Interval robotics

Chapter 5: Robust observers

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1 State estimation

$$\begin{cases} \mathbf{x}(k+1) &= \mathbf{f}_k(\mathbf{x}(k), \mathbf{n}(k)) \\ \mathbf{y}(k) &= \mathbf{g}_k(\mathbf{x}(k)), \end{cases}$$

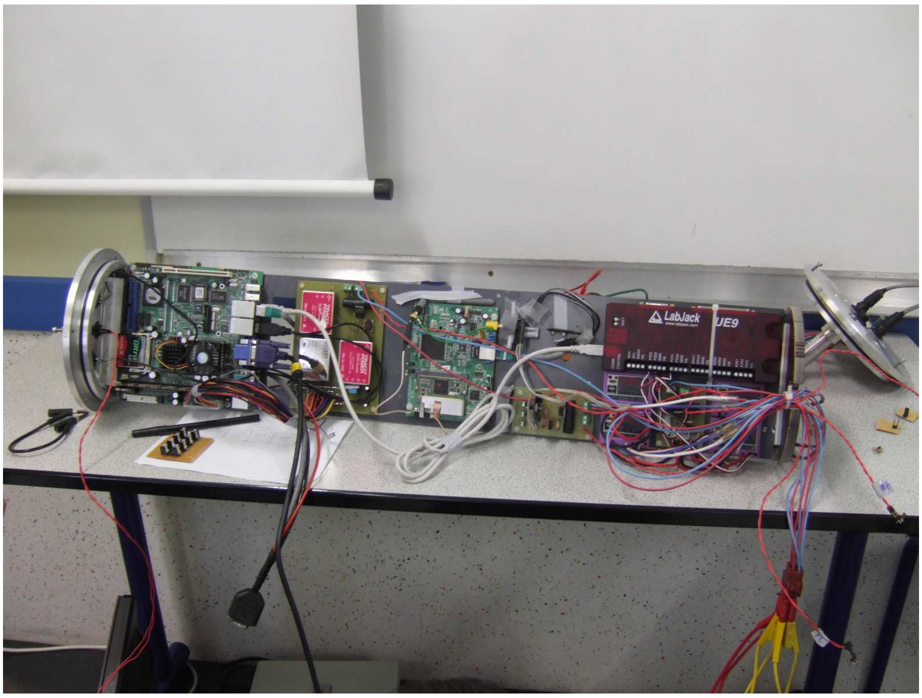
with $\mathbf{n}(k) \in \mathbb{N}(k)$ and $\mathbf{y}(k) \in \mathbb{Y}(k)$.

