_r &= u_2 \\ \dot{v} &= f_s\sin\delta_s - f_r\sin\delta_r - v \\ \dot{\omega} &= (1-\cos\delta_s)\,f_s - \cos\delta_r . f_r - \omega \\ f_s &= \cos(\theta+\delta_s) - v\sin\delta_s \\ f_r &= v\sin\delta_r . \end{cases}$$

In a cruising phase

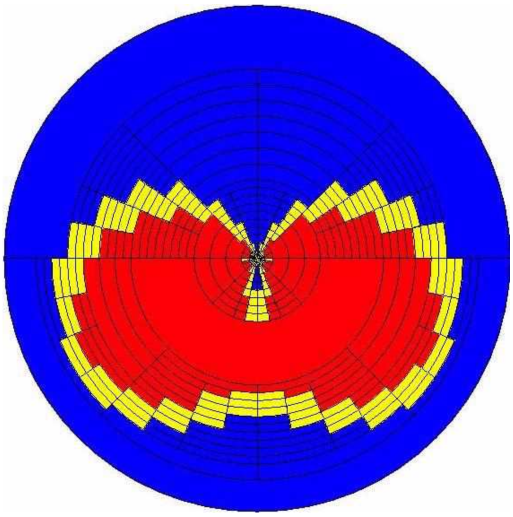
$$\dot{\theta} = 0, \dot{\delta}_s = 0, \dot{\delta}_r = 0, \dot{v} = 0, \dot{\omega} = 0.$$

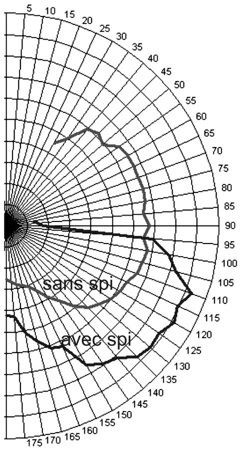
i.e.,

$$\begin{cases} 0 &= \omega \\ 0 &= u_1 \\ 0 &= u_2 \\ 0 &= f_s \sin \delta_s - f_r \sin \delta_r - v \\ 0 &= (1 - \cos \delta_s) f_s - \cos \delta_r . f_r - \omega \\ f_s &= \cos(\theta + \delta_s) - v \sin \delta_s \\ f_r &= v \sin \delta_r . \end{cases}$$

The polar diagram is

$$\begin{aligned}\mathbb{S}_y = \{(\theta, v) \quad & \mid \exists f_s, \delta_s, f_r, \delta_r, \\ & f_s \sin \delta_s - f_r \sin \delta_r - v = 0 \\ & (1 - \cos \delta_s)\, f_s - \cos \delta_r f_r = 0 \\ & f_s = \cos(\theta + \delta_s) - v \sin \delta_s \\ & f_r = v \sin \delta_r \,\}\end{aligned}$$




5
10
15
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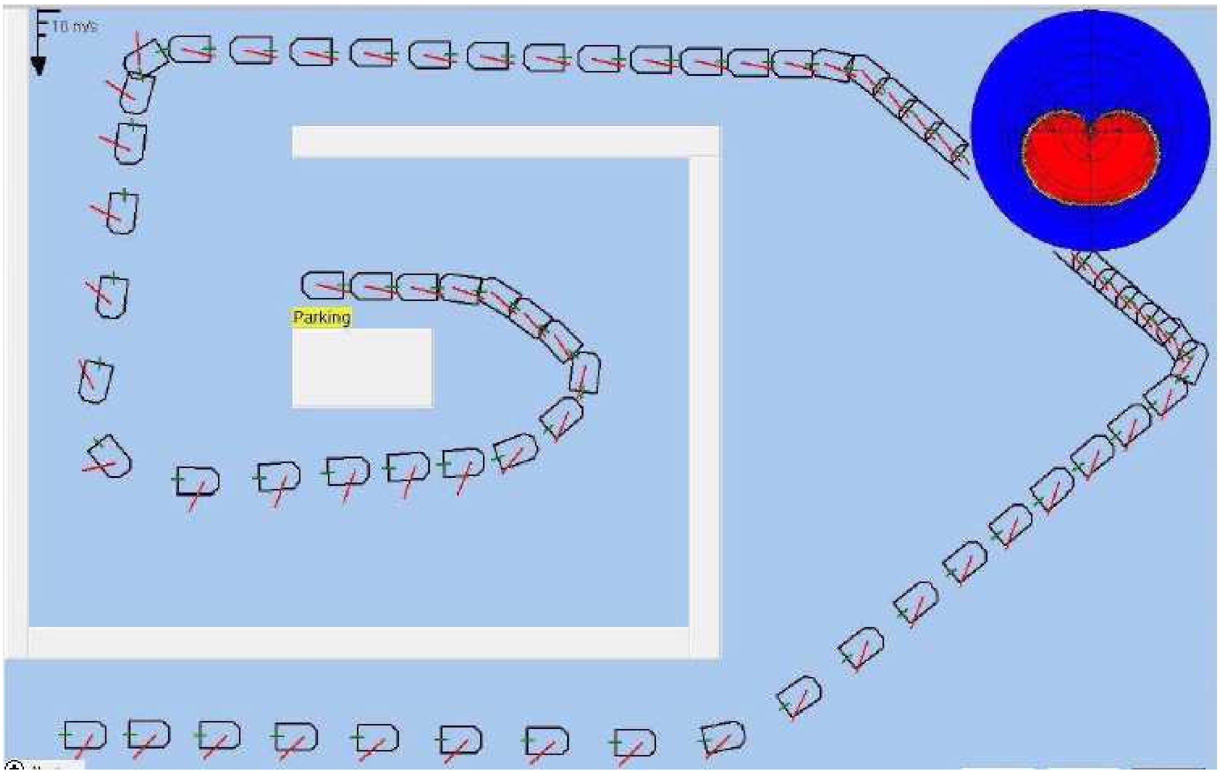
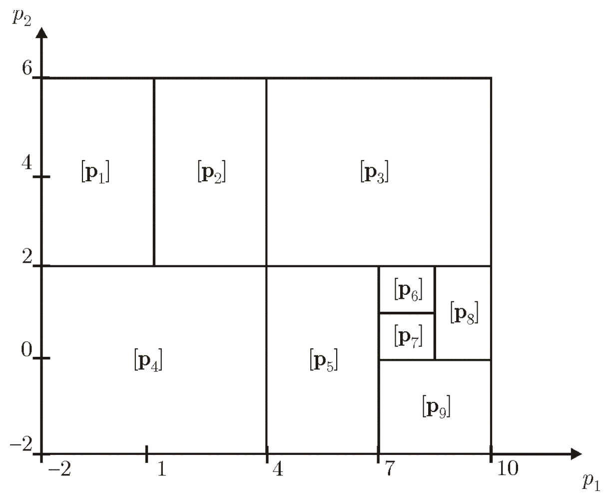
160
165
170
175
sans spi
avec spi

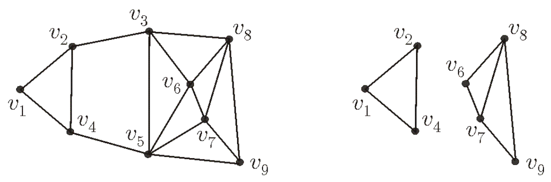

10 m/s
Parking

3 Interval and graphs

3.1 Path planning

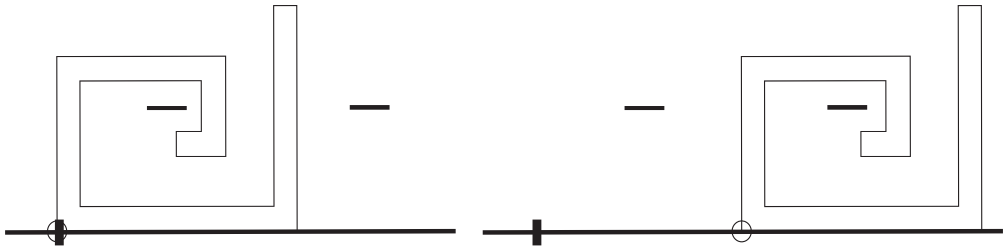


p_2
6
4
2
0
−2
−2
1
4
7
10
p_1
$[\mathbf{p}_1]$
$[\mathbf{p}_2]$
$[\mathbf{p}_3]$
$[\mathbf{p}_4]$
$[\mathbf{p}_5]$
$[\mathbf{p}_6]$
$[\mathbf{p}_7]$
$[\mathbf{p}_8]$
$[\mathbf{p}_9]$

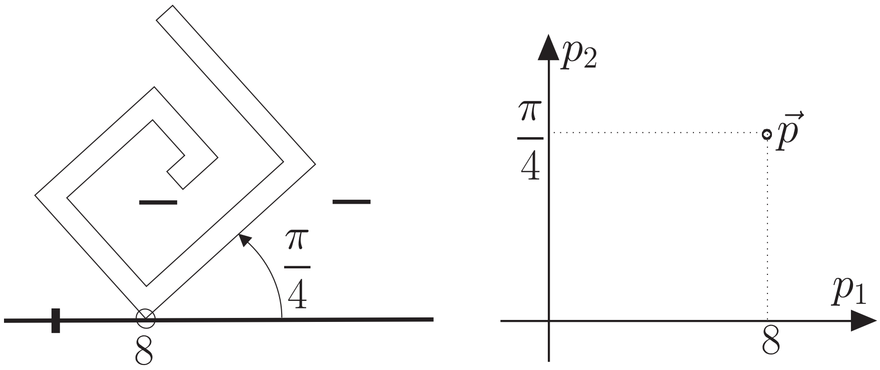


v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8
v_9
v_1
v_2
v_4
v_6
v_7
v_8
v_9

(a) (b)

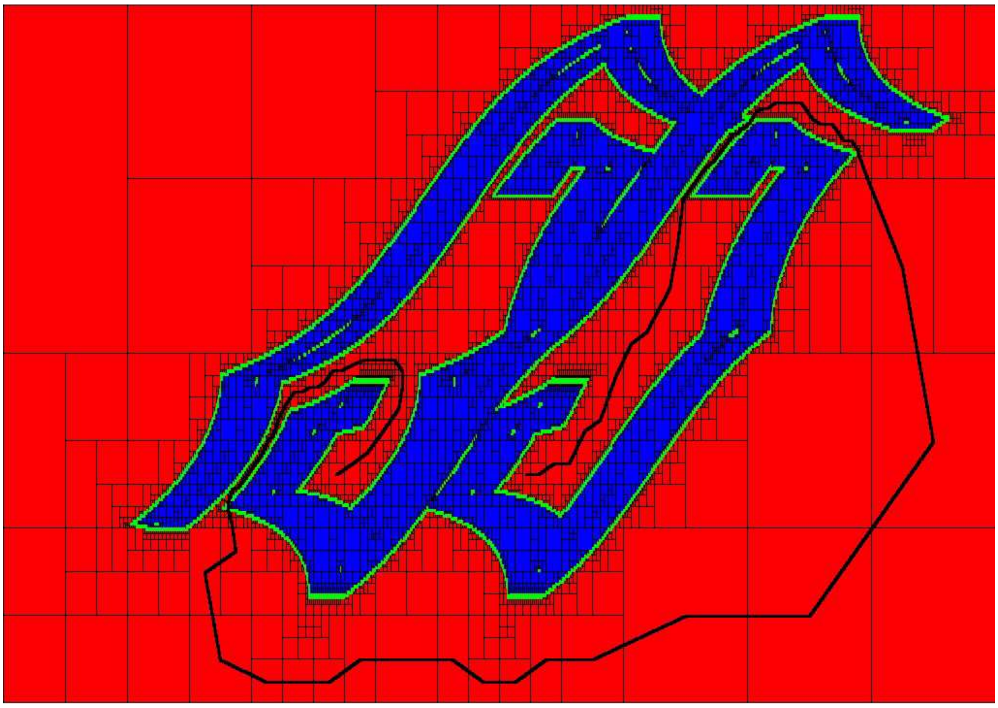


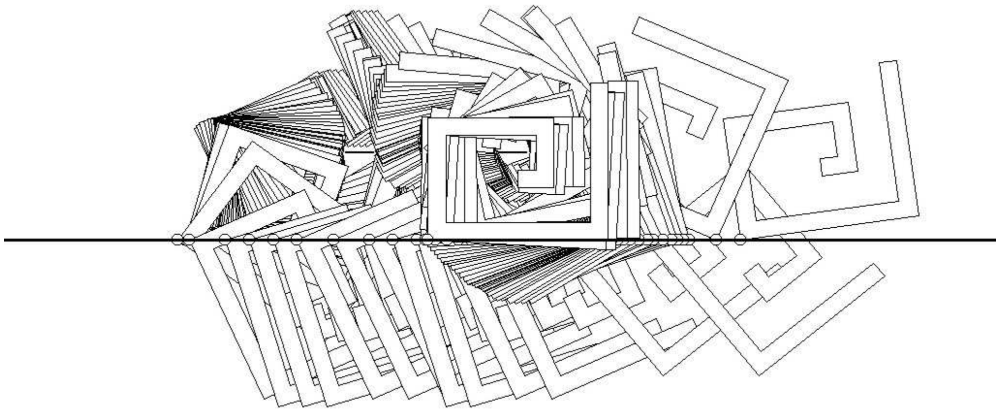
Initial configuration: $\vec{p} = (0\ 0)^{\mathrm{T}}$ Goal configuration: $\vec{p} = (17\ 0)^{\mathrm{T}}$

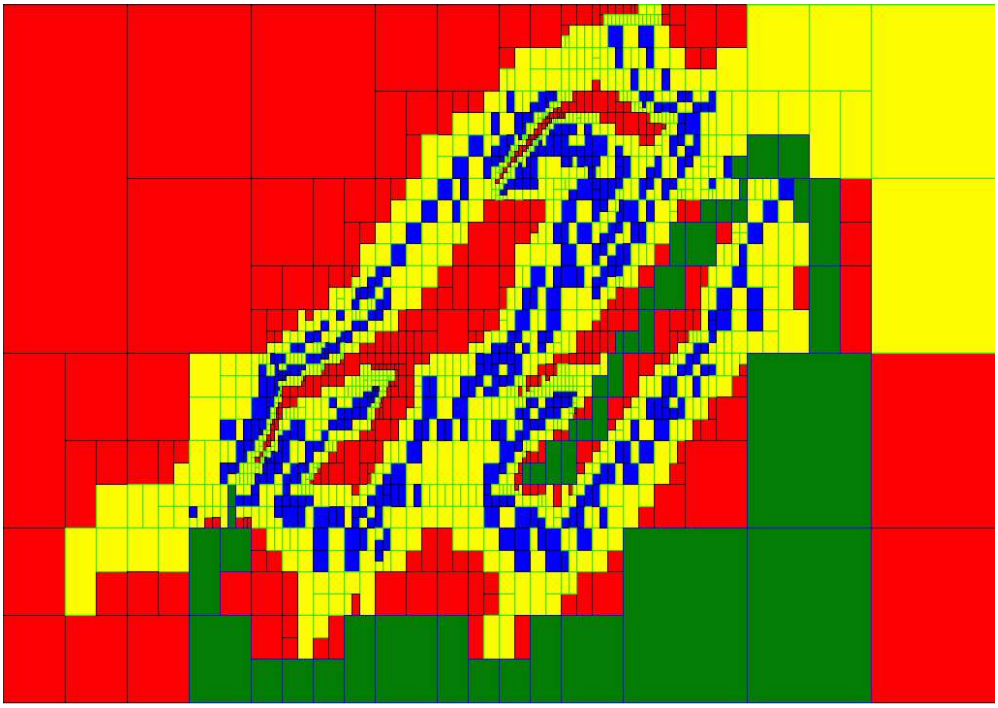


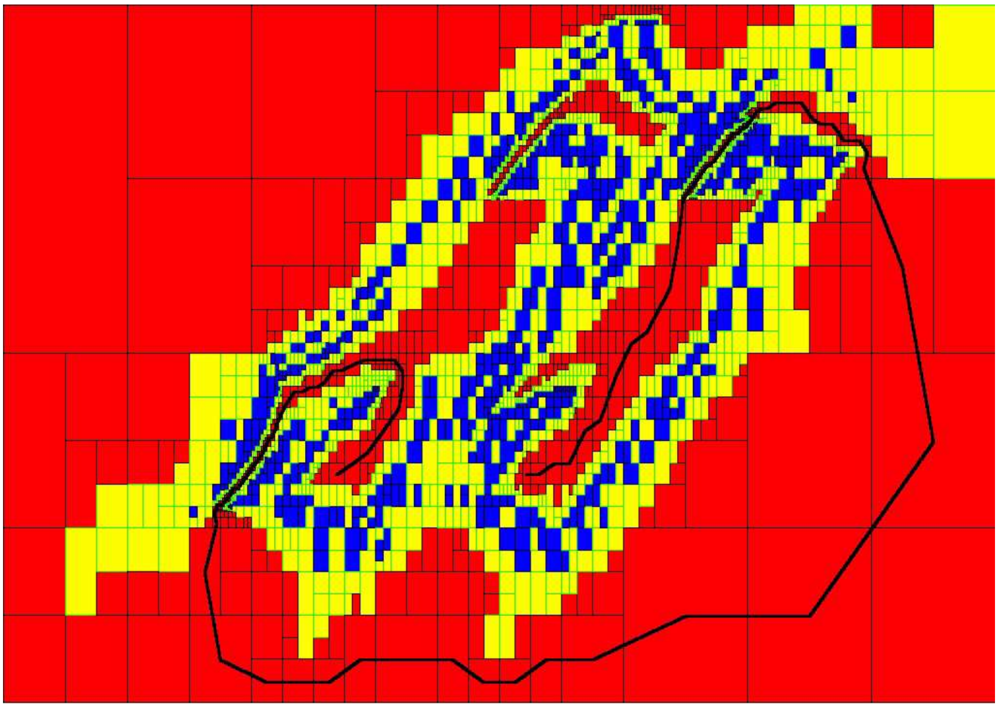
Room

Configuration space









3.2 Counting connected components

(Collaboration with N. Delanoue and B. Cottenceau)

