Interval analysis and sailboat robotics

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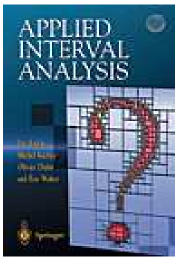
November 7 2012, Puerto Alegra, Brazil



1 Interval analysis

Problem. Given $f : \mathbb{R}^n \to \mathbb{R}$, a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}]\,, f\left(\mathbf{x}\right) \geq 0.$$

Interval arithmetic can solve efficiently this problem.

Interval arithmetic

$$\begin{aligned}
[-1,3]+[2,5] &= [1,8], \\
[-1,3]\cdot[2,5] &= [-5,15], \\
\mathsf{abs}\,([-7,1]) &= [0,7]
\end{aligned}$$

If f is given

Algorithm f(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: y)
$z := x_1$;
for $k := 0$ to 100
$z := (\cos x_2) \cdot (\sin(z) + kx_3)$;
next;
$y := \sin(zx_1)$;

Its interval extension is

Algorithm $[f](\text{in: } [\mathbf{x}] = ([x_1], [x_2], [x_3]), \text{ out: } [y])$

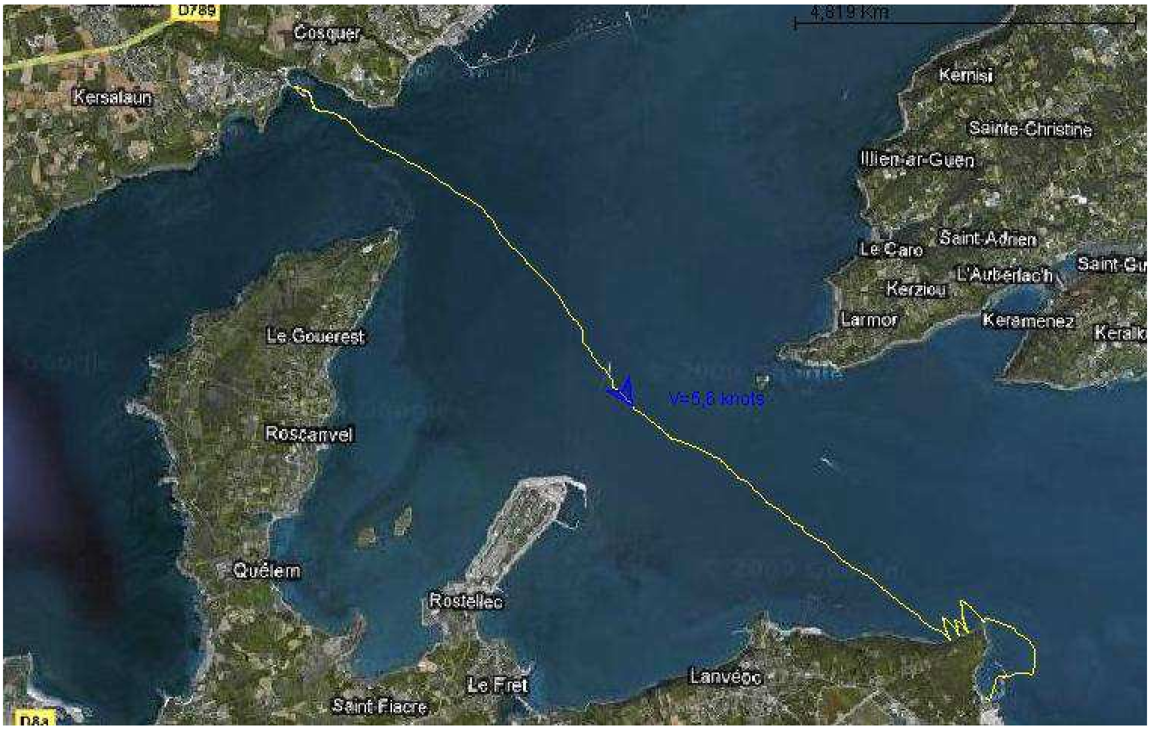
$[z] := [x_1]$;
for $k := 0$ to 100
  $[z] := (\cos [x_2]) \cdot (\sin ([z]) + k \cdot [x_3])$;
next;
$[y] := \sin([z] \cdot [x_1])$;

Theorem (Moore, 1970)

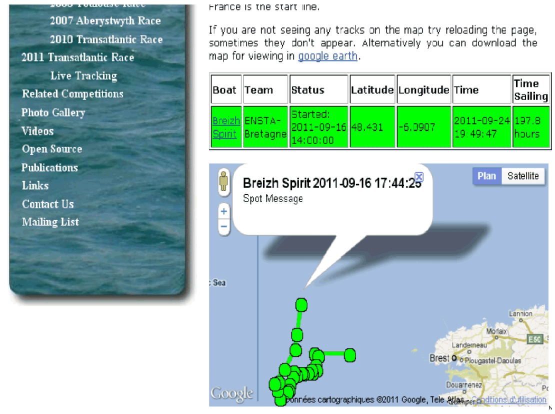
$$[f]\,([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}]\,,\, f\,(\mathbf{x}) \geq 0$$

2 Sailboat robotics

BREIZH
SPIRIT
ENSIETA



D789
Cosquer
Kersalaun
4,819 Km
Kernisi
Sainte-Christine
Illien-ar-Guen
Le Caro
Saint-Adrien
Kerziou
L'Auberlac'h
Saint-Gu
Larmor
Keramenez
Keralk
Le Gouerest
V=5,6 knots
Roscanvel
Quélern
Rostellec
Lanvéoc
Le Fret
Saint Fiacre
D8a