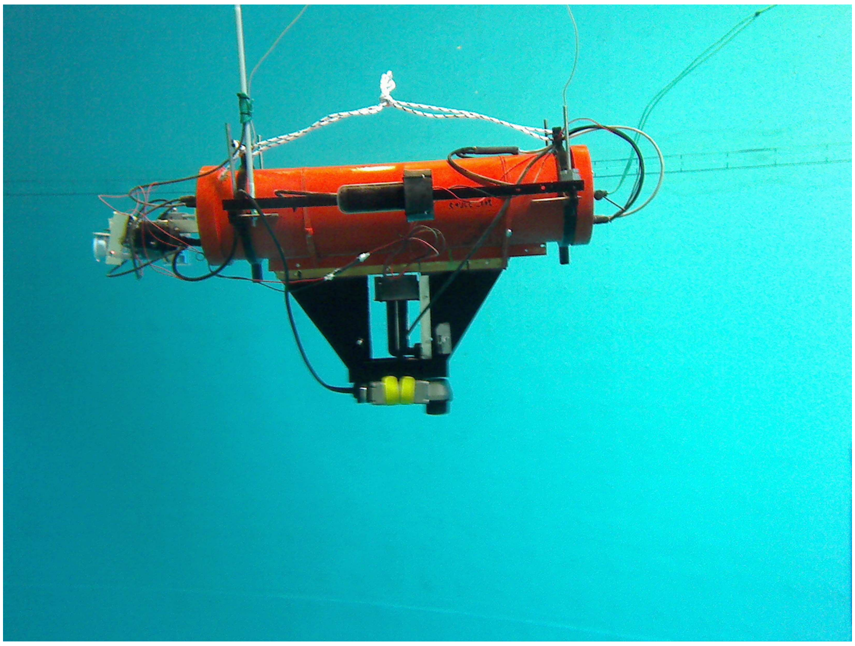
Without outliers

$$\mathbb{X}(k+1) = \mathbf{f}_k\left(\mathbb{X}(k), \mathbb{N}\left(k\right)\right) \quad \cap \quad \mathbf{g}_{k+1}^{-1}\left(\mathbb{Y}(k+1)\right).$$

2 SAUC'E

Portsmouth, July 12-15, 2007.


LabJack
UE9



3 Robust observer

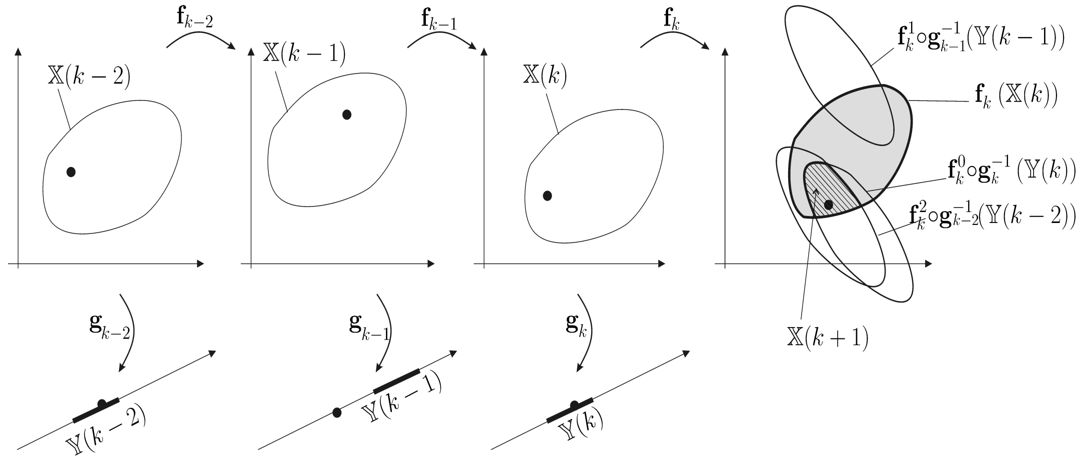
Define

$$\begin{cases} \mathbf{f}_{k:k}\left(\mathbb{X}\right) & \stackrel{\text{def}}{=} \mathbb{X} \\ \mathbf{f}_{k_1:k_2+1}\left(\mathbb{X}\right) & \stackrel{\text{def}}{=} \mathbf{f}_{k_2}(\mathbf{f}_{k_1:k_2}\left(\mathbb{X}\right), \mathbb{N}\left(k_2\right)),\ k_1 \leq k_2. \end{cases}$$

The set $\mathbf{f}_{k_1:k_2}\left(\mathbb{X}\right)$ represents the set of all $\mathbf{x}\left(k_2\right)$, consistent with $\mathbf{x}\left(k_1\right) \in \mathbb{X}$.

Consider the set state estimator

$$\begin{cases} \mathbb{X}(k) & = & \mathbf{f}_{0:k}\left(\mathbb{X}(0)\right) \quad \text{if } k < m, \ \text{(initialization step)} \\ \mathbb{X}(k) & = & \mathbf{f}_{k-m:k}\left(\mathbb{X}(k-m)\right) \cap \\ & & \displaystyle\bigcap_{i\in\{1,\dots,m\}}^{\{q\}} \mathbf{f}_{k-i:k} \circ \mathbf{g}_{k-i}^{-1}\left(\mathbb{Y}(k-i)\right) \quad \text{if } k \geq m \end{cases}$$



