A simple model for the location of Saturn's F ring

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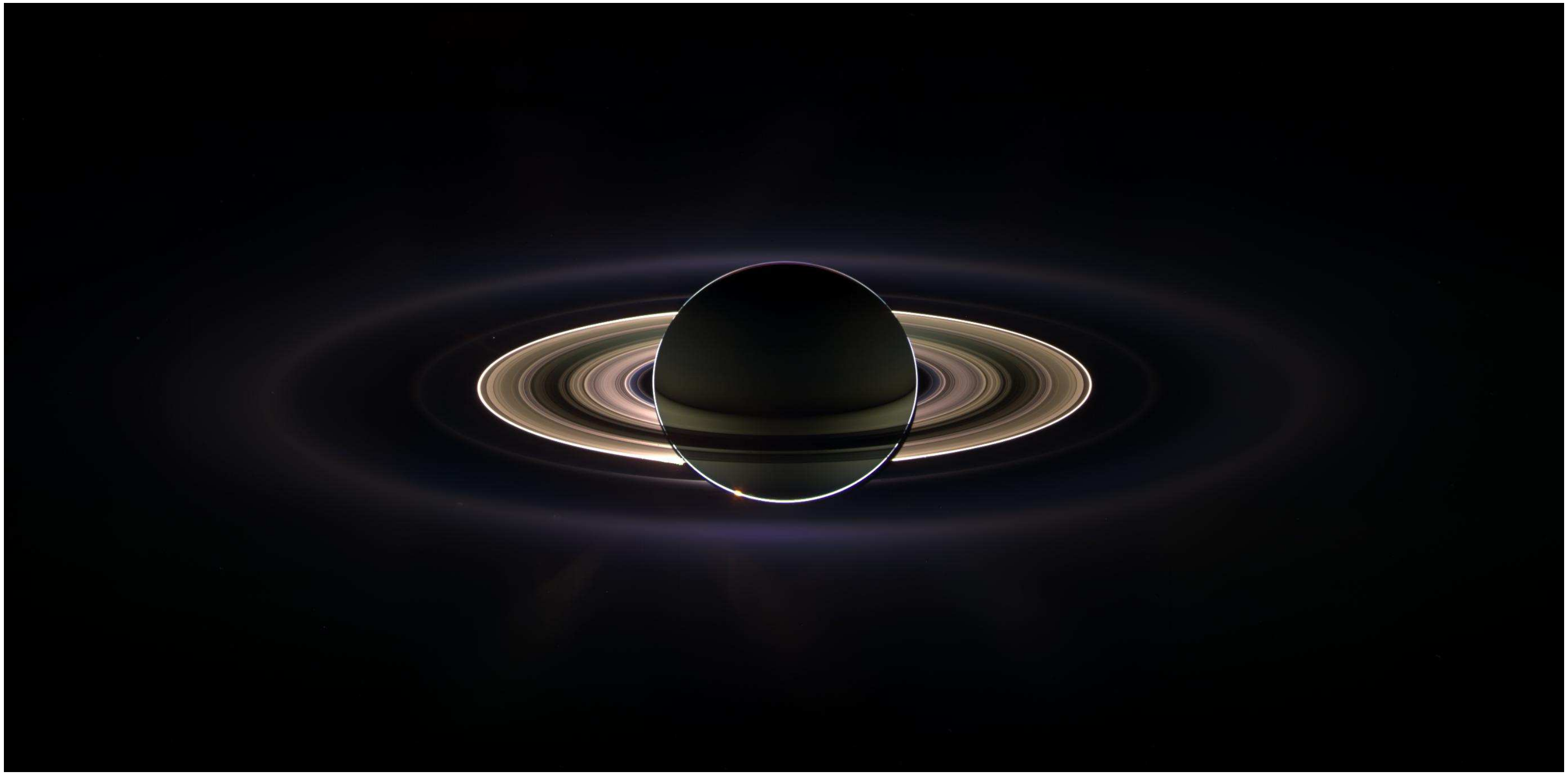
Observations

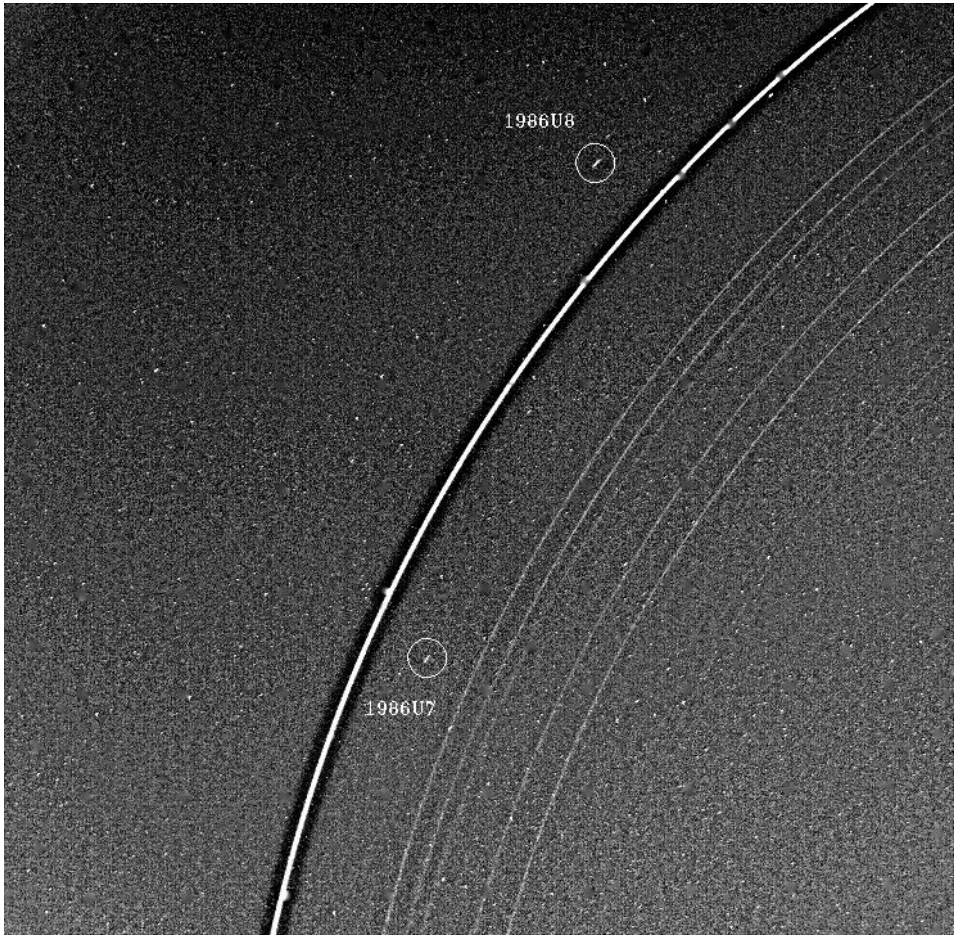


Saturn rings
PIA08329 (NASA/JPL/Space Science Institute)

Observations

Uranus rings
PIA01977 (NASA/JPL/Space Science Institute)

