Set-membership target search and tracking within an unknown cluttered area using cooperating UAVs equipped with vision systems

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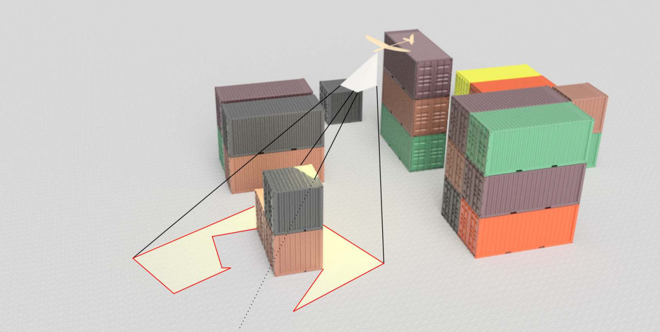
Problem

Localization

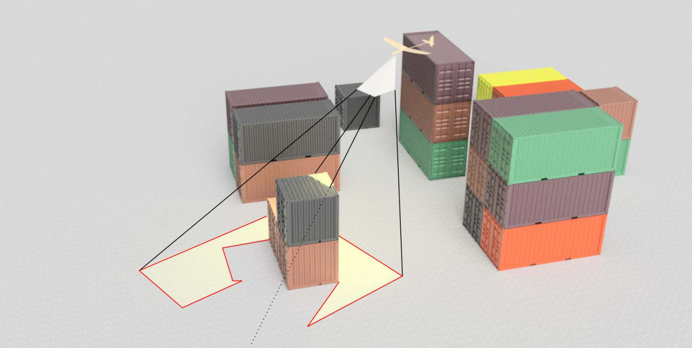
- of partially hidden targets
- in an unknown cluttered environment
- using a fleet of collaborative UAVs



Difficulties encountered

UAVs have limited ability to detect targets due to

- limited field of view
- presence of obstacles



Related work

- Search and track problem: Collecting information and defining exploration strategies
- Common hypotheses
  - Measurment noise modeled by realization of (Gaussian) random variables
  - Outliers or decoys accounted for by false alarm probabilities
- Various search strategies [13, 10]
  - Optimal flight path design
  - Distributed MPC [14]
  - Game-theoretic approaches

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Related work

- Search process: usually based on probabilistic approaches
- Performance usually sensitive to *a* priori information on pdfs describing
  - Process noise
  - Measurement noise
- Alternative set membership approaches [1, 3, 7]
  - Only noise bounds considered
  - Point estimates → set estimates
  - Simplified measurement model in [7]

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Proposed approach

Here, consider

- Obstacles with unknown location
- UAVs equipped with optical seekers and computer vision system (CVS)
- Target detected and identified when located within field of view of seeker
- Set-membership estimation technique as in [7, 5]

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Outline

Agenda

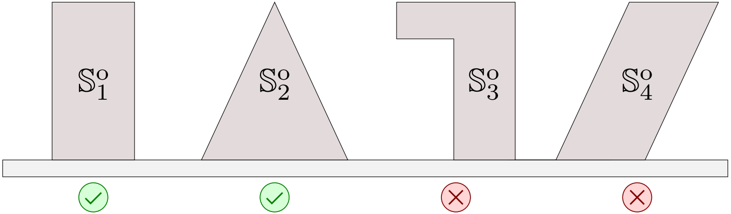
Environment

Unknown **R**egion **of I**nterest (**RoI**) $\mathbb{X}_0$ cluttered with static obstacles

- Flat ground $\mathbb{X}_\mathrm{g}$
- Unknown but structured obstacle

Assumption related to obstacle shape $\mathbb{S}^\mathrm{o}_m$, $m \in \{1, \dots, N^\mathrm{o}\}$

$$\forall \boldsymbol{x} \in \mathbb{S}^\mathrm{o}_m, \forall \lambda \in [0,1], \lambda \boldsymbol{x} + (1-\lambda)\, \boldsymbol{p}_\mathrm{g}(\boldsymbol{x}) \in \mathbb{S}^\mathrm{o}_m$$



Targets

N^{t} mobile ground targets

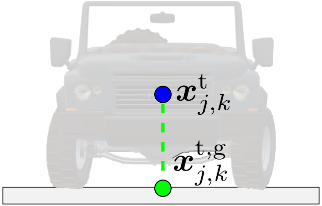
For target $j \in \{1, \ldots, N^{\mathrm{t}}\}$, state $\mathbf{x}^{\mathrm{t}}_{j,k}$

- Orientation, speed, **location of center of gravity** $\boldsymbol{x}^{\mathrm{t}}_{j,k} \in \mathbb{R}^3$
- **Target location:** projection of $\boldsymbol{x}^{\mathrm{t}}_{j,k}$ on ground, $\boldsymbol{x}^{\mathrm{t,g}}_{j,k} = \boldsymbol{p}_{\mathrm{g}}\left(\boldsymbol{x}^{\mathrm{t}}_{j,k}\right)$

Target dynamic

$$\boldsymbol{x}^{\mathrm{t,g}}_{j,k+1} = \mathbf{f}^{\mathrm{t}}\left(\boldsymbol{x}^{\mathrm{t,g}}_{j,k}, \boldsymbol{v}^{\mathrm{t}}_{j,k}\right)$$

with state perturbation $\boldsymbol{v}^{\mathrm{t}}_{j,k} \in [\boldsymbol{v}^{\mathrm{t}}]$.

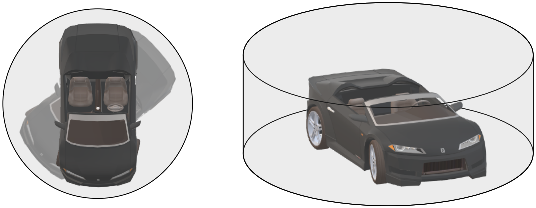


Target - Shape

3D target shape $\mathbb{S}^{\mathrm{t}}\left(\mathbf{x}^{\mathrm{t}}_{j,k}\right)$, usually unknown...
...but target category is known, *i.e.*, cars, bus...

Assumption: Target shape contained in known cylinder $\mathbb{C}^{\mathrm{t}}\left(\boldsymbol{x}^{\mathrm{t,g}}_{j,k}\right)$

$$\mathbb{S}^{\mathrm{t}}\left(\mathbf{x}^{\mathrm{t}}_{j,k}\right) \subset \mathbb{C}^{\mathrm{t}}\left(\boldsymbol{x}^{\mathrm{t,g}}_{j,k}\right)$$



Target - Interaction

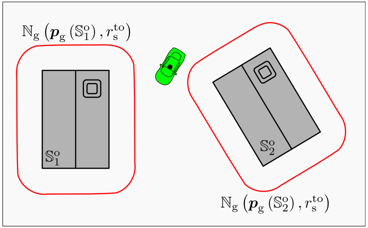
Assumption: Targets avoid collisions with obstacles and other targets

r-ground neighborhood of set $\mathbb{S} \subset \mathbb{X}_{\mathrm{g}}$

$$\mathbb{N}_{\mathrm{g}}(\mathbb{S}, r) = \{\boldsymbol{x} \in \mathbb{X}_{\mathrm{g}} \mid d(\boldsymbol{x}, \mathbb{S}) \leqslant r\}$$

Target-**O**bstacle safety distance $r_{\mathrm{s}}^{\mathrm{to}}$

$$\boldsymbol{x}_{j,k}^{\mathrm{t,g}} \notin \mathbb{N}_{\mathrm{g}}\left(\boldsymbol{p}_{\mathrm{g}}(\mathbb{S}_m^{\mathrm{o}}), r_{\mathrm{s}}^{\mathrm{to}}\right)$$



