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Quantification des erreurs de troncation dans le domaine temporel pour la réinitialisation des intégrateurs fractionnaires

Séminaire ROBEX, ENSTA Bretagne, Brest, France

Andreas Rauh, 18 Octobre 2021

Contents

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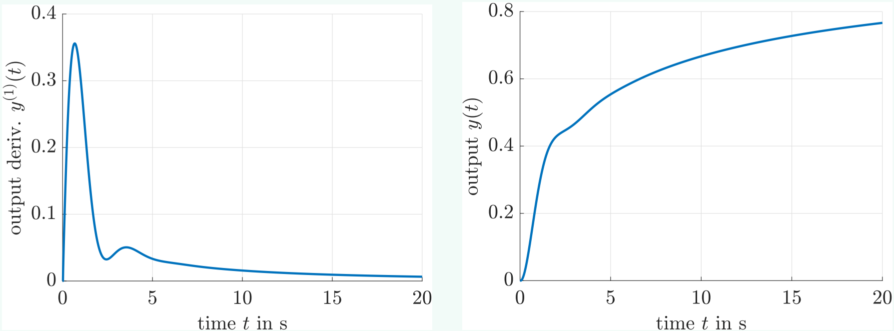
A Time-Domain Point of View

Comparison of “velocity” $y^{(1)}(t)$ and “position” $y(t)$



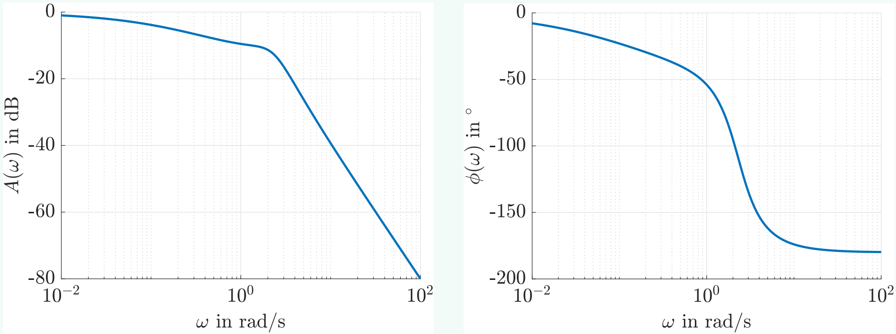
A Time-Domain Point of View

Observations

- The time response seems to be partially faster and partially slower than an exponential
- The time response seems to be impossible to be explained by a linear first- or second-order model
- Do we really need to identify a nonlinear representation?

A Frequency-Domain Point of View

Amplitude and phase response



A Frequency-Domain Point of View

Observations

- Amplitude response does not only show slopes of $\pm 20\,\mathrm{dB}$ per frequency decade
- The phase response seems to have a characteristic value not only around -90° and -180°, but also at $\approx -54^\circ$
- Is it necessary to expand the amplitude and phase responses into series of classical first- and second-order transfer function blocks?

Resolution

Considered system model

$$y^{(2)}(t) + 3y^{(0.6)}(t) + y(t) = u(t)$$

Fractional-order attenuation term

- The system model is linear
- However, not all contained derivatives have an integer order

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Resolution

Considered system model

$$y^{(2)}(t) + 3y^{(0.6)}(t) + y(t) = u(t)$$

Fractional-order attenuation term

– Representation that lies between the dependency of stress ($\sigma(t)$) and strain ($\epsilon(t)$) according to Newton's law

$$\sigma(t) = \eta \frac{\mathrm{d}\epsilon(t)}{\mathrm{d}t} \quad \text{with the viscosity} \quad \eta$$

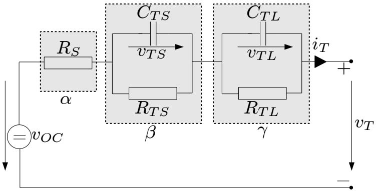
and Hooke's law

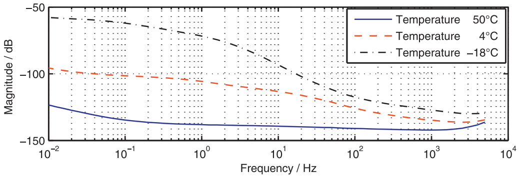
$$\sigma(t) = E\epsilon(t) \quad \text{with the modulus of elasticity} \quad E$$

Modeling and Identification by Means of Impedance Spectroscopy

Typical amplitude and phase response vs. integer-order system models

- Amplitude variations do not correspond to integer multiples of $\pm 20\mathrm{dB}$ per frequency decade
- Phase variations do not correspond to integer multiples of $\pm\frac{\pi}{2}$ per frequency decade
- Thevenin equivalent circuit consisting of series connections of resistors and RC sub-models
- Approximation of frequency response characteristics over a finite bandwidth



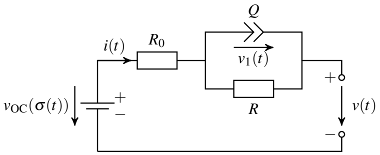


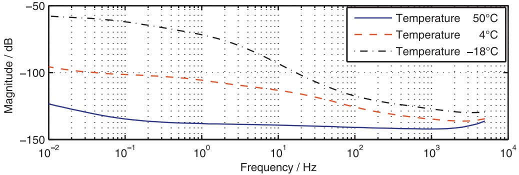
Modeling and Identification by Means of Impedance Spectroscopy

Typical amplitude and phase response vs. fractional-order system models

- Generalization of the Thevenin equivalent circuit by a replacement of capacitors (and possibly inductors) by respective constant phase elements
- Generalization of frequency response functions by expressions of the form

$$\frac{b_0 + \ldots + b_m \, (\jmath\omega)^{m\nu}}{a_0 + \ldots + a_n \, (\jmath\omega)^{n\nu}} \;, \quad m, n \in \mathbb{N}_0 \;, \quad \nu \in (0, 1)$$