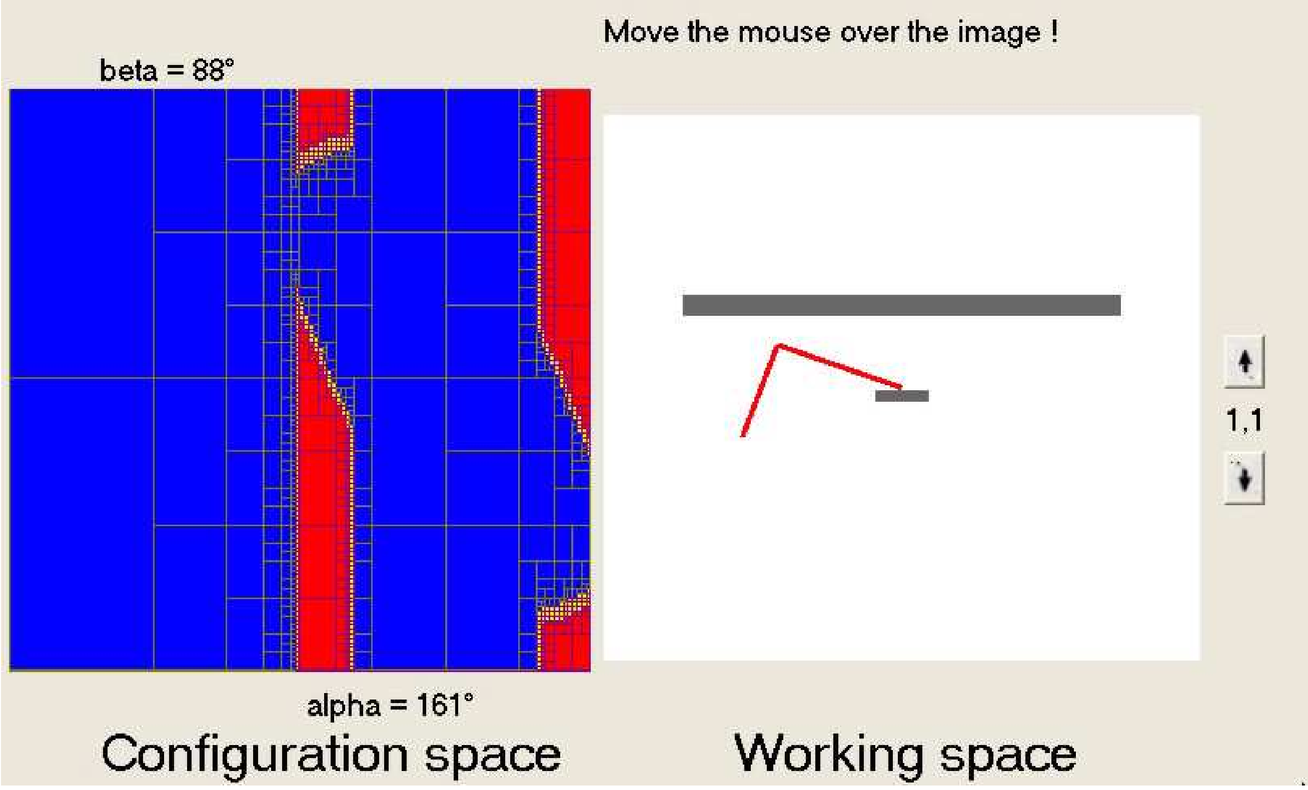
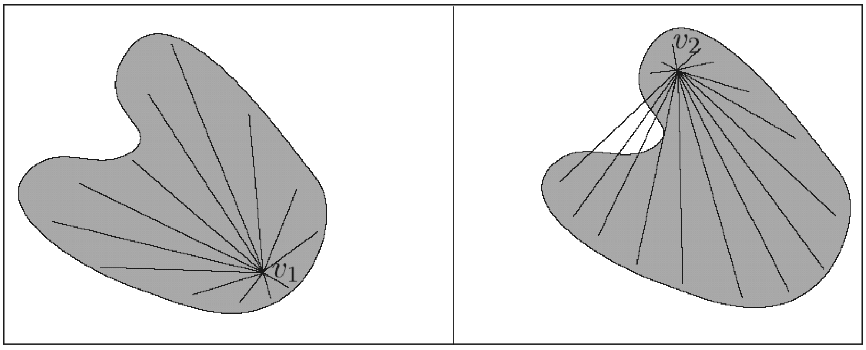


Figure 1:

The point $\mathbf{v}$ is a *star* for $\mathbb{S} \subset \mathbb{R}^n$ if $\forall \mathbf{x} \in \mathbb{S}, \forall \alpha \in [0, 1]$, $\alpha\mathbf{v} + (1 - \alpha)\mathbf{x} \in \mathbb{S}.$.



$\mathbf{v}_1$ is a star for $\mathbb{S}$ whereas $\mathbf{v}_2$ is not

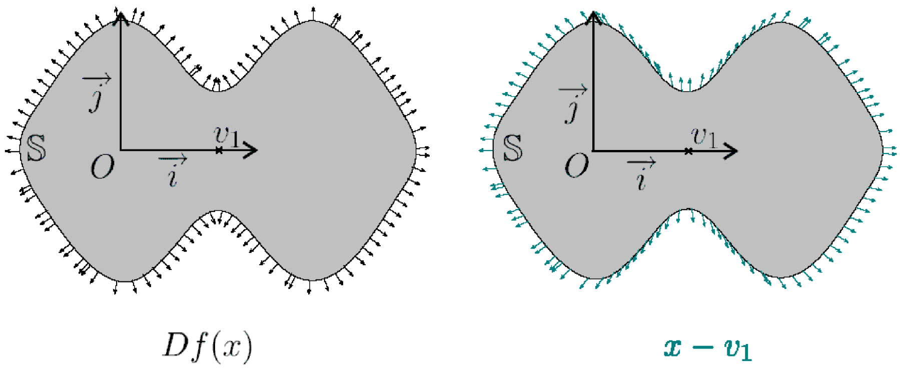
The set $\mathbb{S} \subset \mathbb{R}^n$ is *star-shaped* is there exists $\mathbf{v}$ such that $\mathbf{v}$ is a star for $\mathbb{S}$.

Theorem: Define the set

$$\mathbb{S} \stackrel{\text{def}}{=} \{\mathbf{x} \in [\mathbf{x}] | f(\mathbf{x}) \leq 0\}$$

where f is differentiable. We have the following implication

$$\left\{\mathbf{x} \in [\mathbf{x}] \mid f(\mathbf{x}) = 0, \frac{df}{d\mathbf{x}}(\mathbf{x}).(\mathbf{x} - \mathbf{v}) \leq 0\right\} = \emptyset \Rightarrow \mathbf{v} \text{ is a star}$$



If $\mathbf{v}$ is a star for $\mathbb{S}_1$ and a star for $\mathbb{S}_2$ then it is a star for $\mathbb{S}_1 \cap \mathbb{S}_2$ and for $\mathbb{S}_1 \cup \mathbb{S}_2$.

Consider a subpaving $\mathcal{P} = \{[\mathbf{p}_1], [\mathbf{p}_2], \ldots\}$ covering $\mathbb{S}$.
The relation $\mathcal{R}$ defined by

$$[\mathbf{p}]\mathcal{R}[\mathbf{q}] \Leftrightarrow \mathbb{S} \cap [\mathbf{p}] \cap [\mathbf{q}] \neq \emptyset$$

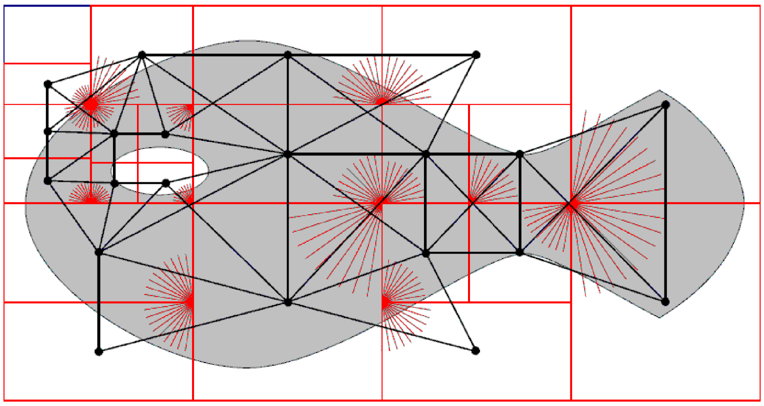
is *star-spangled graph* of the set $\mathbb{S}$ if

$$\forall [\mathbf{p}] \in \mathcal{P}, \mathbb{S} \cap [\mathbf{p}] \text{ is star-shaped.}$$

For instance, a star-spangled graph for the set

$$\mathbb{S} \stackrel{\text{def}}{=} \left\{ (x,y) \in \mathbb{R}^2 \mid \left(\begin{array}{c} x^2 + 4y^2 - 16 \\ 2\sin x - \cos y + y^2 - \frac{3}{2} \\ -(x+\frac{5}{2})^2 - 4(y-\frac{2}{5})^2 + \frac{3}{10} \end{array} \right) \leq 0 \right\}$$

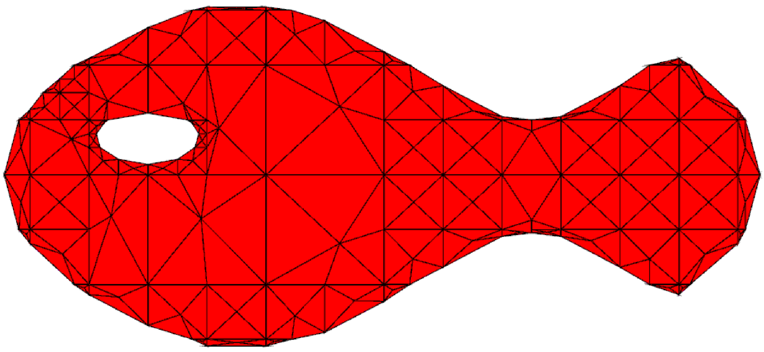
is



For each $[\mathbf{p}]$ of the paving $\mathcal{P}$, a common star located at the corner of $[\mathbf{p}]$ (represented in red) has been found for all three constraints.

Theorem: The number of connected components of the star-spangled graph of $\mathbb{S}$ is equal to that of $\mathbb{S}$.

An extension of this approach has also been developed with N. Delanoue to compute a triangulation homeomorphic to $\mathbb{S}$.



4 Contractors

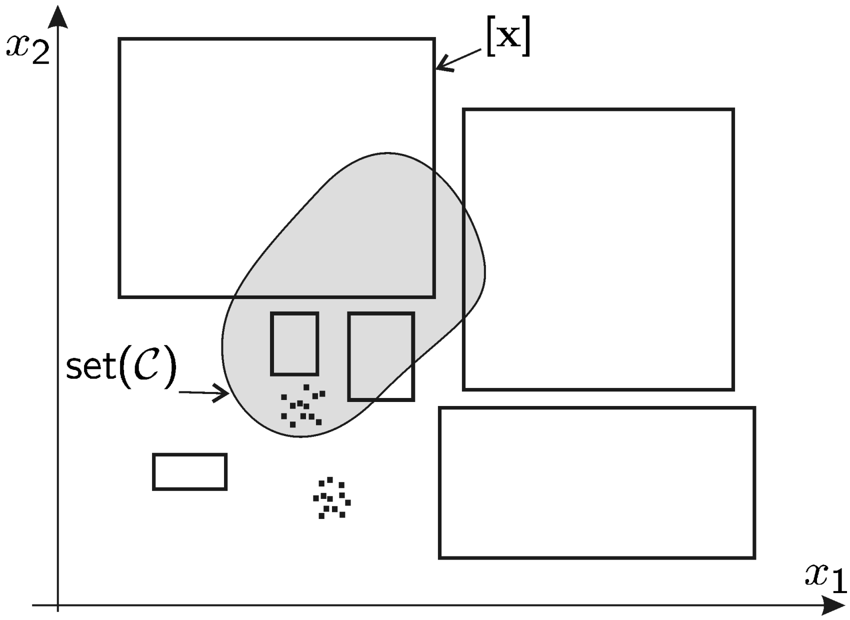
To characterize $\mathbb{X} \subset \mathbb{R}^n$, bisection algorithms bisect all boxes in all directions and become inefficient. Interval methods can still be useful if

- the solution set $\mathbb{X}$ is small (optimization problem, solving equations),
- contraction procedures are used as much as possible,
- bisections are used only as a last resort.

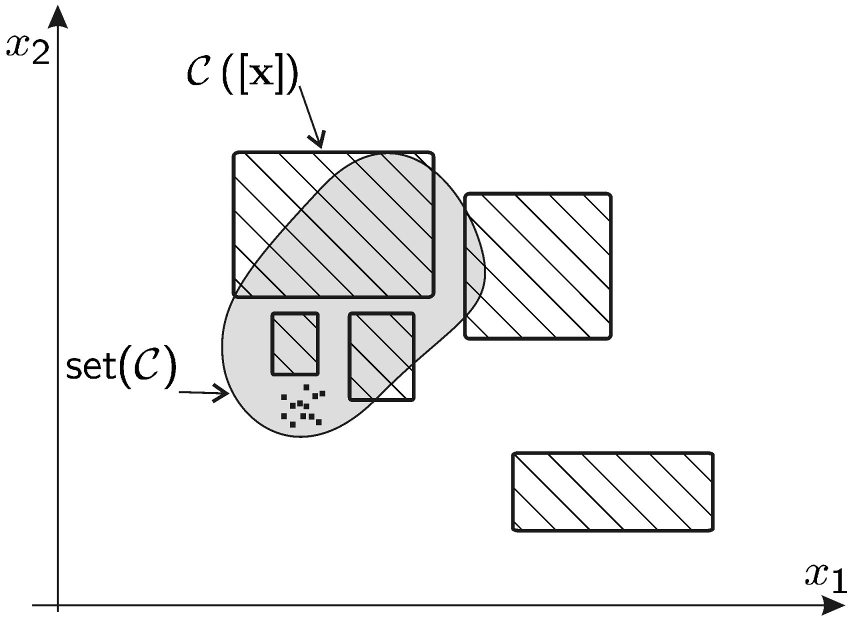
4.1 Definition

The operator $\mathcal{C}_{\mathbb{X}} : \mathbb{IR}^n \to \mathbb{IR}^n$ is a *contractor* for $\mathbb{X} \subset \mathbb{R}^n$ if

$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \left\{ \begin{array}{ll} \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)}, \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(completeness)}. \end{array} \right.$$



x_2
[x]
set($\mathcal{C}$)
x_1



x_2
$\mathcal{C}([\mathbf{x}])$
set($\mathcal{C}$)
x_1

$\mathcal{C}_{\mathbb{X}}$ is *monotonic* if	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset \mathcal{C}_{\mathbb{X}}([\mathbf{y}])$
$\mathcal{C}_{\mathbb{X}}$ is *minimal* if	$\forall[\mathbf{x}] \in \mathbb{IR}^n,\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathbb{X}]$
$\mathcal{C}_{\mathbb{X}}$ is *thin* if	$\forall\mathbf{x} \in \mathbb{R}^n,\ \mathcal{C}_{\mathbb{X}}(\{\mathbf{x}\}) = \{\mathbf{x}\} \cap \mathbb{X}$
$\mathcal{C}_{\mathbb{X}}$ is *idempotent* if	$\forall[\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}\left(\mathcal{C}_{\mathbb{X}}([\mathbf{x}])\right) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}])$.

$\mathcal{C}_{\mathbb{X}}$ is said to be *convergent* if

$$[\mathbf{x}](k) \to \mathbf{x} \quad \Rightarrow \quad \mathcal{C}_{\mathbb{X}}([\mathbf{x}]\,(k)) \to \{\mathbf{x}\} \cap \mathbb{X}.$$

4.2 Projection of constraints

Let x, y, z be 3 variables such that

$$\begin{aligned} x &\in [-\infty, 5], \\ y &\in [-\infty, 4], \\ z &\in [6, \infty], \\ z &= x + y. \end{aligned}$$

The values < 2 for x, < 1 for y and > 9 for z are inconsistent.

To *project* a constraint (here, $z = x + y$), is to compute the smallest intervals which contains all consistent values.

For our example, this amounts to project onto x, y and z the set

$$\mathbb{S} = \{(x, y, z) \in [-\infty, 5] \times [-\infty, 4] \times [6, \infty] \mid z = x + y\}\,.$$

4.3 Numerical method for projection

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}$$

The contractor associated with $z = x + y$ is.

Algorithm pplus(inout: $[z], [x], [y]$)
$[z] := [z] \cap ([x] + [y])$;
$[x] := [x] \cap ([z] - [y])$;
$[y] := [y] \cap ([z] - [x])$.

The projection procedure developed for plus can be extended to other ternary constraints such as mult: $z = x * y$, or equivalently

$$\text{mult} \triangleq \left\{(x, y, z) \in \mathbb{R}^3 \mid z = x * y\right\}.$$

The resulting projection procedure becomes

Algorithm pmult(inout: $[z], [x], [y]$)
1 $[z] := [z] \cap ([x] * [y])$;
2 $[x] := [x] \cap ([z] * 1/[y])$;
3 $[y] := [y] \cap ([z] * 1/[x])$.

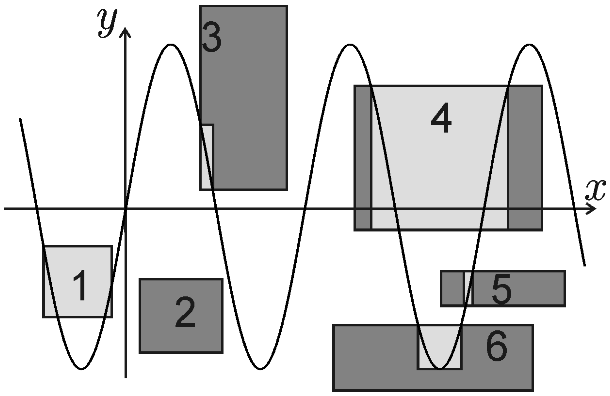
Consider the binary constraint

$$\exp \triangleq \{(x, y) \in \mathbb{R}^n | y = \exp(x)\}.$$

The associated contractor is

Algorithm pexp(inout: $[y], [x]$)
1 $[y] := [y] \cap \exp([x])$;
2 $[x] := [x] \cap \log([y])$.

Any constraint for which such a projection procedure is available will be called a *primitive constraint*.



Projection of the sine constraint

4.4 Constraint propagation

A CSP (Constraint Satisfaction Problem) is composed of

1) a set of variables $\mathcal{V} = \{x_1, \ldots, x_n\}$,

2) a set of constraints $\mathcal{C} = \{c_1, \ldots, c_m\}$ and

3) a set of interval domains $\{[x_1], \ldots, [x_n]\}$.

Principle of propagation techniques: contract $[\mathbf{x}] = [x_1] \times \cdots \times [x_n]$ as follows:

$$((((((([\mathbf{x}] \sqcap c_1) \sqcap c_2) \sqcap \ldots) \sqcap c_m) \sqcap c_1) \sqcap c_2) \ldots,$$

until a steady box is reached.