Target-**T**arget safety distance $r_{\mathrm{s}}^{\mathrm{tt}}$

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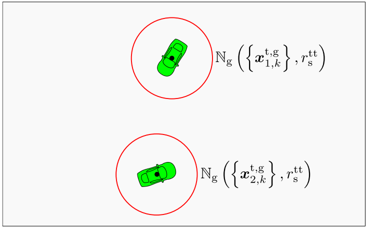
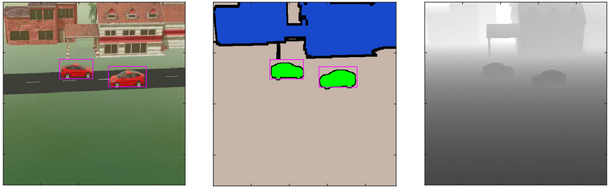


Measurements

N^{u} UAVs with state $\mathbf{x}^{\mathrm{u}}_{i,k}$, $i \in \{1, \ldots, N^{\mathrm{u}}\}$, with embedded computer vision system providing

- Image $\mathbf{I}_{i,k}$
- Depth map $\mathbf{D}_{i,k}$ [9]
- Labeled pixels $\mathbf{L}_{i,k}$ [4]
- Bounding boxes around detected targets [11]



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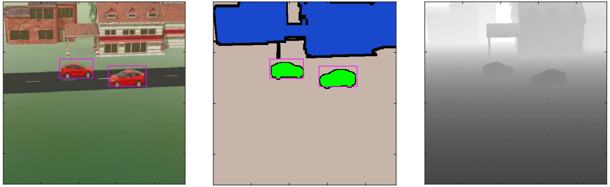


How can this type of information be exploited by a set-membership estimator?

Agenda

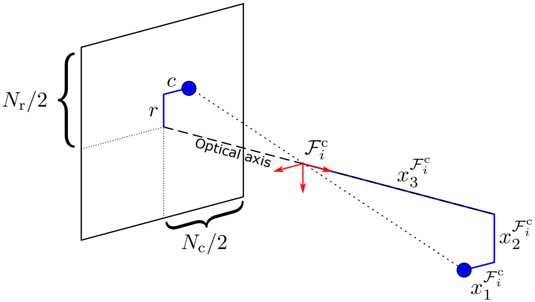
CVS - Camera model

Pinhole model without distortion *[2]*

Camera known parameters:

- optical center $\boldsymbol{x}_i^{\mathrm{c}}$
- Resolution $N_{\mathrm{c}} \times N_{\mathrm{r}}$
- focal length f_{c}, f_{r}
- horizontal/vertical aperture

Frame attached to UAV i camera: $\mathcal{F}_i^{\mathrm{c}}$



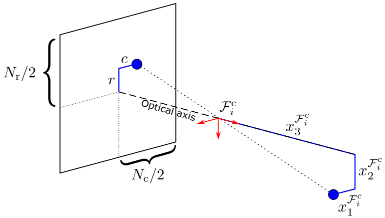
CVS - Camera model

Using the pinhole model, we can

- Project a point $\boldsymbol{x}$ onto CCD array

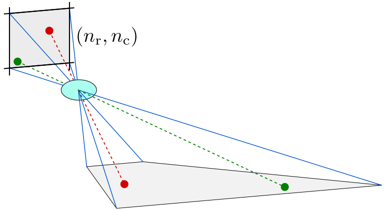
$$\boldsymbol{p}_{\mathcal{F}_i^{\mathrm{c}}}\left(\boldsymbol{x}^{\mathcal{F}_i^{\mathrm{c}}}\right) = \mathbf{K}\boldsymbol{x}^{\mathcal{F}_i^{\mathrm{c}}} / x_3^{\mathcal{F}_i^{\mathrm{c}}} = (c, r)^T$$

  K being matrix of camera intrinsic parameters



- Model light ray illuminating (c, r) by

$$\boldsymbol{v}(c, r) = \frac{1}{\nu(c, r)} \begin{pmatrix} \left(\frac{N_{\mathrm{c}}}{2} - c\right)/f_{\mathrm{c}} \\ \left(\frac{N_{\mathrm{r}}}{2} - r\right)/f_{\mathrm{r}} \\ 1 \end{pmatrix}$$



Set of light rays illuminating pixel $(n_{\mathrm{r}}, n_{\mathrm{c}})$: $\mathcal{V}_i(n_{\mathrm{r}}, n_{\mathrm{c}})$

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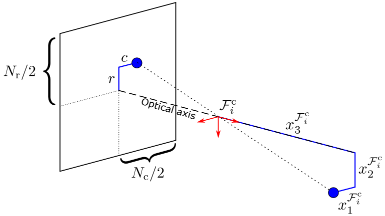
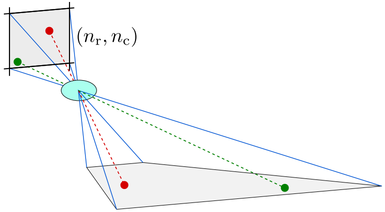




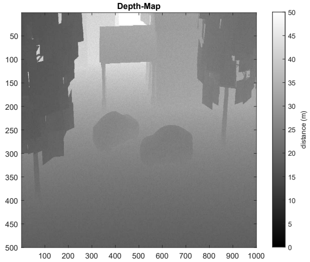
Set of light rays illuminating pixel $(n_{\mathrm{r}}, n_{\mathrm{c}})$: $\mathcal{V}_i(n_{\mathrm{r}}, n_{\mathrm{c}})$

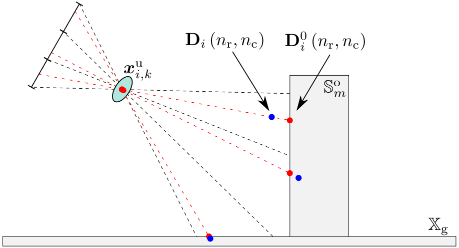
CVS - Depth map model

Hypotheses: $\mathbf{D}_i\left(n_\mathrm{r}, n_\mathrm{c}\right) = \mathbf{D}^0_{i,k}\left(n_\mathrm{r}, n_\mathrm{c}\right)(1+w)$

- range acquisition $\mathbf{D}^0_{i,k}\left(n_\mathrm{r}, n_\mathrm{c}\right) \in \left\{\rho\left(\boldsymbol{x}^\mathrm{c}_{i,k}, \boldsymbol{v}\right) \mid \boldsymbol{v} \in \mathcal{V}_{i,k}\left(n_\mathrm{r}, n_\mathrm{c}\right)\right\}$
- unknown but bounded noise $w \in [\underline{w}, \overline{w}]$

where $\rho\left(\boldsymbol{x}^\mathrm{c}_{i,k}, \boldsymbol{v}\right)$ is the distance between UAV i and environment along $\boldsymbol{v}$





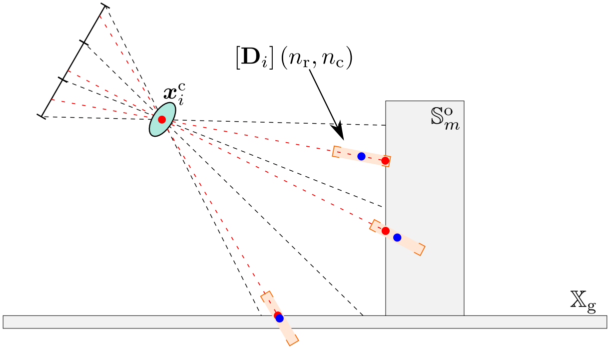
CVS - Depth map model

Using interval analysis

$$[\mathbf{D}_{i,k}]\,(n_\mathrm{r}, n_\mathrm{c}) = \left[\frac{1}{1+\overline{w}}, \frac{1}{1+\underline{w}}\right] \mathbf{D}_{i,k}\,(n_\mathrm{r}, n_\mathrm{c})$$

such that

$$\mathbf{D}^0_{i,k}\,(n_\mathrm{r}, n_\mathrm{c}) \in [\mathbf{D}_{i,k}]\,(n_\mathrm{r}, n_\mathrm{c})$$

