Interval robotics

Chapter 7: Loop detection

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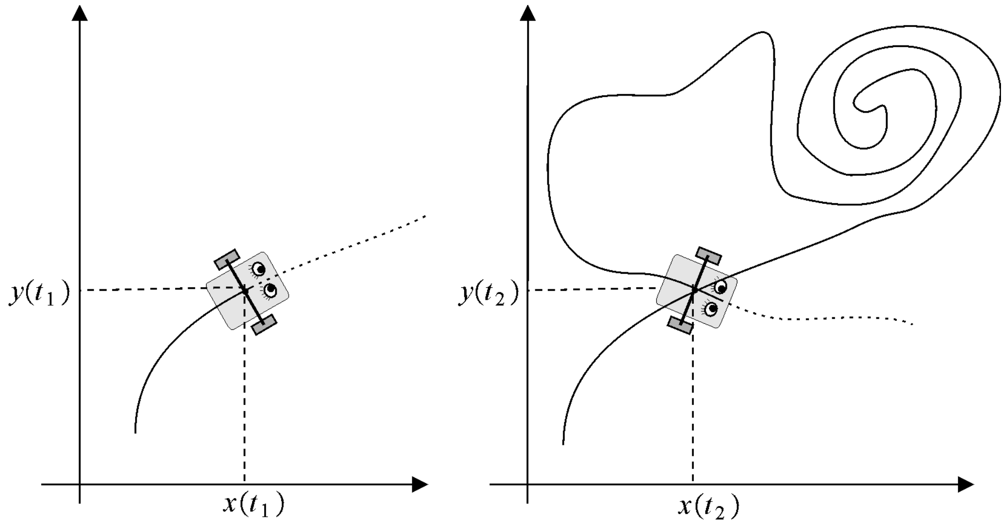
1 Time plane



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A robot trajectory with one single loop

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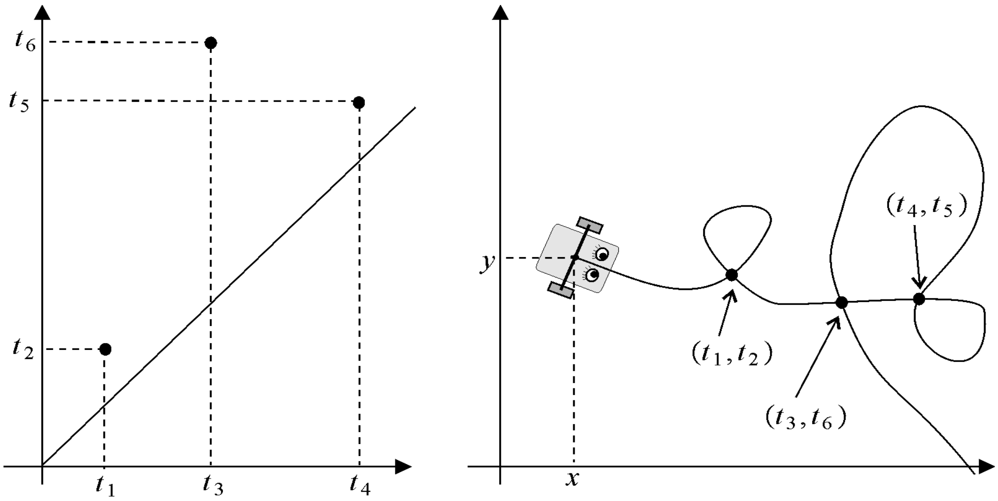


Left: t-plane; Right: trajectory

2 Kernel

Consider a mapping $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$.

$$\ker \mathbf{f} = \{\mathbf{f}\left(\mathbf{x}\right) = \mathbf{0}\} = \mathbf{f}^{-1}\left(\mathbf{0}\right)$$

The kernel of an interval function $[\mathbf{f}] : \mathbb{R}^n \longrightarrow \mathbb{IR}^n$ is

$$\mathbb{X} = \mathsf{ker}\,[\mathbf{f}] = \bigcup_{\mathbf{f}\in[\mathbf{f}]} \mathsf{ker}\,\mathbf{f} = \{\mathbf{x} \in [\mathbf{x}] \subset \mathbb{R}^n \mid \mathbf{0} \in [\mathbf{f}]\,(\mathbf{x})\}\,.$$