$\mathbf{f}_{k-2}$
$\mathbf{f}_{k-1}$
$\mathbf{f}_{k}$
$\mathbb{X}(k-2)$
$\mathbb{X}(k-1)$
$\mathbb{X}(k)$
$\mathbf{f}_k^1 \circ \mathbf{g}_{k-1}^{-1}(\mathbb{Y}(k-1))$
$\mathbf{f}_k\,(\mathbb{X}(k))$
$\mathbf{f}_k^0 \circ \mathbf{g}_k^{-1}\,(\mathbb{Y}(k))$
$\mathbf{f}_k^2 \circ \mathbf{g}_{k-2}^{-1}(\mathbb{Y}(k-2))$
$\mathbb{X}(k+1)$
$\mathbf{g}_{k-2}$
$\mathbf{g}_{k-1}$
$\mathbf{g}_{k}$
$\mathbb{Y}(k-2)$
$\mathbb{Y}(k-1)$
$\mathbb{Y}(k)$

We assume
(i) within any time window of length m we have less than q outliers and
(ii) $\mathbb{X}(0)$ contains $\mathbf{x}\,(0)$, then $\mathbb{X}(k)$ encloses $\mathbf{x}\,(k)$.

What is the probability of this assumption ?

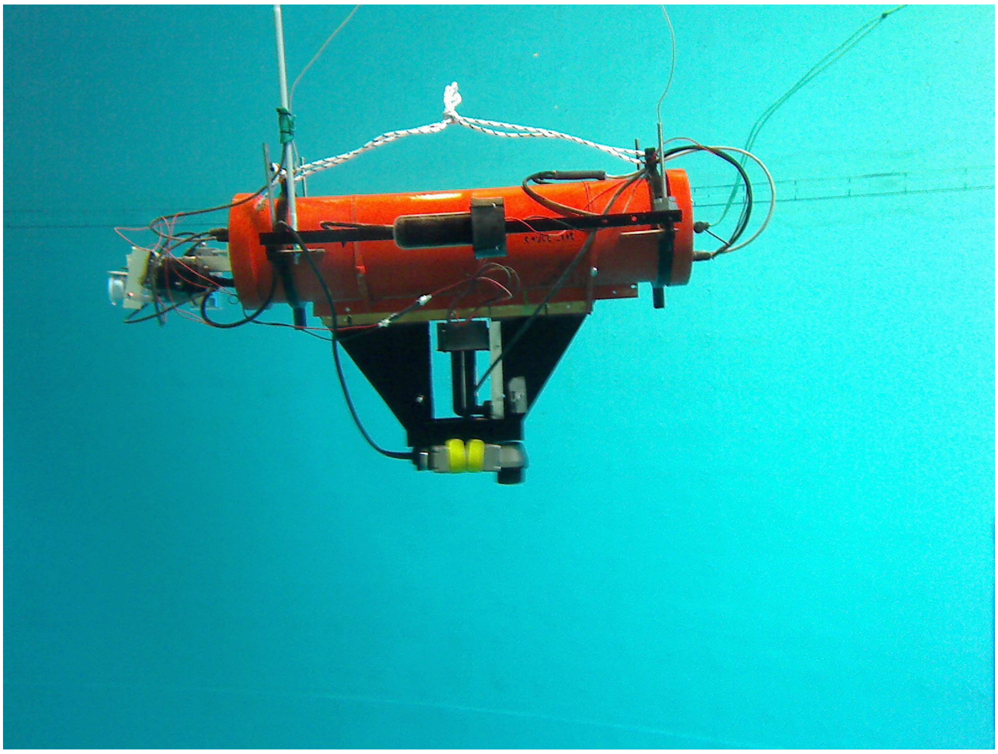
Theorem. Consider the sequence of sets $\mathbb{X}(0), \mathbb{X}(1), \ldots$ built by the set observer. We have

$$\Pr\left(\mathbf{x}\left(k\right) \in \mathbb{X}(k)\right) \geq \alpha \; * \; \Pr\left(\mathbf{x}\left(k-1\right) \in \mathbb{X}(k-1)\right)$$

where

$$\alpha = \sqrt[m]{\sum_{i=m-q}^{m} \frac{m!\ \pi^{i}.\left(1-\pi\right)^{m-i}}{i!\left(m-i\right)!}}.$$