Initial Value Problem with Interval Uncertainty

Definition of the initial value problem (IVP)

– Given set of ordinary differential equations (ODEs)

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t))$$

with smooth right-hand sides

– Uncertain initial conditions

$$\mathbf{x}(0) \in [\mathbf{x}_0] := [\mathbf{x}(0)] = [\underline{\mathbf{x}}(0)\ ;\ \overline{\mathbf{x}}(0)]$$

– Component-wise definition of interval vectors $[\mathbf{x}] = \begin{bmatrix}[x_1] & \dots & [x_n]\end{bmatrix}^T$ with the vector entries $[x_i] = [\underline{x}_i\ ;\ \overline{x}_i]$, $\underline{x}_i \leq x_i \leq \overline{x}_i$, $i = 1, \dots, n$

Generalization for fractional differential equations with derivative orders $0 < \nu \leq 1$?

Exponential Enclosure Technique

Definition of the state enclosure

- Representation of contracting state enclosures by using

$$\mathbf{x}^*(t) \in [\mathbf{x}_e](t) := \exp([\mathbf{\Lambda}] \cdot t) \cdot [\mathbf{x}_e](0)$$

with $0 \notin [x_{e,i}](0)$, $[\mathbf{x}_e](0) = [\mathbf{x}_0]$ for the diagonal matrix

$$[\mathbf{\Lambda}] := \operatorname{diag}\{[\lambda_i]\} \ , \quad i = 1, \ldots, n$$

with element-wise negative real entries λ_i

- Definition of the interval matrix exponential

$$\exp([\mathbf{\Lambda}] \cdot t) := \operatorname{diag}\{\exp([\lambda_1] \cdot t), \ldots, \exp([\lambda_n] \cdot t)\}$$

Exponential Enclosure Technique

Interval-valued iteration scheme

– Picard iteration

$$\mathbf{x}^*(t) \in [\mathbf{x}_e]^{\langle\kappa+1\rangle}(t) := [\mathbf{x}_0] + \int_0^t \mathbf{f}\left([\mathbf{x}_e]^{\langle\kappa\rangle}(s)\right) \mathrm{d}s$$

– Resulting iteration formula

$$[\lambda_i]^{\langle\kappa+1\rangle} := \frac{f_i\left(\exp\left([\mathbf{\Lambda}]^{\langle\kappa\rangle} \cdot [0\,;\,T]\right) \cdot [\mathbf{x}_e](0)\right)}{\exp\left([\lambda_i]^{\langle\kappa\rangle} \cdot [0\,;\,T]\right) \cdot [x_{e,i}](0)}\ ,\ i = 1, \ldots, n$$

with the guaranteed state enclosure at the point $t = T$

$$\mathbf{x}^*(T) \in [\mathbf{x}_e](T) := \exp\left([\mathbf{\Lambda}] \cdot T\right) \cdot [\mathbf{x}_e](0)$$

Fractional Differential Equations (1)

Fractional differential equations (FDEs) of Caputo type

– Generally nonlinear pseudo state equations

$$\mathbf{x}^{(\nu)}(t) = \mathbf{f}\left(\mathbf{x}(t)\right)$$

– Initial conditions specified at $t = 0$ (temporally constant initialization)

Exact solution of the FDE for the linear case

– Scalar, linear fractional pseudo state equation

$$x^{(\nu)}(t) = \lambda \cdot x(t)$$

with the initial condition $x_0 = x(0)$ that also describes the entire past of the pseudo state

Fractional Differential Equations (2)

Exact solution of the FDE for the linear case

- Scalar, linear fractional pseudo state equation

$$x^{(\nu)}(t) = \lambda \cdot x(t)$$

with the initial condition $x_0 = x(0)$ that also describes the entire past of the pseudo state

- Explicit solution

$$x(t) = E_{\nu,1}\left(\lambda t^{\nu}\right) \cdot x(0)$$

depending on the two-parameter Mittag-Leffler function

$$E_{\nu,\beta}(\zeta) = \sum_{i=0}^{\infty} \frac{\zeta^i}{\Gamma\left(\nu i + \beta\right)}$$

with the general argument $\zeta \in \mathbb{C}$ and the parameters $\nu \in \mathbb{R}_+$, $\beta \in \mathbb{R}$

Fractional Differential Equations (3)

Note 1

For $\nu = 1$, the two-parameter Mittag-Leffler function turns into the classical exponential function according to the solution

$$x(t) = E_{1,1}\left(\lambda t^{1}\right) \cdot x(0)$$

with

$$E_{1,1}(\zeta) = \sum_{i=0}^{\infty} \frac{\zeta^{i}}{\Gamma\left(i+1\right)} = e^{\zeta}$$

Note 2

FDE systems are characterized by an **infinite memory of previous states**, i.e., subdividing the integration time horizon into short slices $[t_k \; ; \; t_{k+1}]$, where the state at $t = t_{k+1}$ serves as the initialization for the following slice, is prone to approximation errors

Iterative Solution Procedure for Fractional System Models (1)

Generalization of exponential enclosures to the FDE case

– Definition of the pseudo state enclosures

$$[\mathbf{x}_e]^{\langle\kappa\rangle}(t) = \mathbf{E}_{\nu,1}\left([\mathbf{\Lambda}]^{\langle\kappa\rangle} \cdot t^{\nu}\right) \cdot [\mathbf{x}_e](0)$$