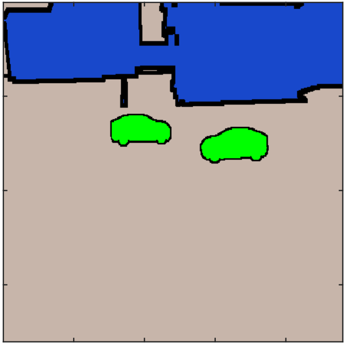


CVS - Pixel Labels

Pixel labeled either

- Ground $\mathcal{Y}^{\mathrm{g}}_{i,k}$
- Target $\mathcal{Y}^{\mathrm{t}}_{i,k}$
- Obstacle $\mathcal{Y}^{\mathrm{o}}_{i,k}$
- Unknown $\mathcal{Y}^{\mathrm{n}}_{i,k}$

Model relating pixel labels to environment needed



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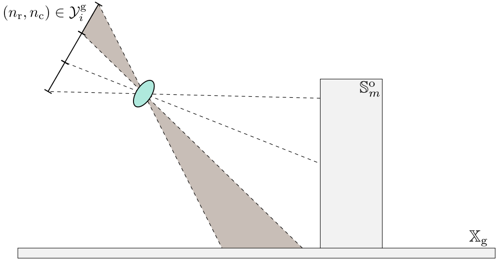
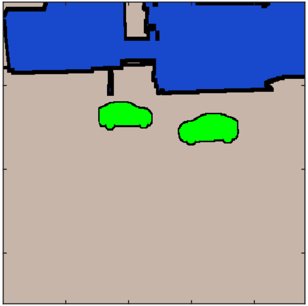


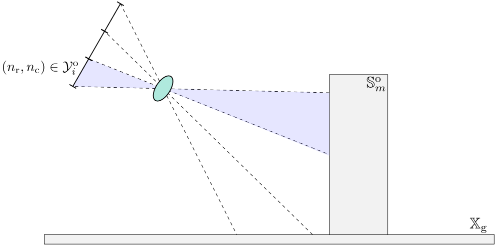


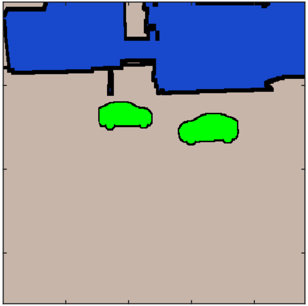
Hypothesis: If pixel $(n_r, n_c) \in \mathcal{Y}_{i,k}^g$ labeled Ground, then

$$\forall \boldsymbol{v} \in \mathcal{V}_i\left(n_r, n_c\right), \rho\left(\boldsymbol{x}_i^c, \boldsymbol{v}\right) = d_{\boldsymbol{v}}\left(\boldsymbol{x}_i^c, \mathbb{X}_g\right)$$

where $d_{\boldsymbol{v}}\left(\boldsymbol{x}_i^u, \mathbb{X}_g\right)$ is the distance between UAV and Ground along $\boldsymbol{v}$

CVS - Pixel Labels

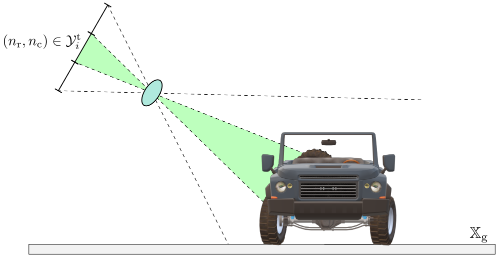


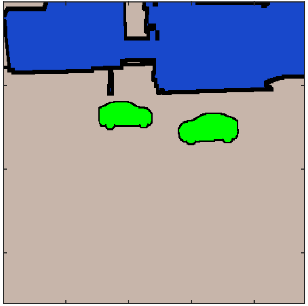


Hypothesis: If pixel $(n_\mathrm{r}, n_\mathrm{c}) \in \mathcal{Y}_i^\mathrm{o}$ labeled Obstacle, then $\exists m \in \mathcal{N}^\mathrm{o}$ such that

$$\forall \boldsymbol{v} \in \mathcal{V}_i\left(n_\mathrm{r}, n_\mathrm{c}\right), \rho\left(\boldsymbol{x}_i^\mathrm{c}, \boldsymbol{v}\right) = d_{\boldsymbol{v}}\left(\boldsymbol{x}_i^\mathrm{c}, \mathbb{S}_m^\mathrm{o}\right)$$

CVS - Pixel Labels





Hypothesis: If pixel $(n_\mathrm{r}, n_\mathrm{c}) \in \mathcal{Y}_i^\mathrm{t}$ labeled Target, then $\exists j \in \mathcal{N}^\mathrm{t}$ such that

$$\forall \boldsymbol{v} \in \mathcal{V}_i\left(n_\mathrm{r}, n_\mathrm{c}\right), \rho\left(\mathbf{x}_i^\mathrm{u}, \boldsymbol{v}\right) = d_{\boldsymbol{v}}\left(\boldsymbol{x}_i^\mathrm{u}, \mathbb{S}_j^\mathrm{t}\left(\mathbf{x}_j^\mathrm{t}\right)\right)$$

But: No target direct identification from single pixels

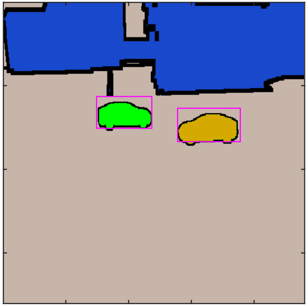
CVS - Bounding boxes

Consider $\mathcal{Y}^{\mathrm{t}}_{i,j} \subset \mathcal{Y}^{\mathrm{t}}_{i}$; for $(n_{\mathrm{r}}, n_{\mathrm{c}}) \in \mathcal{Y}^{\mathrm{t}}_{i,j}$

$$\forall \boldsymbol{v} \in \mathcal{V}_i\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right), \rho\left(\mathbf{x}^{\mathrm{u}}_i, \boldsymbol{v}\right) = d_{\boldsymbol{v}}\left(\boldsymbol{x}^{\mathrm{u}}_i, \mathbb{S}^{\mathrm{t}}_j\left(\mathbf{x}^{\mathrm{t}}_j\right)\right)$$

If target j identified, *i.e.*, $j \in \mathcal{D}^{\mathrm{t}}_i$, then we assume

- $\mathcal{Y}^{\mathrm{t}}_{i,j} \neq \emptyset$
- CVS provides box $\left[\mathcal{Y}^{\mathrm{t}}_{i,j}\right]$ for target j
- $\mathcal{Y}^{\mathrm{t}}_{i,j} \cap \left[\mathcal{Y}^{\mathrm{t}}_{i,j}\right] \neq \emptyset$

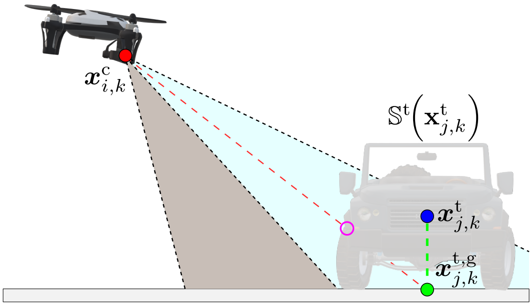


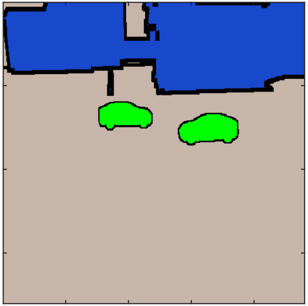
CVS - Negative information

Hypothesis:

$$\boldsymbol{x}_j^{\mathrm{t,g}} \in \mathbb{F}\left(\mathbf{x}_i^{\mathrm{u}}\right) \implies \left[\boldsymbol{x}_i^{\mathrm{c}}, \boldsymbol{x}_j^{\mathrm{t,g}}\right[\cap \mathbb{S}^{\mathrm{t}}\left(\mathbf{x}_j^{\mathrm{t}}\right) \neq \emptyset$$

Consequently, Ground-labeled pixels cannot contain $\boldsymbol{x}_j^{\mathrm{t,g}}$, $j \in \{1, \ldots, N^{\mathrm{t}}\}$.