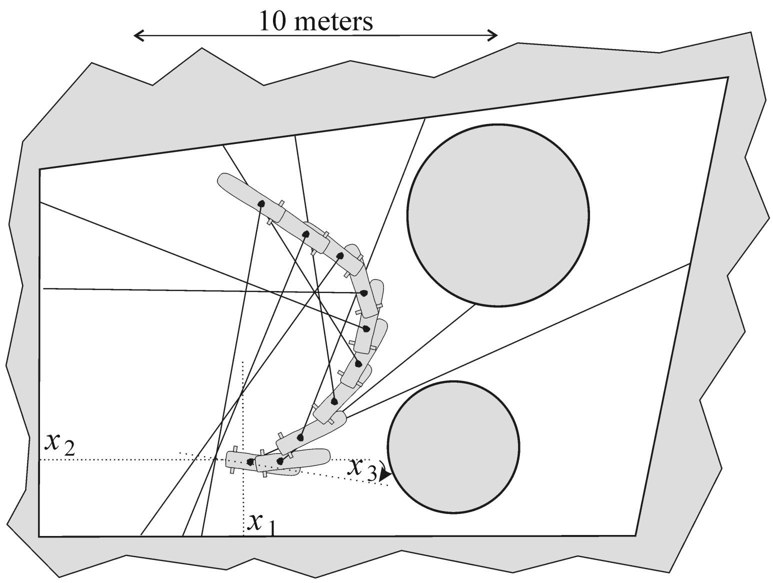
4 Underwater localization



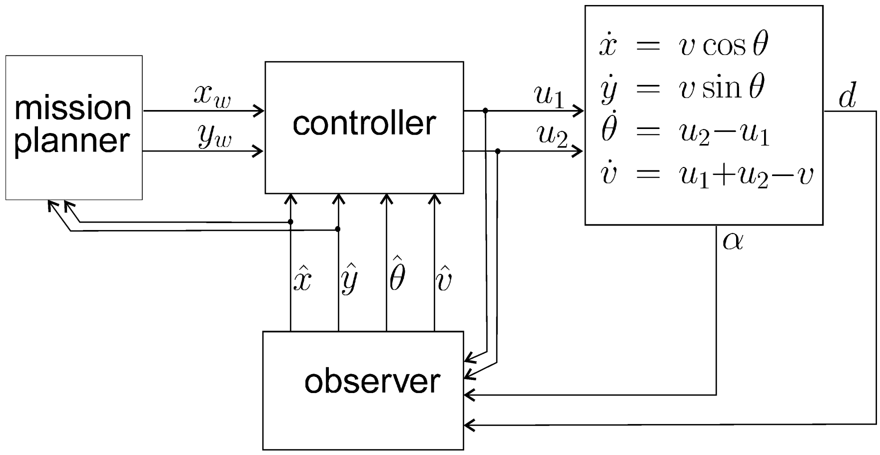
SAUCISSE inside a swimming pool

The robot evolution is

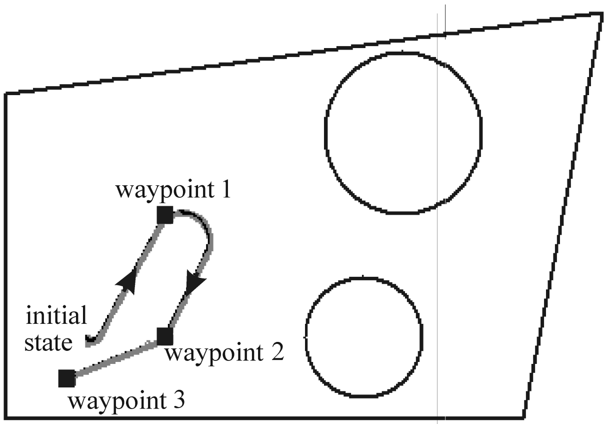
$$\begin{cases} \dot{x}_1 &= x_4 \cos x_3 \\ \dot{x}_2 &= x_4 \sin x_3 \\ \dot{x}_3 &= u_2 - u_1 \\ \dot{x}_4 &= u_1 + u_2 - x_4, \end{cases}$$