Observations

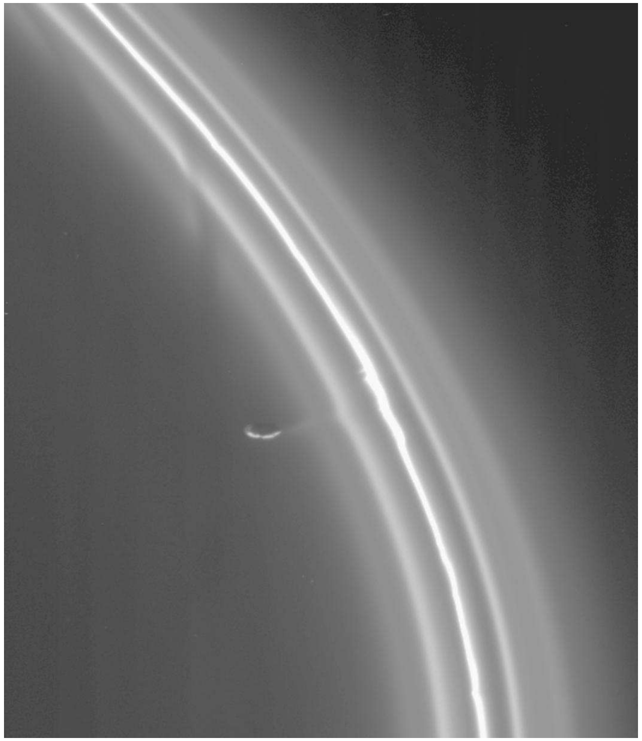


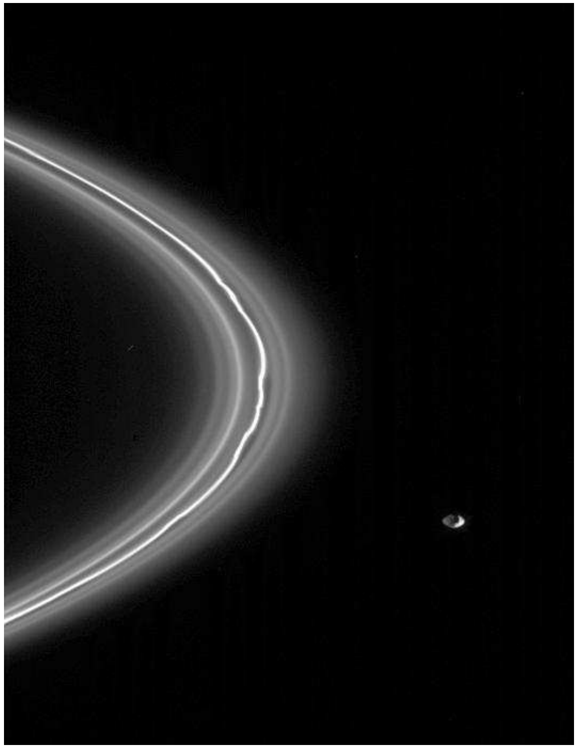
Uranus rings
PIA01976 (NASA/JPL/Space Science Institute)

Observations

Saturn's F rings
PIA02283 (NASA/JPL/Space Science Institute)

Observations

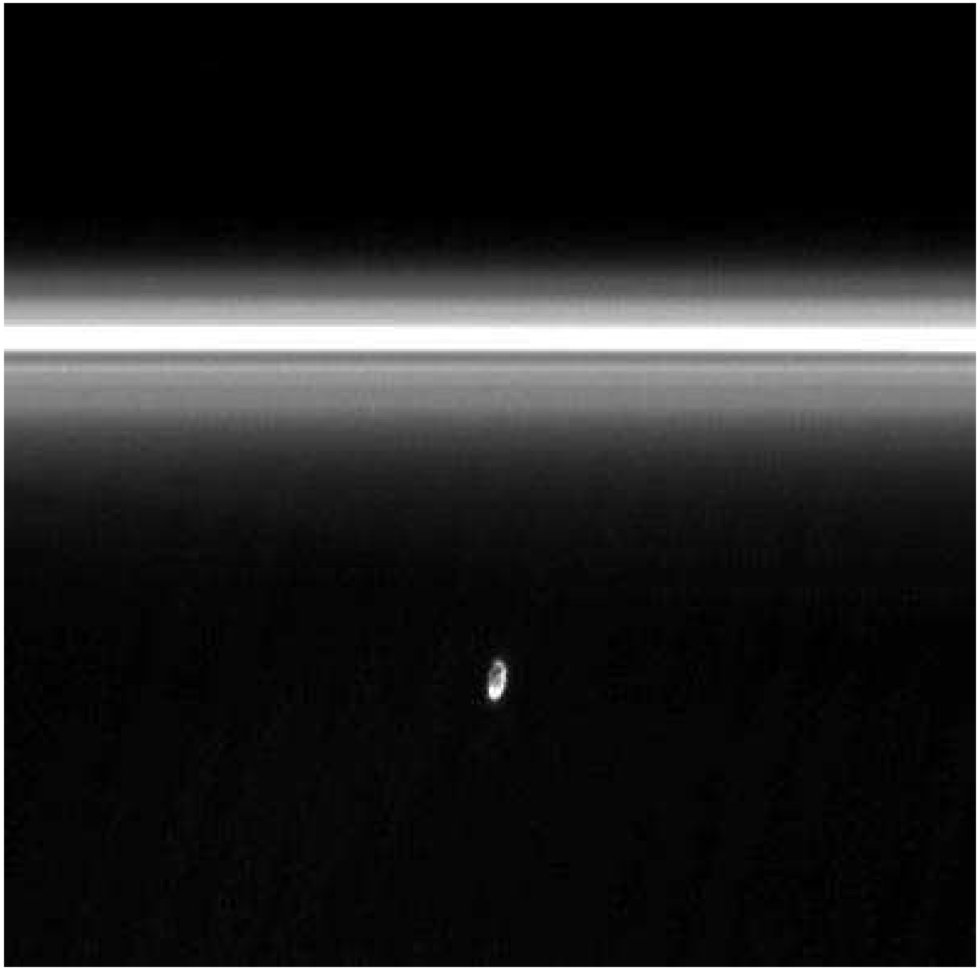




Saturn’s F ring, Prometheus and Pandora

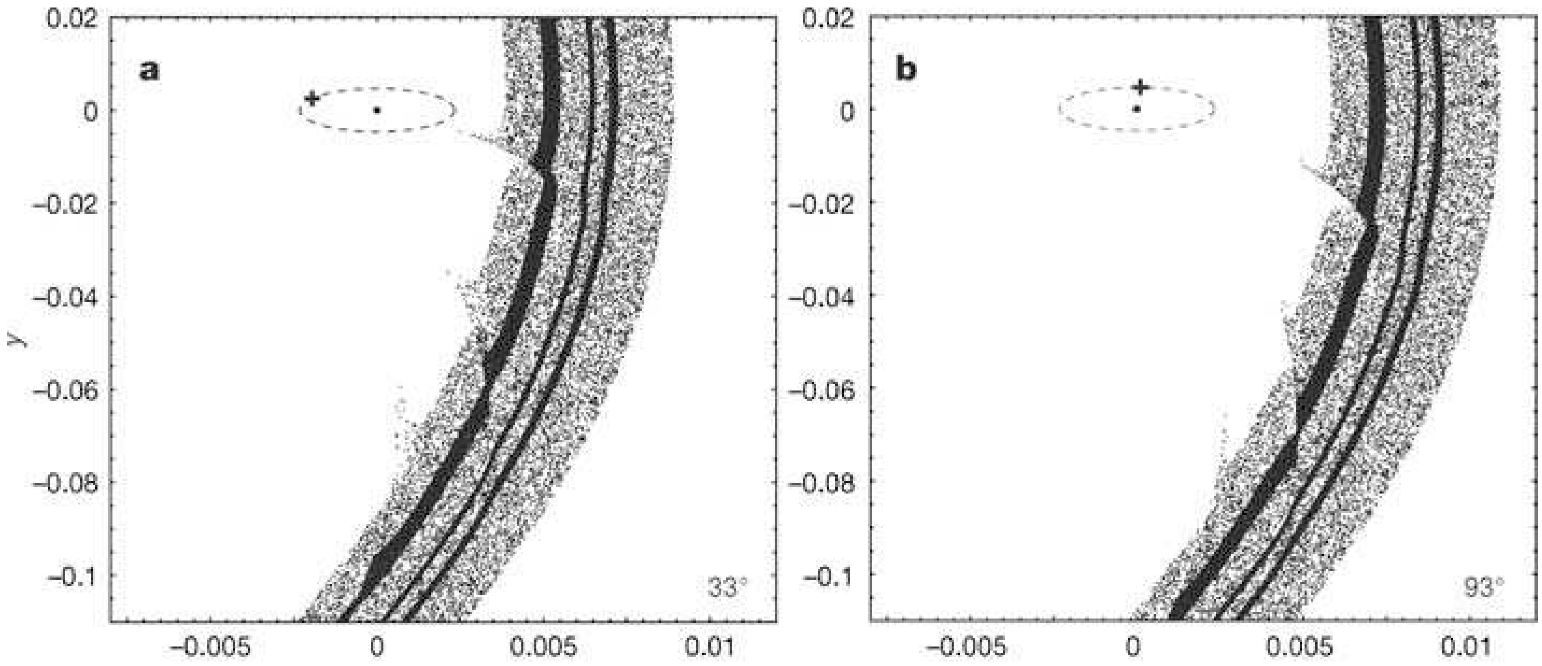
PIA06143, PIA07523 (NASA/JPL/Space Science Institute)