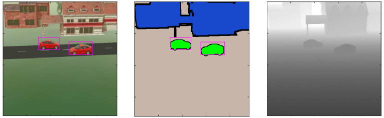




Problem formulation

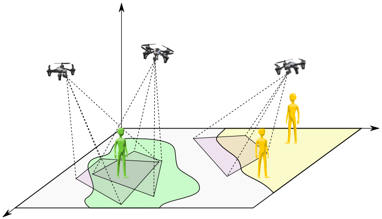
UAV i exploits CVS measurements to

- **detect** and **identify** targets
- **localize** identified targets
- **update** its knowledge (targets and obstacles)



UAV i evaluates at time t_k

- $\overline{\mathbb{X}}^{\mathrm{t}}_{i,k}$ containing locations of targets to detect
- $\mathbb{X}^{\mathrm{t}}_{i,j,k}$ containing target j location
- $\mathcal{L}^{\mathrm{t}}_{i,k}$: list of identified targets



Then, UAV i updates its trajectory to minimize

$$\Phi\left(\mathcal{X}^{\mathrm{t}}_{i,k}, \overline{\mathbb{X}}^{\mathrm{t}}_{i,k}\right) = \phi\left(\overline{\mathbb{X}}^{\mathrm{t}}_{i,k} \cup \bigcup_{j\in\mathcal{L}^{\mathrm{t}}_{i,k}} \mathbb{X}^{\mathrm{t}}_{i,j,k}\right)$$

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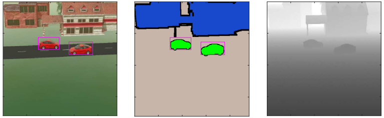

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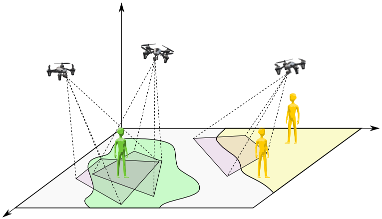

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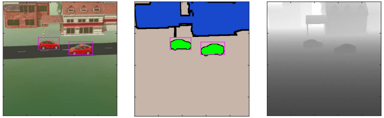

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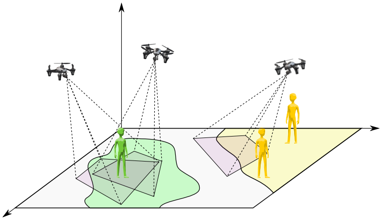

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Agenda

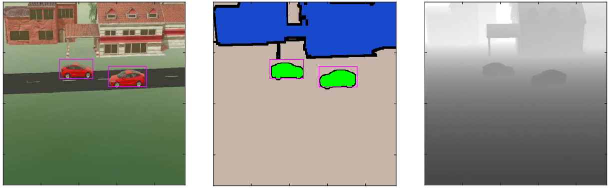
Set-membership estimator

UAV i exploits at time t_k available CVS measurements such as

- depth-map $[\mathbf{D}_{i,k}]$
- pixels labeled Ground $\mathcal{Y}^{\mathrm{g}}_{i,k}$, Obstacle $\mathcal{Y}^{\mathrm{o}}_{i,k}$, Target $\mathcal{Y}^{\mathrm{t}}_{i,k}$
- detected and identified targets $\mathcal{D}^{\mathrm{t}}_{i,k}$ and associated bounding box $\left[\mathcal{Y}^{\mathrm{t}}_{i,j}\right]$

to characterize

- set $\mathbb{X}^{\mathrm{t,m}}_{i,j,k}$ containing location of identified target j,
- sets free of targets,
- while updating environmental knowledge



Time index k omitted in what follows

Set-membership estimator

UAV i exploits at time t_k available CVS measurements such as

- depth-map $[\mathbf{D}_{i,k}]$
- pixels labeled Ground $\mathcal{Y}^{\mathrm{g}}_{i,k}$, Obstacle $\mathcal{Y}^{\mathrm{o}}_{i,k}$, Target $\mathcal{Y}^{\mathrm{t}}_{i,k}$
- detected and identified targets $\mathcal{D}^{\mathrm{t}}_{i,k}$ and associated bounding box $\left[\mathcal{Y}^{\mathrm{t}}_{i,j}\right]$

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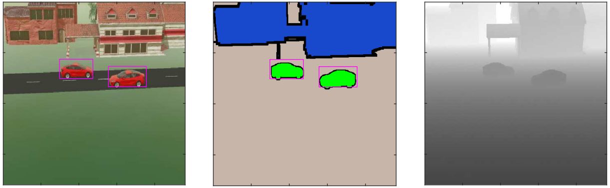
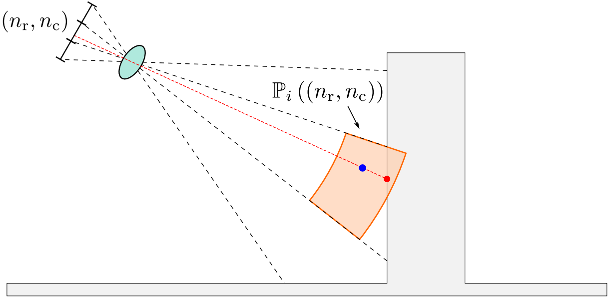


Time index k omitted in what follows

Target Localization

Using Depth-map $\mathbf{D}_i\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)$, consider

$$\mathbb{P}_i\left(\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)\right)=\left\{\boldsymbol{x} \in \mathbb{X}_0 \mid \exists \boldsymbol{v} \in \mathcal{V}_i\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right), d_{\boldsymbol{v}}\left(\boldsymbol{x}_i^{\mathrm{u}}, \boldsymbol{x}\right) \in\left[\mathbf{D}_i\right]\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)\right\}$$



$\mathbb{P}_i\left(\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)\right)$ contains points of environment which

- may have illuminated pixel $\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)$
- have a distance to UAV i consistent with $\mathbf{D}_i\left(n_{\mathrm{r}}, n_{\mathrm{c}}\right)$

Target Localization

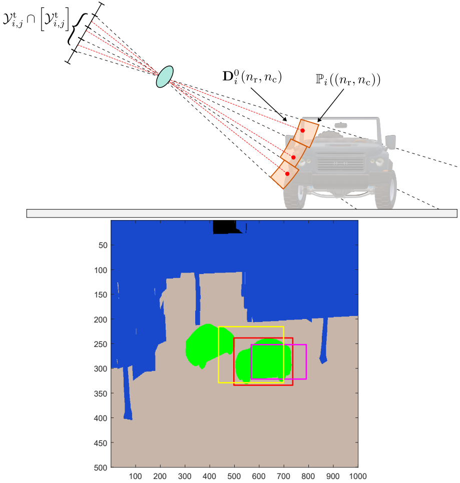
Consider identified target j, *i.e.*, $j \in \mathcal{D}_i^{\mathrm{t}}$

Consider

$$\mathbb{P}_{i,j}^{\mathrm{t}} = \left\{ \mathbb{P}_i \left((n_{\mathrm{r}}, n_{\mathrm{c}}) \right) \mid (n_{\mathrm{r}}, n_{\mathrm{c}}) \in \left[\mathcal{Y}_{i,j}^{\mathrm{t}} \right] \cap \mathcal{Y}_i^{\mathrm{t}} \right\}$$

Since $\mathcal{Y}_{i,j}^{\mathrm{t}} \cap \left[\mathcal{Y}_{i,j}^{\mathrm{t}} \right] \neq \emptyset$, then $\mathbb{P}_{i,j}^{\mathrm{t}} \cap \mathbb{S}_j^{\mathrm{t}} \left(\mathbf{x}_j^{\mathrm{t}} \right) \neq \emptyset$

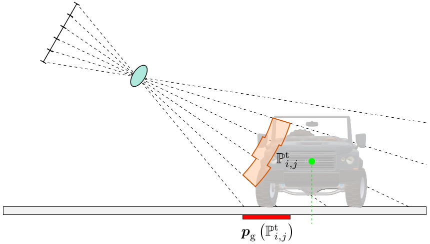
$\Rightarrow$ **Robust to bad bounding box**