2007 Aberystwyth Race
2010 Transatlantic Race
2011 Transatlantic Race
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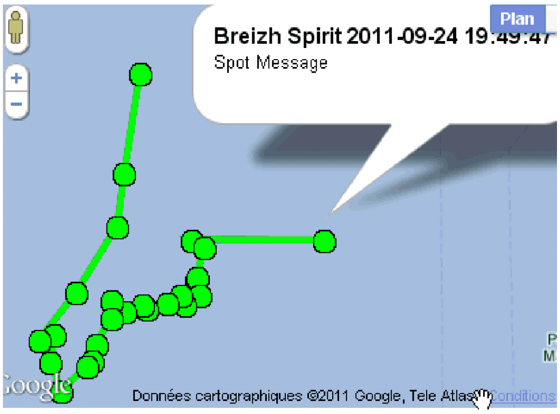
France is the start line.

If you are not seeing any tracks on the map try reloading the page, sometimes they don't appear. Alternatively you can download the map for viewing in google earth.

Boat	Team	Status	Latitude	Longitude	Time	Time Sailing
Breizh Spirit	ENSTA-Bretagne	Started: 2011-09-16 14:00:00	48.431	-6.0907	2011-09-24 19:49:47	197.8 hours



3 Vaimos

Vaimos (IFREMER and ENSTA)

The robot satisfies a state equation

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{u}\right).$$

With the controller $\mathbf{u} = \mathbf{g}\left(\mathbf{x}\right)$, the robot satisfies an equation of the form

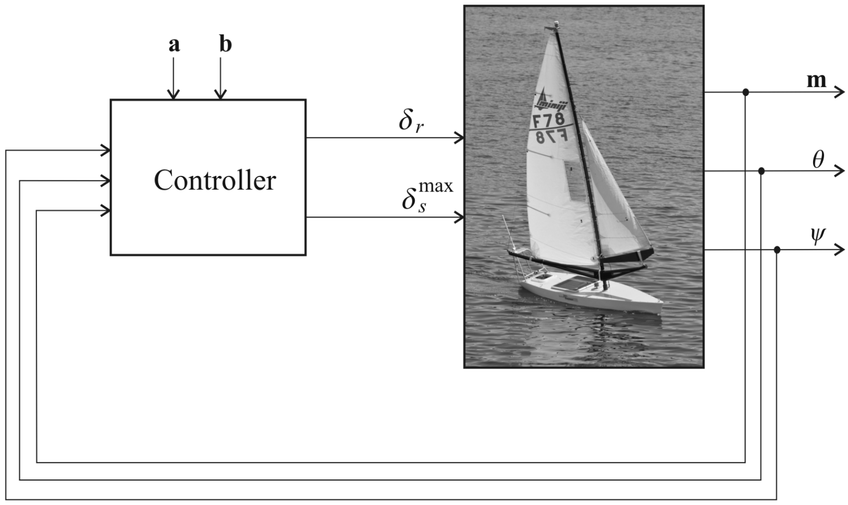
$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}\right).$$

With all uncertainties, the robot satisfies.

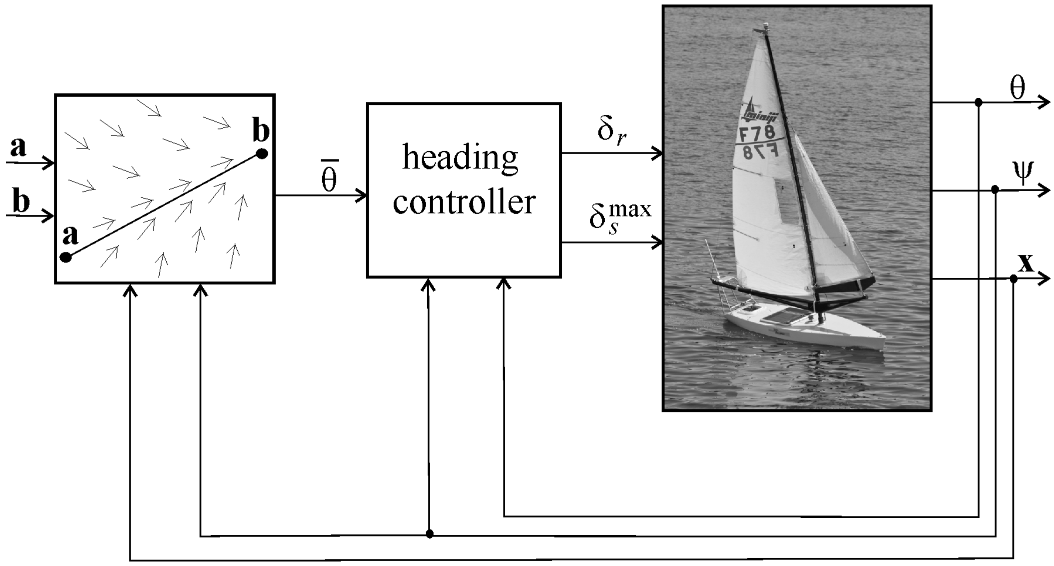
$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$$

which is *a differential inclusion*.