Observations



Structure in Saturn’s F ring
PIA08397 (NASA/JPL/Space Science Institute)

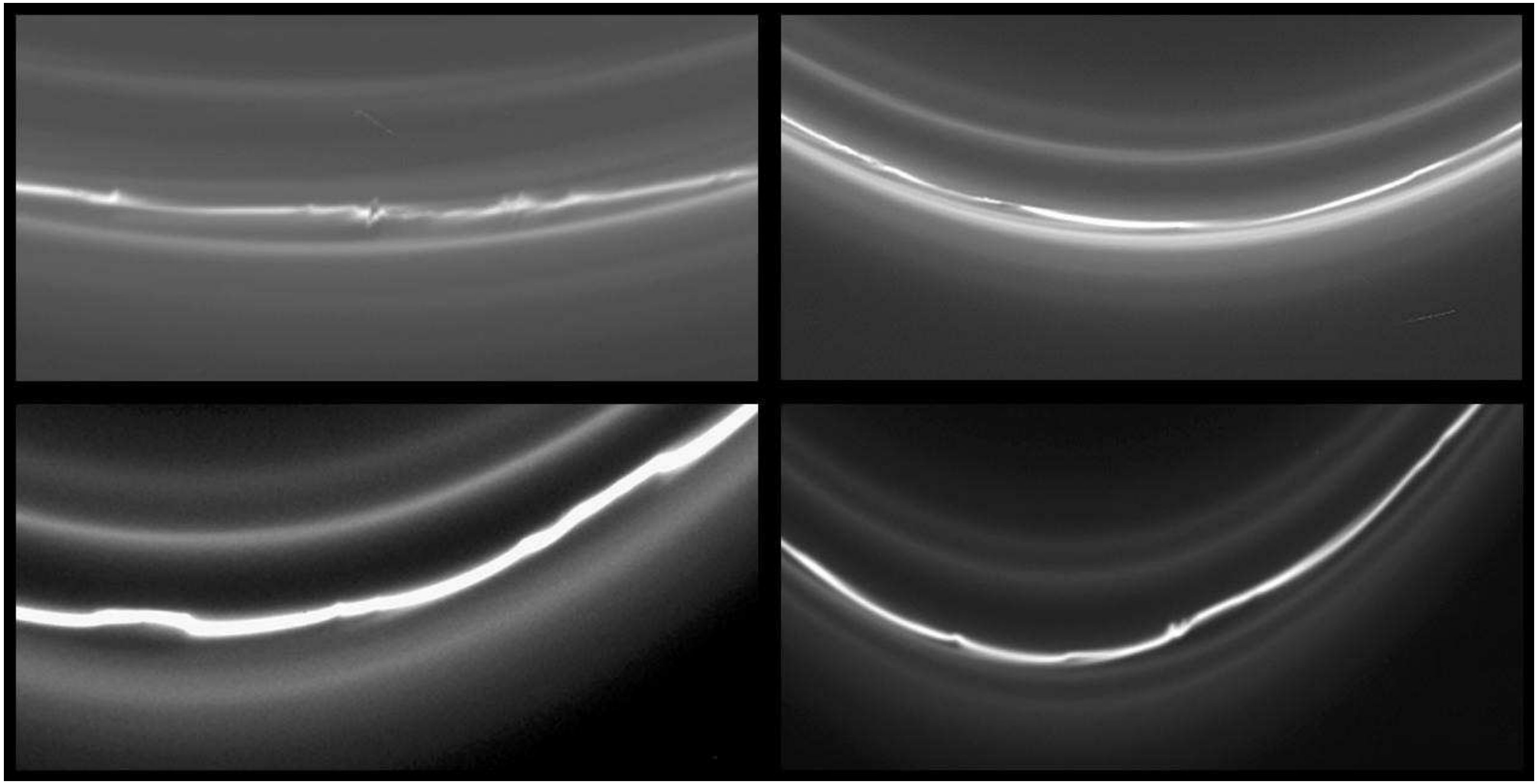
Observations



Close approaches by Prometheus

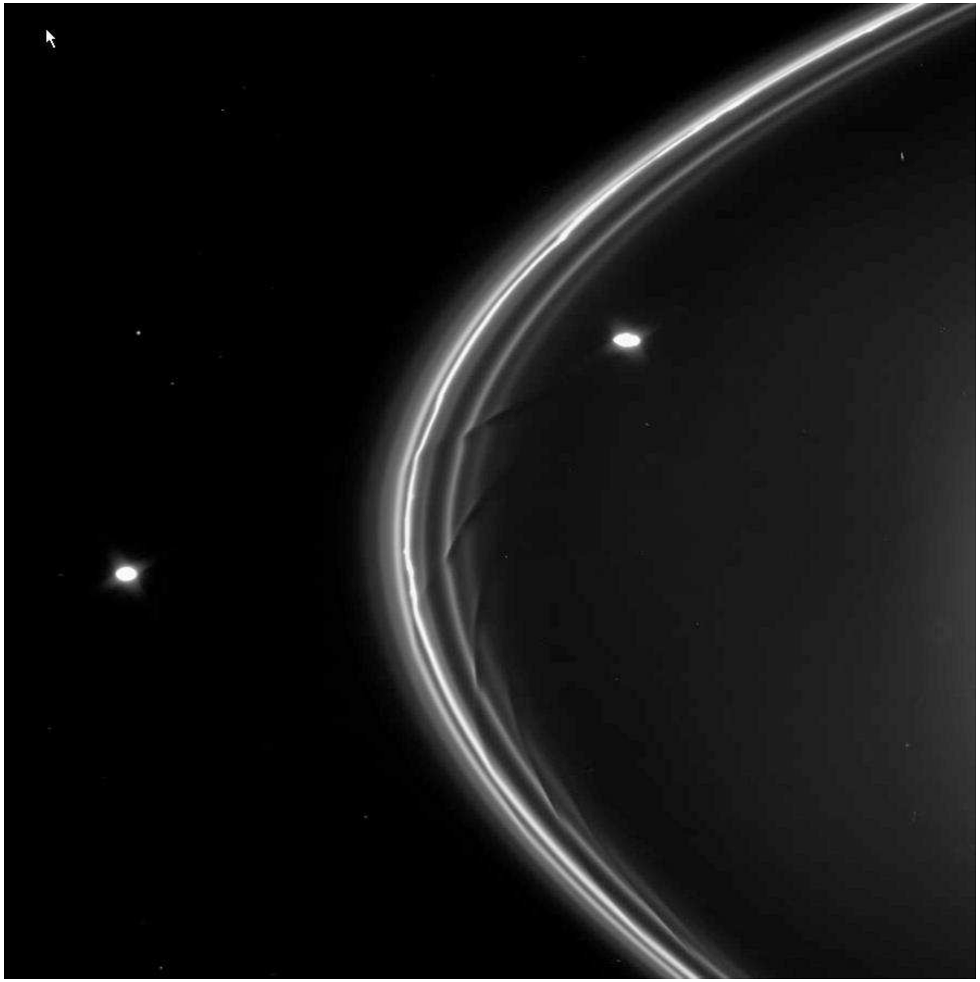
S.M. Giuliatti Winter, C.D. Murray, M. Gordon, Planetary and Space Science **48**, 817 (2000); Murray C.D., et al, Nature 437, 1326 (2005).