Target Localization

2D estimation: projection of $\mathbb{P}^{\mathrm{t}}_{i,j}$ on the ground

- $\boldsymbol{p}_{\mathrm{g}}(\mathbb{X})$: projection on the ground of a set $\mathbb{X}$



$\boldsymbol{p}_{\mathrm{g}}\left(\mathbb{P}^{\mathrm{t}}_{i,j}\right)$ has no guarantee to contain $\boldsymbol{x}^{\mathrm{t,g}}_{j}$

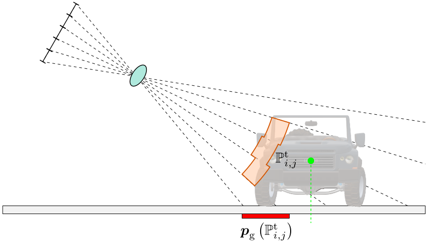
- $\mathbb{P}^{\mathrm{t}}_{i,j}$ obtained from points at vehicle surface
- $\boldsymbol{x}^{\mathrm{t}}_{j}$ is inside the vehicle

$\Rightarrow$ Account for target shape

Target Localization

2D estimation: projection of $\mathbb{P}^{\mathrm{t}}_{i,j}$ on the ground

- $\boldsymbol{p}_{\mathrm{g}}(\mathbb{X})$: projection on the ground of a set $\mathbb{X}$



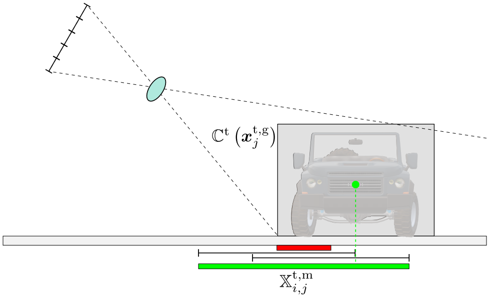
$\boldsymbol{p}_{\mathrm{g}}\left(\mathbb{P}^{\mathrm{t}}_{i,j}\right)$ has no guarantee to contain $\boldsymbol{x}^{\mathrm{t,g}}_{j}$

- $\mathbb{P}^{\mathrm{t}}_{i,j}$ obtained from points at vehicle surface
- $\boldsymbol{x}^{\mathrm{t}}_{j}$ is inside the vehicle

$\Rightarrow$ Account for target shape

Target Localization

Proposition: Since $\mathbb{S}^{\mathrm{t}}\left(\mathbf{x}_j^{\mathrm{t}}\right) \subset \mathbb{C}^{\mathrm{t}}\left(\boldsymbol{x}_j^{\mathrm{t,g}}\right)$: if $\boldsymbol{x} \in \mathbb{S}^{\mathrm{t}}\left(\mathbf{x}_j^{\mathrm{t}}\right)$, then $\boldsymbol{x}_j^{\mathrm{t,g}} \in \mathbb{C}^{\mathrm{t}}\left(\left\{\boldsymbol{p}_{\mathrm{g}}\left(\boldsymbol{x}\right)\right\}\right)$.



Consequently, since $\mathbb{P}_{i,j}^{\mathrm{t}} \cap \mathbb{S}_j^{\mathrm{t}}\left(\mathbf{x}_j^{\mathrm{t}}\right) \neq \emptyset$, one has

$$\boldsymbol{x}_j^{\mathrm{t,g}} \in \mathbb{X}_{i,j}^{\mathrm{t,m}} = \bigcup_{\boldsymbol{x} \in \boldsymbol{p}_{\mathrm{g}}\left(\mathbb{P}_{i,j}^{\mathrm{t}}\right)} \boldsymbol{p}_{\mathrm{g}}\left(\mathbb{C}^{\mathrm{t}}\left(\{\boldsymbol{x}\}\right)\right)$$

Set free of target

UAV i exploits

- pixels labeled Ground
- pixels labeled Obstacle
- set estimate $\mathbb{X}_{i,j}^{\mathrm{t,m}}$

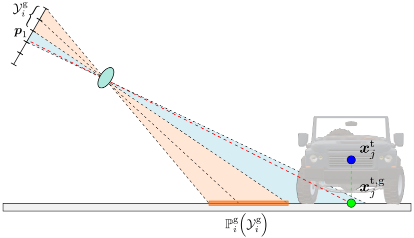
as negative information to characterize

- set $\mathbb{P}_i^{\mathrm{g}}\left(\mathcal{Y}_i^{\mathrm{g}}\right)$ that cannot contain any target location at time t_k
- set $\underline{\mathbb{X}}_i^{\mathrm{o}}$ that never contain any target location
- set $\underline{\mathbb{X}}_{i,j}^{\mathrm{t,m}}$ that cannot contain the location of targets in the vicinity of target j

Set free of target - Ground

By combing

- pixels labeled ground $\mathcal{Y}_i^{\mathrm{g}}$
- flat ground $\mathbb{X}_{\mathrm{g}}$
- UAV field of view $\mathbb{F}_i\left(\mathbf{x}_i^{\mathrm{u}}\right)$

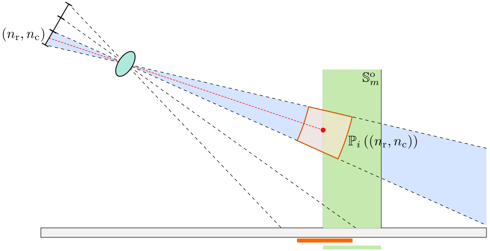


UAV i characterizes a set free of targets

$$\mathbb{P}_i^{\mathrm{g}}\left(\mathcal{Y}_i^{\mathrm{g}}\right) = \left\{\boldsymbol{x} \in \mathbb{F}_i\left(\mathbf{x}_i^{\mathrm{u}}\right) \cap \mathbb{X}_{\mathrm{g}} \mid \boldsymbol{p}_{\mathrm{c}}\left(\mathbf{x}_i^{\mathrm{u}}, \boldsymbol{x}\right) \in \mathcal{Y}_i^{\mathrm{g}}\right\}$$

$\boldsymbol{p}_{\mathrm{c}}\left(\mathbf{x}_i^{\mathrm{u}}, \boldsymbol{x}\right)$ being the projection on CCD array of $\boldsymbol{x}$

Set free of target - Obstacle



For any pixel $(n_\mathrm{r}, n_\mathrm{c}) \in \mathcal{Y}^\mathrm{o}_i$ labeled Obstacle, one proves that $\exists m \in \mathcal{N}^\mathrm{o}$ such that

$$\mathbb{P}_i\left((n_\mathrm{r}, n_\mathrm{c})\right) \cap \mathbb{S}^\mathrm{o}_m \neq \emptyset$$

We assumed a Target-Obstacle safety distance r^to_s

$$\boldsymbol{x}^{\mathrm{t,g}}_{j,k} \notin \mathbb{N}_\mathrm{g}\left(\boldsymbol{p}_\mathrm{g}\left(\mathbb{S}^\mathrm{o}_m\right), r^\mathrm{to}_\mathrm{s}\right)$$

Set free of target - Obstacle

UAV i characterizes

$$\mathbb{S}\left((n_r, n_c), r_\mathrm{s}^\mathrm{to}\right) = \bigcap_{\boldsymbol{x} \in \boldsymbol{p}_\mathrm{g}(\mathbb{P}_i((n_\mathrm{r}, n_\mathrm{c})))} \mathbb{N}_\mathrm{g}\left(\{\boldsymbol{x}\}, r_\mathrm{s}^\mathrm{to}\right)$$