– Diagonal matrix containing the evaluation of the scalar Mittag-Leffler function

$$\mathbf{E}_{\nu,1}\left([\mathbf{\Lambda}]^{\langle\kappa\rangle} \cdot t^{\nu}\right)$$

with

$$[\mathbf{\Lambda}] := \mathrm{diag}\left\{[\lambda_i]\right\} \ , \quad i = 1, \ldots, n$$

Iterative Solution Procedure for Fractional System Models (2)

Element-wise notation of the fixed-point iteration

$$\operatorname{diag}\left\{[\lambda_i]^{\langle\kappa+1\rangle}\right\} \cdot \underbrace{\mathbf{E}_{\nu,1}\left([\mathbf{\Lambda}]^{\langle\kappa+1\rangle} \cdot [t]^{\nu}\right) \cdot [\mathbf{x}_e](0)}_{[\mathbf{x}_e]^{\langle\kappa+1\rangle}([t]) \subseteq [\mathbf{x}_e]^{\langle\kappa\rangle}([t])}$$
$$=: \mathbf{f}\left(\mathbf{E}_{\nu,1}\left([\mathbf{\Lambda}]^{\langle\kappa\rangle} \cdot [t]^{\nu}\right) \cdot [\mathbf{x}_e](0)\right)$$

Detailed derivation

- Rauh, Andreas; Kersten, Julia; Aschemann, Harald: *Interval-Based Verification Techniques for the Analysis of Uncertain Fractional-Order System Models*, 2020 European Control Conference (ECC), Saint Petersburg, Russia, pp. 1853–1858, 2020. DOI: 10.23919/ECC51009.2020.9143758
- Rauh, Andreas; Kersten, Julia: *Toward the Development of Iteration Procedures for the Interval-Based Simulation of Fractional-Order Systems*, Acta Cybernetica, 2020. DOI: 10.14232/actacyb.285660.
- Rauh, Andreas; Kersten, Julia: *Verification and Reachability Analysis of Fractional-Order Differential Equations Using Interval Analysis*, Proceedings 6th International Workshop on Symbolic-Numeric Methods for Reasoning about CPS and IoT (SNR 2020), Electronic Proceedings in Theoretical Computer Science 331, pp. 18–32. 2021. DOI: 10.4204/EPTCS.331.2

Iterative Solution Procedure for Fractional System Models (2)

Element-wise notation of the fixed-point iteration

$$\operatorname{diag}\left\{[\lambda_i]^{\langle\kappa+1\rangle}\right\} \cdot \underbrace{\mathbf{E}_{\nu,1}\left([\boldsymbol{\Lambda}]^{\langle\kappa+1\rangle} \cdot [t]^{\nu}\right) \cdot [\mathbf{x}_e]\,(0)}_{[\mathbf{x}_e]^{\langle\kappa+1\rangle}([t]) \subseteq [\mathbf{x}_e]^{\langle\kappa\rangle}([t])}$$

$$=: \mathbf{f}\left(\mathbf{E}_{\nu,1}\left([\boldsymbol{\Lambda}]^{\langle\kappa\rangle} \cdot [t]^{\nu}\right) \cdot [\mathbf{x}_e]\,(0)\right)$$

Necessity for an interval extension of Mittag-Leffler functions

- Rauh, Andreas; Kersten, Julia; Aschemann, Harald: *Interval-Based Verification Techniques for the Analysis of Uncertain Fractional-Order System Models*, 2020 European Control Conference (ECC), Saint Petersburg, Russia, pp. 1853–1858, 2020. DOI: 10.23919/ECC51009.2020.9143758
- Rauh, Andreas; Jaulin Luc: *Novel Techniques for a Verified Simulation of Fractional-Order Differential Equations*. Fractal and Fractional, 5(1):17, 2021. DOI: 10.3390/fractalfract5010017
- Rauh, Andreas; Jaulin, Luc; Malti, Rachid: *Verification Techniques for Fractional Differential Equations with Bounded Uncertainty*. 2e Journées Virtuelles de la SAGIP (online), 2021. DOI: 10.13140/RG.2.2.27052.80002.

Consideration of the Infinite Memory Property

Quantification of temporal truncation errors (Podlubny)

$$\left| {}_{t_k}\mathcal{D}_t^{\nu}\mathbf{x}(t) - {}_{t_k+T}\mathcal{D}_t^{\nu}\mathbf{x}(t) \right| \leq \frac{\boldsymbol{\mathcal{X}} T^{-\nu}}{|\Gamma(1-\nu)|} =: \boldsymbol{\mu}$$

with the component-wise defined supremum of the reachable states according to

$$\mathcal{X}_i = \sup_{t \in [t_k \ ; \ t_{k+1}]} |x_i(t)| \ , \quad i \in \{1, \ldots, n\}$$

and a subsequent inflation of the right-hand side of the system model

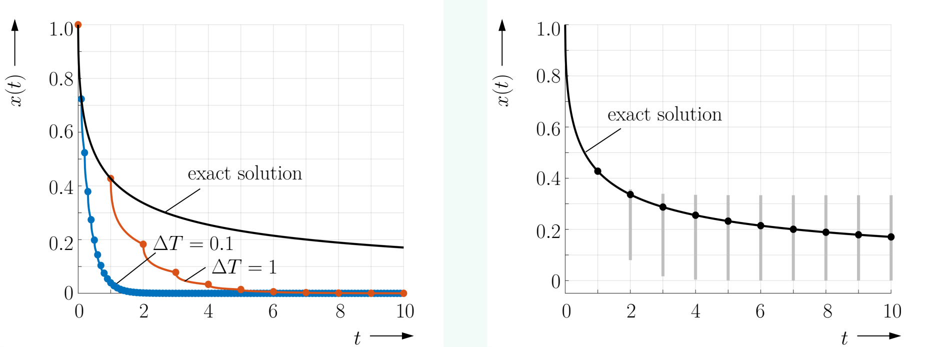
$$\tilde{\mathbf{f}}\left(\mathbf{x}(t)\right) := \mathbf{f}\left(\mathbf{x}(t)\right) + [-\boldsymbol{\mu} \ ; \ \boldsymbol{\mu}]$$