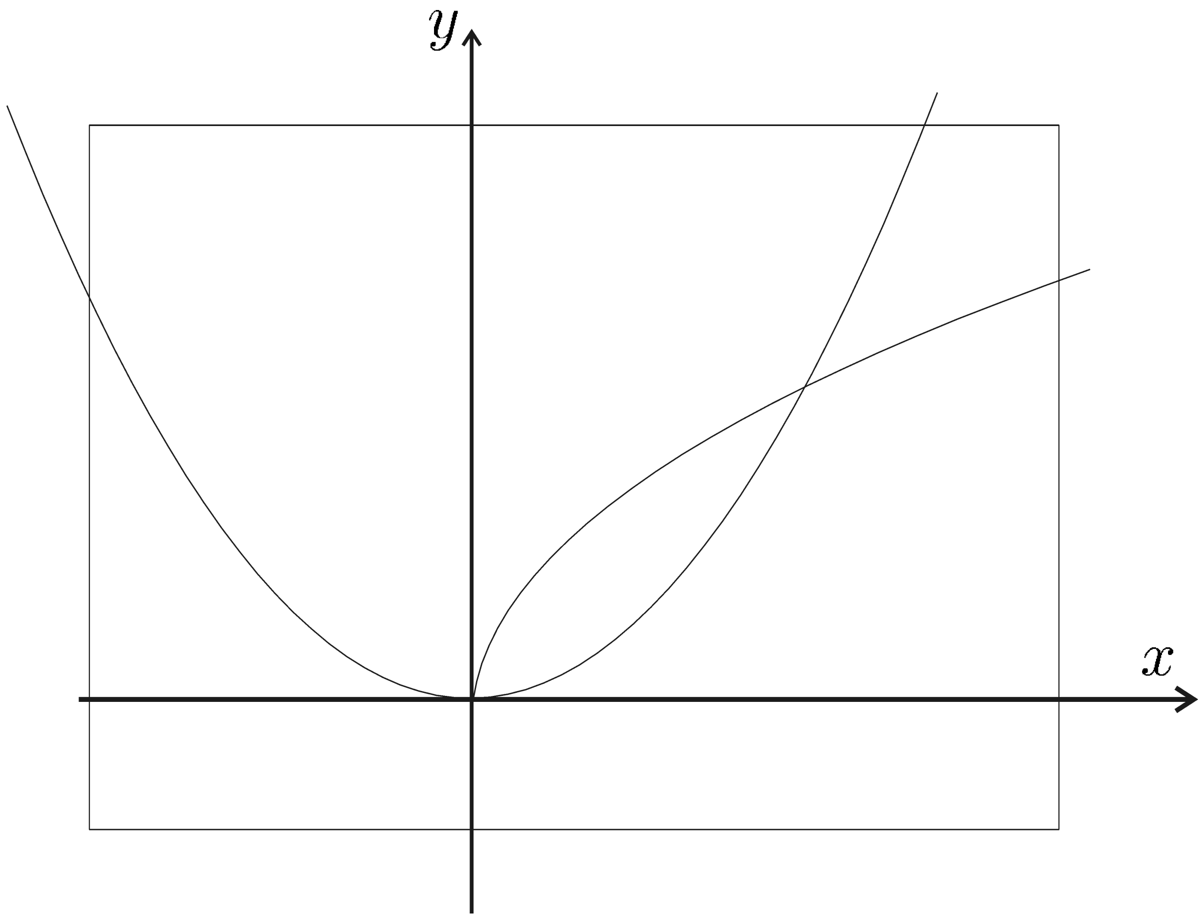
Example. Consider the system of two equations.

$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

We can build two contractors

$$\mathcal{C}_1 : \left\{ \begin{array}{l} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{array} \right. \quad \text{associated to } y = x^2$$

$$\mathcal{C}_2 : \left\{ \begin{array}{l} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{array} \right. \quad \text{associated to } y = \sqrt{x}$$



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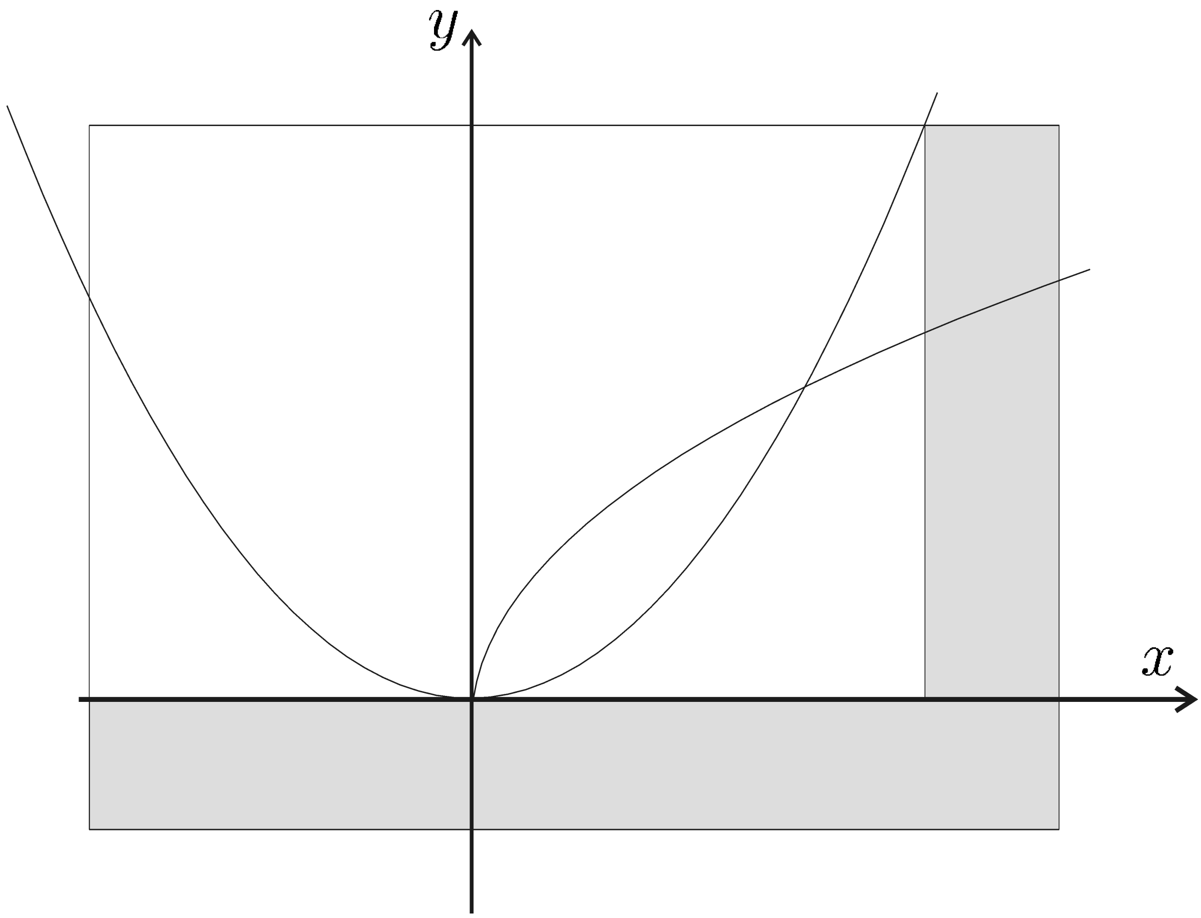
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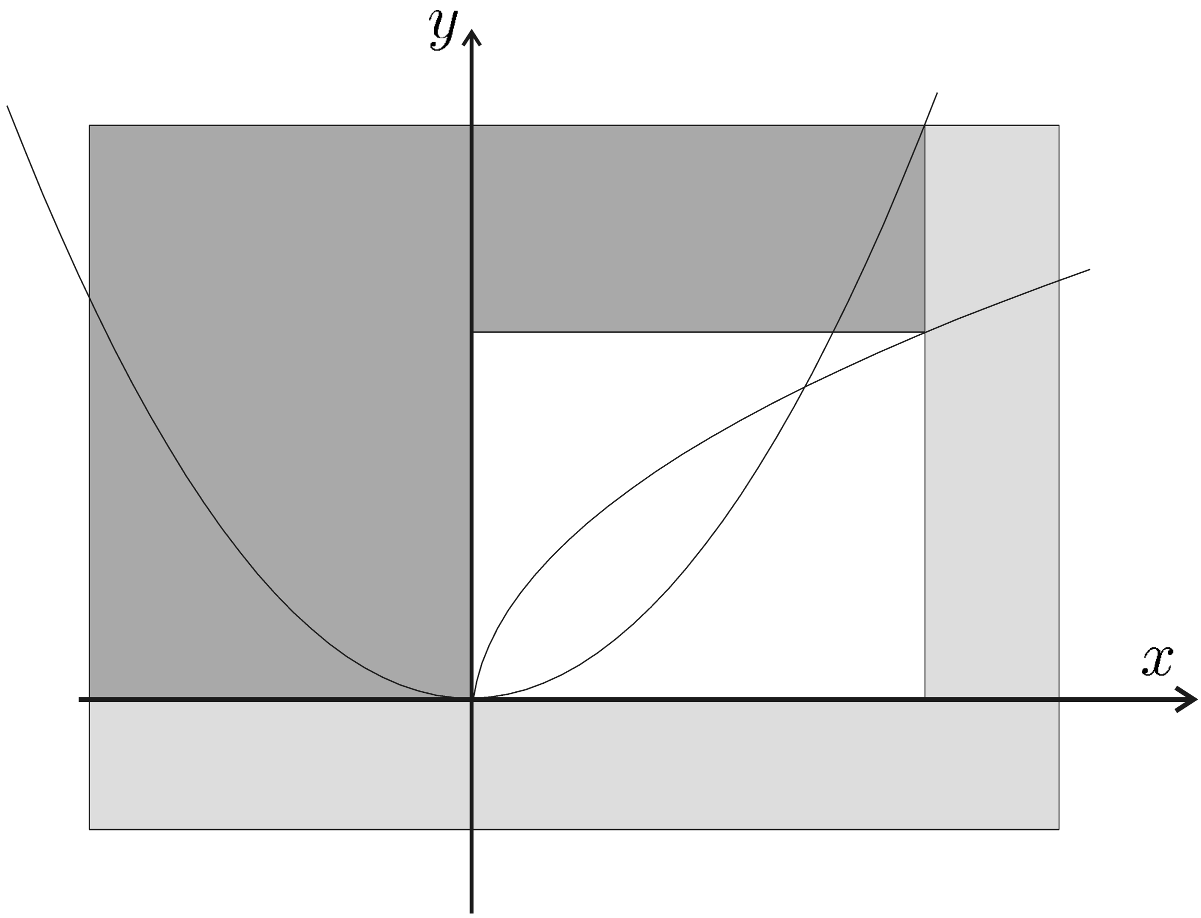

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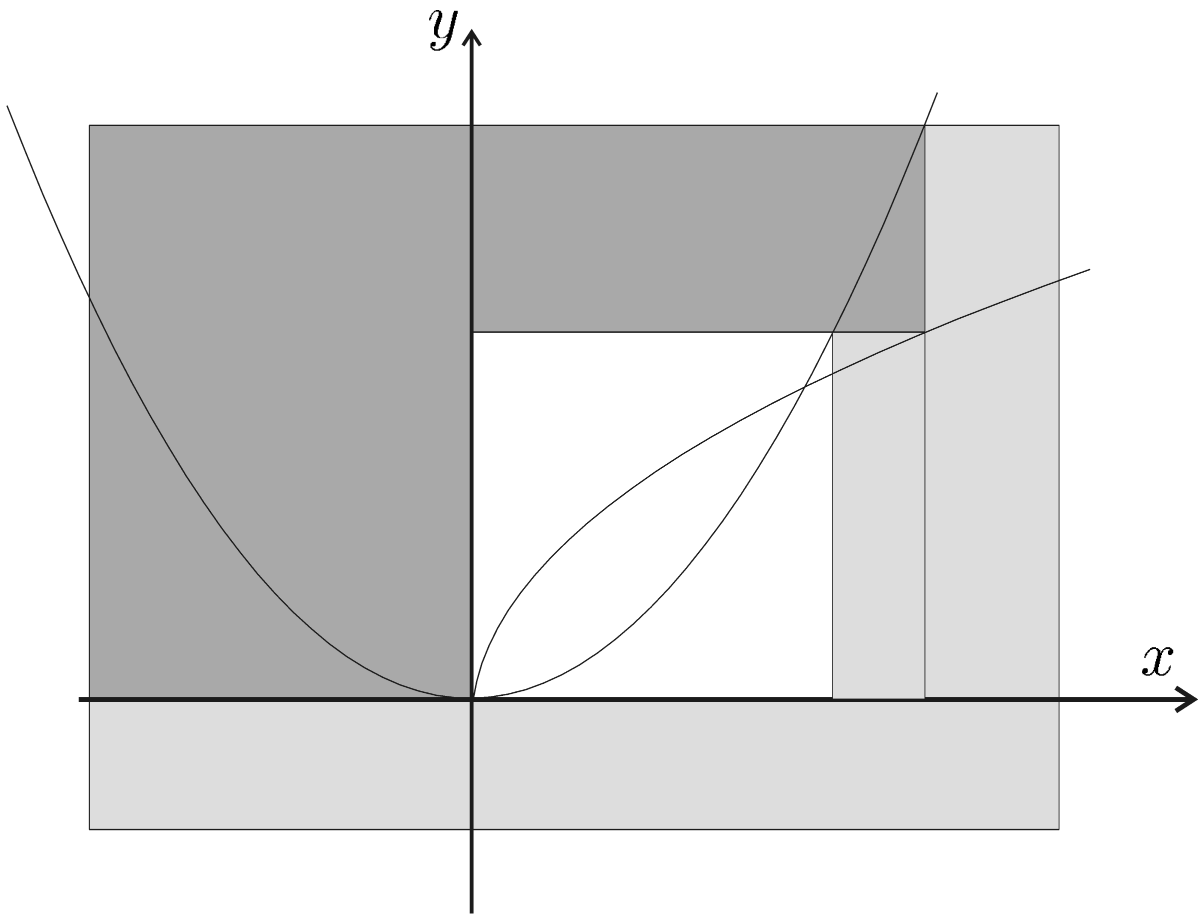
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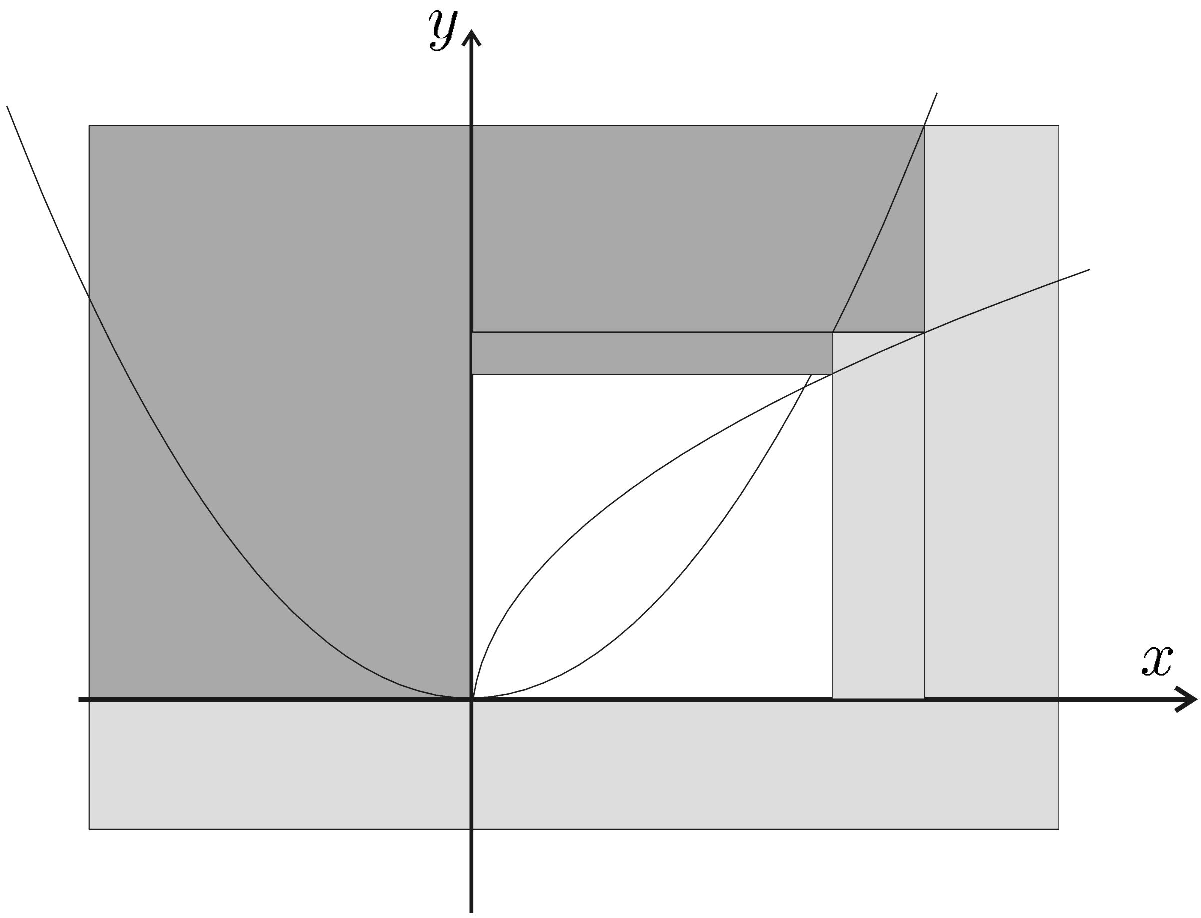