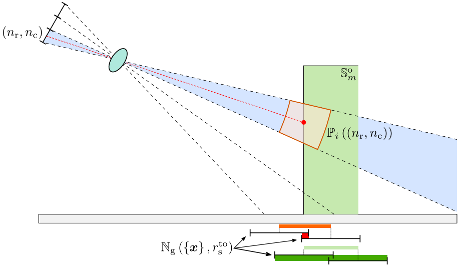
One is able to prove that

$$\mathbb{S}\left((n_r, n_c), r_\mathrm{s}^\mathrm{to}\right) \subset \mathbb{N}_\mathrm{g}\left(\boldsymbol{p}_\mathrm{g}\left(\mathbb{S}_m^\mathrm{o}\right), r_\mathrm{s}^\mathrm{to}\right)$$

Consequently, the set estimate

$$\mathbb{X}_i^\mathrm{o} = \bigcup_{(n_r, n_c) \in \mathcal{Y}_i^\mathrm{o}} \mathbb{S}\left((n_r, n_c), r_\mathrm{s}^\mathrm{to}\right)$$

cannot contain any target location



Set free of target - Obstacle

UAV i characterizes

$$\underline{\mathbb{S}}\left((n_r, n_c), r_{\mathrm{s}}^{\mathrm{to}}\right) = \bigcap_{\boldsymbol{x} \in \boldsymbol{p}_{\mathrm{g}}(\mathbb{P}_i((n_{\mathrm{r}}, n_{\mathrm{c}})))} \mathbb{N}_{\mathrm{g}}\left(\{\boldsymbol{x}\}, r_{\mathrm{s}}^{\mathrm{to}}\right)$$

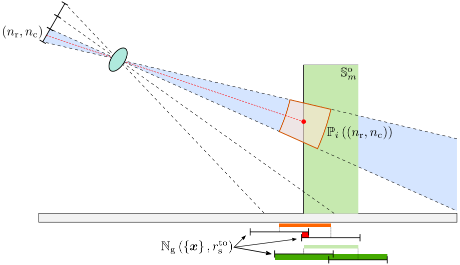
One is able to prove that

$$\underline{\mathbb{S}}\left((n_r, n_c), r_{\mathrm{s}}^{\mathrm{to}}\right) \subset \mathbb{N}_{\mathrm{g}}\left(\boldsymbol{p}_{\mathrm{g}}\left(\mathbb{S}_m^{\mathrm{o}}\right), r_{\mathrm{s}}^{\mathrm{to}}\right)$$

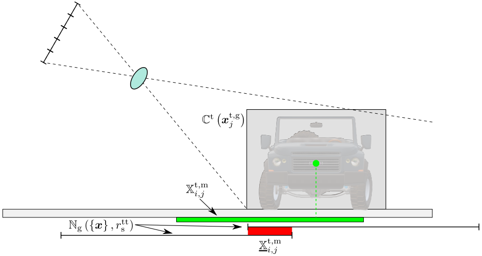
Consequently, the set estimate

$$\underline{\mathbb{X}}_i^{\mathrm{o}} = \bigcup_{(n_r, n_c) \in \mathcal{Y}_i^{\mathrm{o}}} \underline{\mathbb{S}}\left((n_r, n_c), r_{\mathrm{s}}^{\mathrm{to}}\right)$$

cannot contain any target location



Set free of target - Target



We assumed

$$\boldsymbol{x}_\ell^{\mathrm{t,g}} \notin \mathbb{N}_{\mathrm{g}}\left(\left\{\boldsymbol{x}_j^{\mathrm{t,g}}\right\}, r_{\mathrm{s}}^{\mathrm{tt}}\right)$$

But, we only know that $\boldsymbol{x}_j^{\mathrm{t,g}} \in \mathbb{X}_{i,j}^{\mathrm{t,m}}$

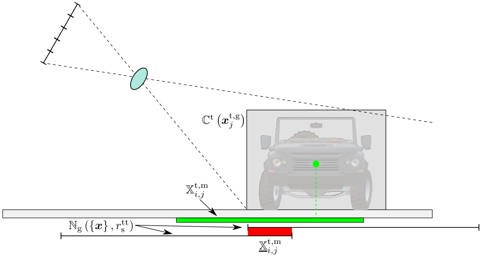
Thus, with

$$\underline{\mathbb{X}}_{i,j}^{\mathrm{t,m}} = \bigcap_{\boldsymbol{x}\in\mathbb{X}_{i,j}^{\mathrm{t,m}}} \mathbb{N}_{\mathrm{g}}\left(\{\boldsymbol{x}\}, r_{\mathrm{s}}^{\mathrm{tt}}\right)$$

one has $\underline{\mathbb{X}}_{i,j}^{\mathrm{t,m}} \subset \mathbb{N}_{\mathrm{g}}\left(\left\{\boldsymbol{x}_j^{\mathrm{t,g}}\right\}, r_{\mathrm{s}}^{\mathrm{tt}}\right)$

$\Rightarrow$ Consequently, $\underline{\mathbb{X}}_{i,j}^{\mathrm{t,m}}$ cannot contain any target location except $\boldsymbol{x}_j^{\mathrm{t,g}}$

Set free of target - Target



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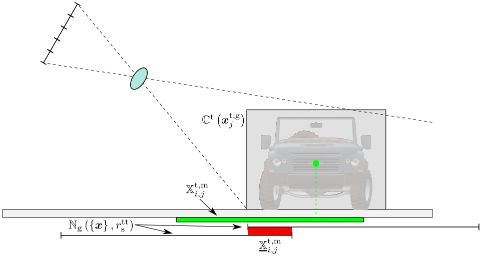
Set free of target - Target



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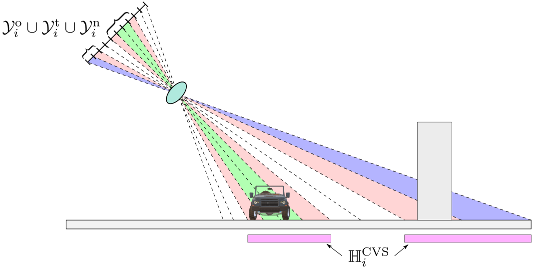
$\Rightarrow$ Consequently, $\underline{\mathbb{X}}_{i,j}^{\mathrm{t,m}}$ cannot contain any target location except $\boldsymbol{x}_j^{\mathrm{t,g}}$

Hidden portion of the ground

The portion of the ground hidden behind

- an obstacle
- a target
- an unidentified object

cannot be observed by UAV i



To approximate the hidden portion of the ground, UAV i evaluates

$$\mathbb{H}_i^{\mathrm{CVS}} = \mathbb{P}_i^{\mathrm{g}} \left(\mathcal{Y}_i^{\mathrm{o}} \cup \mathcal{Y}_i^{\mathrm{t}} \cup \mathcal{Y}_i^{\mathrm{n}} \right)$$

Recursive set-membership target location estimator

Adaptation of the recursive set-membership state estimator proposed in [6]

Initialization:

- List of identified targets $\mathcal{L}^{\mathrm{t}}_{i,0} = \emptyset$
- List of set estimates related to identified targets $\mathcal{X}^{\mathrm{t}}_{i,0} = \emptyset$
- Set containing unidentified targets $\overline{\mathbb{X}}^{\mathrm{t}}_{i,0} = \mathbb{X}_{\mathrm{g}}$
- Neighborhood of obstacles $\mathbb{X}^{\mathrm{o}}_{i,0} = \emptyset$

The estimator consists of

- **Prediction:** $k-1 \rightarrow k \mid k-1$
- **Correction** from **CVS measurements**: $k \mid k-1 \rightarrow k \mid k$
- **Correction** after communication with **neighboring UAVs**: $k \mid k \rightarrow k$

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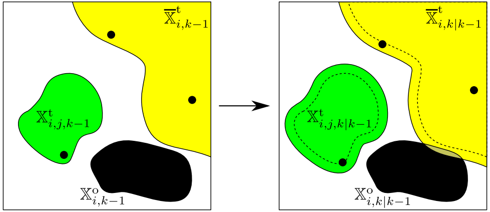
- **Correction** after communication with **neighboring UAVs**: $k \mid k \rightarrow k$

Prediction of the evolution

$k-1 \to k \mid k-1 \to k \mid k \to k$

UAV i characterizes

$$\mathbb{X}^{\mathrm{t}}_{i,j,k|k-1} = \left\{ \mathbf{f}^{\mathrm{t}}\left(\boldsymbol{x}, \boldsymbol{v}\right) \in \mathbb{X}_{\mathrm{g}} \mid \boldsymbol{x} \in \mathbb{X}^{\mathrm{t}}_{i,j,k-1}, \boldsymbol{v} \in \left[\boldsymbol{v}^{\mathrm{t}}\right] \right\}$$



After prediction, UAV i obtains $\mathbb{X}^{\mathrm{t}}_{i,j,k|k-1}$, $\overline{\mathbb{X}}^{\mathrm{t}}_{i,k|k-1}$, and $\mathbb{X}^{\mathrm{o}}_{i,k|k-1}$

- Obstacles are static: $\mathbb{X}^{\mathrm{o}}_{i,k|k-1} = \mathbb{X}^{\mathrm{o}}_{i,k-1}$.