Note

The given bounds $\boldsymbol{\mu}$ are conservative, especially due to the fact that they are temporally constant

Visualization of the Effect of Truncation Errors

Uncertain FDE model $x^{(0.5)}(t) = -x(t)$, $x(t) = 1$ for $t \leq 0$



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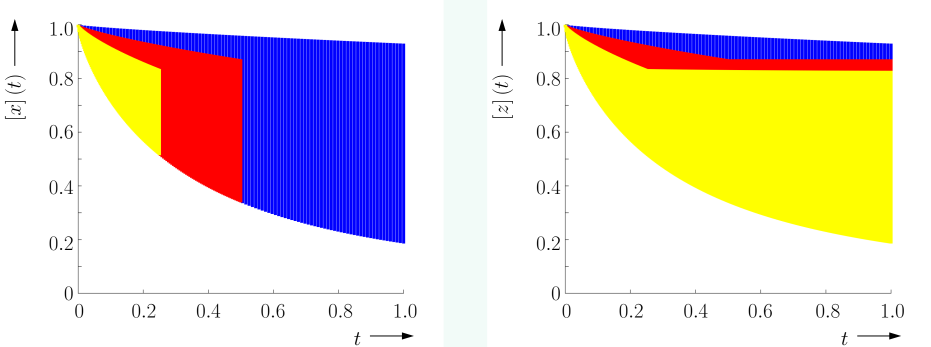
Illustrating Example: Mittag-Leffler-Type Enclosures

Uncertain FDE model $x^{(\nu)}(t) = p \cdot x^3(t)$

$[x](0) = [0.99\ ;\ 1.0]$, $[p] = [-2\ ;\ -1.99]$, $[\nu] = [0.8\ ;\ 0.81]$



Interval Observer for Improved Bounds of Truncation Errors (1)

Reference model: *virtual measurements* for a cooperative dynamic system model

- Pseudo state enclosures

$$\mathbf{x}(t) \in [\mathbf{v}(t)\ ;\ \mathbf{w}(t)] \quad \text{with} \quad {}_0\mathcal{D}_t^{(\nu)}\mathbf{x}(t) = \mathbf{f}\left(\mathbf{x}(t)\right)$$

- Without loss of generality: temporally constant initialization

$$\mathbf{x}(t) \in [\mathbf{x}_0]\ ,\quad \dot{\mathbf{x}}(t) = \mathbf{0} \quad \text{for} \quad t \leq 0$$

Bounding systems (Cooperativity of the system model)

$$\begin{aligned} {}_0\mathcal{D}_t^{(\nu)}\mathbf{v}(t) &= \mathbf{f}_v\left(\mathbf{v}(t)\right)\ ,\quad \mathbf{v}(t \leq 0) = \underline{\mathbf{x}}_0 \quad \text{and} \\ {}_0\mathcal{D}_t^{(\nu)}\mathbf{w}(t) &= \mathbf{f}_w\left(\mathbf{w}(t)\right)\ ,\quad \mathbf{w}(t \leq 0) = \overline{\mathbf{x}}_0 \end{aligned}$$

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Interval Observer for Improved Bounds of Truncation Errors (2)

Observer-based approach

- Denote the truncation error bounds at $t = T > 0$ by $[-\boldsymbol{\mu}_T \ ; \ \boldsymbol{\mu}_T]$
- Augmented observer model: $\mathbf{z}(t) = \begin{bmatrix} \tilde{\mathbf{v}}^T(t) & \tilde{\mathbf{w}}^T(t) & \boldsymbol{\mu}_v^T(t) & \boldsymbol{\mu}_w^T(t) \end{bmatrix}^T$

$$
{}_T\mathcal{D}_t^{(\nu)}\mathbf{z}(t) = \begin{bmatrix} \mathbf{f}_v\left(\tilde{\mathbf{v}}(t)\right) + \boldsymbol{\mu}_v(t) \\ \mathbf{f}_w\left(\tilde{\mathbf{w}}(t)\right) + \boldsymbol{\mu}_w(t) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \mathbf{H} \cdot \begin{bmatrix} \mathbf{v}(t) - \tilde{\mathbf{v}}(t) \\ \mathbf{w}(t) - \tilde{\mathbf{w}}(t) \end{bmatrix} \quad \text{with} \quad \mathbf{z}(T) = \begin{bmatrix} \mathbf{v}(T) \\ \mathbf{w}(T) \\ -\boldsymbol{\mu}_T \\ \boldsymbol{\mu}_T \end{bmatrix}
$$

- Choose the observer gain $\mathbf{H}$ such that $[\mathbf{v}(t) \ ; \ \mathbf{w}(t)] \subseteq [\tilde{\mathbf{v}}(t) \ ; \ \tilde{\mathbf{w}}(t)]$

Applicable to (periodically) reset the fractional integrator for the differential equation model

Interval Observer for Improved Bounds of Truncation Errors (2)