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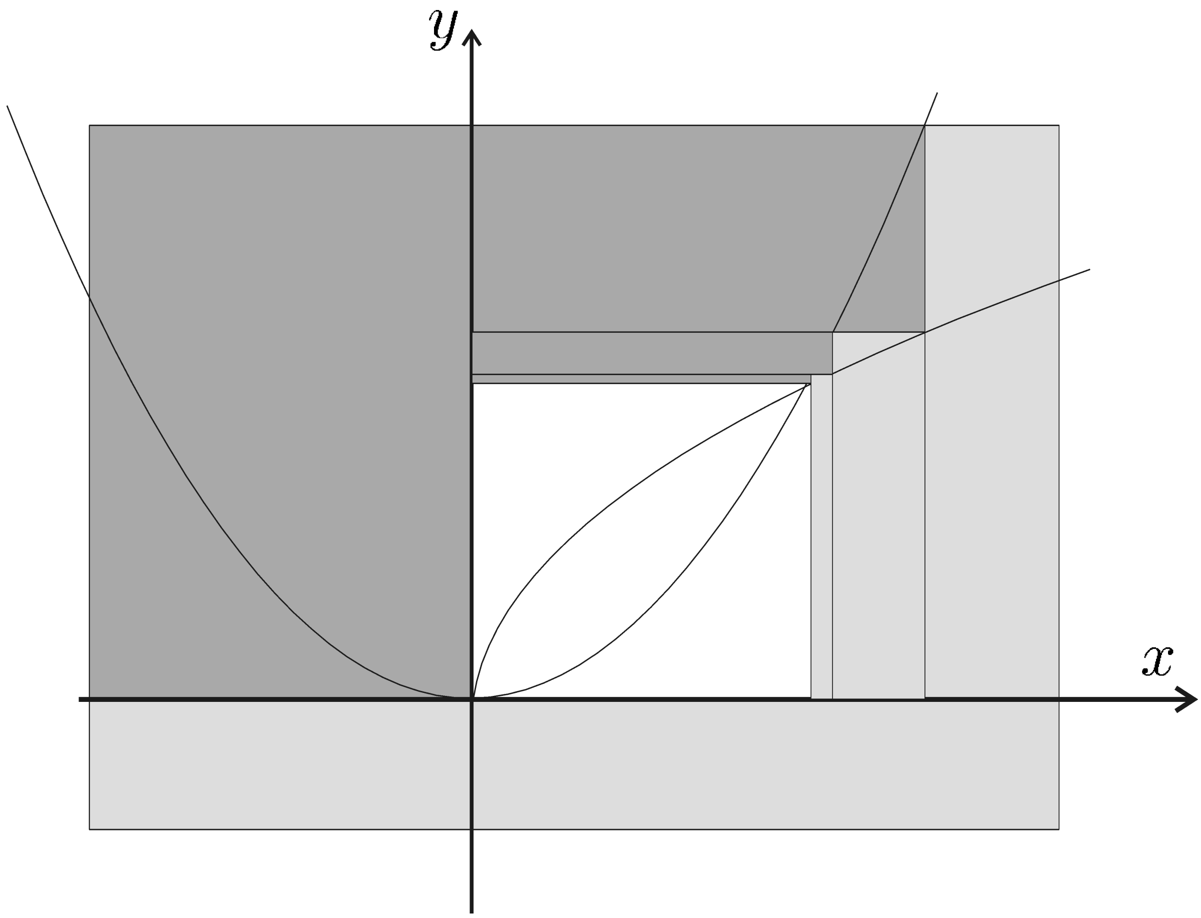

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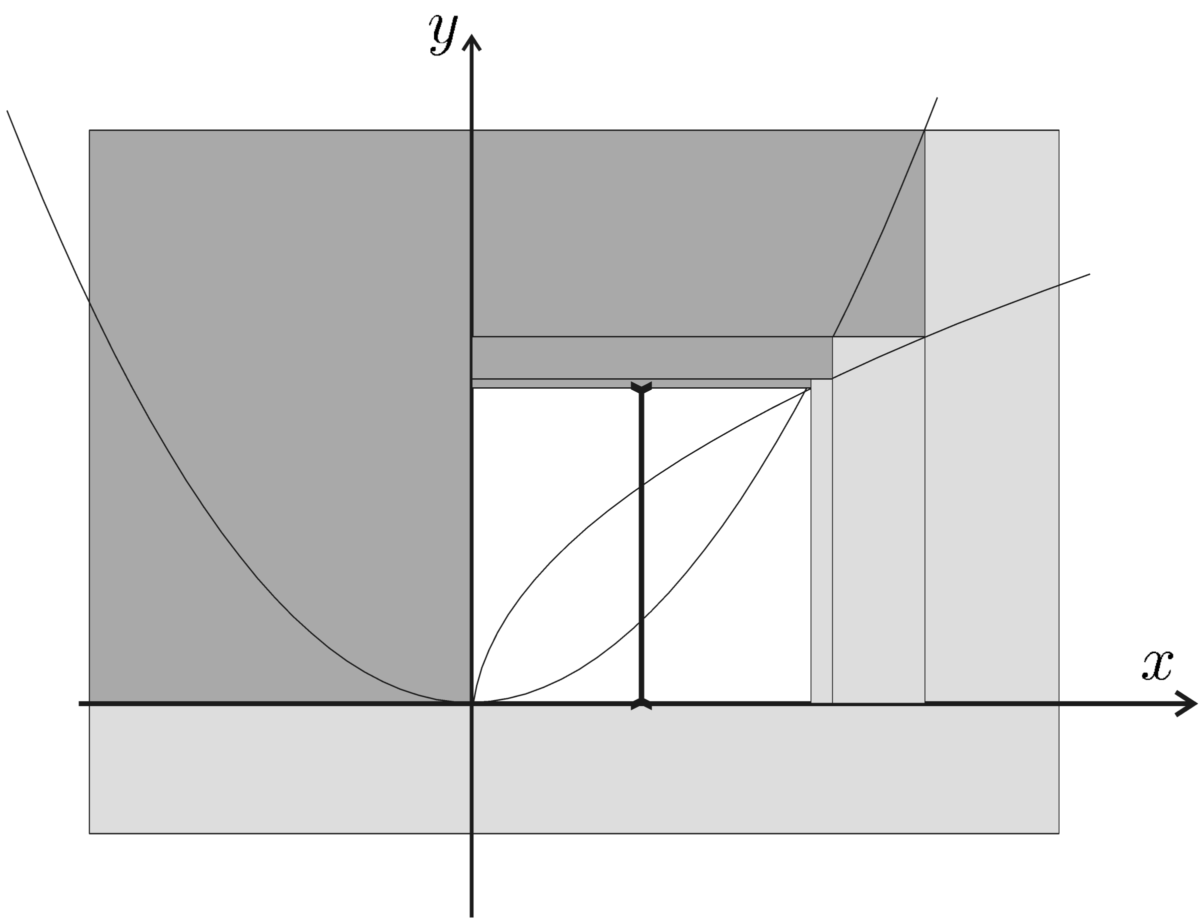

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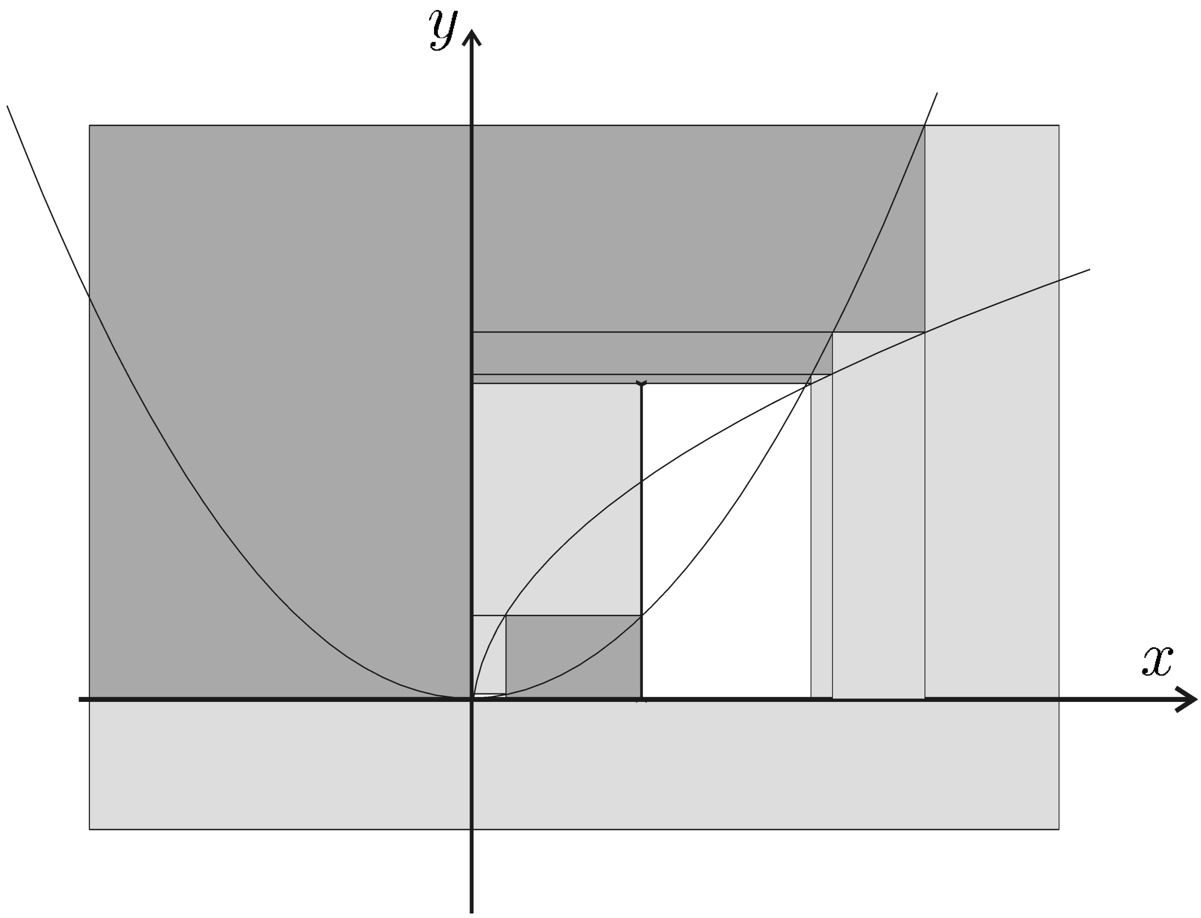

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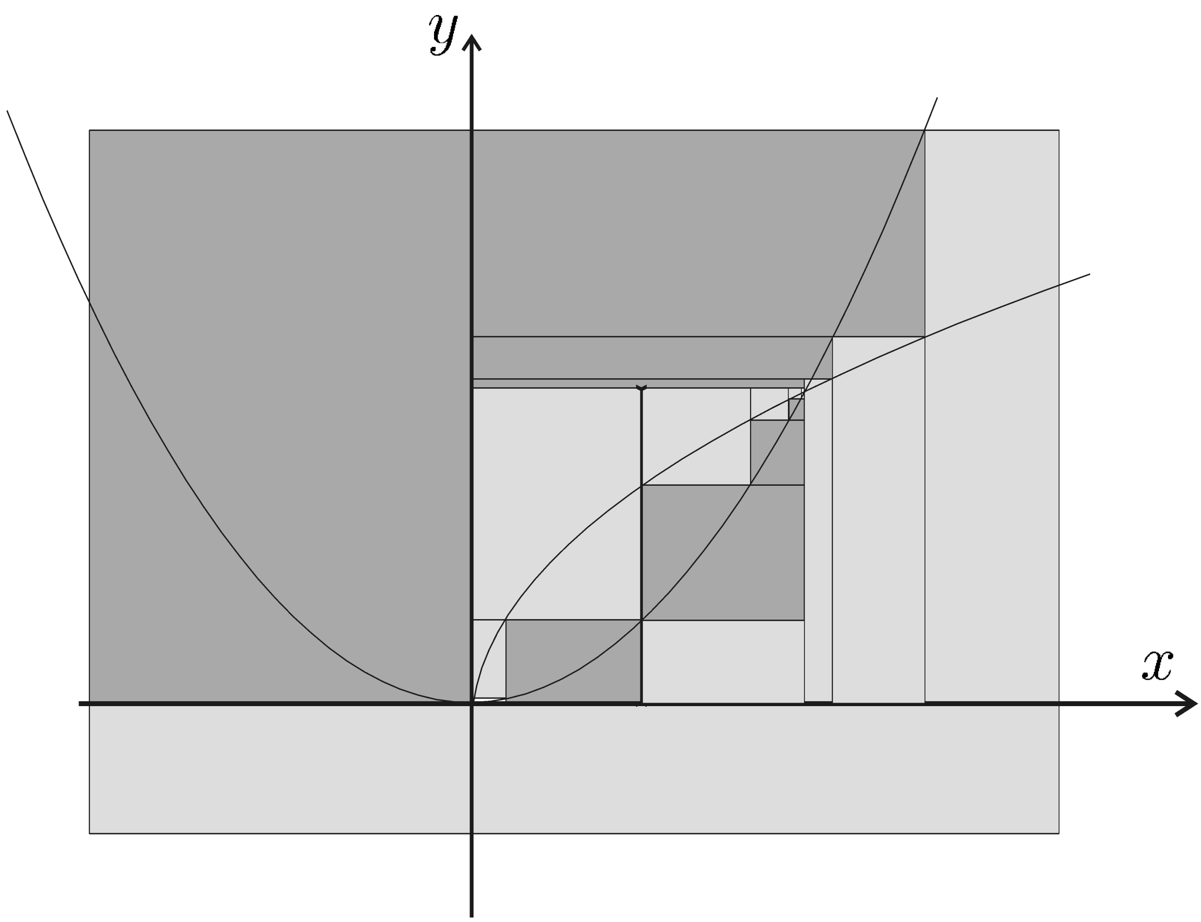


y
x

4.5 Local consistency

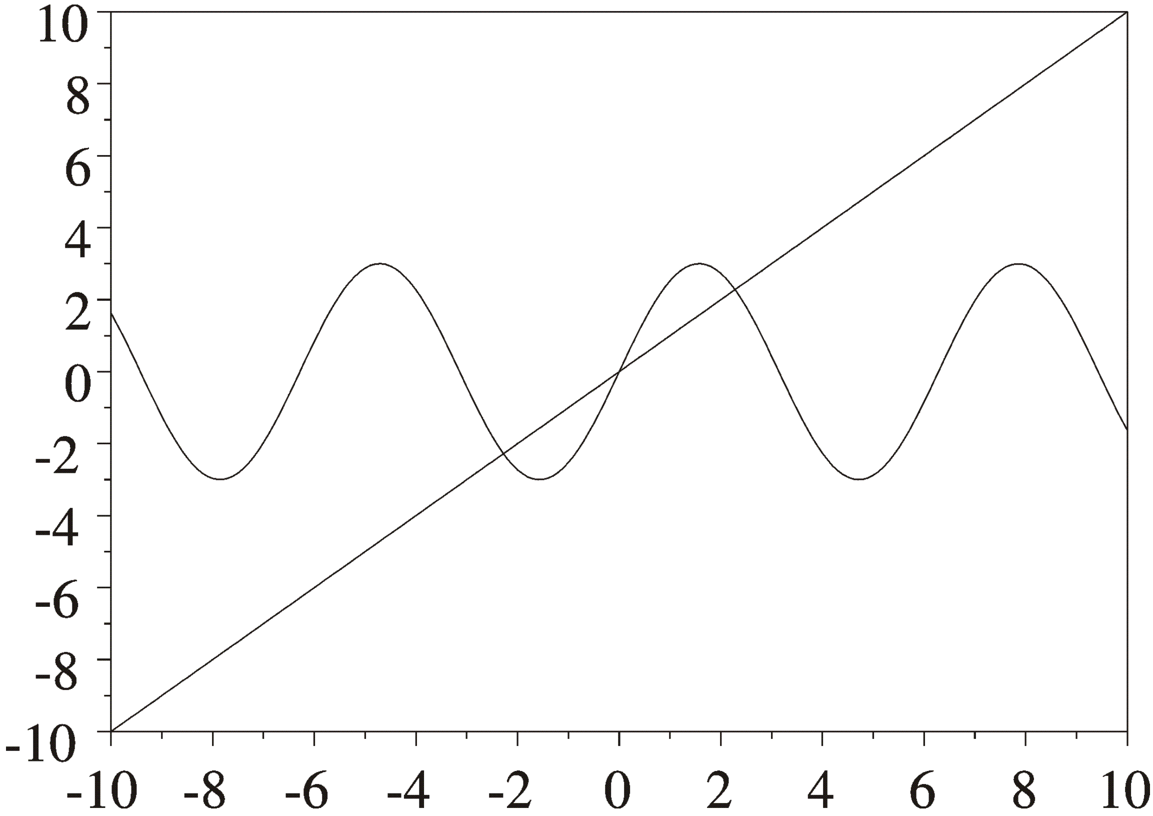
If $\mathcal{C}^*_{\mathbb{S}_1}$ and $\mathcal{C}^*_{\mathbb{S}_2}$ are two minimal contractors for $\mathbb{S}_1$ and $\mathbb{S}_2$ then

$$\mathcal{C}_{\mathbb{S}} = \mathcal{C}^*_{\mathbb{S}_1} \circ \mathcal{C}^*_{\mathbb{S}_2} \circ \mathcal{C}^*_{\mathbb{S}_1} \circ \mathcal{C}^*_{\mathbb{S}_2} \circ \dots$$

is a contractor for $\mathbb{S} = \mathbb{S}_1 \cap \mathbb{S}_2$, but it is not always optimal. This is the *local consistency effect.*

Exemple. Consider the system

$$\begin{cases} y &= 3\sin(x) \\ y &= x \end{cases} \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$