4 Line following



Controller of a sailboat robot



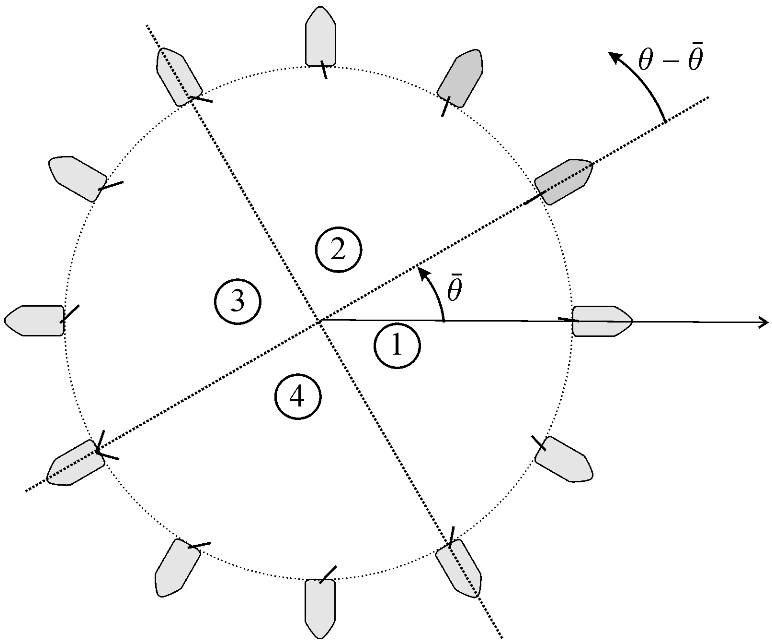
a
b
a
b
$\bar{\theta}$
heading
controller
δ_r
$\delta_s^{\max}$
θ
ψ
x

Heading controller

$$\left\{\begin{array}{lll}\delta_r & = & \left\{\begin{array}{ll}\delta_r^{\mathsf{max}}.\,\mathsf{sin}\left(\theta-\bar{\theta}\right) & \text{if } \mathsf{cos}\left(\theta-\bar{\theta}\right)\geq 0\\ \delta_r^{\mathsf{max}}.\mathsf{sign}\left(\mathsf{sin}\left(\theta-\bar{\theta}\right)\right) & \text{otherwise}\end{array}\right.\\ \delta_s^{\mathsf{max}} & = & \frac{\pi}{2}.\left(\frac{\mathsf{cos}(\psi-\bar{\theta})+1}{2}\right).\end{array}\right.$$

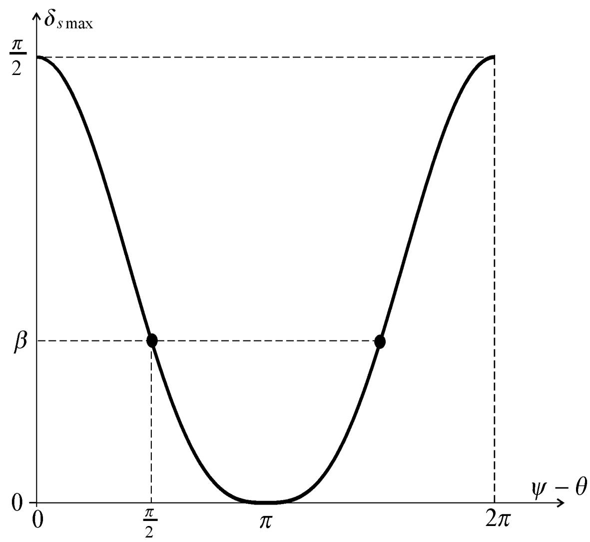
Rudder

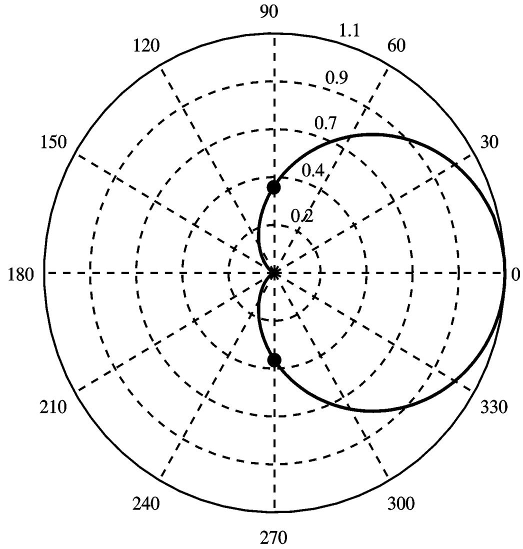
$$\delta_r = \begin{cases} \delta_r^{\max}.\sin\left(\theta - \bar{\theta}\right) & \text{if } \cos\left(\theta - \bar{\theta}\right) \geq 0 \\ \delta_r^{\max}.\text{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right) & \text{otherwise} \end{cases}$$