Observer-based approach

- Denote the truncation error bounds at $t = T > 0$ by $[-\boldsymbol{\mu}_T \ ; \ \boldsymbol{\mu}_T]$
- Augmented observer model: $\mathbf{z}(t) = \begin{bmatrix} \tilde{\mathbf{v}}^T(t) & \tilde{\mathbf{w}}^T(t) & \boldsymbol{\mu}_v^T(t) & \boldsymbol{\mu}_w^T(t) \end{bmatrix}^T$

$$
{}_T\mathcal{D}_t^{(\nu)}\mathbf{z}(t) = \begin{bmatrix} \mathbf{f}_v\left(\tilde{\mathbf{v}}(t)\right) + \boldsymbol{\mu}_v(t) \\ \mathbf{f}_w\left(\tilde{\mathbf{w}}(t)\right) + \boldsymbol{\mu}_w(t) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \mathbf{H} \cdot \begin{bmatrix} \mathbf{v}(t) - \tilde{\mathbf{v}}(t) \\ \mathbf{w}(t) - \tilde{\mathbf{w}}(t) \end{bmatrix} \quad \text{with} \quad \mathbf{z}(T) = \begin{bmatrix} \mathbf{v}(T) \\ \mathbf{w}(T) \\ -\boldsymbol{\mu}_T \\ \boldsymbol{\mu}_T \end{bmatrix}
$$

- Choose the observer gain $\mathbf{H}$ such that $[\mathbf{v}(t) \ ; \ \mathbf{w}(t)] \subseteq [\tilde{\mathbf{v}}(t) \ ; \ \tilde{\mathbf{w}}(t)]$

Basis for the implementation of predictor–corrector state estimators (continuous-time dynamics with discrete-time measurements)

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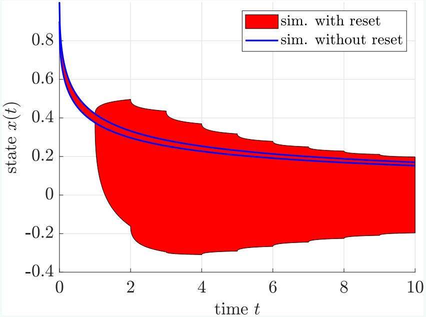
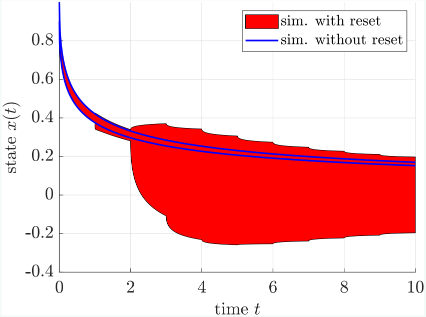
Illustrating Example: Observer Approach

Uncertain FDE model $x^{(0.5)}(t) = -x(t) - p \cdot x^3(t)$, $x(t) \in [0.9\ ;\ 1.0]$ for $t \leq 0$, $p \in [0.1\ ;\ 0.2]$, integrator reset for $t = T, 2T, 3T, \ldots$ with $T = 1$

Constant error bounds $\boldsymbol{\mu}_T$



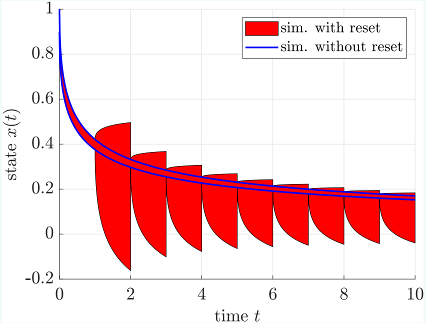
Observer-based enhancement of $\boldsymbol{\mu}_v$, $\boldsymbol{\mu}_w$



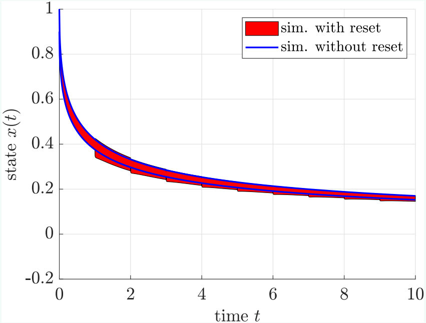
Illustrating Example: Observer Approach

Uncertain FDE model $x^{(0.5)}(t) = -x(t) - p \cdot x^3(t)$, $x(t) \in [0.9 \; ; \; 1.0]$ for $t \leq 0$, $p \in [0.1 \; ; \; 0.2]$, **Pseudo-state reinitialization with exact bounds**

Constant error bounds $\boldsymbol{\mu}_T$



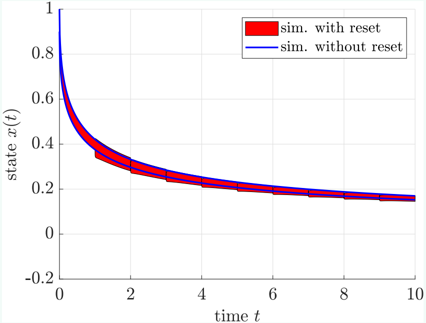
Observer-based enhancement of $\boldsymbol{\mu}_v$, $\boldsymbol{\mu}_w$



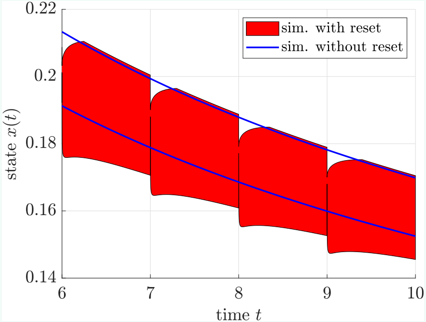
Illustrating Example: Observer Approach

Predictor–corrector state estimator, true state $\hat{x}^{(0.5)}(t) = -\hat{x}(t) - 0.15 \cdot \hat{x}^3(t)$, uncertain measurement $y_{\mathrm{m}}(kT) \in \hat{x}(kT) + 0.001 \cdot [-1\ ;\ 1]$, $k \in \mathbb{N}$

Observer-based enhancement of $\boldsymbol{\mu}_v$, $\boldsymbol{\mu}_w$



Enlarged view



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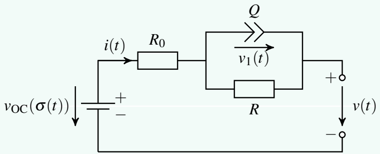
Simplified Battery Model (1)