Problem. Find an inner and an outer approximation of $\mathbb{X}$:

$$\mathbb{X}^{-} \subset \mathbb{X} \subset \mathbb{X}^{+}.$$

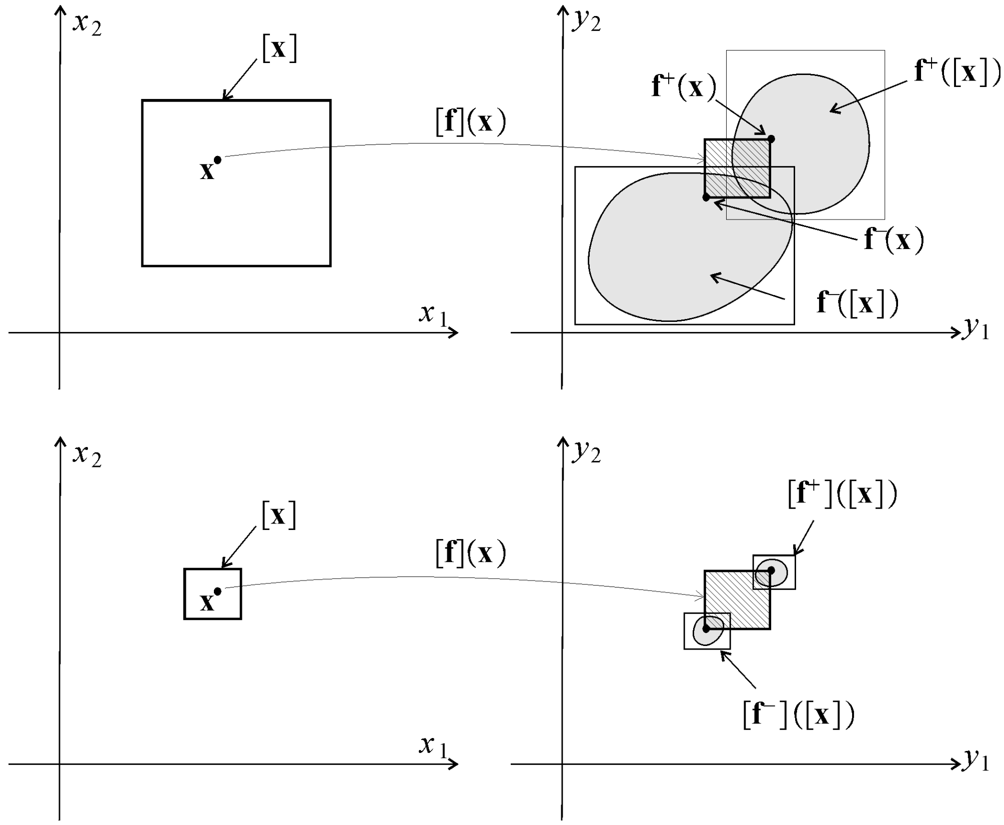
As a consequence,

$$\ker \mathbf{f}^{*} \subset \mathbb{X}^{+}.$$

3 Characterization of the kernel

Denote by $\mathbf{f}^{-}(\mathbf{x})$ and $\mathbf{f}^{+}(\mathbf{x})$ upper and lower bounds of $[\mathbf{f}](\mathbf{x})$, i.e,

$$\forall \mathbf{x},\ \ [\mathbf{f}](\mathbf{x}) = \left[\mathbf{f}^{-}(\mathbf{x}), \mathbf{f}^{+}(\mathbf{x})\right].$$



Inclusion functions associated with $\mathbf{f}^-(\mathbf{x})$ and $\mathbf{f}^+(\mathbf{x})$

Define

$$\begin{aligned}\left[\mathbf{f}^{\subset}\right]([\mathbf{x}]) &= \left[\mathsf{ub}\left(\mathbf{f}^{-}([\mathbf{x}])\right), \mathsf{lb}\left(\mathbf{f}^{+}([\mathbf{x}])\right)\right]\\ \left[\mathbf{f}^{\supset}\right]([\mathbf{x}]) &= \left[\mathsf{lb}\left(\mathbf{f}^{-}([\mathbf{x}])\right), \mathsf{up}\left(\mathbf{f}^{+}([\mathbf{x}])\right)\right]\end{aligned}$$