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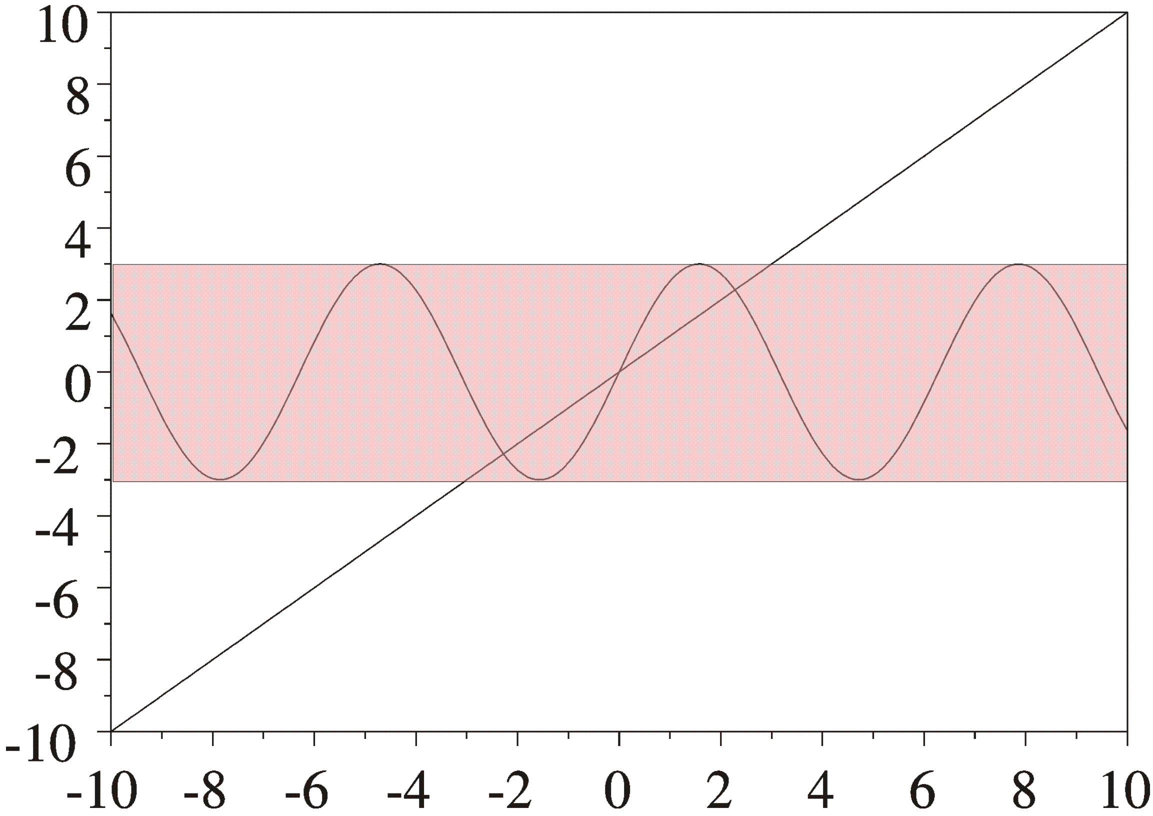
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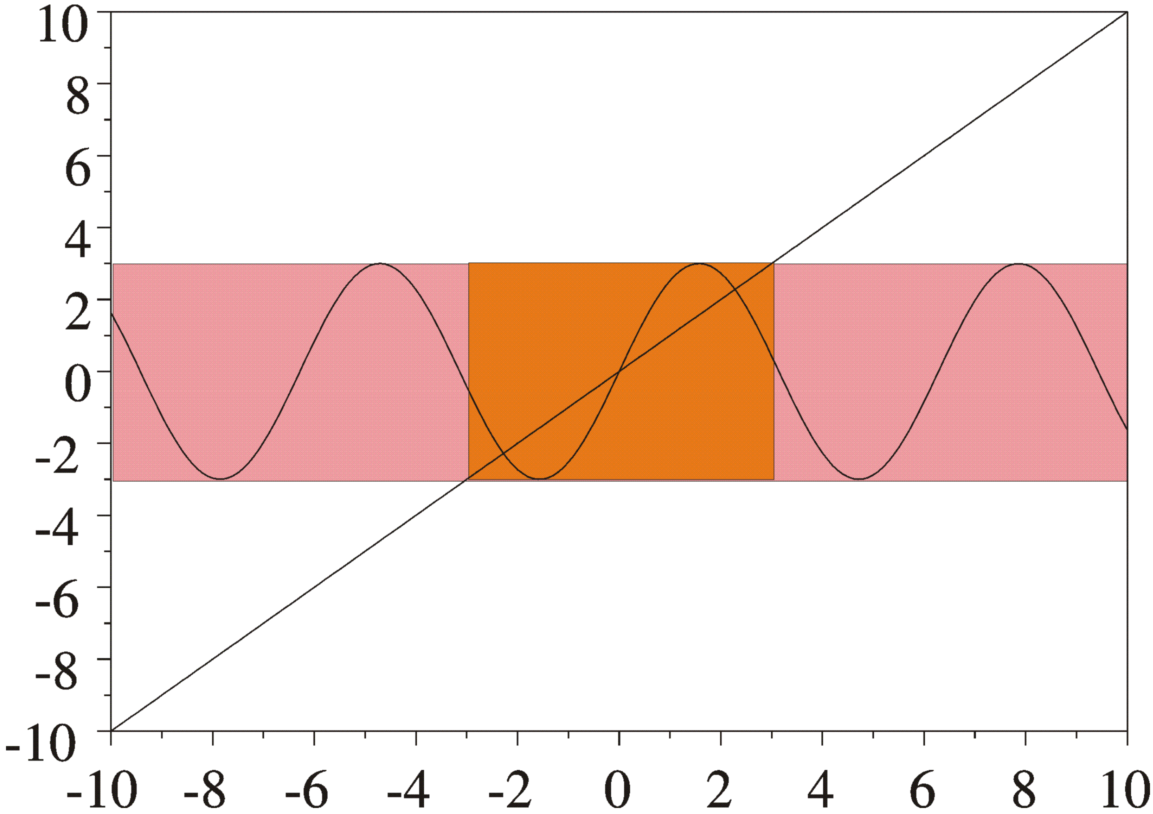
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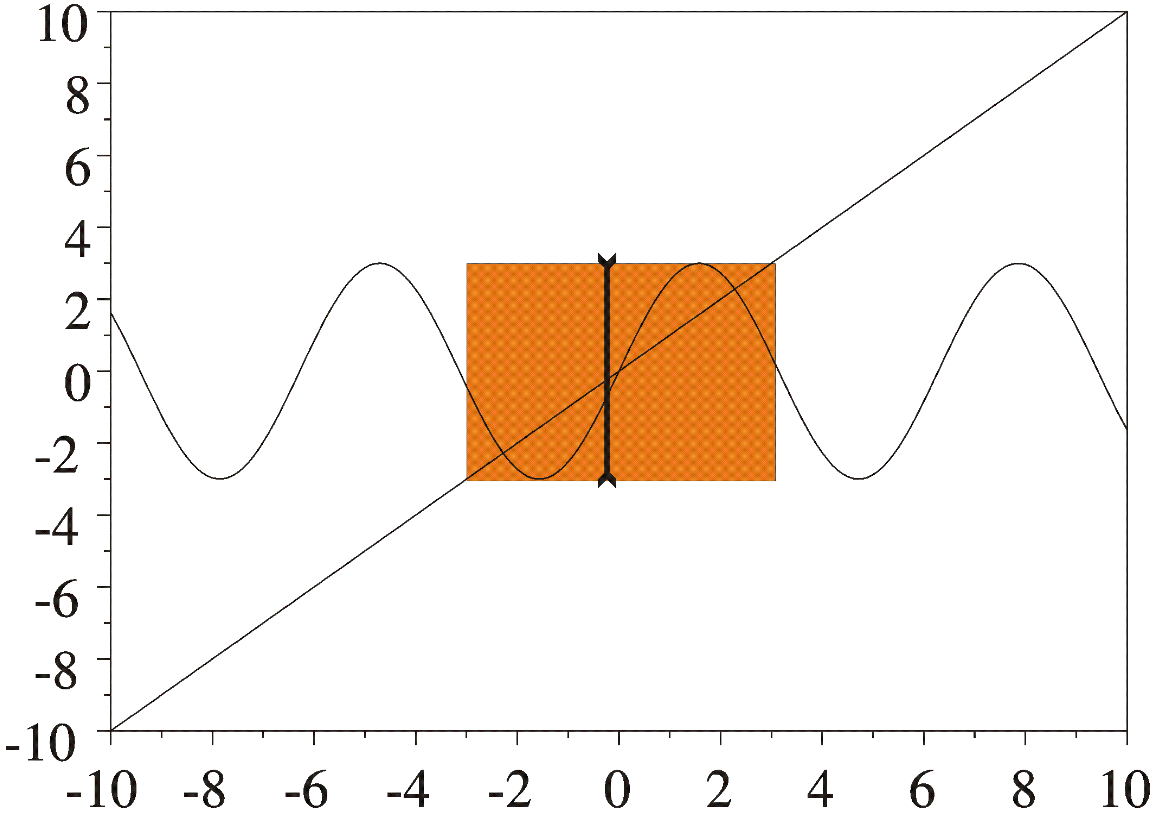

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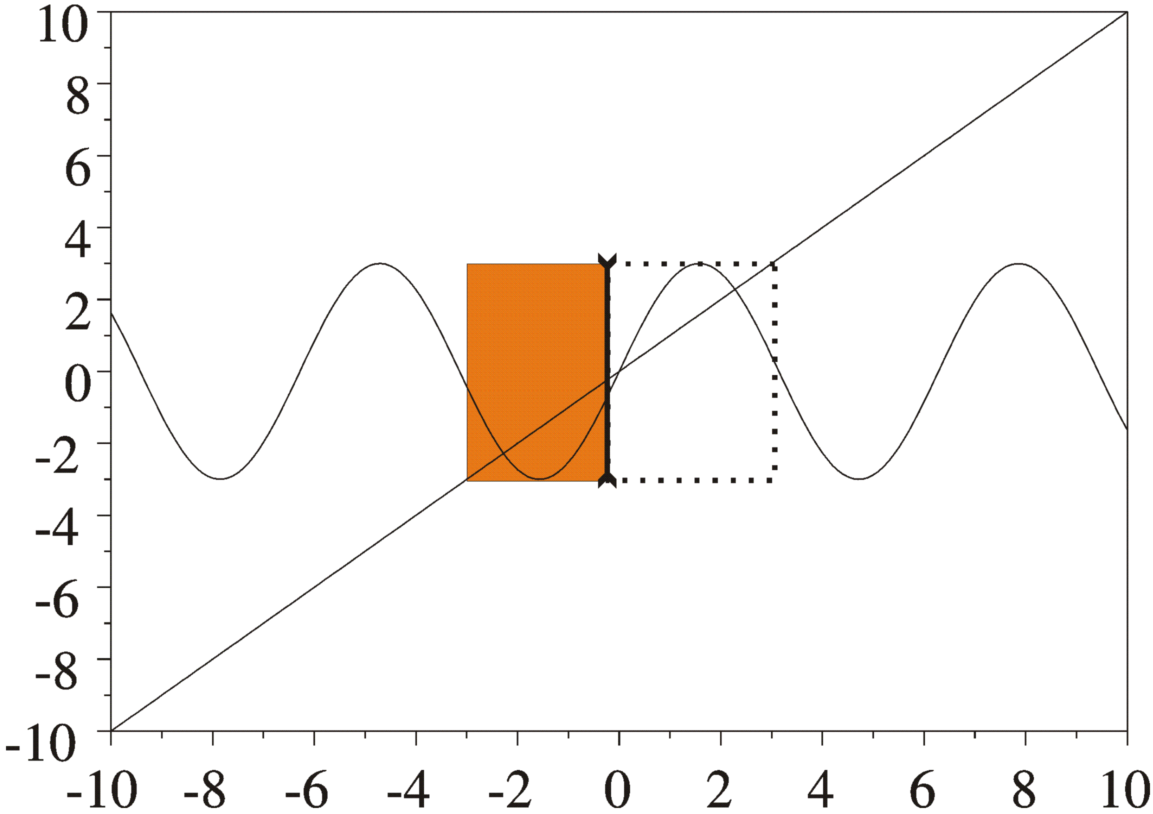
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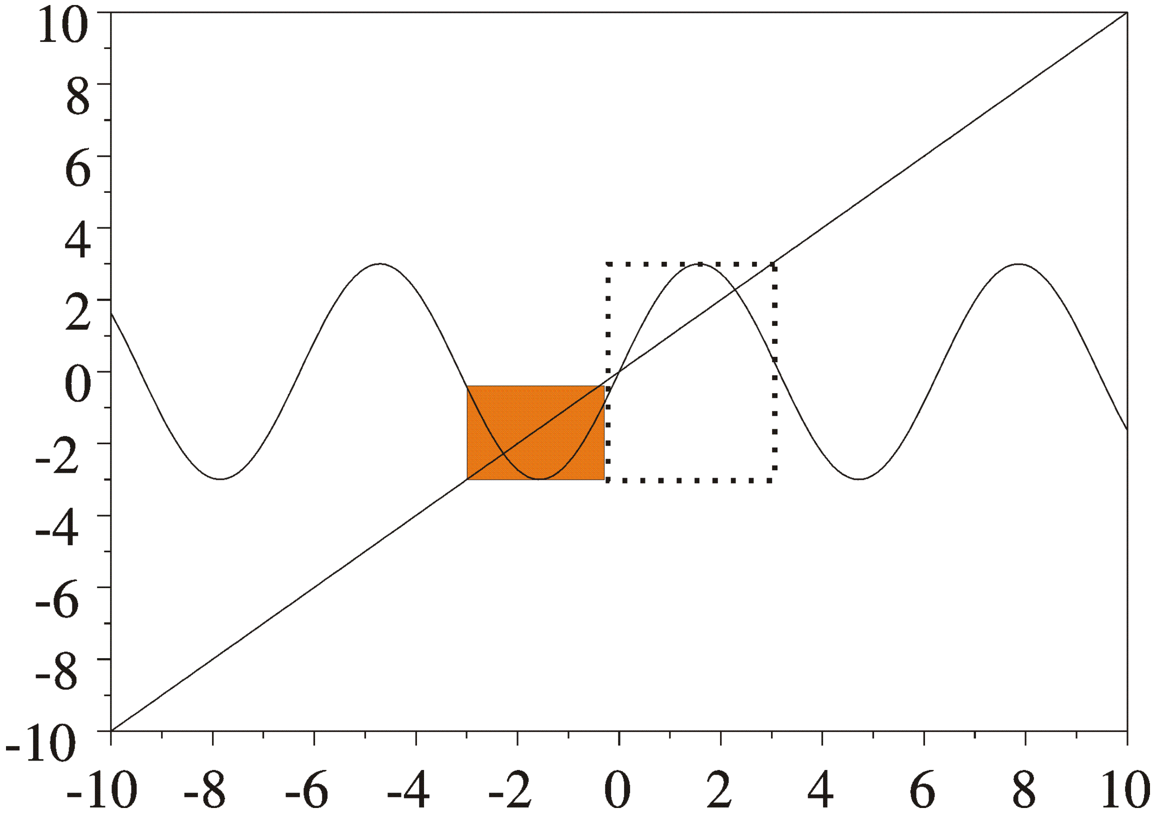

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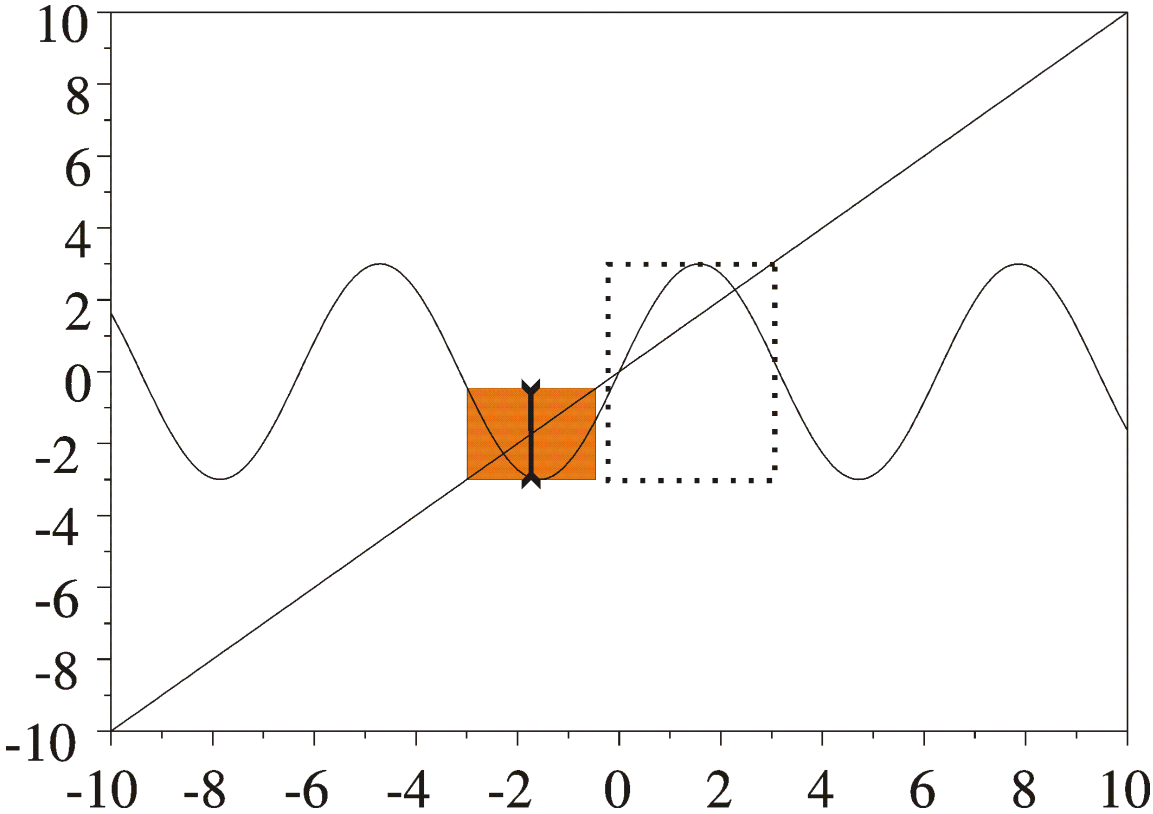

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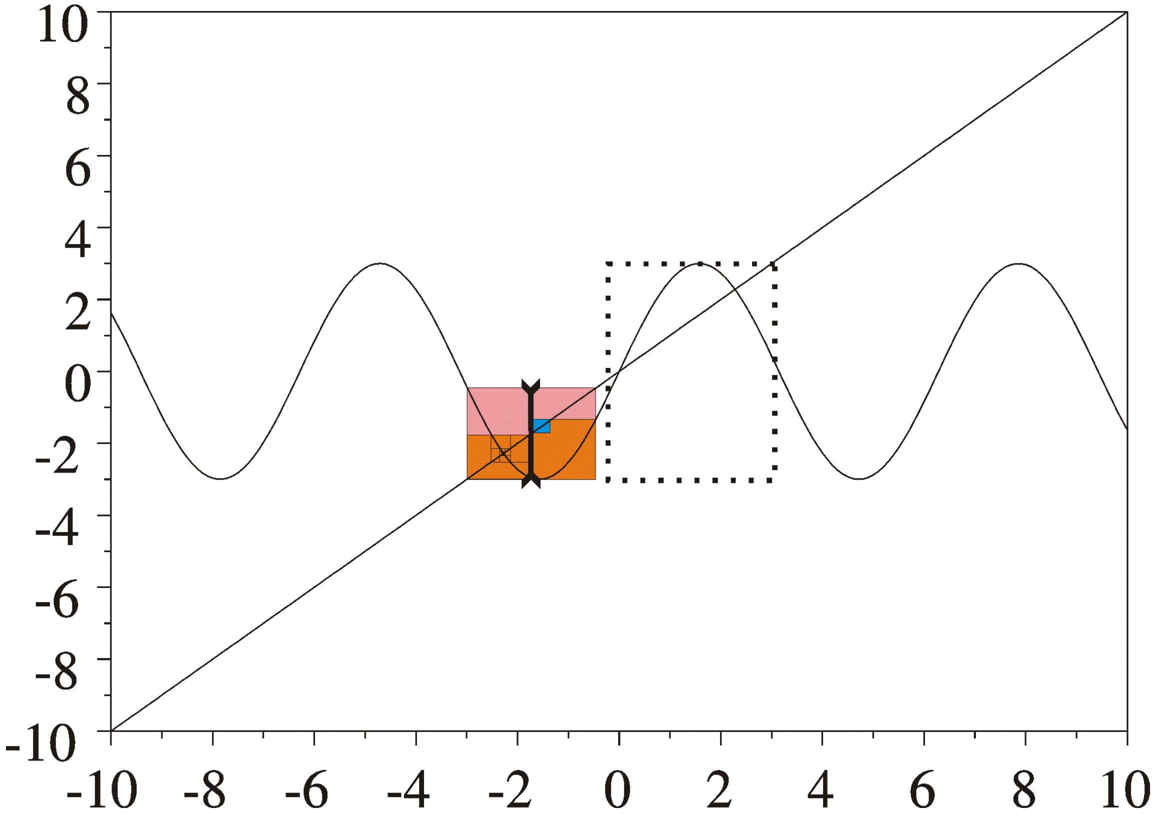