Fractional-order equivalent circuit model



Mathematical formulation

– Pseudo state vector: $\mathbf{x}(t) = \begin{bmatrix} \sigma(t) & {}_0\mathcal{D}_t^{0.5}\sigma(t) & v_1(t) \end{bmatrix}^T \in \mathbb{R}^3$

Parameters identified experimentally

J. Reuter, E. Mank, H. Aschemann, A. Rauh: *Battery state observation and condition monitoring using online minimization*, 21st International Conference on Methods and Models in Automation and Robotics (MMAR), Miedzyzdroje, Poland, pp. 1223–1228, 2016. DOI: 10.1109/MMAR.2016.7575313

Simplified Battery Model (2)

Mathematical formulation

- State equations

$$ {}_0\mathcal{D}_t^{0.5}\mathbf{x}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{b}\cdot i(t) $$

  with the system and input matrices

$$ \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{\eta_1 \cdot \mathrm{sign}(i(t))}{3600 C_\mathrm{N}} & 0 & 0 \\ 0 & 0 & -\frac{1}{RQ} \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ -\frac{\eta_0}{3600 C_\mathrm{N}} \\ \frac{1}{Q} \end{bmatrix} $$

- Controlled discharging process with $i(t) = -\mathbf{k}^T \cdot \mathbf{x}(t)$
- Terminal voltage

$$ v(t) = \begin{bmatrix} \sum_{k=0}^{4} c_k \sigma^{k-1}(t) & 0 & -1 \end{bmatrix} \cdot \mathbf{x}(t) + \left(-R_0 + d_0 e^{d_1 \sigma(t)}\right) \cdot i(t) $$

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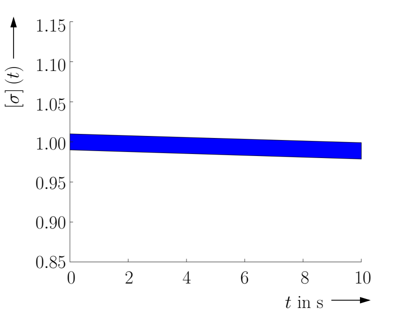
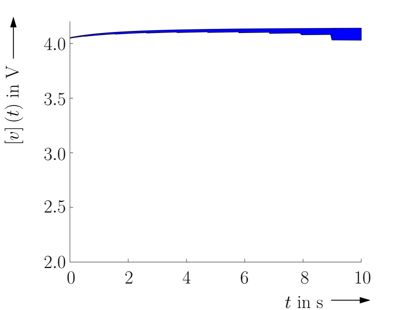
Mittag-Leffler Enclosures (Non-Cooperative Control Parameterization)

Prediction of state-of-charge and terminal voltage (control with pole assignment)

State-of-charge



Terminal voltage



Simplified Battery Model (cont'd)

Mathematical formulation: Specific parameterization in closed-loop operation

- State equations

$$ {}_0\mathcal{D}_t^{0.5}\mathbf{x}(t) = \mathbf{A}_\mathrm{C} \cdot \mathbf{x}(t) $$

  with

$$ \mathbf{A}_\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix} \quad \text{and} \quad \begin{aligned} a_{22} &\in [-0.00022000000000 \,;\ -0.00017999999999] \\ a_{32} &\in [\ \ 0.09755718517445 \,;\ \ \ 0.11923655965767] \\ a_{33} &\in [-0.53155619600024 \,;\ -0.43490961490928] \end{aligned} $$

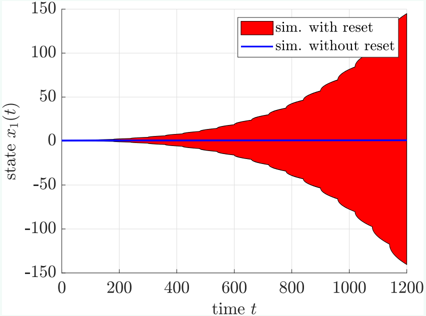
- Uncertain initial conditions $\mathbf{x}(0) = \begin{bmatrix} 0.5 & 0.01 & 0.1 \end{bmatrix}^T \cdot [0.9 \,;\ 1.1]$
- Terminal voltage

$$ v(t) = \begin{bmatrix} \sum_{k=0}^{4} c_k \sigma^{k-1}(t) & 0 & -1 \end{bmatrix} \cdot \mathbf{x}(t) + \left(-R_0 + d_0 e^{d_1 \sigma(t)}\right) \cdot i(t) $$

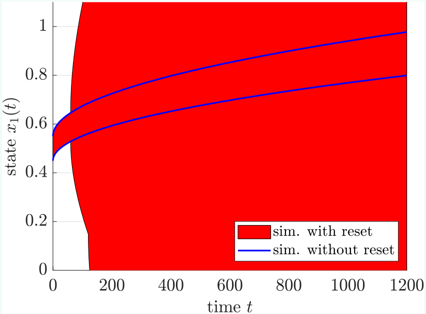
Observer-Based Bounds of Truncation Errors

Simulation with temporally constant error bounds

State $x_1(t)$ (state of charge)



State $x_1(t)$ (state of charge)