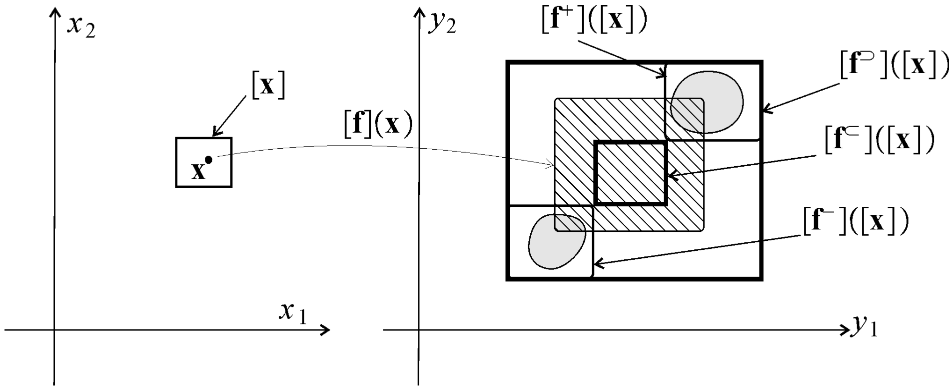


Illustration of the inclusion $\left[\mathbf{f}^{\subset}\right]([\mathbf{x}]) \subset [\mathbf{f}](\mathbf{x}) \subset \left[\mathbf{f}^{\supset}\right]([\mathbf{x}])$

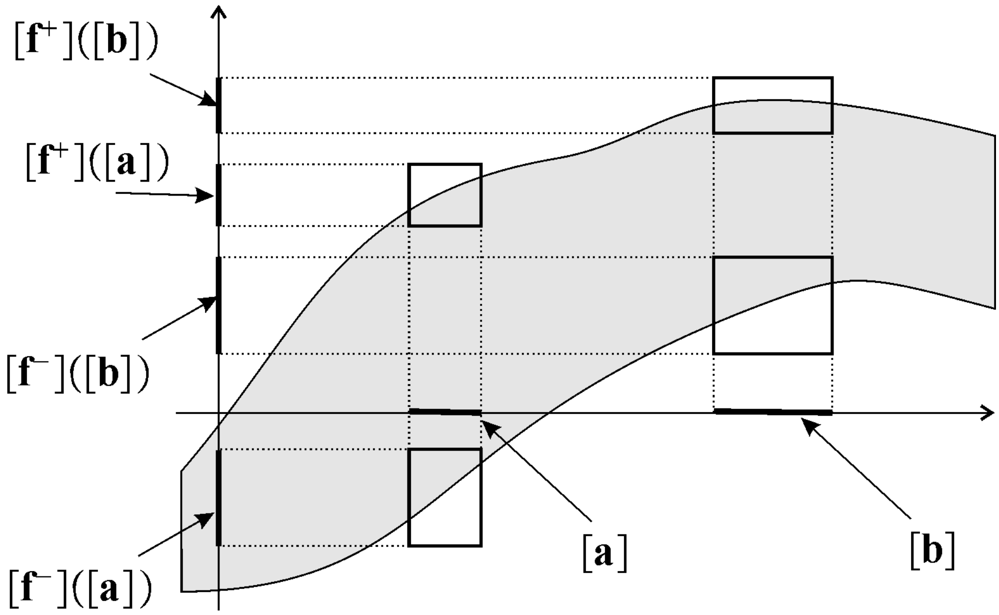
The quantity $\left[\left[\mathbf{f}^{\subset} \right] ([\mathbf{x}]) , \left[\mathbf{f}^{\supset} \right] ([\mathbf{x}]) \right]$ is an interval of the lattice $(\mathbb{IR}^n, \subset)$ equipped with the inclusion.