Observations



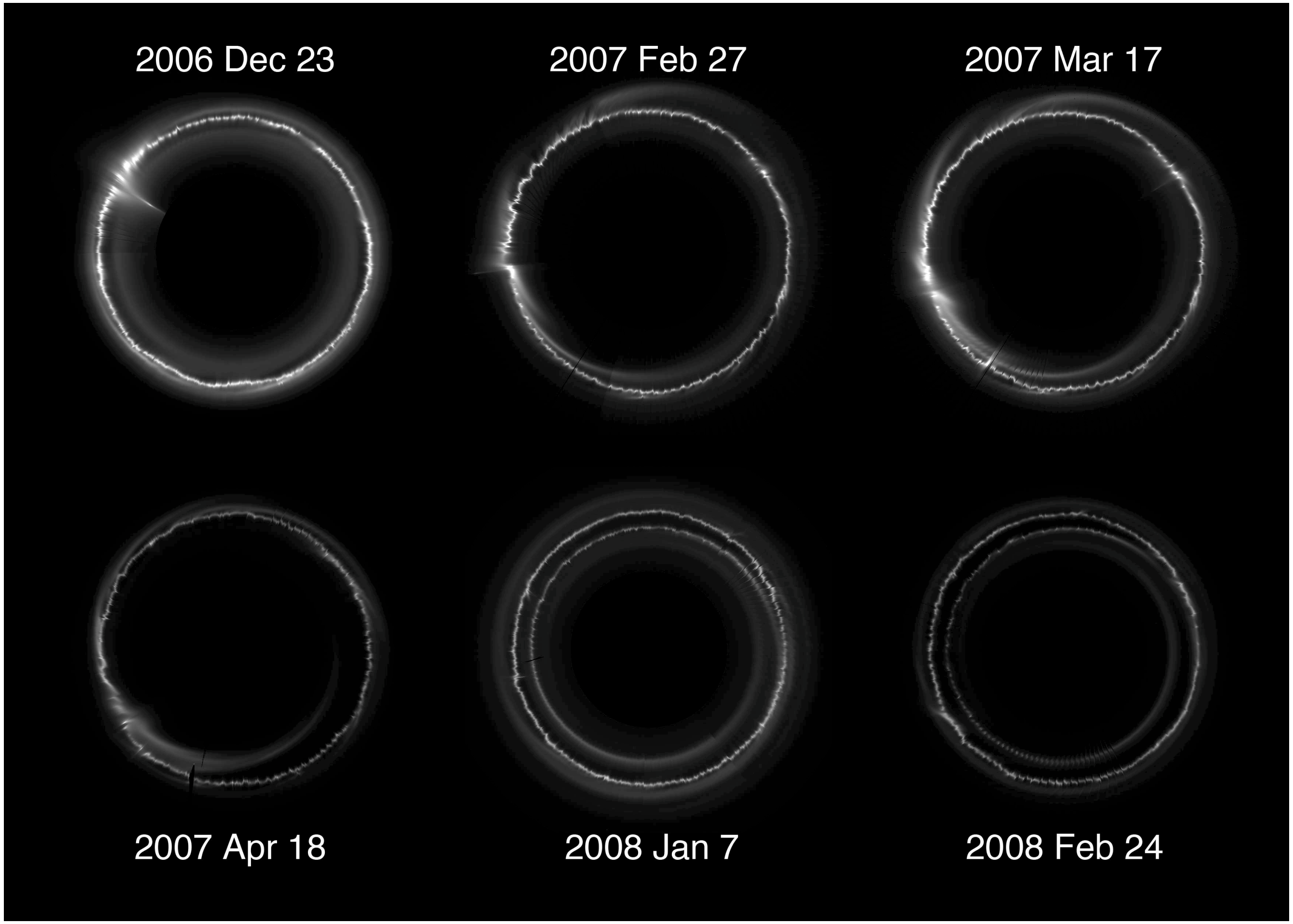
Structure in Saturn’s F ring
PIA07522 (NASA/JPL/Space Science Institute)

Observations



Dynamics in narrow rings: streamers and chanels
PIA07712 (NASA/JPL/Space Science Institute)

Observations



Structure in Saturn's F ring
PIA15505 (NASA/JPL-Caltech/SSI/QMUL)

Open questions

Unfortunately, the models are often idealized (for example, treating all particles as hard spheres of the same size) and cannot yet predict many phenomena in the detail observed by spacecraft (for example, sharp edges). Non-intuitive collective effects give rise to unusual structures.
(...) One such example is the case of shepherding satellites. The F ring is not exactly placed where the shepherding torques would balance. (...) Another issue is that the sharp edges of rings are too sharp!

Larry Esposito, Planetary rings (Cambridge University Press, 2006)

Open questions

Despite being entirely composed of “clumps” and “kinks” so numerous that they cannot be individually tracked (...), the F ring core nevertheless maintains over decadal timescales the shape of a freely precessing eccentric inclined ellipse (...). However, decade-scale time variations in the core’s internal structure, as well as in the surrounding dust, have clearly taken place between the Voyager and Cassini visits (...).

Mathew S. Tiscareno, Planets, Stars and Stellar Systems, Vol 3, 2013

Open questions

Some open issues are:

- Location
- Sharp–edges, narrow and eccentricity
- Multiple components (strands)
- Clumps and arcs
- Kinks and bendings
- Dynamical stability, life times, origin, ...

Scattering approach to narrow planetary rings

A model that includes Saturn (with flattening) + moons + ring-particles system is too complicated

Approx. 1: No interaction among ring particles

Approx. 2: Ring does not influence the motion of Saturn or its moons

$$\mathcal{H}_{\rm N} = \sum_{i=0}^{N-1} \frac{1}{2m_i}(P_{x_i}^2 + P_{y_i}^2) - \sum_{i=1}^{N-1} \frac{G m_0 m_i}{R_{i0}} \left[1 + \frac{J_2}{2}\left(\frac{R_S}{R_{i0}}\right)^2\right] - \sum_{1\le i<j}^{N-1} \frac{G m_i m_j}{R_{ij}} ,$$

$$\mathcal{H}_{\rm rp} = \frac{1}{2}(p_x^2 + p_y^2) - \frac{G m_0}{r_0(t)} \left[1 + \frac{J_2}{2}\left(\frac{R_S}{r_0(t)}\right)^2\right] - \sum_{i=1}^{N-1} \frac{G m_i}{r_i(t)} .$$

$\Rightarrow$ Time-dependent restricted $(N_m + 1) + 1$–body problem

Scattering approach to narrow planetary rings

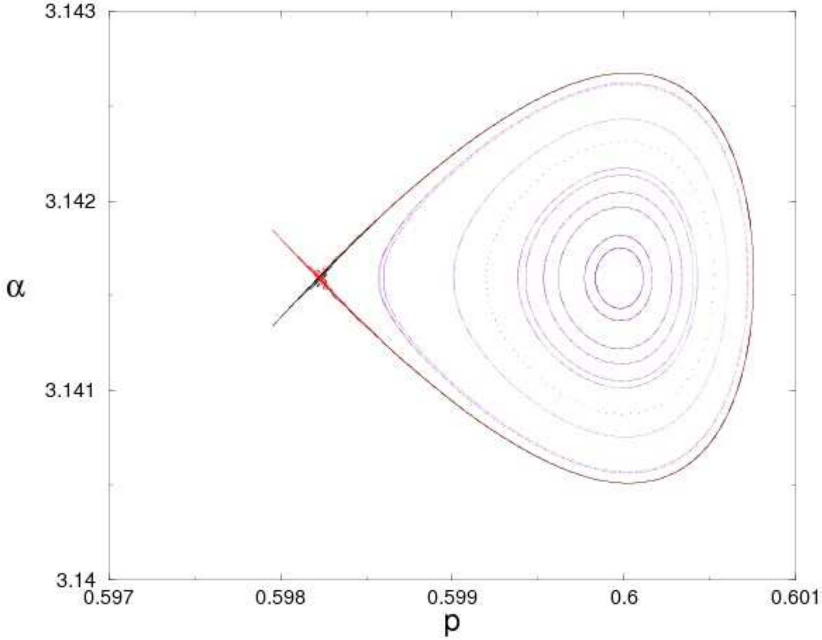
Basic idea:

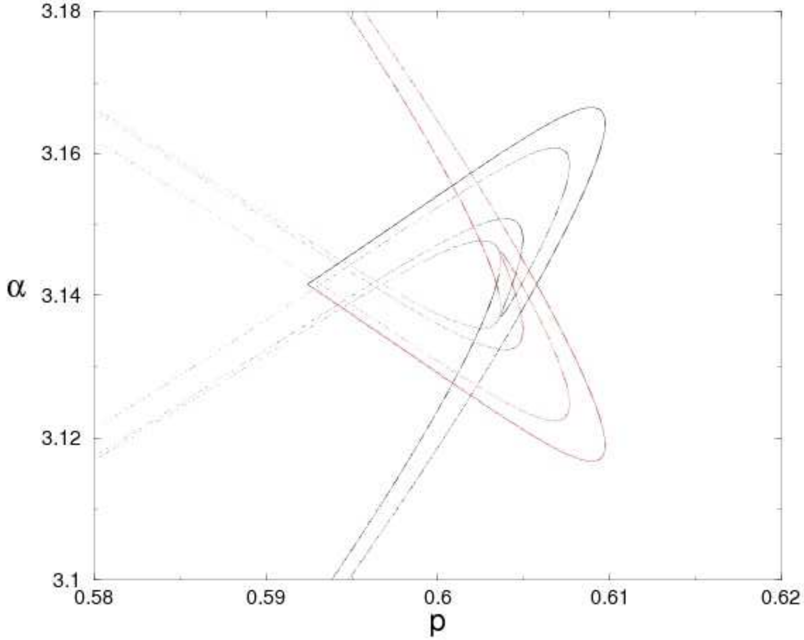
1. Ensemble of non-interacting particles, with almost arbitrary initial conditions, whose dynamics is given by $\mathcal{H}_{\mathrm{rp}}$.
2. Phase–space regions where certain stability assures long-time trapping.
3. Instabilities lead to escape along scattering trajecories.

Rings are obtained by projecting onto the configuration space, at fixed time, the phase-space position of *all particles that are dynamically trapped.*

Scattering approach to narrow planetary rings

Phase space in a co-rotating frame





A model for Saturn's F ring

Our model reads:

$$\mathcal{H}_{\rm N} = \sum_{i=0}^{N-1} \frac{1}{2m_i}(P_{x_i}^2 + P_{y_i}^2) - \sum_{i=1}^{N-1} \frac{Gm_0 m_i}{R_{i0}} \left[1 + \frac{J_2}{2}\left(\frac{R_S}{R_{i0}}\right)^2\right] - \sum_{1\le i<j}^{N-1} \frac{Gm_i m_j}{R_{ij}} ,$$

$$\mathcal{H}_{\rm rp} = \frac{1}{2}(p_x^2 + p_y^2) - \frac{Gm_0}{r_0(t)} \left[1 + \frac{J_2}{2}\left(\frac{R_S}{r_0(t)}\right)^2\right] - \sum_{i=1}^{N-1} \frac{Gm_i}{r_i(t)} .$$

Question: Which Saturn moons shall we consider in order to capture the essence of the location and structural properties of the ring?

A model for Saturn's F ring

Object	a (R_s)	e	m (M_S)
Prometheus	2.312	0.0024	2.4 $\times 10^{-10}$
F Ring	2.324	0.0027	?
Pandora	2.351	0.0042	2.3 $\times 10^{-10}$
Mimas	3.076	0.0196	6.59 $\times 10^{-8}$
Encedalus	3.948	0.0047	1.90 $\times 10^{-7}$
Tethis	4.888	0.0001	1.086$\times 10^{-6}$
Dione	6.262	0.0022	1.927$\times 10^{-6}$
Rhea	8.746	0.0013	4.058$\times 10^{-6}$
Titan	20.27	0.029	2.367$\times 10^{-4}$
Hyperion	24.57	0.123	9.82 $\times 10^{-9}$
Iapetus	29.96	0.028	3.177$\times 10^{-6}$

$R_S = 60268\,\mathrm{km},\ M_S = 5.683 \times 10^{26}\,\mathrm{kg} \approx 95.1 M_\oplus$

JPL/Horizons data.

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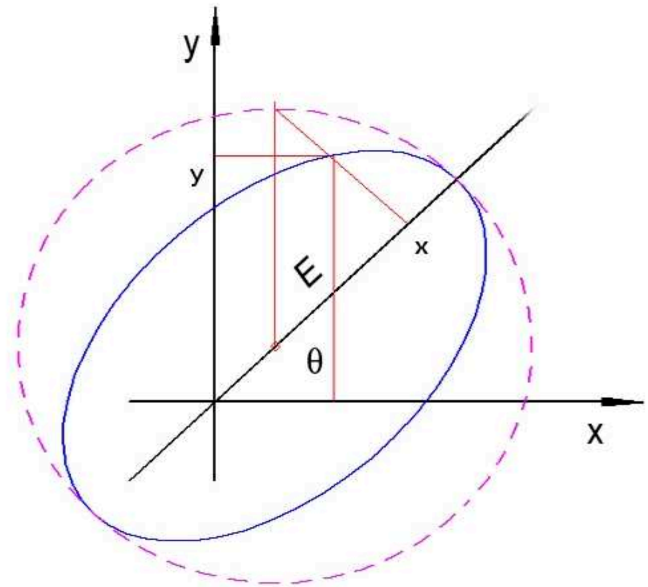
A model for Saturn’s F ring

We are interested in initial conditions (test particles) that remain trapped for very long times, and are somewhat stable.



A model for Saturn's F ring

We are interested in initial conditions (test particles) that remain trapped for very long times, and are somewhat stable.

Ring particles escape through sequences of close approaches with the shepherd moons: the particles may abandon the region confined by the orbits of the shepherd moons or display physical collisions. Either case manifests some instability in the motion of the particles.

But, the dynamics is very slow (sticky regions?)

L. Benet and O. Merlo, Celest. Mech and Dyn. Astron. **103** 209-225 (2009).

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But, the dynamics is very slow (sticky regions?)