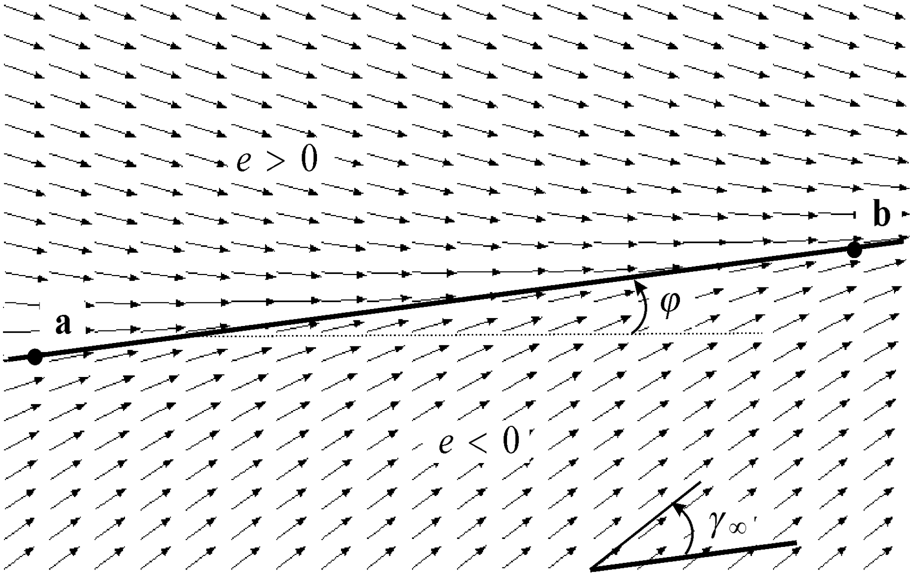
Sail

$$\delta_s^{\max} = \frac{\pi}{2} \cdot \left(\frac{\cos\left(\psi - \bar{\theta}\right) + 1}{2} \right)$$