We have

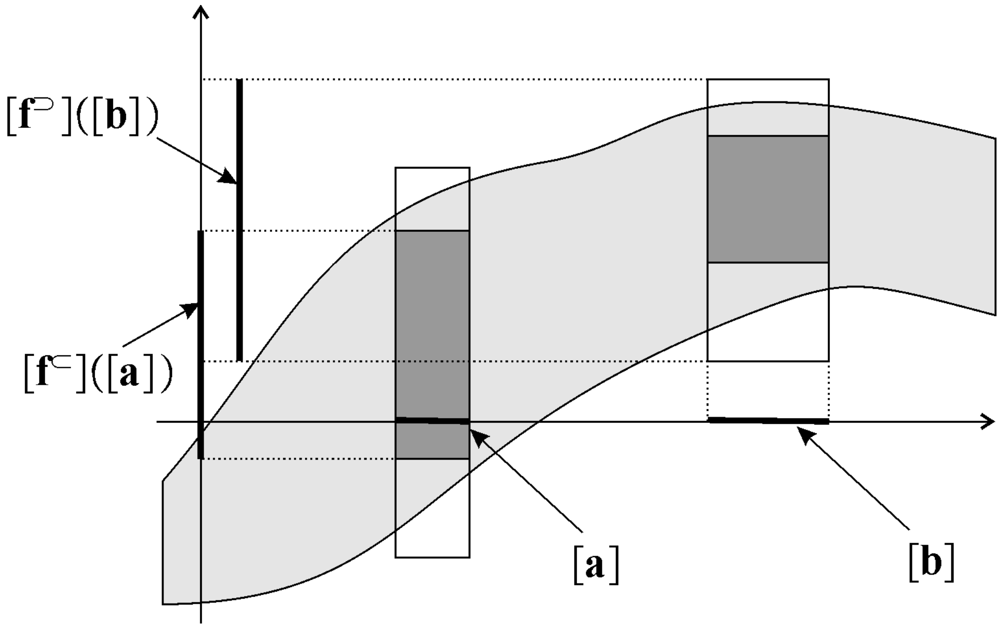
$$\mathbf{x} \in [\mathbf{x}] \Rightarrow \left[\mathbf{f}^{\subset} \right] ([\mathbf{x}]) \subset [\mathbf{f}] (\mathbf{x}) \subset \left[\mathbf{f}^{\supset} \right] ([\mathbf{x}]) .$$

Proposition

$$\left(\begin{array}{lll} \text{(i)} & \mathbf{0} \in \left[\mathbf{f}^{\subset}\right](\left[\mathbf{x}\right]) & \Rightarrow \quad \left[\mathbf{x}\right] \subset \mathbb{X} \\ \text{(ii)} & \mathbf{0} \notin \left[\mathbf{f}^{\supset}\right](\left[\mathbf{x}\right]) & \Rightarrow \quad \left[\mathbf{x}\right] \cap \mathbb{X} = \emptyset \end{array} \right).$$



[f+]([b])
[f+]([a])
[f−]([b])
[f−]([a])
[a]
[b]



$[\mathbf{f}^{\supset}]([\mathbf{b}])$
$[\mathbf{f}^{\subset}]([\mathbf{a}])$
$[\mathbf{a}]$
$[\mathbf{b}]$

4 Loop detection

The robot knows a box $[\mathbf{v}](t)$ which contains $\mathbf{v}(t)$ for each $t \in [0, t_{\mathsf{max}}]$. The loop set is

$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\mathsf{max}}]^2 \mid \exists \mathbf{v} \in [\mathbf{v}], \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0} \right\}$$