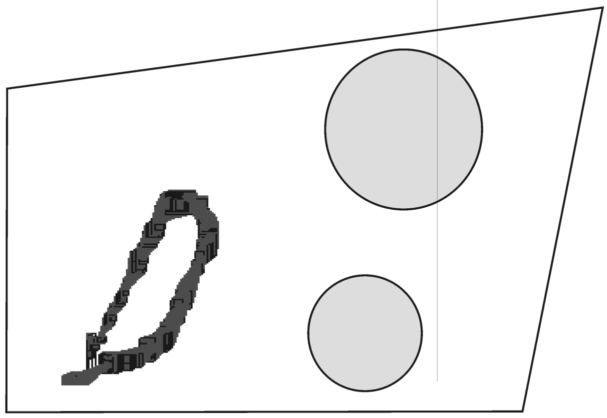


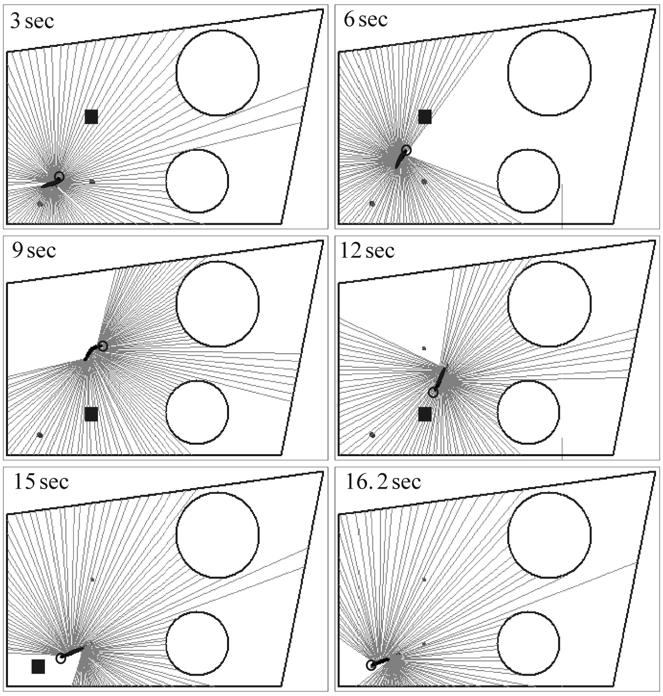
Underwater robot moving inside a pool

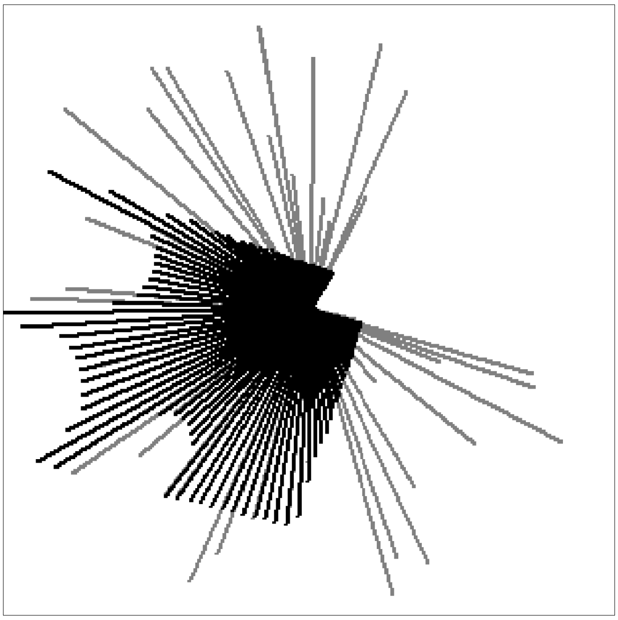


Principle of the control of the underwater robot


waypoint 1
initial
state
waypoint 2
waypoint 3




3 sec
6 sec
9 sec
12 sec
15 sec
16. 2 sec

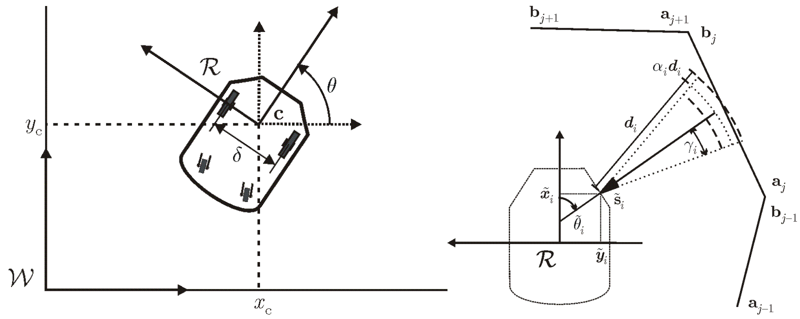


Emmision diagram at time $t = 16.2\,\mathrm{sec}$