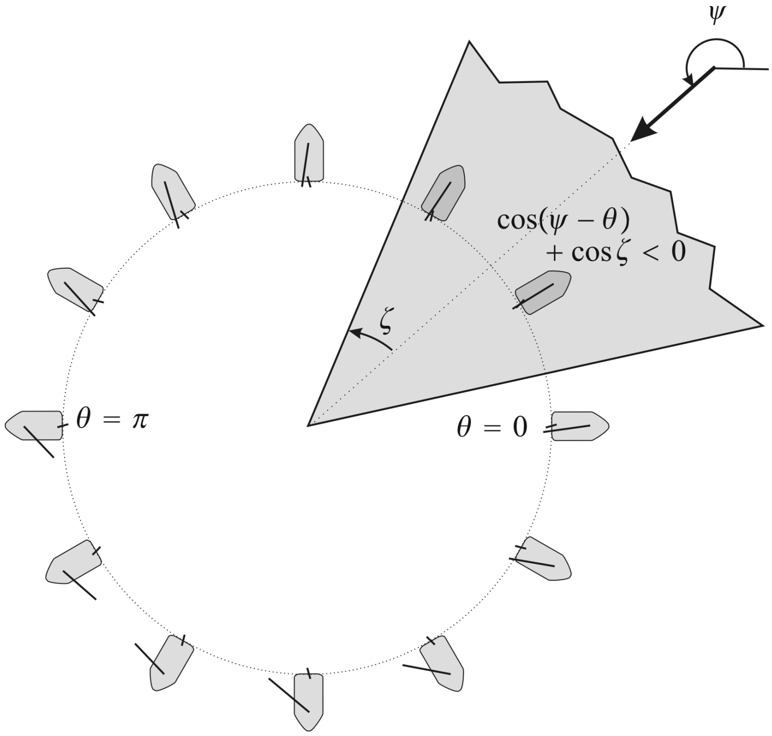


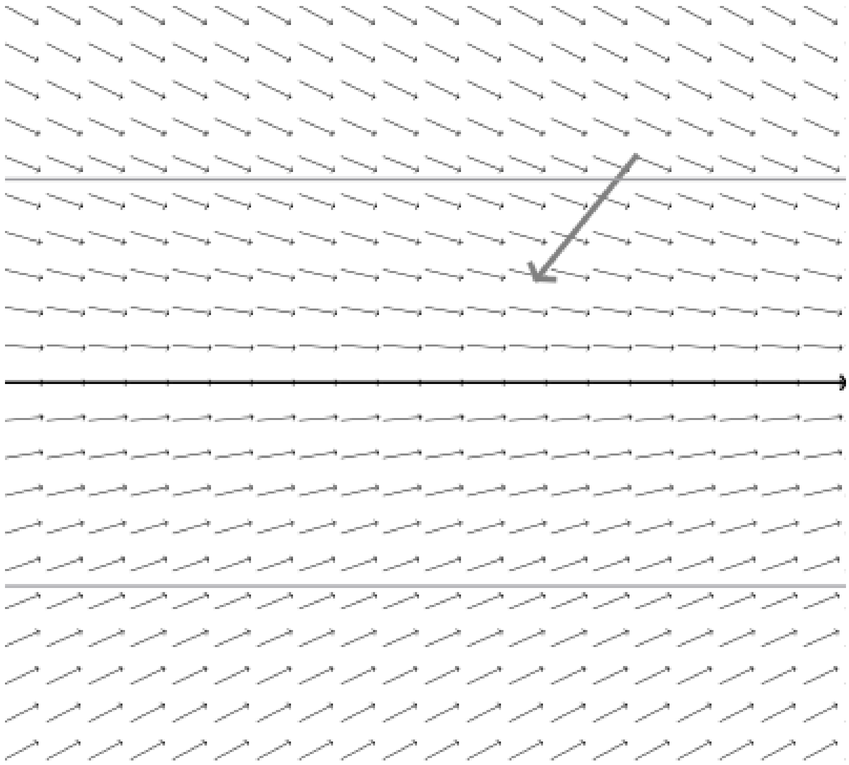
4.1 Vector field



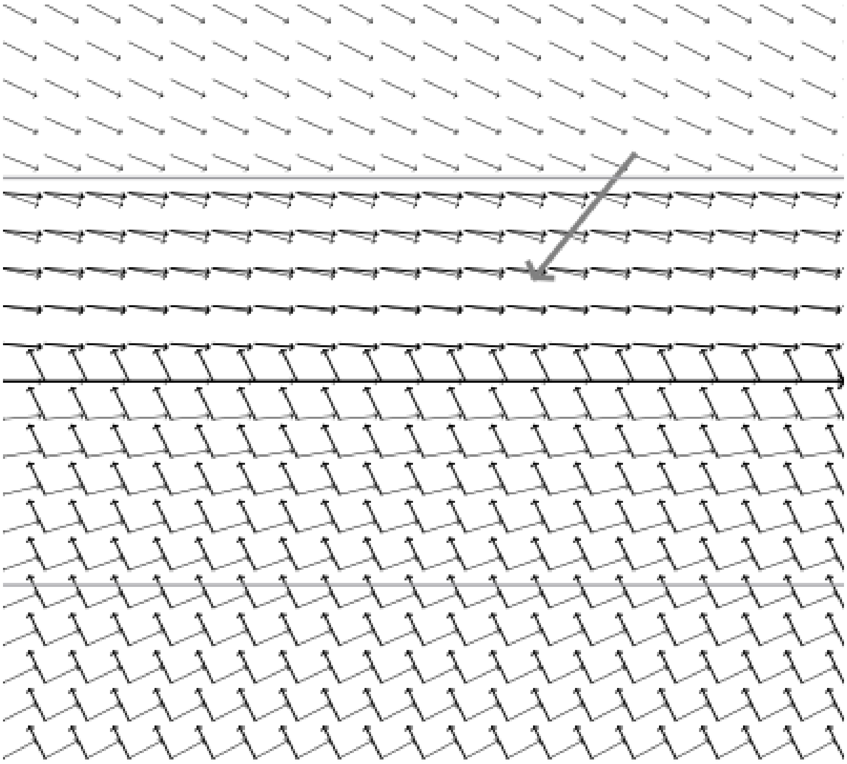
Nominal vector field: $\theta^* = \varphi - \frac{2.\gamma_\infty}{\pi}.\mathrm{atan}\left(\frac{e}{r}\right)$.

A course θ^* may be unfeasible





$$\theta^* = -\frac{2.\gamma_\infty}{\pi}.\mathsf{atan}\left(\frac{e}{r}\right)$$



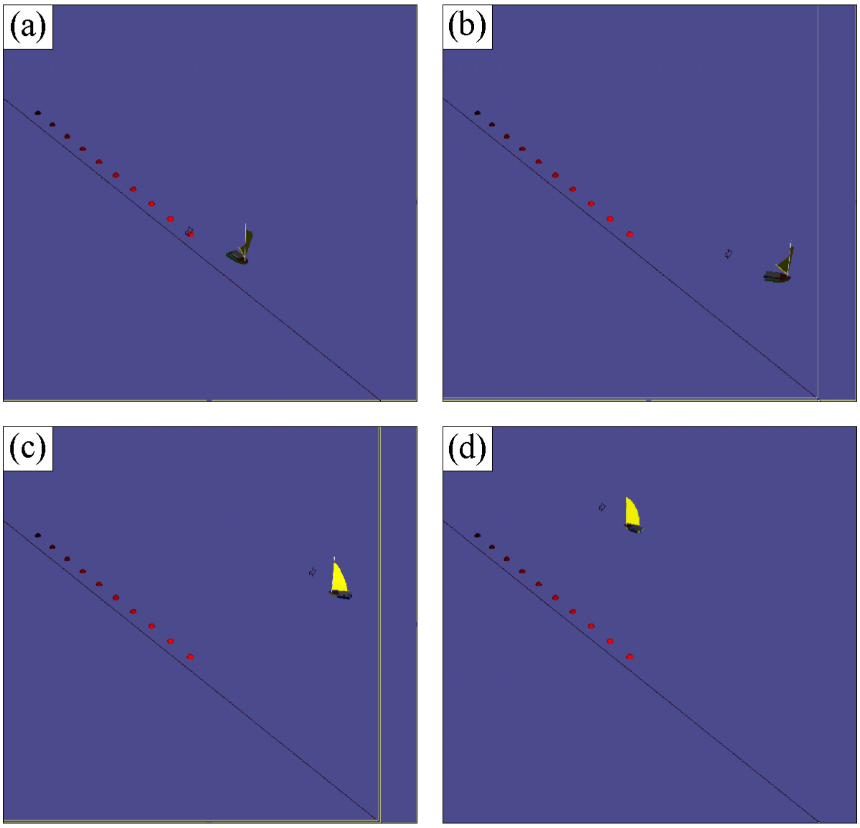
Keep close hauled strategy.

4.2 Controller

Controller in: $\mathbf{m}, \theta, \psi, \mathbf{a}, \mathbf{b}$; out: $\delta_r, \delta_s^{\mathsf{max}}$; inout: q

$e = \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{m}-\mathbf{a}\right)$
if $|e| > \frac{r}{2}$ then $q = \mathrm{sign}(e)$
$\varphi = \mathrm{atan2}(\mathbf{b}-\mathbf{a})$
$\theta^* = \varphi - \frac{2.\gamma_\infty}{\pi}.\mathrm{atan}\left(\frac{e}{r}\right)$
if $\cos(\psi - \theta^*) + \cos\zeta < 0$
    or $(|e| < r$ and $(\cos(\psi - \varphi) + \cos\zeta < 0))$
      then $\bar{\theta} = \pi + \psi - q.\zeta.$
      else $\bar{\theta} = \theta^*$
end
if $\cos\left(\theta - \bar{\theta}\right) \geq 0$ then $\delta_r = \delta_r^{\mathsf{max}}.\sin\left(\theta - \bar{\theta}\right)$
else $\delta_r = \delta_r^{\mathsf{max}}.\mathrm{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right)$
$\delta_s^{\mathsf{max}} = \frac{\pi}{2}.\left(\frac{\cos(\psi - \bar{\theta}) + 1}{2}\right)^q.$