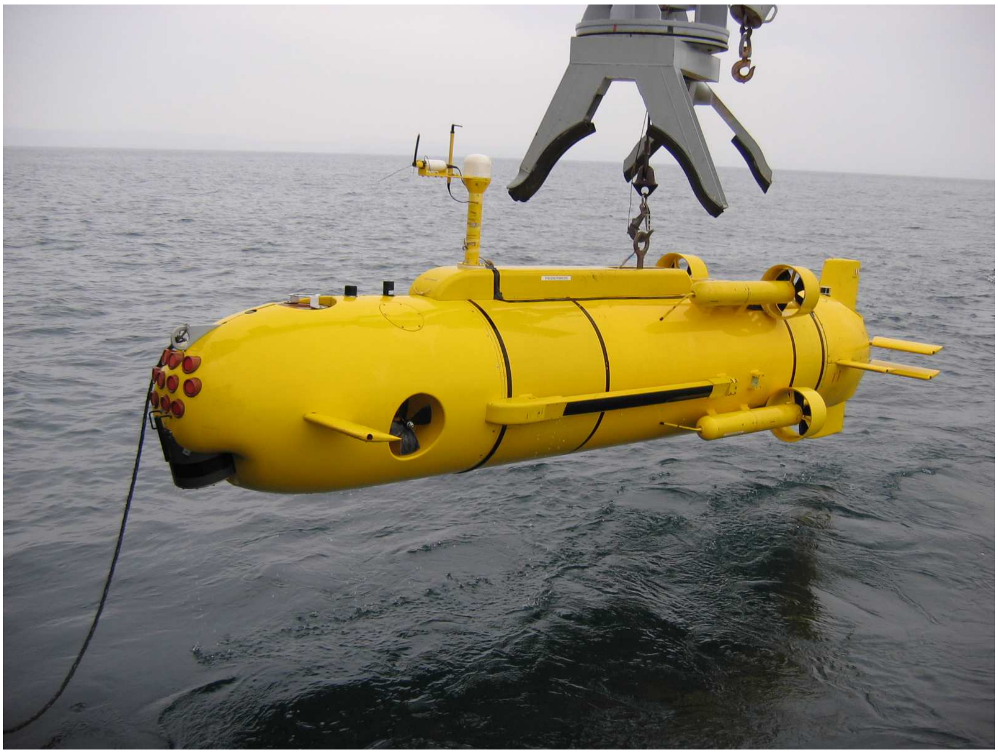
If

$$\left[\mathbf{f}\right](\mathbf{t}) = \left[\int_{t_1}^{t_2} \mathbf{v}^{-}(\tau)d\tau, \int_{t_1}^{t_2} \mathbf{v}^{+}(\tau)d\tau\right]$$