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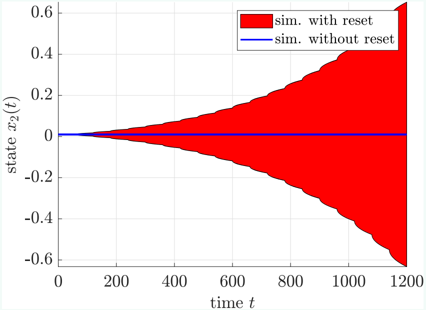
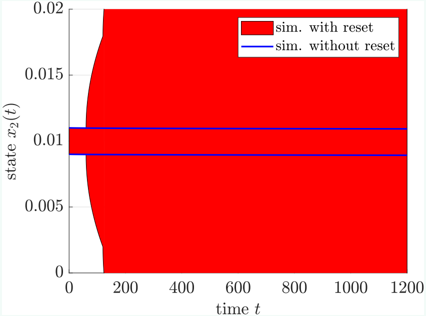
Observer-Based Bounds of Truncation Errors

Simulation with temporally constant error bounds

State $x_2(t)$



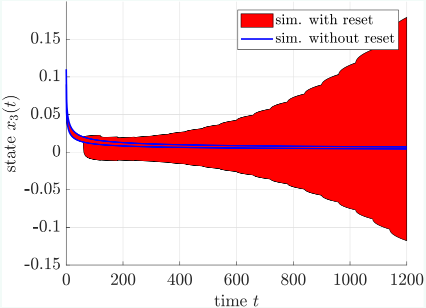
State $x_2(t)$



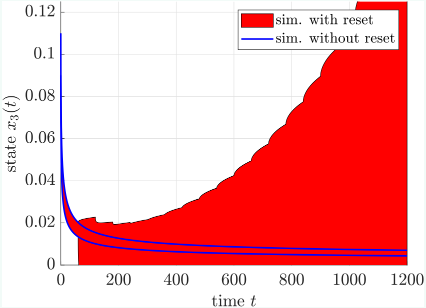
Observer-Based Bounds of Truncation Errors

Simulation with temporally constant error bounds

State $x_3(t)$



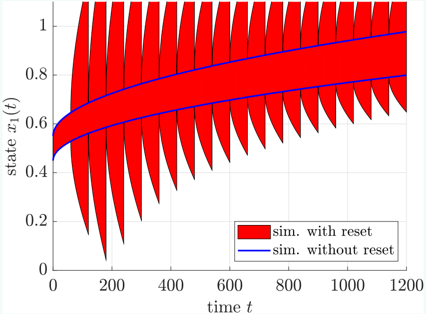
State $x_3(t)$



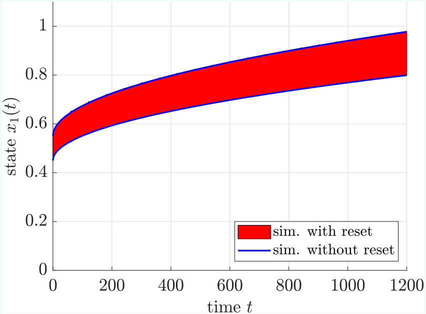
Observer-Based Bounds of Truncation Errors

Constant errors bounds with reset vs. contractor-based observer

State $x_1(t)$ (state of charge)

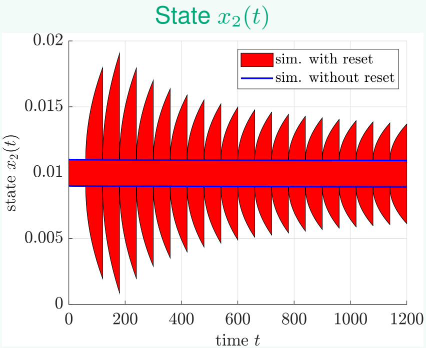


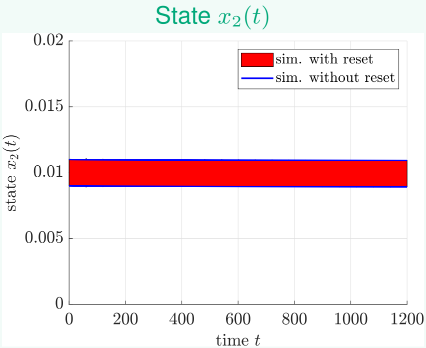
State $x_1(t)$ (state of charge)



Observer-Based Bounds of Truncation Errors

Constant errors bounds with reset vs. contractor-based observer

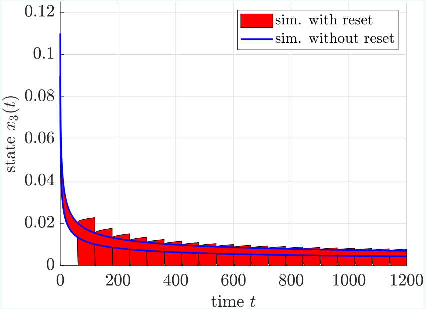




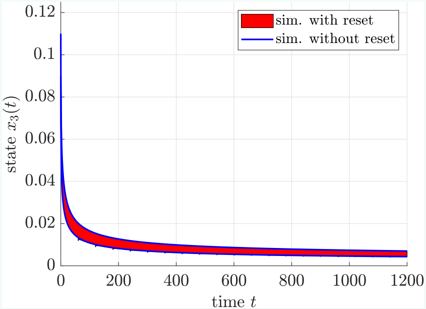
Observer-Based Bounds of Truncation Errors

Constant errors bounds with reset vs. contractor-based observer

State $x_3(t)$



State $x_3(t)$



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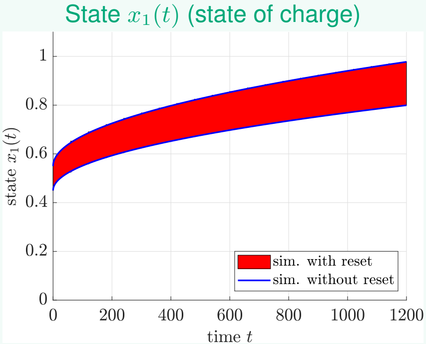
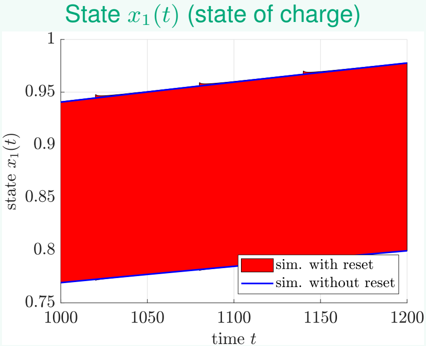
Observer-Based Bounds of Truncation Errors