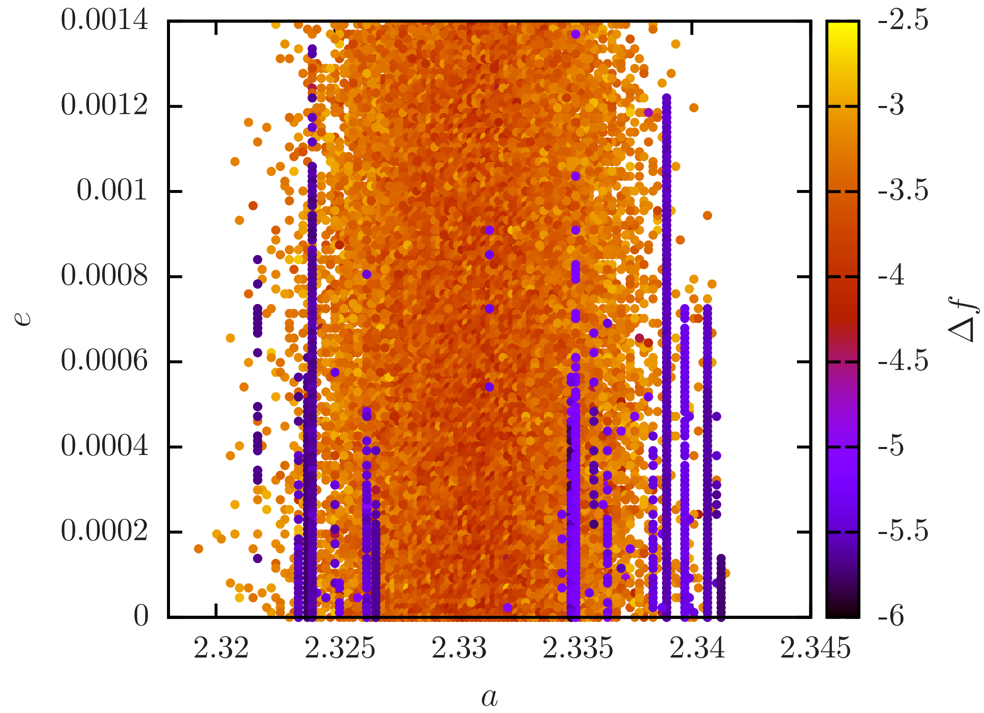
To quantify the stability of the ring particles we perform a frequency type analysis using short segments of the whole orbit. We compute the dynamical indicator

$$\Delta x = \frac{[\mathrm{var}(x)]^{1/2}}{\overline{x}}$$

L. Benet and O. Merlo, Celest. Mech and Dyn. Astron. **103** 209-225 (2009); *ibid*, submitted to Icarus (2016).

Numerical results

Initial-conditions of particles that remain trapped after $1.6 \times 10^6 T_{\mathrm{Prom}}$



Stable localized strips according to the stability indicator

L. Benet and À. Jorba, in Springer Proc. in Maths. and Stats. Vol. 54 (2013), pp. 65-75; submitted to Icarus (2016).

Numerical results

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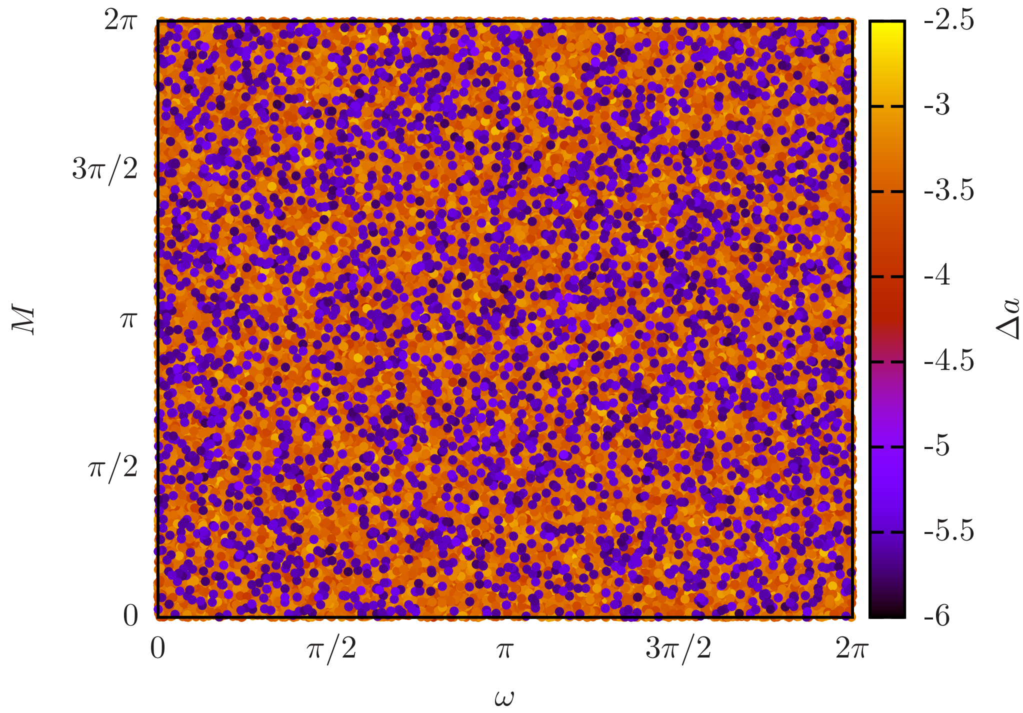
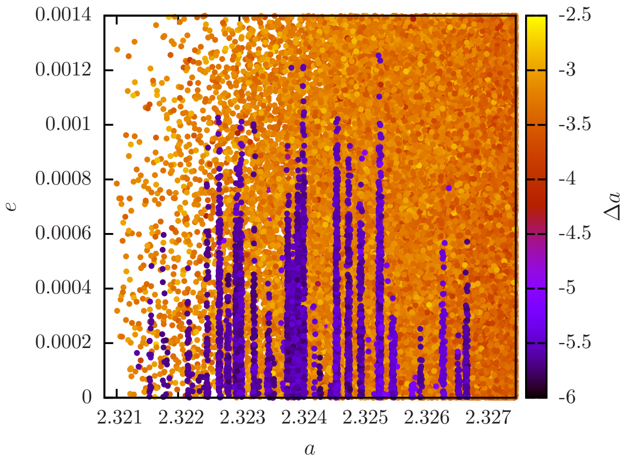


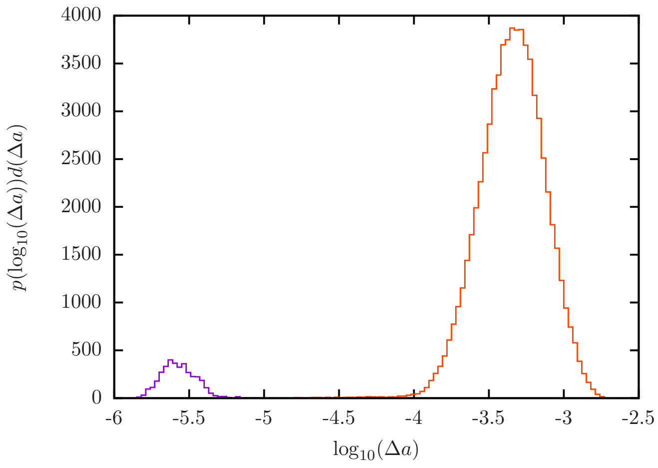
Stable localized strips according to the stability indicator with no correlations among the initial angles