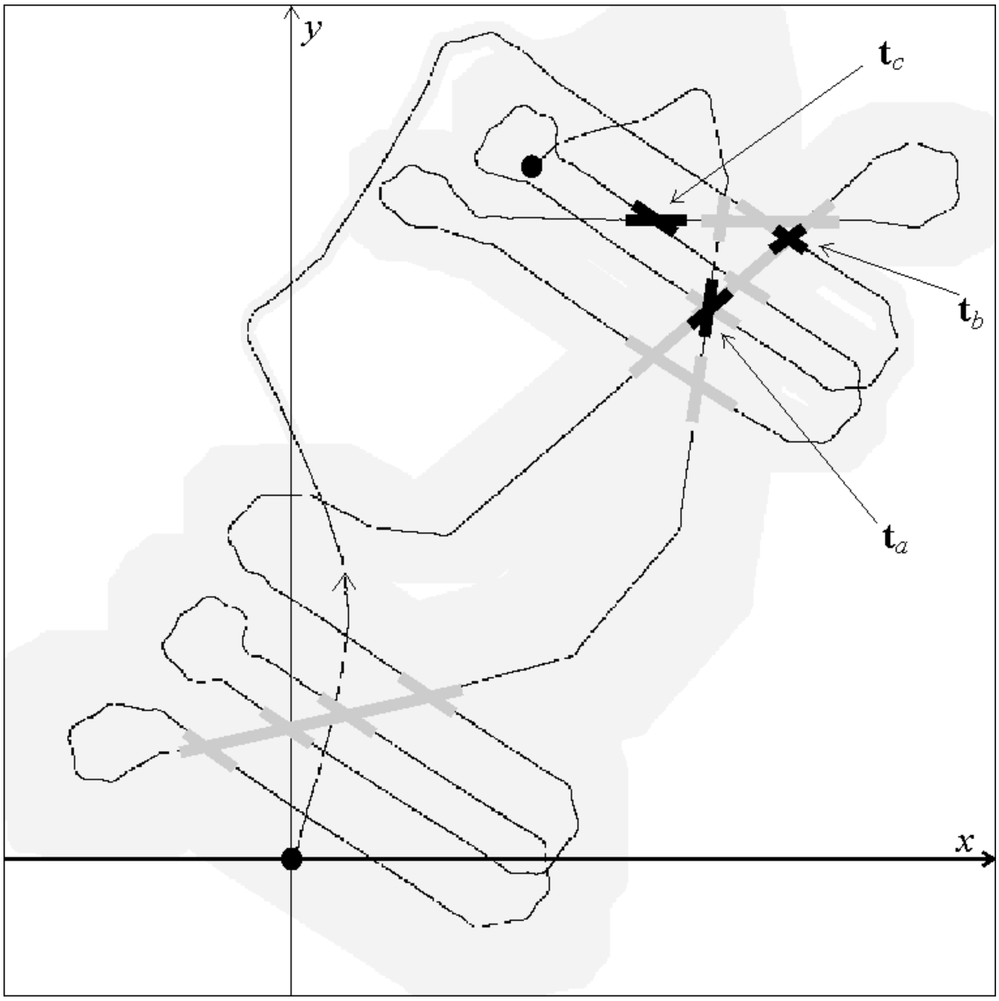
Thus

$$\mathbb{T} = \left\{\mathbf{t} \in [0, t_{\mathsf{max}}]^2, \mathbf{0} \in \left[\mathbf{f}\right](\mathbf{t})\right\} = \mathsf{ker}\left[\mathbf{f}\right].$$

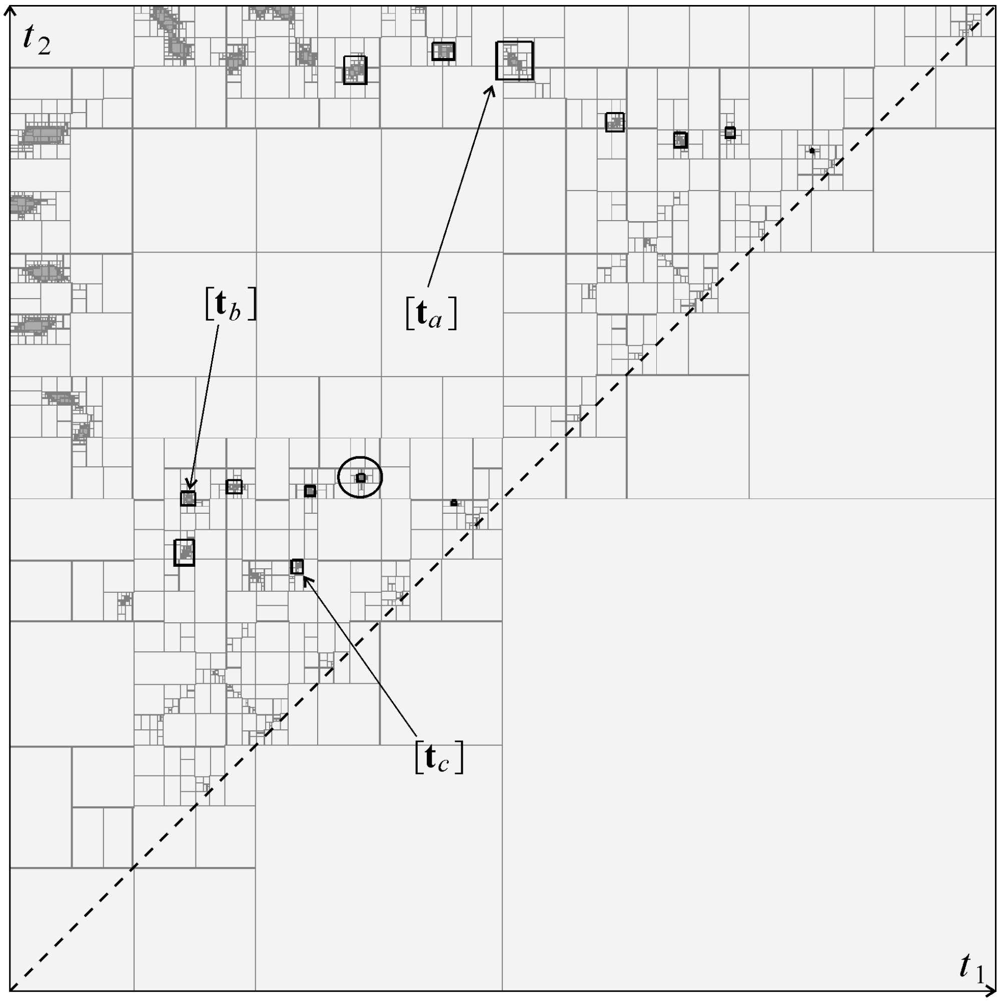
5 Tescase



Redermor, built by GESMA (Groupe d'Etude Sous-Marine de l'Atlantique)



Tube enclosing the trajectory of the robot. The 14 black/gray crosses correspond to detected loops.



Inner and outer approximation of $\mathbb{T}$