5 Validation by simulation


(a)
(b)
(c)
(d)

6 Theoretical validation

When the wind is known, the sailboat with the heading controller is described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

The system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

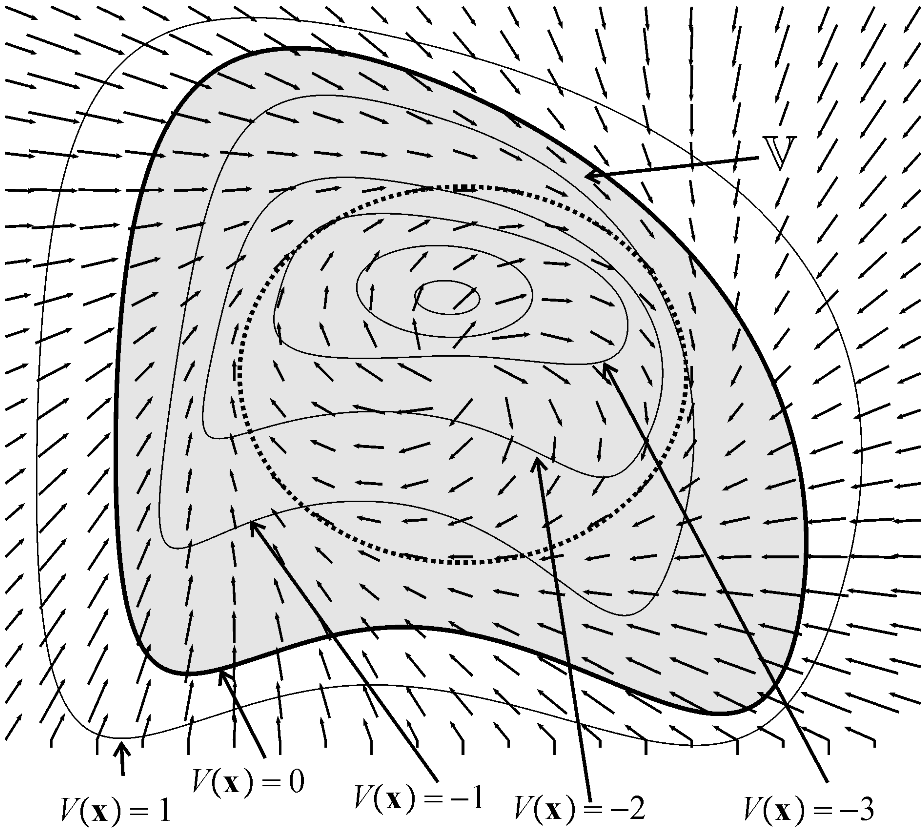
is Lyapunov-stable (1892) is there exists $V(\mathbf{x}) \geq 0$ such that

$$\begin{aligned} \dot{V}(\mathbf{x}) &< 0 \text{ if } \mathbf{x} \neq \mathbf{0}, \\ V(\mathbf{x}) &= 0 \text{ iff } \mathbf{x} = \mathbf{0}. \end{aligned}$$

Definition. Consider a differentiable function $V(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$. The system is V-stable* if

$$\left(V(\mathbf{x}) \geq 0 \;\Rightarrow\; \dot{V}(\mathbf{x}) < 0\right).$$

*Jaulin, Le Bars (2012). An interval approach for stability analysis; Application to sailboat robotics. IEEE TRO.



$\mathbb{V}$
$V(\mathbf{x}) = 1$
$V(\mathbf{x}) = 0$
$V(\mathbf{x}) = -1$
$V(\mathbf{x}) = -2$
$V(\mathbf{x}) = -3$

Theorem. If the system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ is V-stable then

(i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
(ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0,\ V(\mathbf{x}(t+\tau)) < 0$.

Now,

$$
\begin{aligned}
& \left(V(\mathbf{x}) \geq 0 \;\Rightarrow\; \dot{V}(\mathbf{x}) < 0\right) \\
\Leftrightarrow\; & \left(V(\mathbf{x}) \geq 0 \Rightarrow \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) < 0\right) \\
\Leftrightarrow\; & \forall \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) < 0 \text{ or } V(\mathbf{x}) < 0 \\
\Leftrightarrow\; & \neg\left(\exists \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) \geq 0 \text{ and } V(\mathbf{x}) \geq 0\right) \\
\Leftrightarrow\; & -\min\left(\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) < 0, V(\mathbf{x})\right) \geq 0
\end{aligned}
$$