L. Benet and À. Jorba, submitted to Icarus (2016).

Numerical results

A detail of a region after $3.2 \times 10^6 T_{\text{Prom}}$

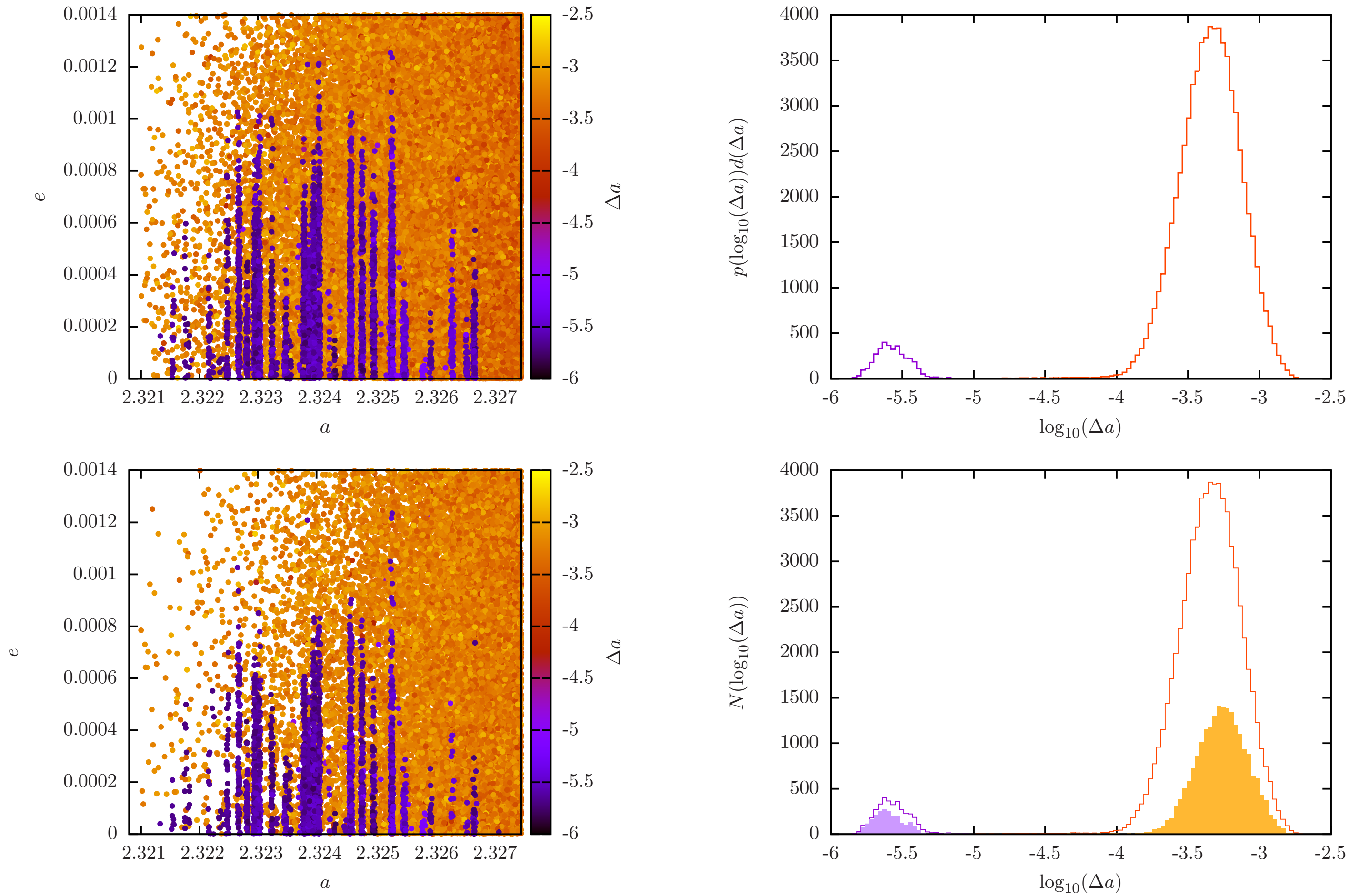




L. Benet and À. Jorba, submitted to Icarus (2016).

Numerical results

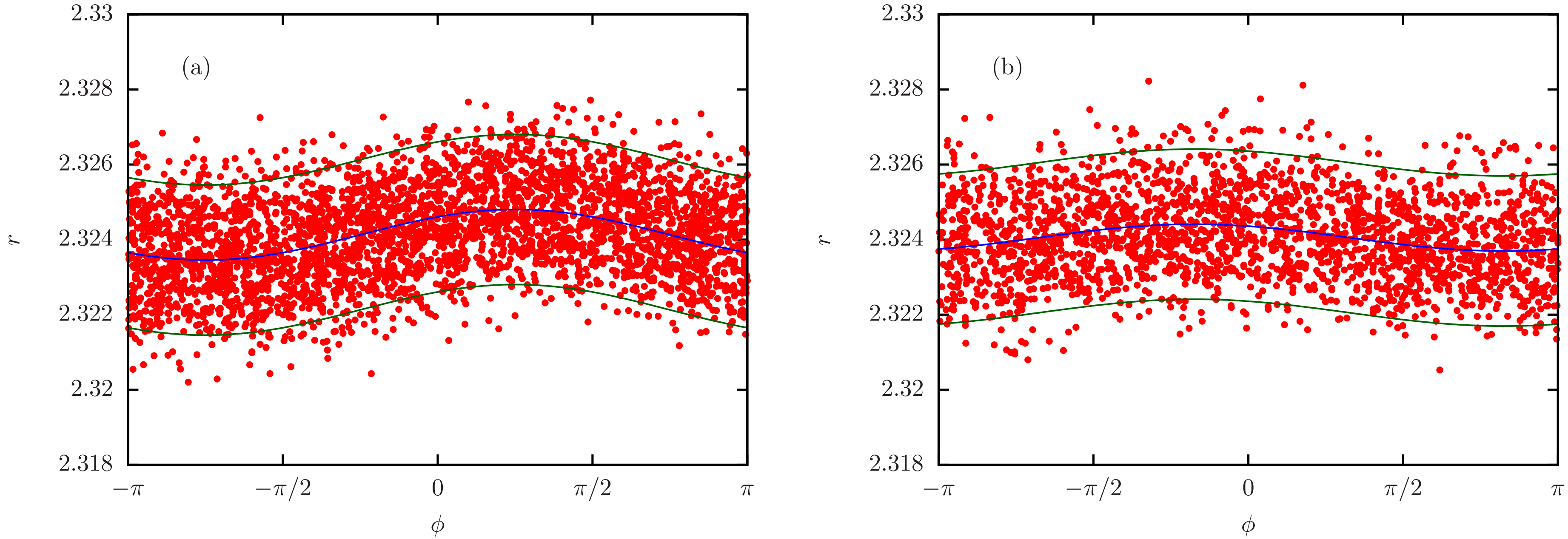
A detail of a region after $3.2 \times 10^6 T_{\mathrm{Prom}}$ and $6.0 \times 10^6 T_{\mathrm{Prom}}$



L. Benet and À. Jorba, submitted to Icarus (2016).

Numerical results

The ring formed by the most stable trapped orbits ($\Delta a < 10^{-5}$)