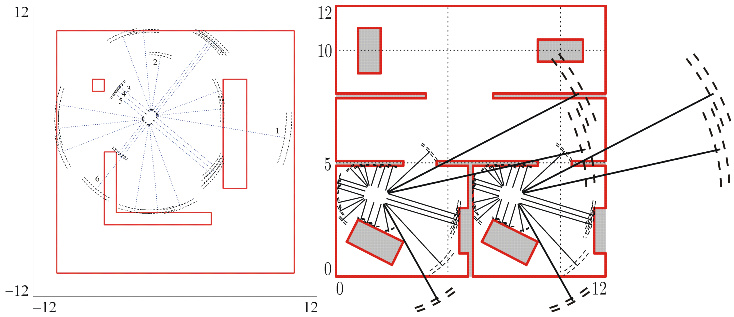
t(sec)	$\Pr(\mathbf{x} \in \mathbb{X})$	Outliers
3.0	≥ 0.965	58
6.0	≥ 0.932	50
9.0	≥ 0.899	42
12.0	≥ 0.869	51
15.0	≥ 0.838	51
16.2	≥ 0.827	49

5 Indoor localization

The robot is equipped with 24 ultrasonic telemetric sensors



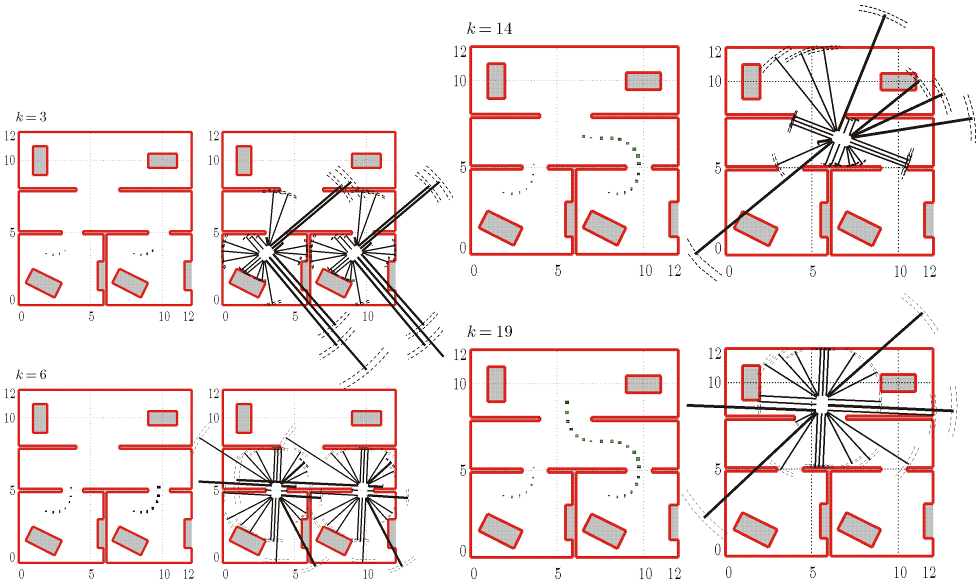
Sivia computes the set of all consistent poses