Contractor-based observer

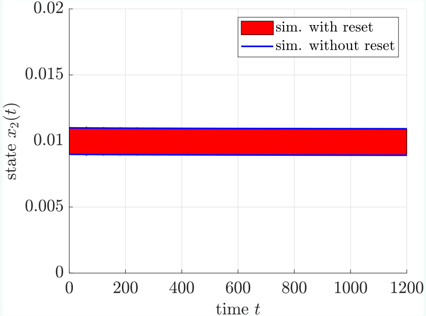




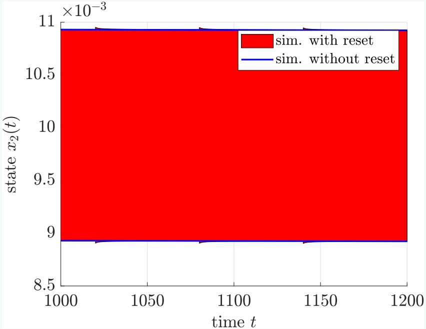
Observer-Based Bounds of Truncation Errors

Contractor-based observer

State $x_2(t)$



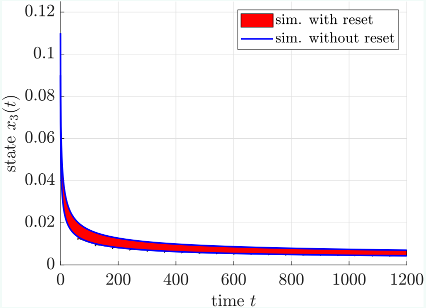
State $x_2(t)$



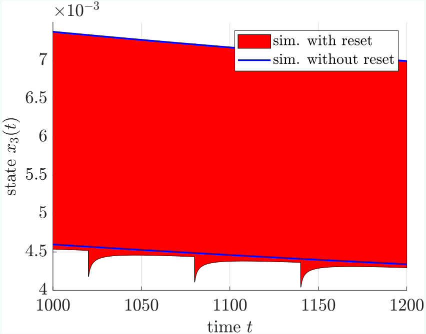
Observer-Based Bounds of Truncation Errors

Contractor-based observer

State $x_3(t)$



State $x_3(t)$



Conclusions and Outlook on Future Work

- Interval techniques for a verified simulation of fractional differential equations
- Applicable for use in predictive control strategies, system identification and state estimation
- Handling of uncertainty in initial conditions and parameters
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Conclusions and Outlook on Future Work

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- Use of quantification of time-domain truncation errors for further predictor–corrector state estimation tasks
- Enhancement of the contractor-based reset for nonlinear output functions
- Development of set-valued parameter identification schemes (including GPU-based parallelization)
- Identification of bounds on the initialization function on finite past time windows

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Related publications

- Rauh, Andreas; Westphal, Ramona; Aschemann, Harald: *Verified Simulation of Control Systems with Interval Parameters Using an Exponential State Enclosure Technique*, Proc. of IEEE Intl. Conference on Methods and Models in Automation and Robotics MMAR 2013, Miedzyzdroje, Poland, 2013. DOI: 10.1109/MMAR.2013.6669913
- Rauh, Andreas; Westphal, Ramona; Aschemann, Harald; Auer, Ekaterina: *Exponential Enclosure Techniques for Initial Value Problems with Multiple Conjugate Complex Eigenvalues*, 16th GAMM-IMACS International Symposium on Scientific Computing, Computer Arithmetic, and Validated Numerics SCAN2014, Würzburg, Germany, 2014. Vol. 9553 of Lecture Notes in Computer Science, pp. 87–122. DOI: 10.1007/978-3-319-31769-4_20
- Rauh, Andreas; Kersten, Julia; Aschemann, Harald: *Techniques for Verified Reachability Analysis of Quasi-Linear Continuous-Time Systems*, Proc. of IEEE Intl. Conference on Methods and Models in Automation and Robotics MMAR 2019, Miedzyzdroje, Poland, 2019. DOI: 10.1109/MMAR.2019.8864648
- Rauh, Andreas; Kersten, Julia; Aschemann, Harald: *Interval-Based Verification Techniques for the Analysis of Uncertain Fractional-Order System Models*, 2020 European Control Conference (ECC), Saint Petersburg, Russia, pp. 1853–1858, 2020. DOI: 10.23919/ECC51009.2020.9143758
- Rauh, Andreas; Kersten, Julia: *Toward the Development of Iteration Procedures for the Interval-Based Simulation of Fractional-Order Systems*, Acta Cybernetica, 2020. DOI: 10.14232/actacyb.285660.
- Rauh, Andreas; Kersten, Julia: *Verification and Reachability Analysis of Fractional-Order Differential Equations Using Interval Analysis*, Proceedings 6th International Workshop on Symbolic-Numeric Methods for Reasoning about CPS and IoT (SNR 2020), Electronic Proceedings in Theoretical Computer Science 331, pp. 18–32. 2021. DOI: 10.4204/EPTCS.331.2
- Rauh, Andreas; Jaulin, Luc: *Novel Techniques for a Verified Simulation of Fractional-Order Differential Equations*, Fractal Fract, 5(1), 17, 2021. DOI: 10.3390/fractalfract5010017

Thank you very much for your attention !