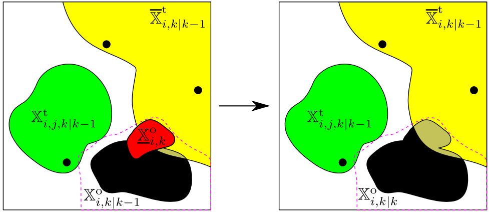
Correction from measurements - obstacles

$k-1 \to k \mid k-1 \to k \mid k \to k$

The set $\underline{\mathbb{X}}^{\mathrm{o}}_{i,k}$ is an inner-approximation of $\bigcup_{m\in\mathcal{N}^{\mathrm{o}}} \mathbb{N}_{\mathrm{g}}\left(\boldsymbol{p}_{\mathrm{g}}\left(\mathbb{S}^{\mathrm{o}}_{m}\right), r^{\mathrm{to}}_{\mathrm{s}}\right)$

Thus, the update is

$$\mathbb{X}^{\mathrm{o}}_{i,k|k} = \mathbb{X}^{\mathrm{o}}_{i,k|k-1} \cup \underline{\mathbb{X}}^{\mathrm{o}}_{i,k}$$



Correction from measurements

- $\mathcal{D}^{\mathrm{t}}_{i,k}$: List of target identified at time t_k
- $\mathcal{L}^{\mathrm{t}}_{i,k-1}$: List of previously identified targets
- Thus: $\mathcal{L}^{\mathrm{t}}_{i,k|k} = \mathcal{L}^{\mathrm{t}}_{i,k-1} \cup \mathcal{D}^{\mathrm{t}}_{i,k}$

Several cases are considered

- Target j is known but not currently identified $\Rightarrow j \in \mathcal{L}^{\mathrm{t}}_{i,k-1} \setminus \mathcal{D}^{\mathrm{t}}_{i,k}$
- Target j is known and currently identified $\Rightarrow j \in \mathcal{L}^{\mathrm{t}}_{i,k-1} \cap \mathcal{D}^{\mathrm{t}}_{i,k}$
- Target j is unknown but currently identified $\Rightarrow j \in \mathcal{D}^{\mathrm{t}}_{i,k} \setminus \mathcal{L}^{\mathrm{t}}_{i,k-1}$
- Target j is unknown and not currently identified $\Rightarrow j \notin \mathcal{L}^{\mathrm{t}}_{i,k-1} \cup \mathcal{D}^{\mathrm{t}}_{i,k}$

Correction from measurements - Case 1

Target j is known but not currently identified $\Rightarrow j \in \mathcal{L}^{\mathrm{t}}_{i,k-1} \setminus \mathcal{D}^{\mathrm{t}}_{i,k}$

Consequently:

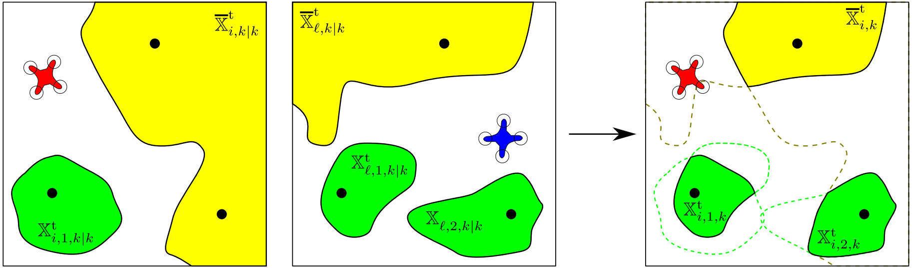
- $\boldsymbol{x}^{\mathrm{t,g}}_{j,k} \in \mathbb{X}^{\mathrm{t}}_{i,j,k|k-1}$
  - $\boldsymbol{x}^{\mathrm{t,g}}_{j,k} \notin \mathbb{P}^{\mathrm{g}}_{i,k}\left(\mathcal{Y}^{\mathrm{g}}_{i,k}\right) \cup \mathbb{X}^{\mathrm{o}}_{i,k|k}$
  - $\boldsymbol{x}^{\mathrm{t,g}}_{j,k} \notin \bigcup_{\ell \in \mathcal{D}^{\mathrm{t}}_{i,k}} \underline{\mathbb{X}}^{\mathrm{t,m}}_{i,\ell,k}$

The correction of $\mathbb{X}^{\mathrm{t}}_{i,j,k|k-1}$ is

$$\mathbb{X}^{\mathrm{t}}_{i,j,k|k} = \mathbb{X}^{\mathrm{t}}_{i,j,k|k-1} \setminus \left(\mathbb{P}^{\mathrm{g}}_{i,k}\left(\mathcal{Y}^{\mathrm{g}}_{i,k}\right) \cup \mathbb{X}^{\mathrm{o}}_{i,k|k} \cup \bigcup_{\ell \in \mathcal{D}^{\mathrm{t}}_{i,k}} \underline{\mathbb{X}}^{\mathrm{t,m}}_{i,\ell,k} \right)$$

Correction after communication with neighbors

Exchange of information between UAV i and UAV ℓ



Mapping - Occupancy-Elevation Map

OEM $\mathcal{M}_{i,k}$ is a regular 2D grid

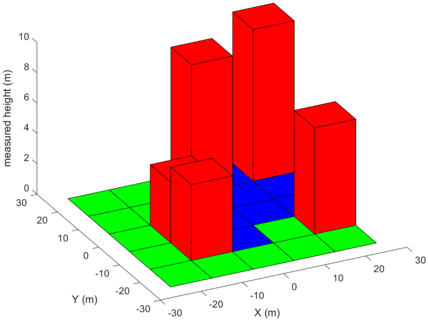
Each cell is characterized by

- a status
- an elevation

Status:

- Unexplored
- Empty: no obstacle
- Occupied: presence of an obstacle

Elevation: approximate obstacle height



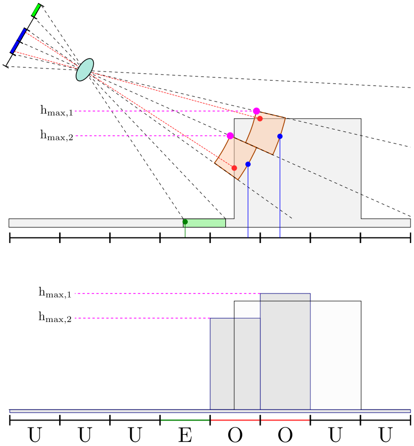
Mapping - OEM

Pixels labeled Obstacles

- Localized the obstacles
- Estimate their height

Pixels labeled Ground

- Detect the absence of obstacles



OEM - Prediction of hidden ground

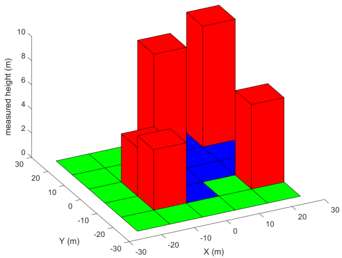
Occupied cells are used to approximate

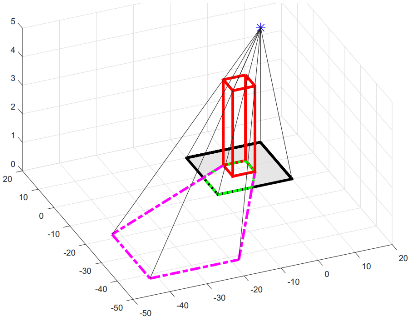
- obstacle shape
- obstacle location

they can also be used to

- evaluate the hidden portion of the ground [12]