If state equation of the robot are given

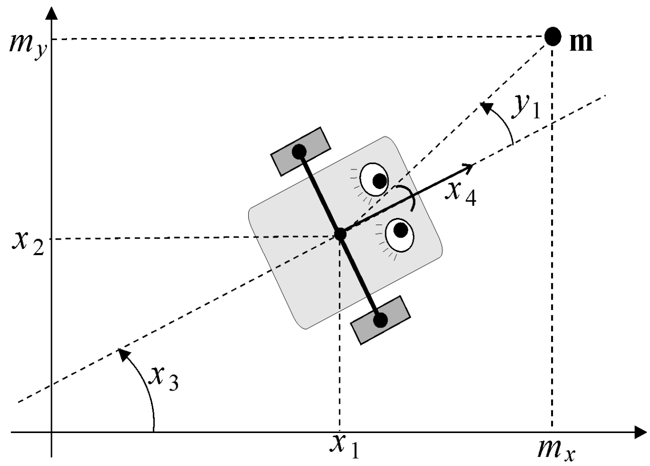
$$\begin{cases} \dot{x} & = & \rho\,\dfrac{\omega_{\mathsf{r}} + \omega_{\mathsf{l}}}{2}\cos\theta, \\ \dot{y} & = & \rho\,\dfrac{\omega_{\mathsf{r}} + \omega_{\mathsf{l}}}{2}\sin\theta \\ \dot{\theta} & = & \rho\,\dfrac{\omega_{\mathsf{r}} - \omega_{\mathsf{l}}}{\delta} \end{cases}$$

a set counterpart of the Kalman filter can be implemented.



$k = 3$
$k = 6$
$k = 14$
$k = 19$

6 Comparison with the Kalman filter



A robot (unicycle type) which measures the angle y_1 corresponding to the mark $\mathbf{m}$

$$
\begin{cases}
\dot{x}_1 & = & x_4 \cos x_3 \\
\dot{x}_2 & = & x_4 \sin x_3 \\
\dot{x}_3 & = & u_1 \\
\dot{x}_4 & = & u_2
\end{cases}
$$

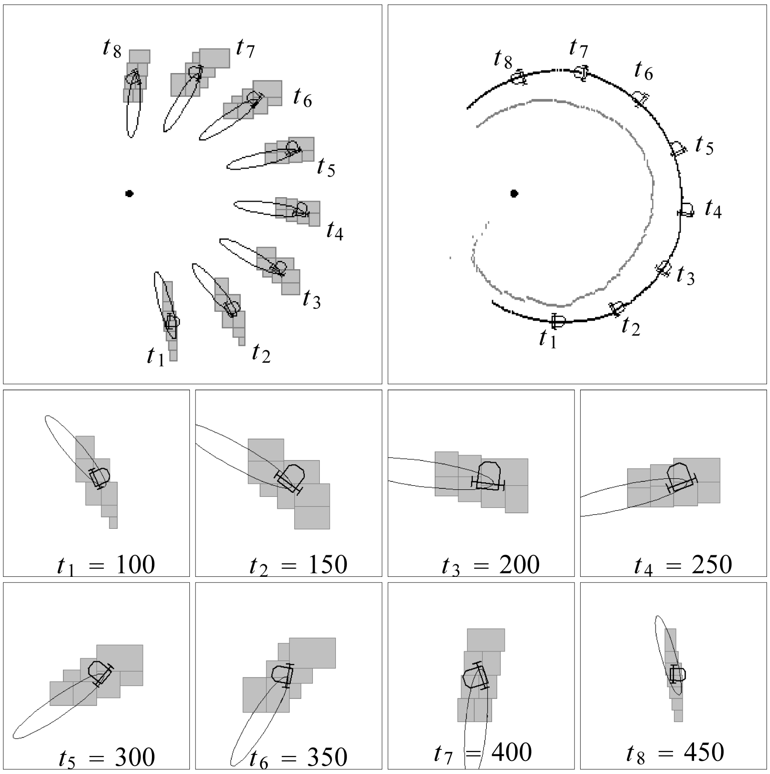
$$
\begin{cases}
y_1 & = & \operatorname{atan2}\left(m_y - x_2, m_x - x_1\right) + x_3, \quad k \in \mathbb{Z} \\
y_2 & = & x_3 \\
y_3 & = & x_4.
\end{cases}
$$