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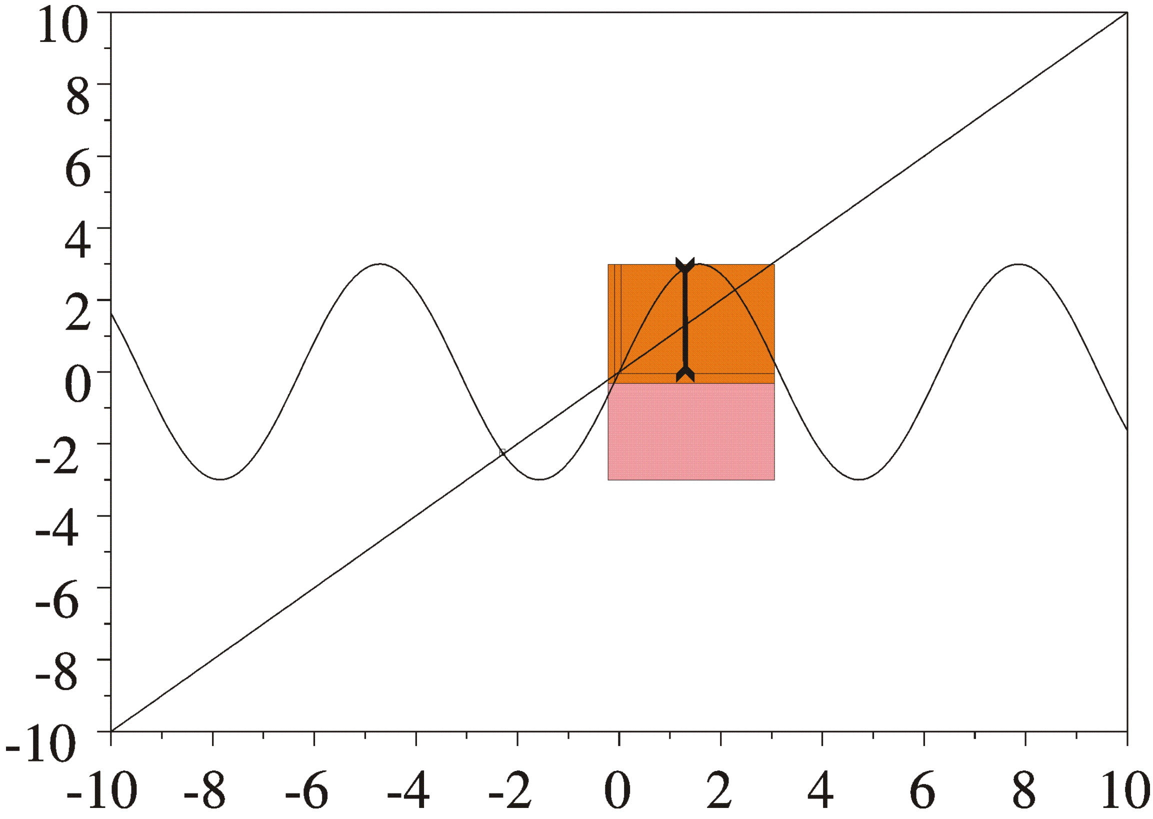

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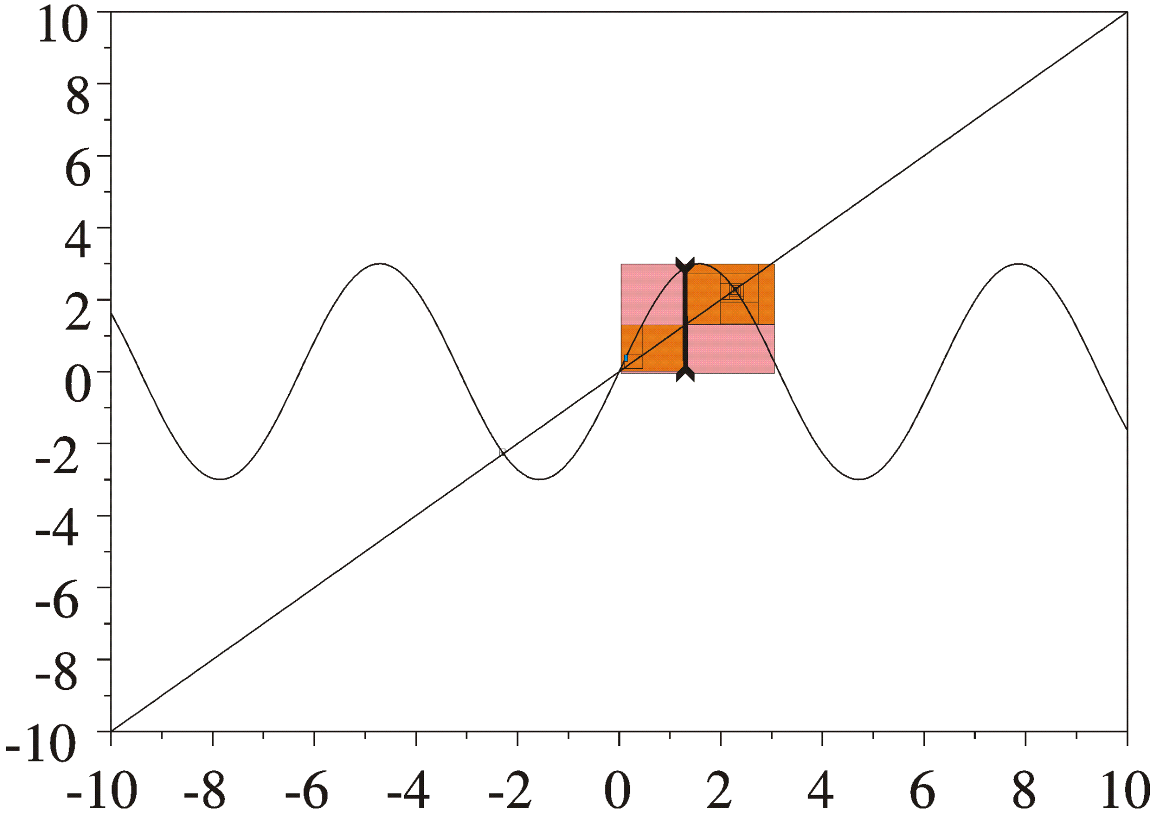

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4.6 Decomposition into primitive constraints

$$x + \mathsf{sin}(xy) \leq 0,$$
$$x \in [-1, 1], y \in [-1, 1], z \in [-1, 1]$$

can be decomposed into

$$\begin{cases} a = xy \\ b = \mathsf{sin}(a) \\ c = x + b \end{cases}, \qquad \begin{array}{ll} x \in [-1, 1] & a \in [-\infty, \infty] \\ y \in [-1, 1] & b \in [-\infty, \infty] \\ z \in [-1, 1] & c \in [-\infty, 0] \end{array}$$

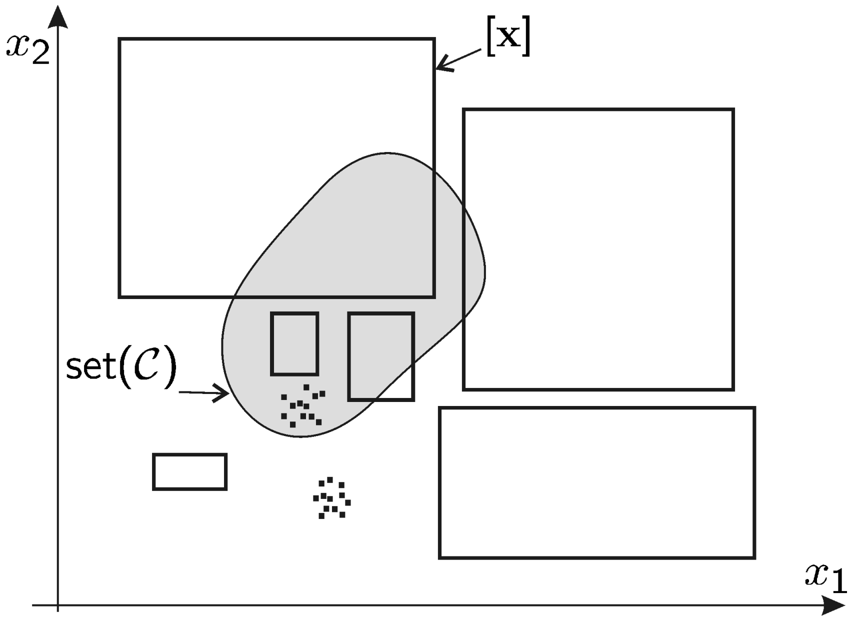
4.7 Set and contractors

A contractor represents a set of $\mathbb{R}^n$. The set associated with a contractor $\mathcal{C}$ is

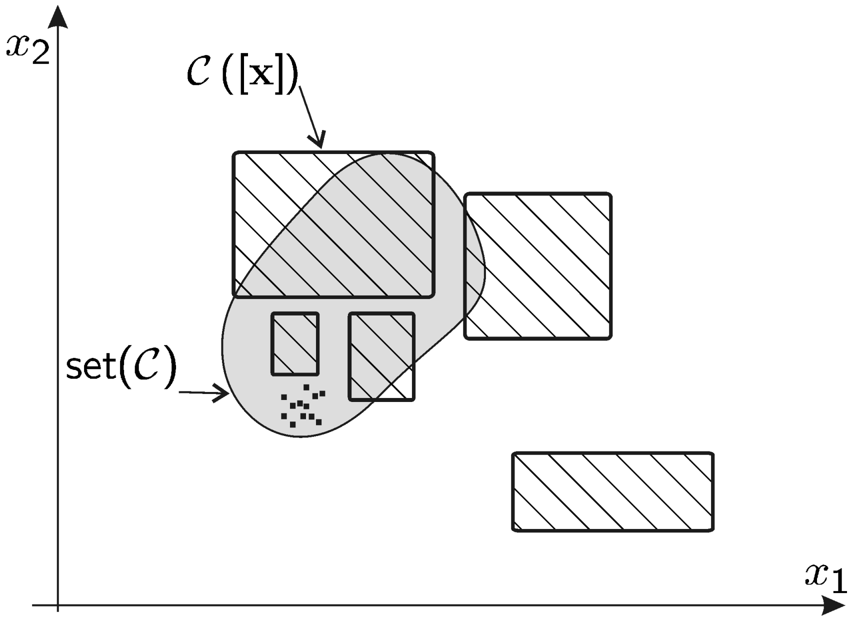
$$\mathrm{set}\left(\mathcal{C}\right)=\left\{\mathbf{x}\in\mathbb{R}^n,\mathcal{C}(\{\mathbf{x}\})=\{\mathbf{x}\}\right\}.$$

Its domain is

$$\mathrm{dom}\left(\mathcal{C}\right)=\left\{\mathbf{x}\in\mathbb{R}^n,\mathcal{C}(\{\mathbf{x}\})=\emptyset\right\}.$$



x_2
$[\mathbf{x}]$
set($\mathcal{C}$)
x_1


x_2
$\mathcal{C}([\mathbf{x}])$
set($\mathcal{C}$)
x_1

For instance, the set associated with the contractor

$$\mathcal{C}_1 \begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} [x_1] \cap ([x_3] - [x_2]) \\ [x_2] \cap ([x_3] - [x_1]) \\ [x_3] \cap ([x_1] + [x_2]) \end{pmatrix}$$

is

$$\text{set}\left(\mathcal{C}_1\right) = \left\{(x_1, x_2, x_3), x_3 = x_1 + x_2\right\}.$$

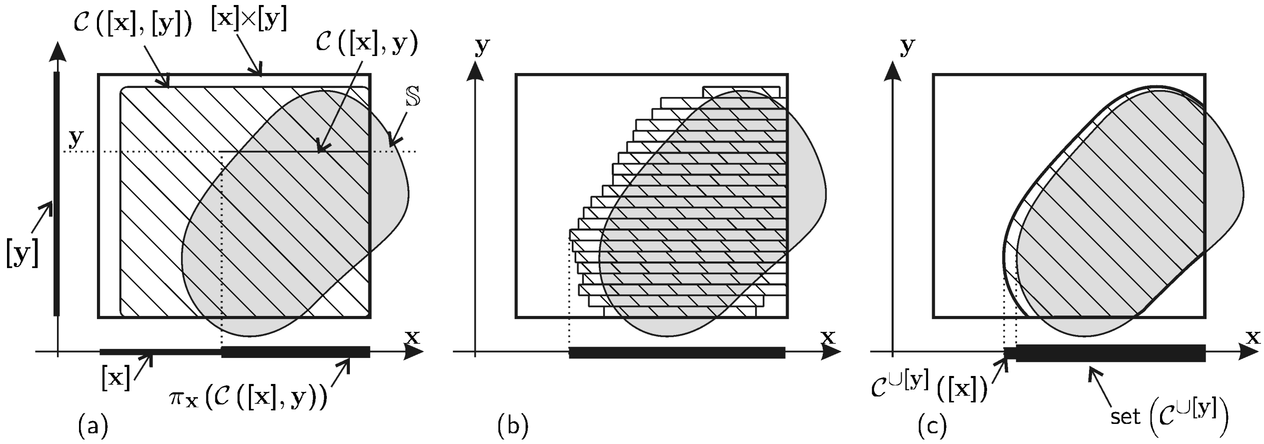
A contractor is also one way to represent one equation $x_3 = x_1 + x_2$.

4.8 Operations on contractors

intersection	$(\mathcal{C}_1 \cap \mathcal{C}_2)([\mathbf{x}]) \stackrel{\text{def}}{=} \mathcal{C}_1([\mathbf{x}]) \cap \mathcal{C}_2([\mathbf{x}])$
union	$(\mathcal{C}_1 \cup \mathcal{C}_2)([\mathbf{x}]) \stackrel{\text{def}}{=} [\mathcal{C}_1([\mathbf{x}]) \cup \mathcal{C}_2([\mathbf{x}])]$
composition	$(\mathcal{C}_1 \circ \mathcal{C}_2)([\mathbf{x}]) \stackrel{\text{def}}{=} \mathcal{C}_1(\mathcal{C}_2([\mathbf{x}]))$
repetition	$\mathcal{C}^{\infty} \stackrel{\text{def}}{=} \mathcal{C} \circ \mathcal{C} \circ \mathcal{C} \circ \dots$
repeat intersection	$\mathcal{C}_1 \sqcap \mathcal{C}_2 = (\mathcal{C}_1 \cap \mathcal{C}_2)^{\infty}$
repeat union	$\mathcal{C}_1 \sqcup \mathcal{C}_2 = (\mathcal{C}_1 \cup \mathcal{C}_2)^{\infty}$

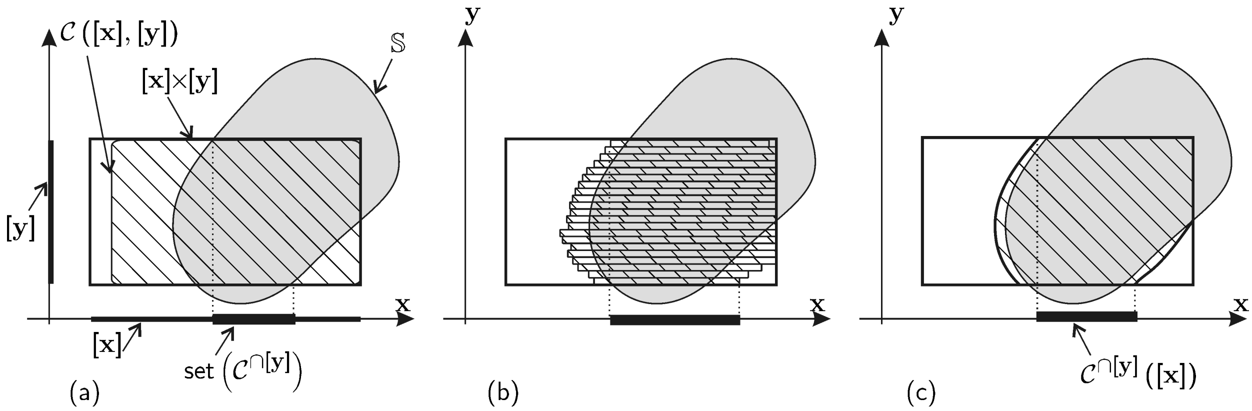
Consider the contractor $\mathcal{C}([\mathbf{x}],[\mathbf{y}])$, where $[\mathbf{x}] \in \mathbb{R}^n$, $[\mathbf{y}] \in \mathbb{R}^p$. We define the contractor

$$\mathcal{C}^{\cup[\mathbf{y}]}([\mathbf{x}]) = \left[\bigcup_{\mathbf{y}\in[\mathbf{y}]} \pi_{\mathbf{x}}\left(\mathcal{C}([\mathbf{x}],\mathbf{y})\right)\right] \quad \text{(projected union)}$$



and also the contractor

$$\mathcal{C}^{\cap[\mathbf{y}]}([\mathbf{x}]) = \bigcap_{\mathbf{y}\in[\mathbf{y}]} \pi_{\mathbf{x}}(\mathcal{C}([\mathbf{x}],\mathbf{y})), \qquad \text{(projected intersectio}$$



(a) (b) (c)

We have

$$\mathsf{set}\left(\mathcal{C}^{\cup[\mathbf{y}]}\right) = \{\mathbf{x}, \exists \mathbf{y} \in [\mathbf{y}], (\mathbf{x}, \mathbf{y}) \in \mathsf{set}(\mathcal{C})\}$$
$$\mathsf{set}\left(\mathcal{C}^{\cap[\mathbf{y}]}\right) = \{\mathbf{x}, \forall \mathbf{y} \in [\mathbf{y}], (\mathbf{x}, \mathbf{y}) \in \mathsf{set}(\mathcal{C})\}.$$

4.9 QUIMPER

The collection of contractors $\{\mathcal{C}_1, \ldots, \mathcal{C}_m\}$ is *complementary* if

$$\text{set}(\mathcal{C}_1) \cap \cdots \cap \text{set}(\mathcal{C}_m) = \emptyset.$$

Quimper is a high-level language for QUick Interval Modeling and Programming in a bounded-ERror context.

Quimper is an interpreted language for set computation.

A Quimper program is a set of complementary contractors.