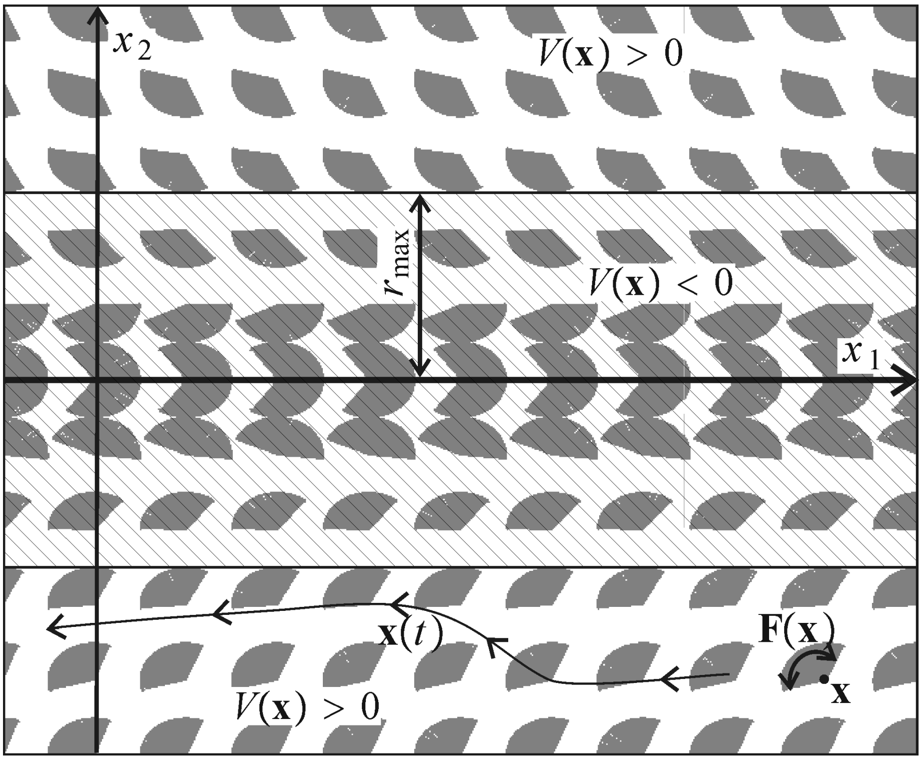
Theorem. We have

$$\left\{ \begin{array}{c} \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) \cdot \mathbf{f}(\mathbf{x}) \geq 0 \\ V(\mathbf{x}) \geq 0 \end{array} \right. \text{ inconsistent } \Leftrightarrow \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \text{ is } V\text{-stable.}$$

Interval method could easily prove the V-stability.

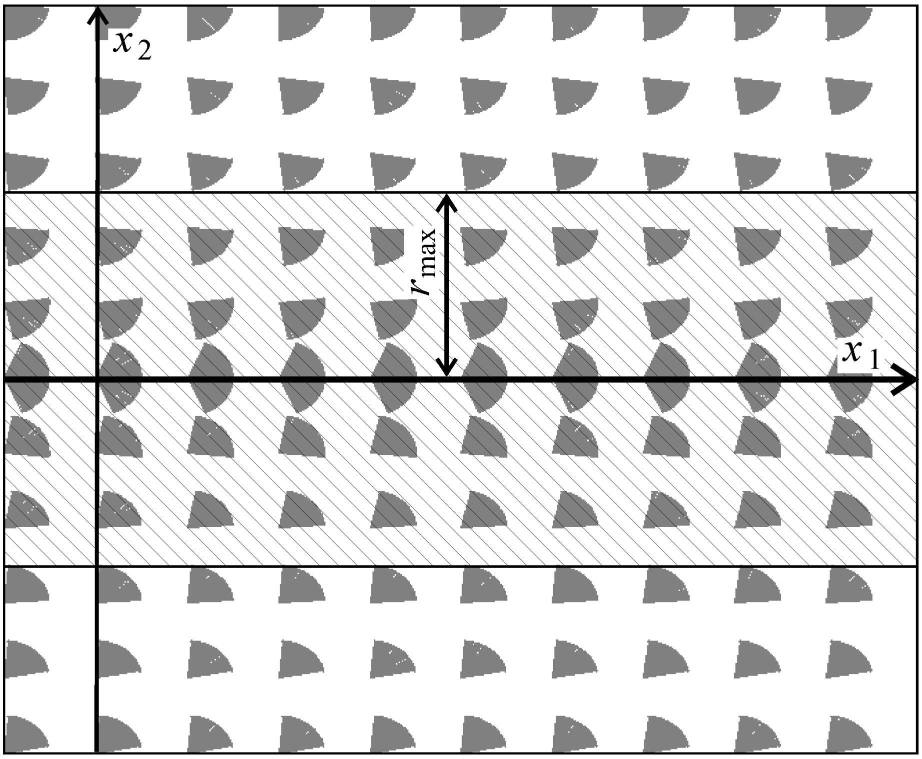
Theorem. We have

$$\left\{ \begin{array}{l} \frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{x}\right).\mathbf{a} \geq 0 \\ \quad \mathbf{a} \in \mathbf{F}\left(\mathbf{x}\right) \\ \quad V(\mathbf{x}) \geq 0 \end{array} \right. \quad \text{inconsistent} \quad \Leftrightarrow \quad \dot{\mathbf{x}} \in \mathbf{F}\left(\mathbf{x}\right) \text{ is } V\text{-stable}$$



Differential inclusion $\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$ for the sailboat.