Scenario 1. The measurement noises as well as the state noises are all Gaussian and centered with a variance of 0.01.

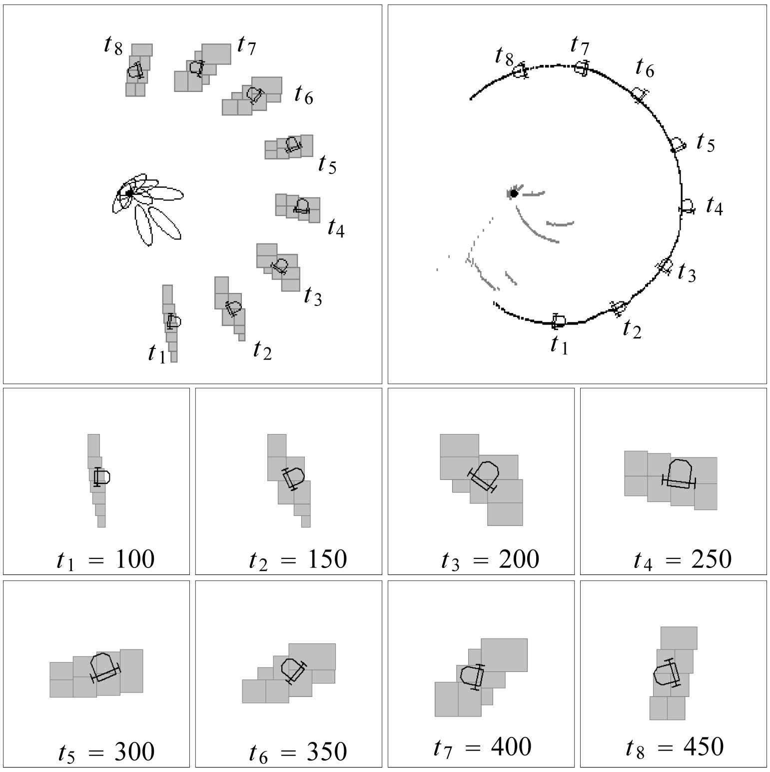
Scenario 2. With a probability of 5%, an outlier for y_1 is generated.

Scenario 3. This scenario is similar to Scenario 1 but a bias of 0.5 is added to y_1.

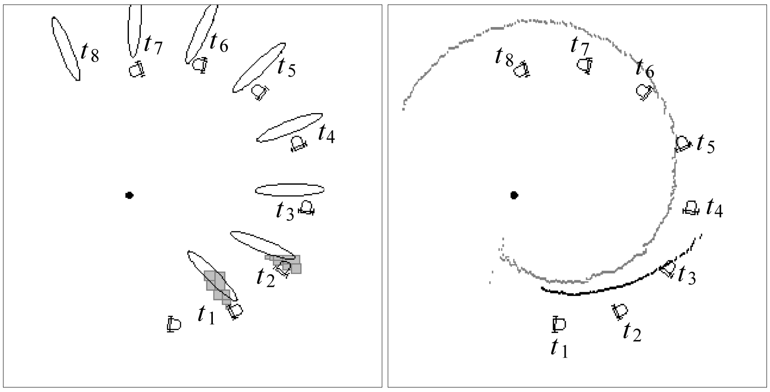
For RSO, $m = 50, q = 10$.



Scenario 1: All noises are Gaussian



Scenario 2: 1% of the data are outliers



Scenario 3. An unknown bias has been added to y_1.