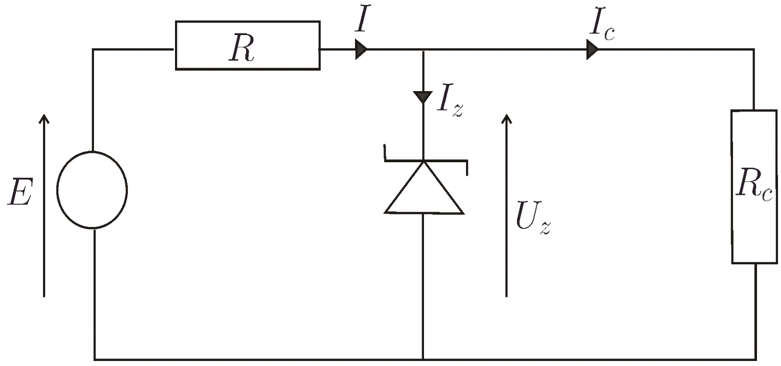
Quimper returns m subpavings, where m is the number of contractors

It is available at

`http://ibex-lib.org/`

5 Application of contractors

5.1 Bounded-error estimation



R
I
I_c
I_z
E
U_z
R_c

It is known that

$$U_z \in [6,7]V,\ r \in [7,8]\Omega,\ U_0 \in [6,6.2]V$$
$$R \in [100,110]\Omega,\ E \in [18,20]V,\ I_z \in [0,\infty]A$$
$$I \in]-\infty,\infty[A,\ I_c \in]-\infty,\infty[A, R_c \in [50,60]\Omega.$$

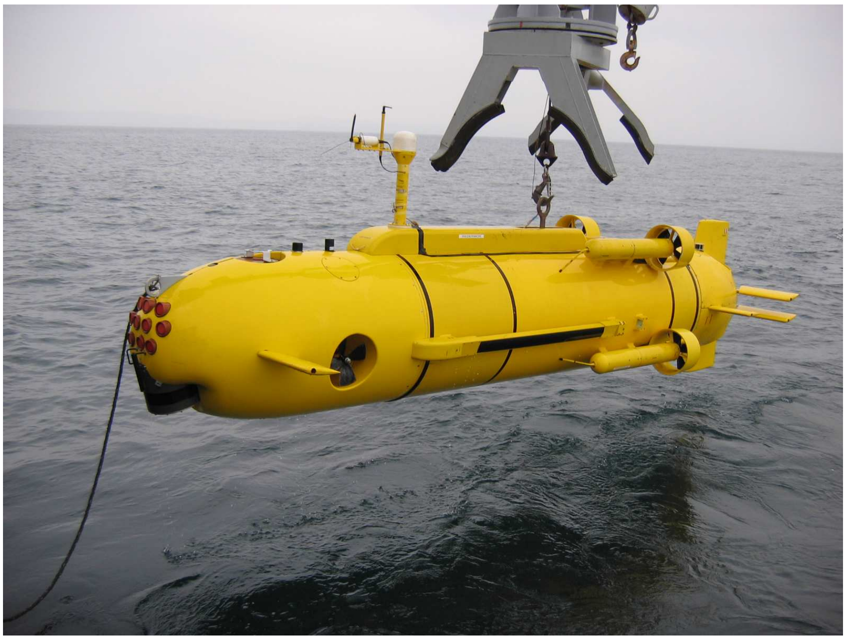
The constraints are

Zener diode	$I_z = \max(0, \frac{U_z - U_0}{r})$,
Ohm rule	$U_z = R_c I_c$,
Current rule	$I = I_c + I_z$,
Voltage rule	$E = RI + U_z$.

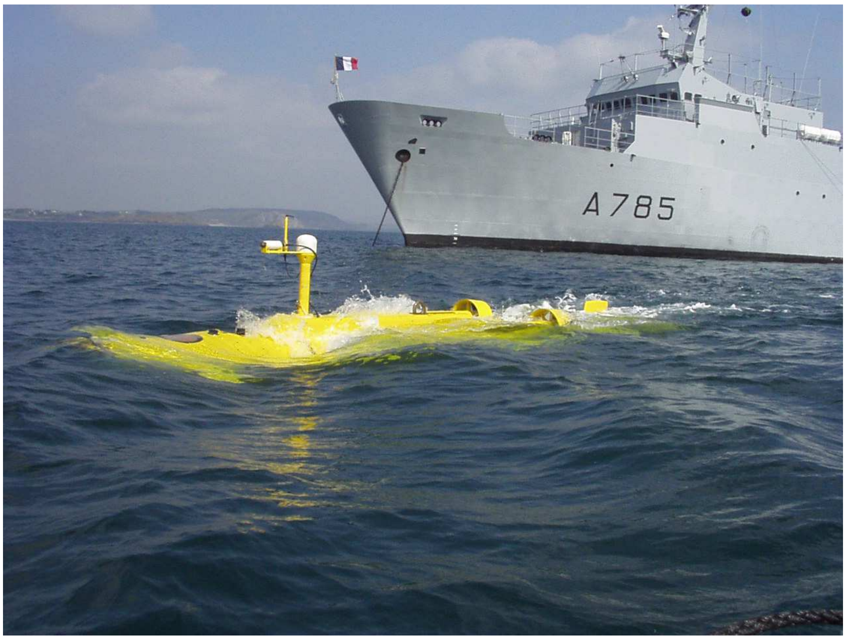
IntervalPeeler contracts the domains into:

$$U_z \in [6,007;6,518], r \in [7,8]\Omega,$$
$$U_0 \in [6,6.2]V, R \in [100,110]\Omega,$$
$$E \in [18,20]V, I_z \in [0.,0.398]A$$
$$I \in [0.11;0.14]A, \; I_c \in [0.1;0,13]A,$$
$$R_c \in [50,60]\Omega$$

5.2 SLAM



Redermor, GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)


A785

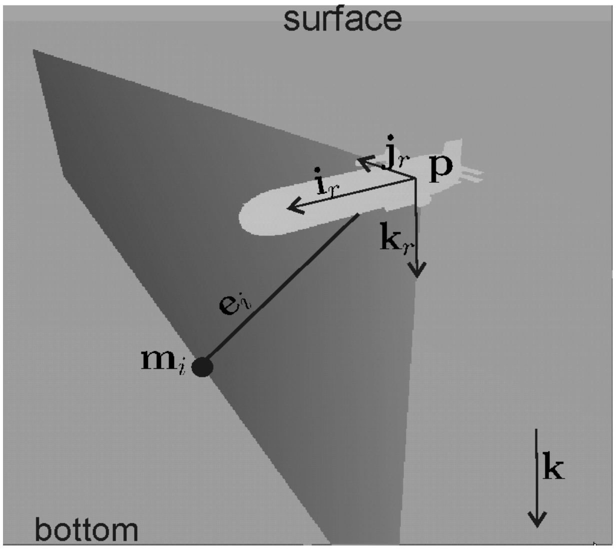
Montrer la simulation

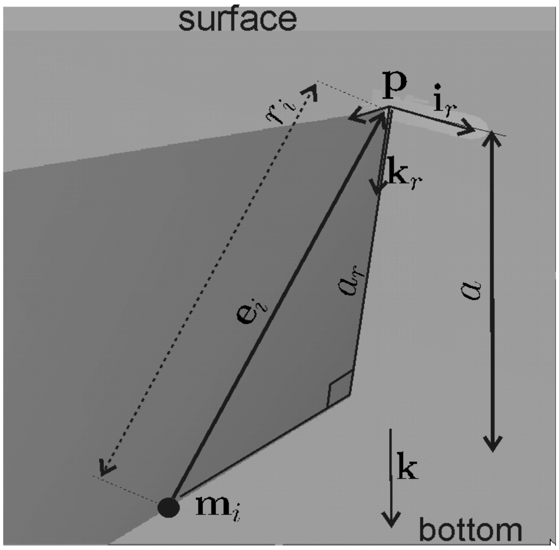
5.2.1 Sensors

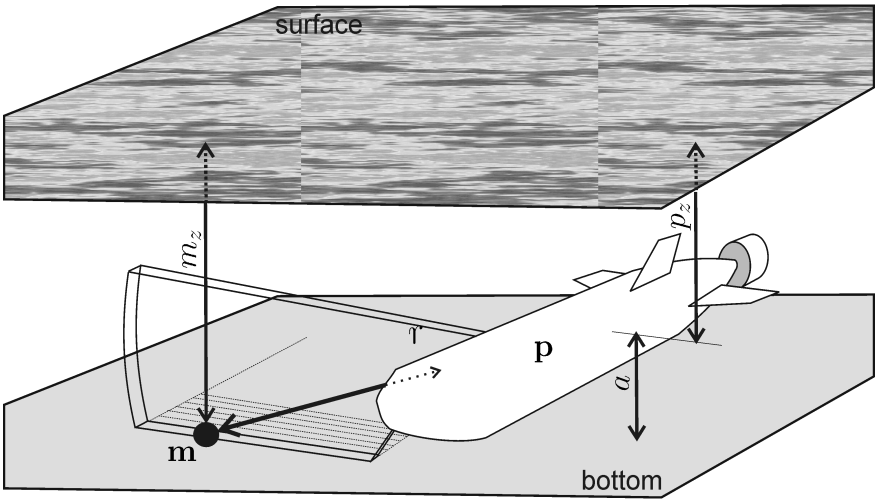
GPS (Global positioning system), only at the surface.

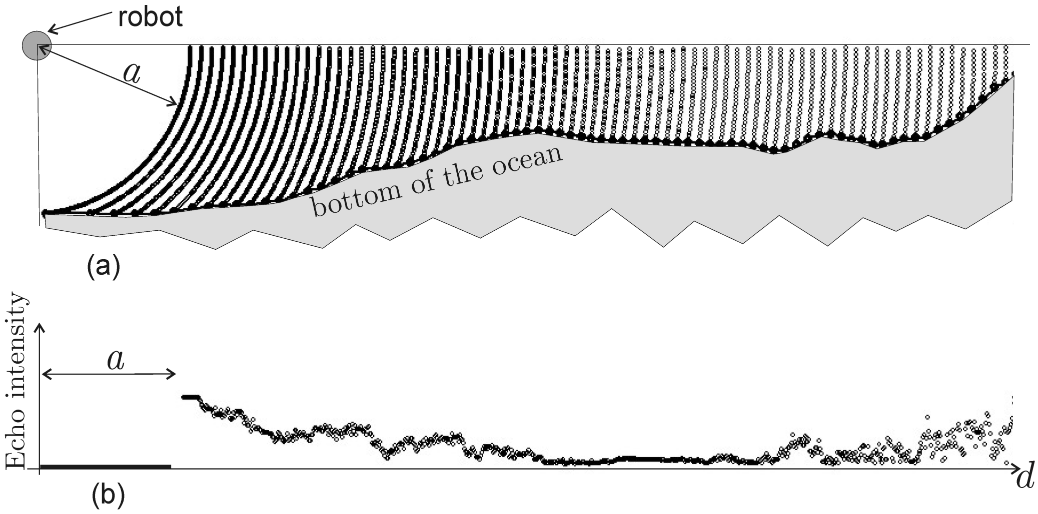
$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m$$

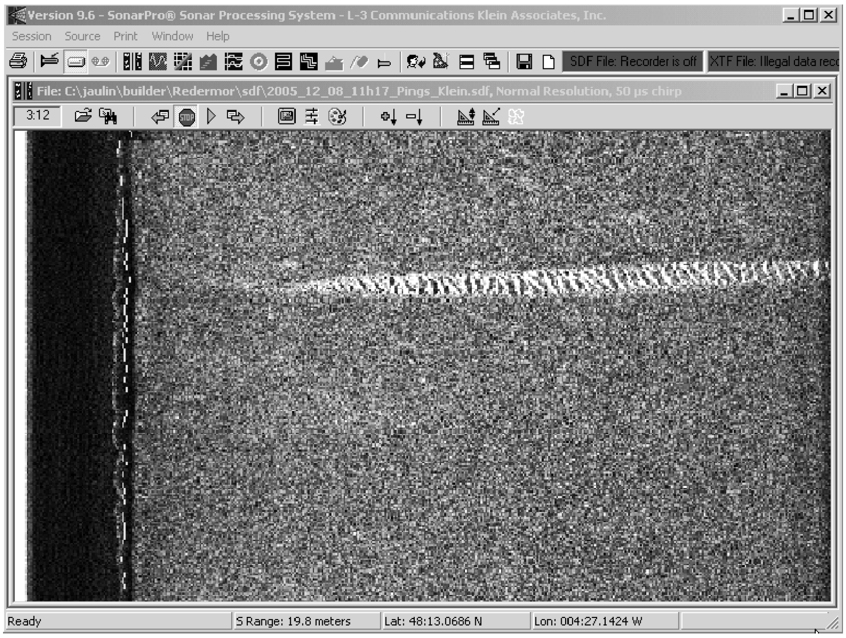
Sonar (KLEIN 5400 side scan sonar).



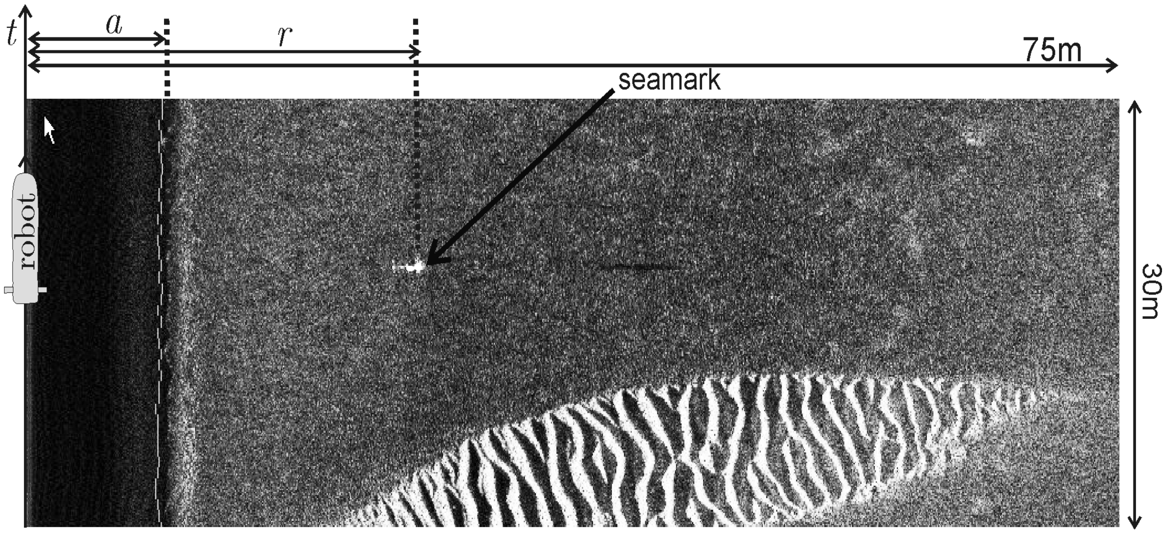



surface
m_z
p_z
r
$\mathbf{p}$
a
$\mathbf{m}$
bottom


robot
a
bottom of the ocean
(a)
Echo intensity
a
d
(b)



Screenshot of SonarPro



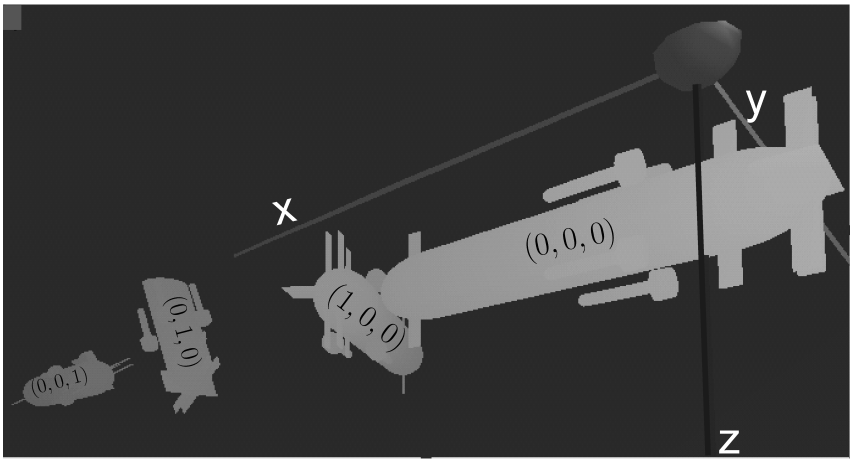
Mine detecttion with SonarPro

Loch-Doppler returns the speed robot $\mathbf{v}_r$.

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] \,.\tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

Inertial central (Octans III from IXSEA).

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



Six mines have been detected.