$$V(x) = x_2^2 - r_{\max}^2.$$



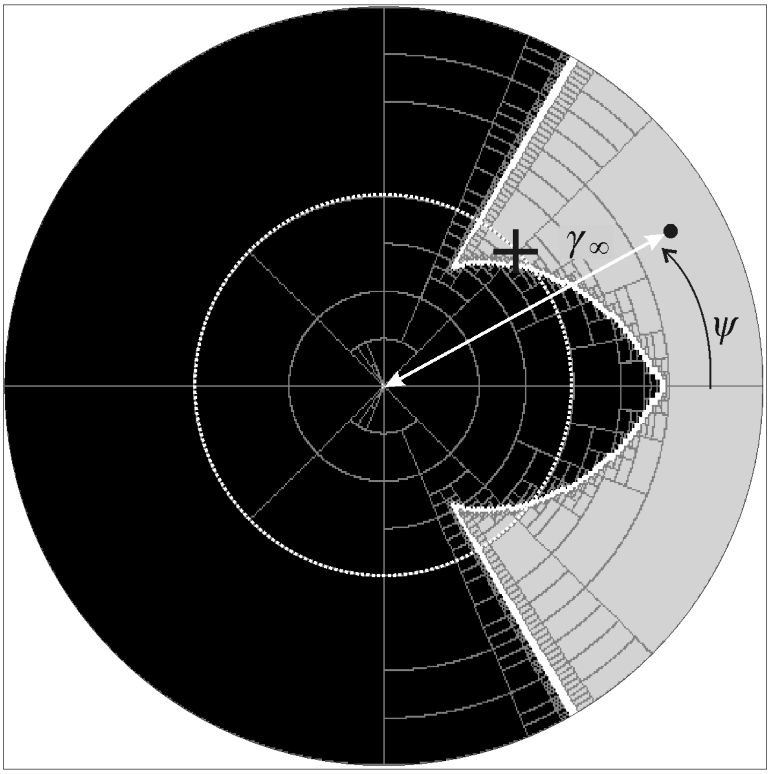
x_2
r_{max}
x_1

7 Parametric case

Consider the differential inclusion

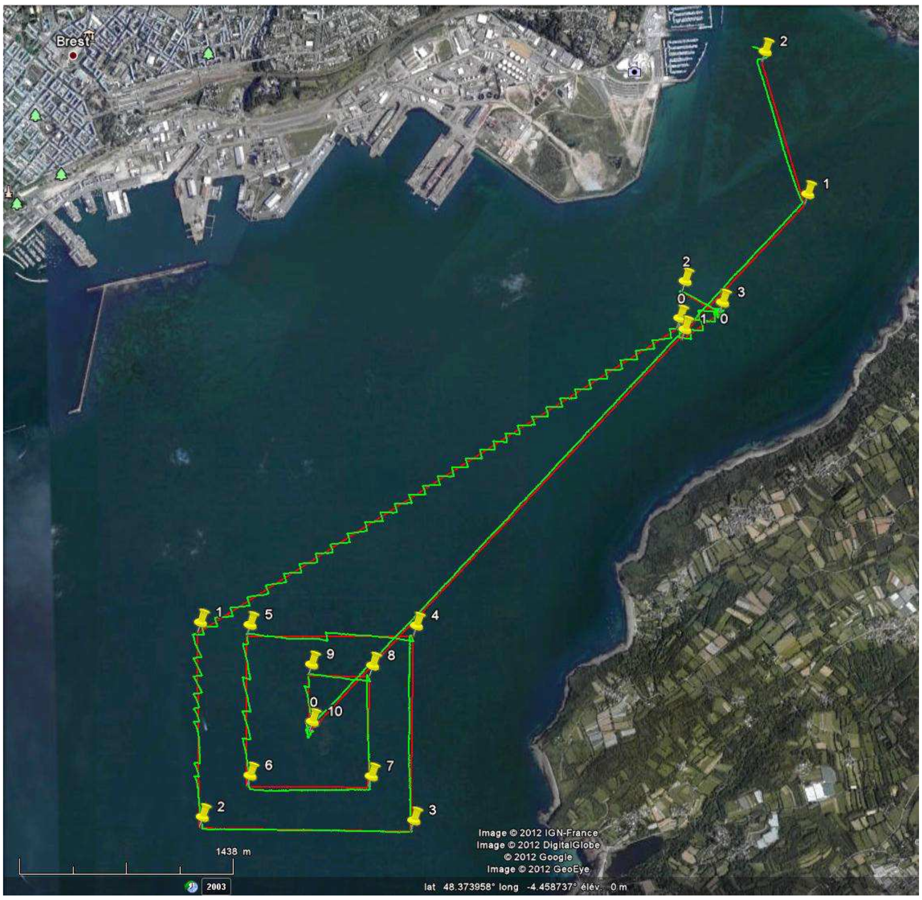
$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x}, \mathbf{p}).$$

We characterize the set $\mathbb{P}$ of all $\mathbf{p}$ such that the system is V-stable.


+
γ_∞
ψ

8 Experimental validation

Brest


Brest
2
1
2
0
3
1
0
1
5
4
9
8
0
10
6
7
2
3
Image © 2012 IGN-France
Image © 2012 DigitalGlobe
© 2012 Google
Image © 2012 GeoEye
1438 m
2003
lat 48.373958° long -4.458737° élév. 0 m

Show Dashboard

Brest-Douarnenez. January 17, 2012, 8am

Brest
PRO
BR D98 505
YAMAHA


F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Le Conquet
Plougastel-Daoulas
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
lat 48.230580° long -4.564526° élév 6 m
Altitude 53.47 km
©2010 Google
Start
vaimos.txt -...
Logs
sftp://root...
Command Pr...
Command Pr...
data
Interface po...
VCtrl (Runni...
VAIMOS aut...
Google Earth
FR
98%
01:42

Montrer la mise à l'eau

Middle of Atlantic ocean

350 km made by Vaimos in 53h, September 6-9, 2012.

Consequence.

It is possible for a sailboat robot to navigate inside a corridor.

Essential, to create circulation rules when robot swarms are considered.

Essential to determine who has to pay in case of accident.

Montrer vidéo de l'expérience en rade