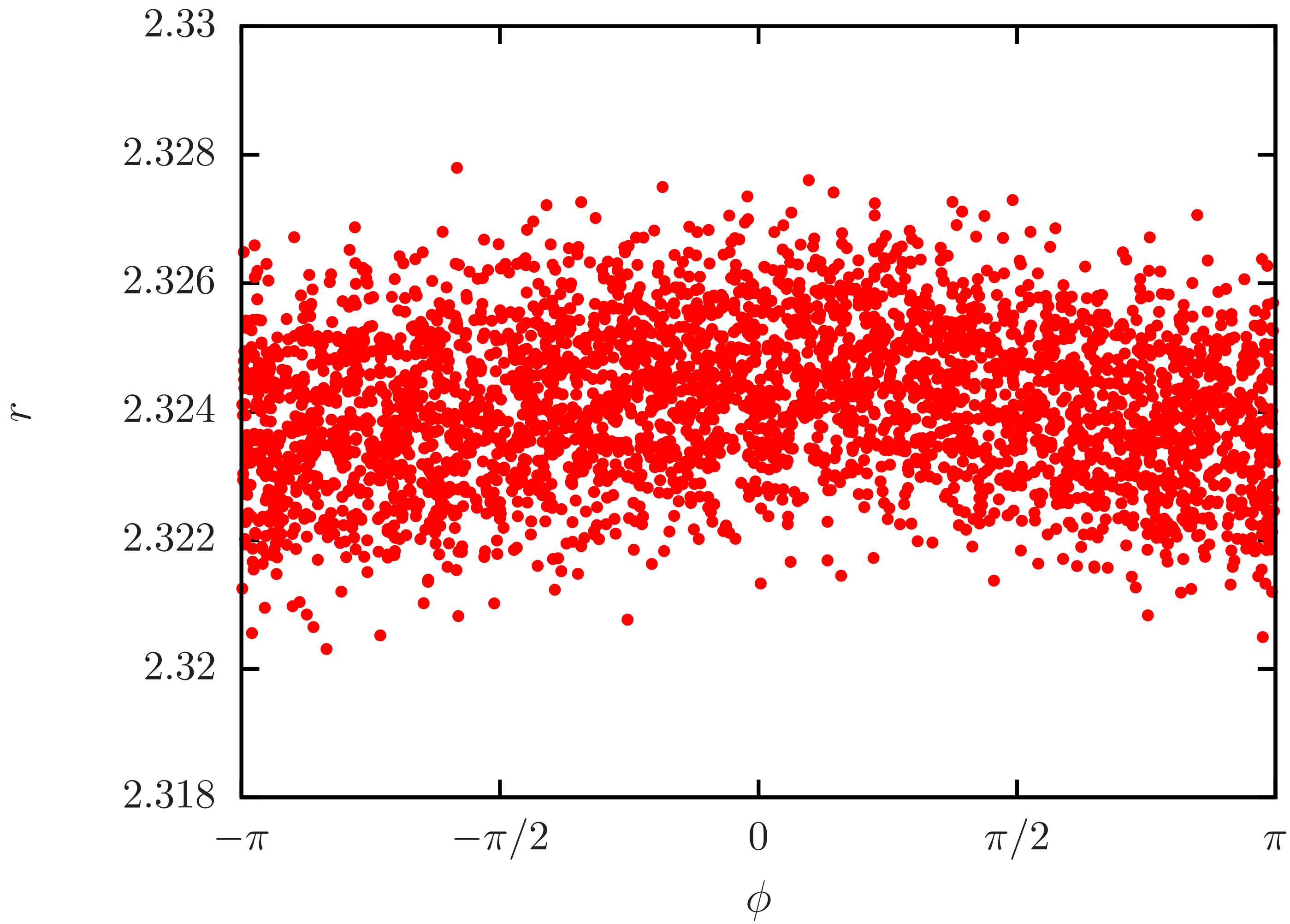


Fitting a Kepler ellipse to all ring particles:

$$a_{\rm fit} = 2.3241 R_S,\ e_{\rm fit}^{(a)} = 2.9 \times 10^{-4}, e_{\rm fit}^{(b)} = 1.5 \times 10^{-4}, \delta r \lesssim 0.004\, R_S \approx 240\,{\rm km}$$

L. Benet and À. Jorba, submitted to Icarus (2016).

Numerical results



L. Benet and À. Jorba, submitted to Icarus (2016).

Confinement

Are there any resonances?

Two conditions must be fulfilled:

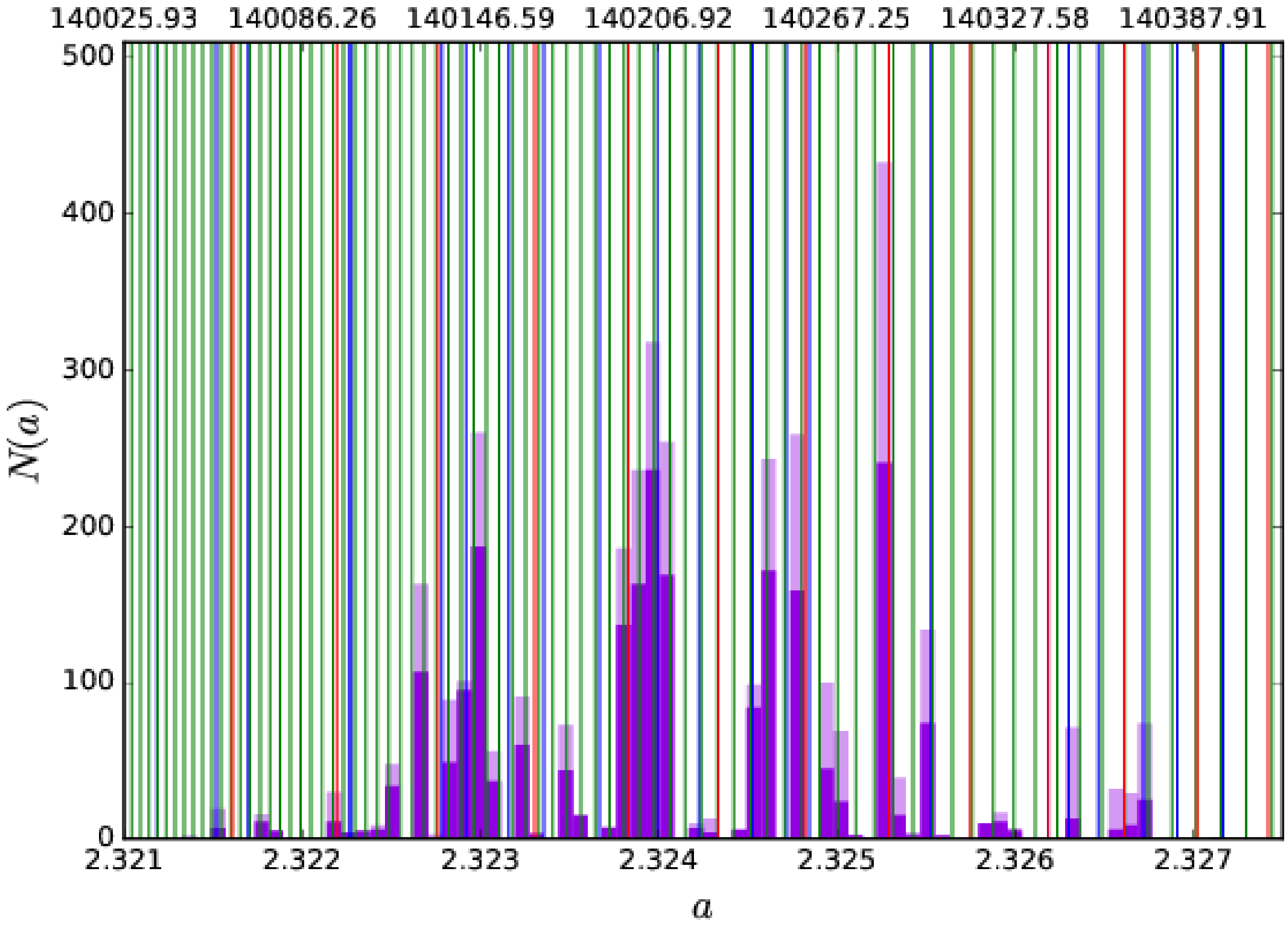
$$\begin{aligned}\sum_{i=1}^{4}\left[k_i f_i + (k_i - l_i)\dot{\varpi}_i\right] &= 0,\\ \sum_{i=1}^{4}(k_i - l_i) &= 0,\end{aligned}$$

where $k_i, l_i \in \mathbb{Z}$.

We shall restrict to resonances such that $\sum_i |l_i| \leq 1$, since the amplitude of the resonance is proportional to $e_i^{|l_i|}$.

Confinement

Are there any resonances?



L. Benet and À. Jorba, submitted to Icarus (2016).

Summary and conclusions

- Our approach focuses on global aspects of phase space.
- There is a set of initial conditions which remains trapped at least $6.0 \times 10^6 T_{\rm Prom}$; the rest escape away mainly due to collisions with the shepherd moons.
- Trapped particles selected according to their stability form a ring which is narrow, eccentric and displays sharp edges.
- The semimajor axis and the width of the ring are in good correspondence with the observations. The eccentricity of the ring is smaller than the observed one.
- The confinement of Saturn's F ring may be understood in terms of some two-body resonances; instabilities seem to processes involving three-body resonances.

Summary and conclusions

Imaging team leader Brad Smith opined in a press conference that the pictures defied the laws of physics; Peter Goldreich later that day commented, “Don’t worry about Newton’s equations; most people don’t realize how many solutions they have” (Elliot and Kerr, 1984). (...) In retrospect, it is not at all surprising that the newest findings show even more dynamic structures!

Larry Esposito, Planetary rings: A post-equinox view (Cambridge University Press, 2014)

Special thanks to Carles Simó and Alex Luque for discussions, and J.M. Mondelo for `furian`.

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