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

5.2.2 Constraints

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t)\mathbf{R}_{\theta}(t)\mathbf{R}_{\varphi}(t),$$

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^{\mathsf{T}}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5]$$

```
//------------------------------------------------
Constants
N = 59996; // Number of time steps
Variables
  R[N-1][3][3], // rotation matrices
  p[N][3], // positions
  v[N-1][3], // speed vectors
  phi[N-1],theta[N-1],psi[N-1]; // Euler angles
  px[N],py[N]; // for display only
//------------------------------------------------
```

```
function R[3][3]=euler(phi,theta,psi)
  cphi = cos(phi);
  sphi = sin(phi);
  ctheta = cos(theta);
  stheta = sin(theta);
  cpsi = cos(psi);
  spsi = sin(psi);
  R[1][1]=ctheta*cpsi;
  R[1][2]=-cphi*spsi+stheta*cpsi*sphi;
  R[1][3]=spsi*sphi+stheta*cpsi*cphi;
  R[2][1]=ctheta*spsi;
  R[2][2]=cpsi*cphi+stheta*spsi*sphi;
  R[2][3]=-cpsi*sphi+stheta*cphi*spsi;
  R[3][1]=-stheta;
  R[3][2]=ctheta*sphi;
  R[3][3]=ctheta*cphi;
end
```

contractor-list rotation

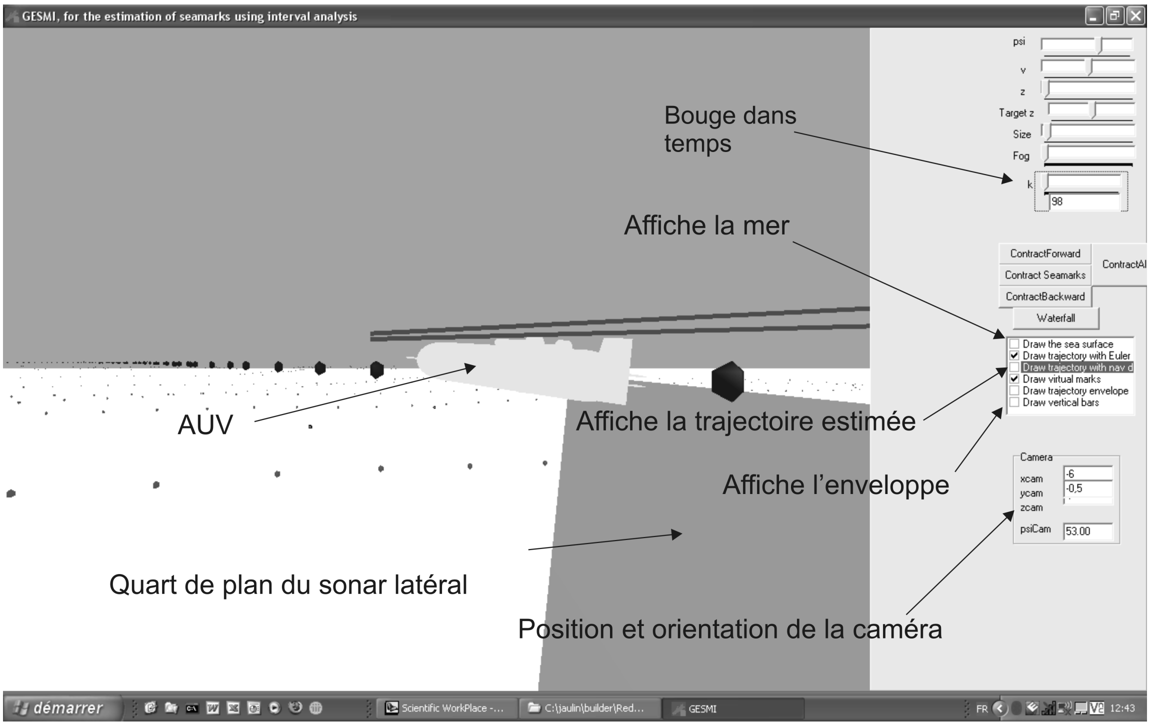
```
  for k=1:N-1;
    R[k]=euler(phi[k],theta[k],psi[k]);
  end
end
//---------------------------------
contractor-list statequ
  for k=1:N-1;
    p[k+1]=p[k]+0.1*R[k]*v[k];
  end
end
//---------------------------------
contractor init
  inter k=1:N-1;
    rotation(k)
  end
end
```

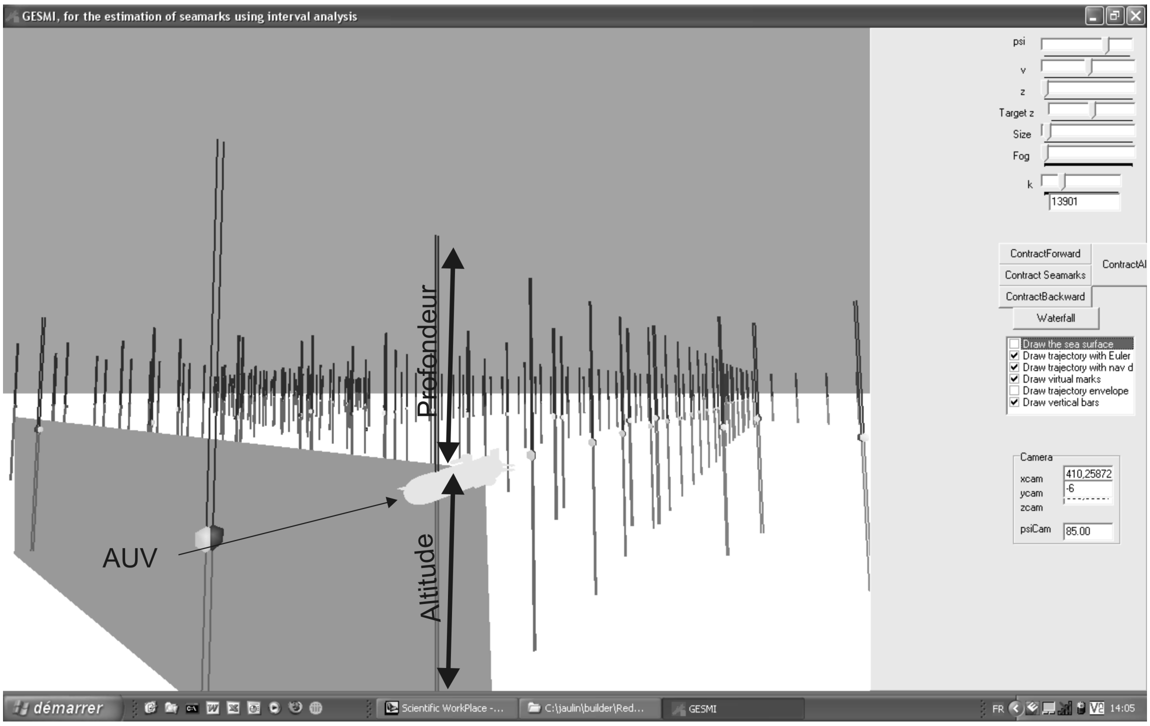
```
contractor fwd
  inter k=1:N-1;
    statequ(k)
  end
end
//--------------------------------
contractor bwd
  inter k=1:N-1;
    statequ(N-k)
  end
end
```

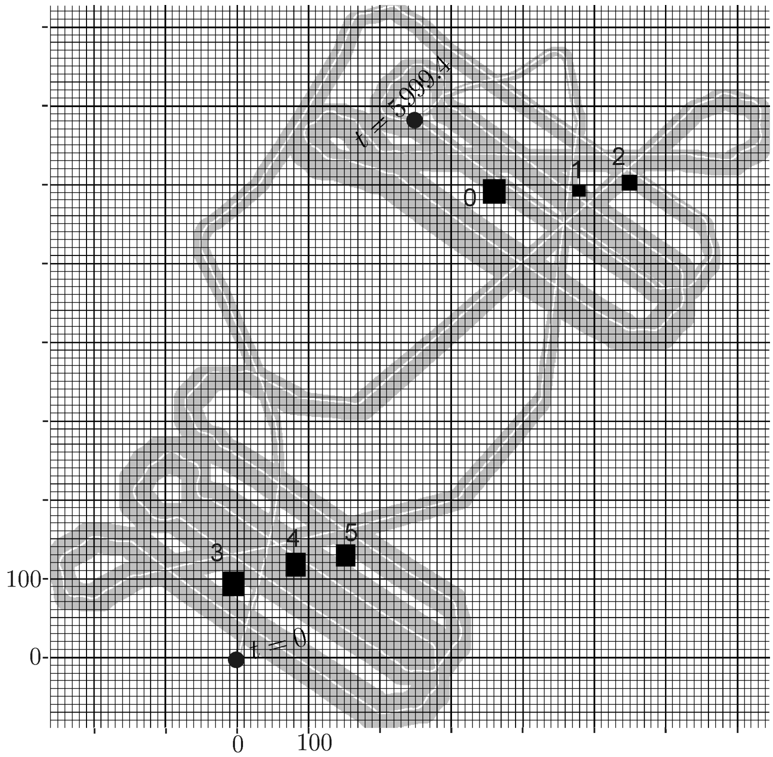
main

```
  p[1] :=read("gps_init.dat");
  v :=read("Quimper_v.dat");
  phi :=read("Quimper_phi.dat");
  theta :=read("Quimper_theta.dat");
  psi :=read("Quimper_psi.dat");
  init;
  fwd;
  bwd;
  column(p,px,1);
  column(p,py,2);
  print("--- Robot positions: ---");
  newplot("gesmi.dat");
  plot(px,py,color(rgb(1,1,1),rgb(0,0,0)));
end
```

5.2.3 GESMI


GESMI, for the estimation of seamarks using interval analysis
Bouge dans temps
Affiche la mer
AUV
Affiche la trajectoire estimée
Affiche l'enveloppe
Quart de plan du sonar latéral
Position et orientation de la caméra
psi
v
z
Target z
Size
Fog
k
98
ContractForward
Contract Seamarks
ContractBackward
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
ycam
zcam
psiCam
53.00
démarrer
GESMI
12:43

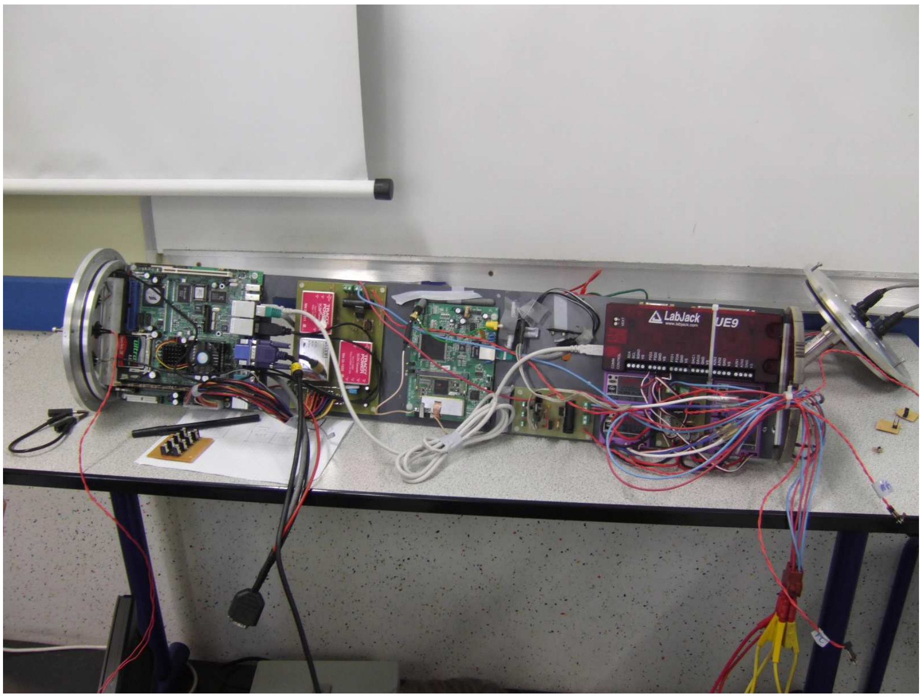

GESMI, for the estimation of seamarks using interval analysis
Profondeur
Altitude
AUV
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
ContractAl
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw trajectory with nav d
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
ycam
zcam
psiCam
410,25872
-6
85.00
démarrer
Scientific WorkPlace -...
C:\jaulin\builder\Red...
GESMI
FR
14:05

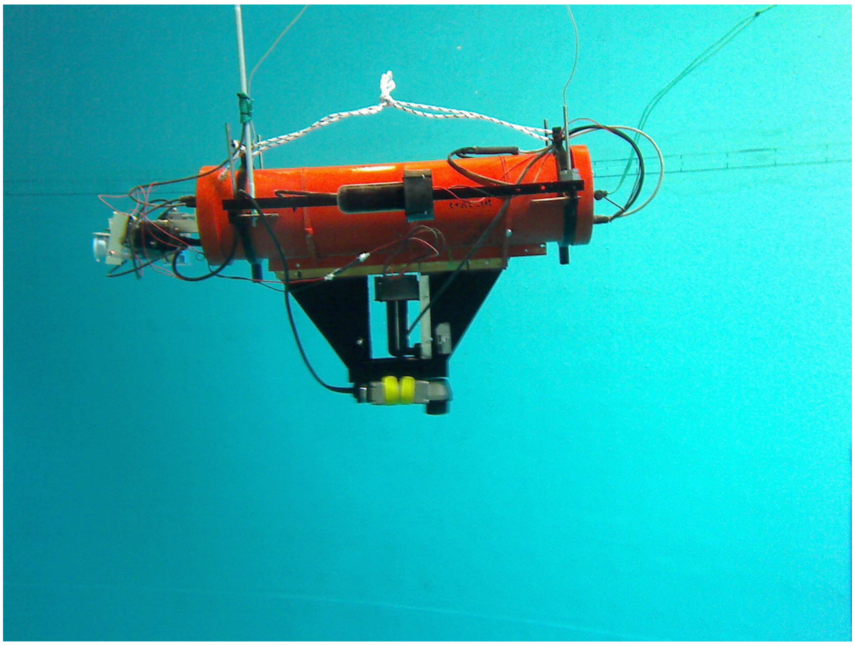


t = 5999.4
0
1
2
3
4
5
t = 0
100
0
0
100

5.3 Robust state estimation

Portsmouth, July 12-15, 2007.


LabJack
UE9



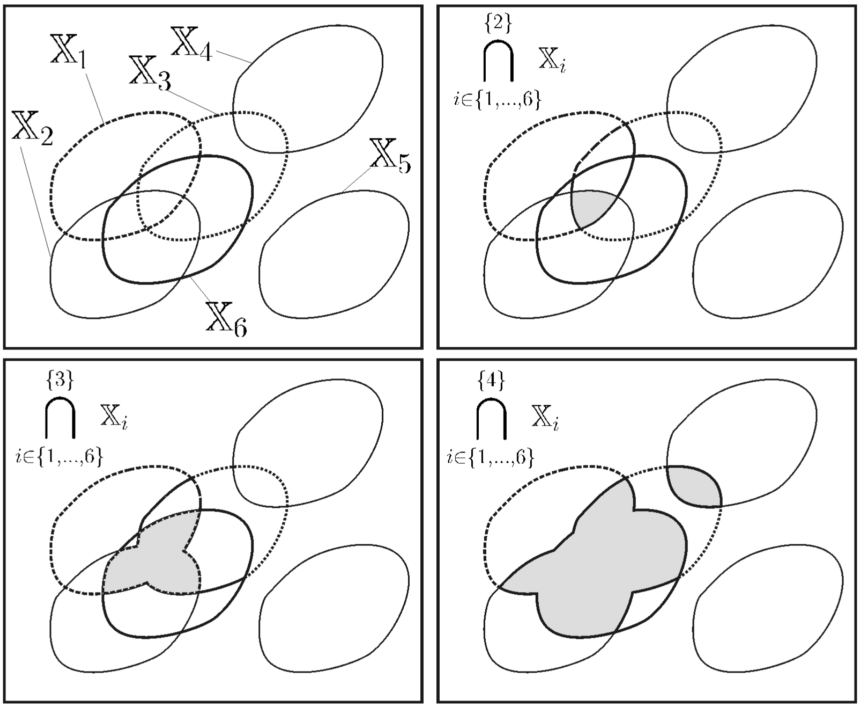
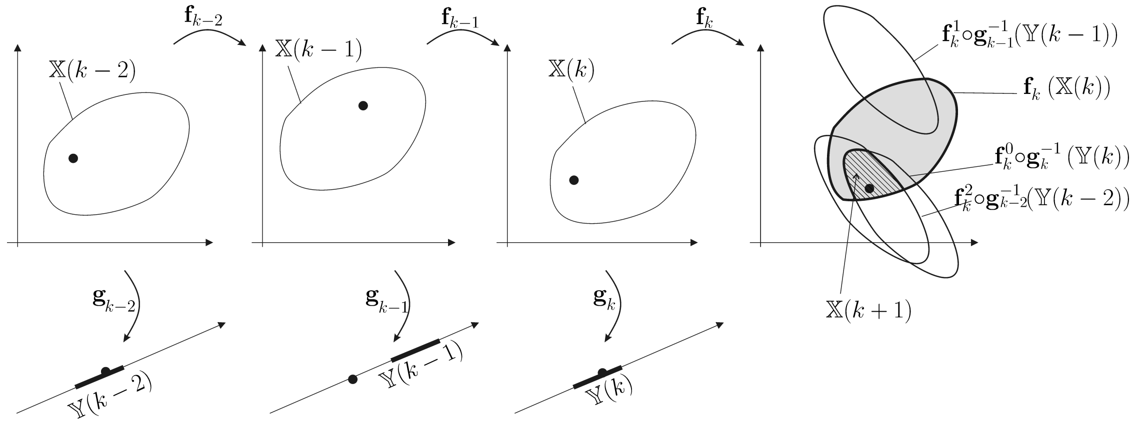


Illustration (in gray) of the q-relaxed intersection the 6 sets $\mathbb{X}_1, \ldots, \mathbb{X}_6$ where $q \in \{2, 3, 4\}$

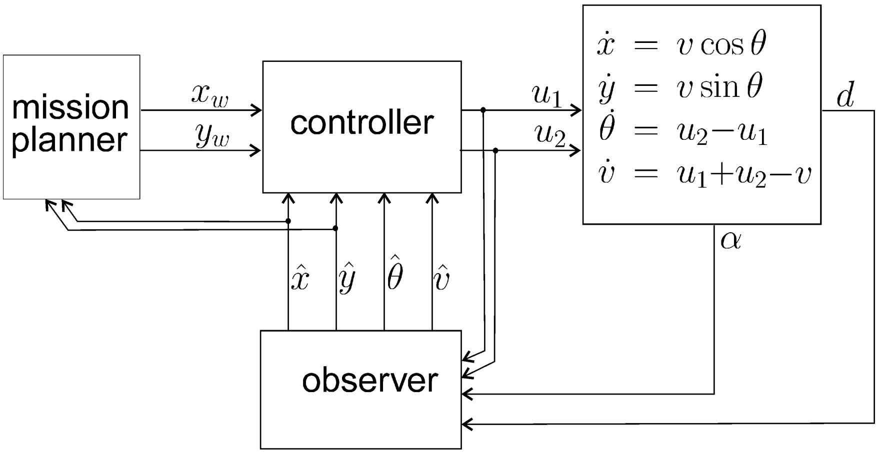


The feasible set for the state vector $\mathbb{X}(k+1)$, assuming at most $q = 1$ outlier, can be defined recursively from $\mathbb{X}(k)$ and from the data sets $\mathbb{Y}(k), \mathbb{Y}(k-1), \mathbb{Y}(k-2)$.

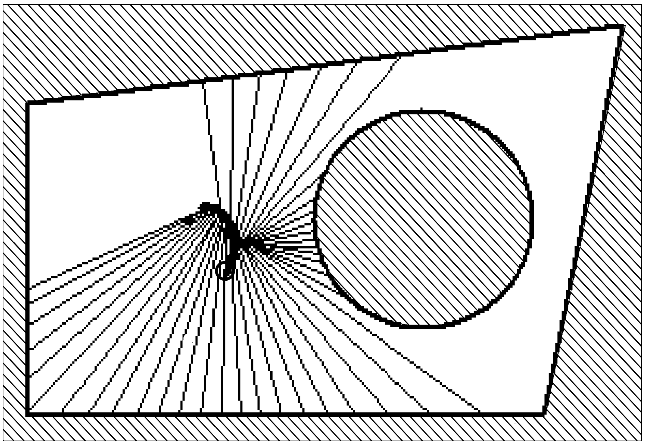
Assumption. Within any time window of length ℓ there are less than q outliers.

The set of feasible state can be computed recursively by

$$\mathbb{X}_{k+1} = \mathbf{f}_k\left(\mathbb{X}_k\right) \cap \bigcap_{i \in \{0,\dots,\ell\}}^{\{q\}} \mathbf{f}_k \circ \mathbf{f}_{k-1} \circ \dots \circ \mathbf{f}_{k-i} \circ \mathbf{g}_{k-i}^{-1}\left(\mathbb{Y}_{k-i}\right).$$



Principle of the control of the underwater robot



Superposition of the poses of the robot