UAV i uses its OEM to predict $\mathbb{H}_i^{\mathrm{CVS}}$ by evaluating $\mathbb{H}_i^{\mathrm{OEM}}\left(\mathbf{x}_i^{\mathrm{u}}\right)$





Agenda

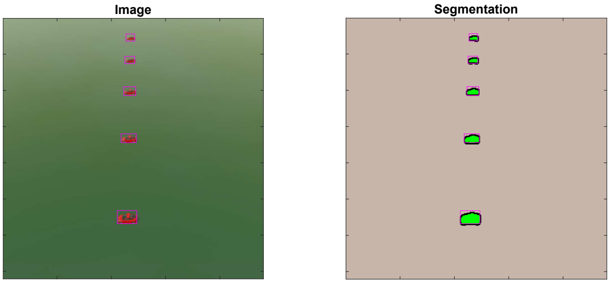
Simulations

Simulation conditions

- Targets: 5 identical cars
- 1 UAV
- Processed image 360×480
- depth-map noise: 1%

Accuracy of localization in function of:

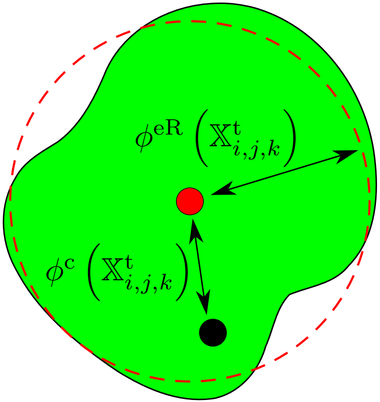
- distance to UAV
- depth-map noise



Metrics

Metrics:

- $\phi(\mathbb{X})$ area of $\mathbb{X}$
- $\phi^{\mathrm{eR}}\left(\mathbb{X}^{\mathrm{t}}_{i,j,k}\right) = \sqrt{\frac{\phi\left(\mathbb{X}^{\mathrm{t}}_{i,j,k}\right)}{\pi}}$
- $\phi^{\mathrm{c}}\left(\mathbb{X}^{\mathrm{t}}_{i,j,k}\right) = \left\|\boldsymbol{c}\left(\mathbb{X}^{\mathrm{t}}_{i,j,k}\right) - \boldsymbol{x}^{\mathrm{t,g}}_{j,k}\right\|$



Simulation - Set estimates

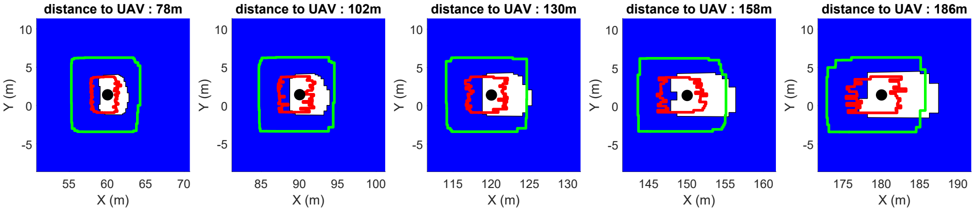
One single measurement, no obstacle

$$\overline{\mathbb{X}}^{\mathrm{t}}_{i,k|k-1} = \mathbb{X}_{\mathrm{g}} \qquad \mathbb{P}^{\mathrm{t}}_{i,j,k}$$

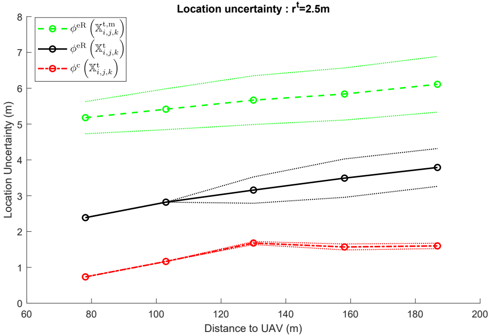
$$\mathbb{P}^{\mathrm{g}}_{i,k}\left(\mathcal{Y}^{\mathrm{g}}_{i,k}\right) \qquad \mathbb{X}^{\mathrm{t,m}}_{i,j,k}$$

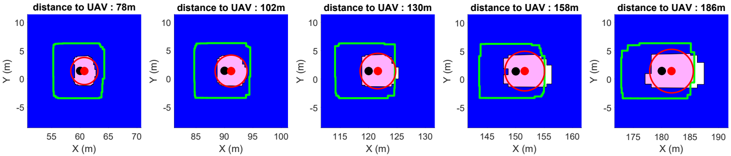
Estimated target location:

$$\mathbb{X}^{\mathrm{t}}_{i,j} = \mathbb{X}^{\mathrm{t,m}}_{i,j} \cap \left(\mathbb{X}_{\mathrm{g}} \setminus \mathbb{P}^{\mathrm{g}}_{i}\left(\mathcal{Y}^{\mathrm{g}}_{i}\right)\right)$$



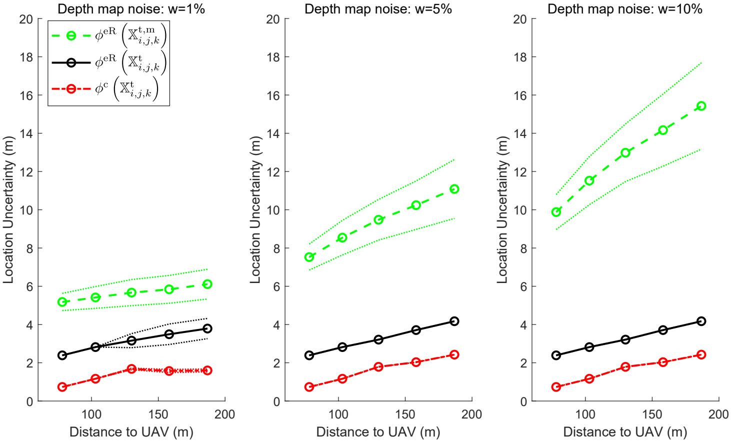
Simulation - Performance





Simulation - Depth map noise

Evaluation of the impact on the depth-map noise on the localization performance

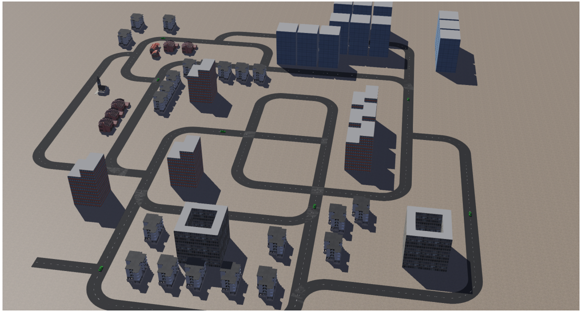


Agenda

Simulations

Recursive estimation algorithm may then be applied.

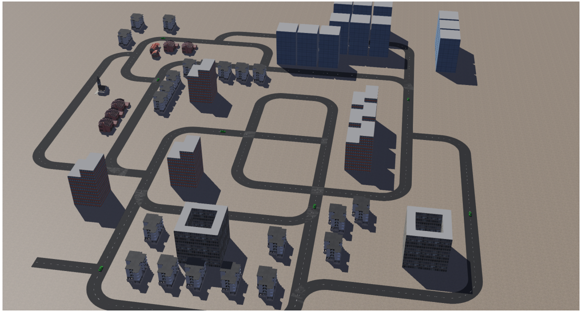
Simulations via Webots and Matlab



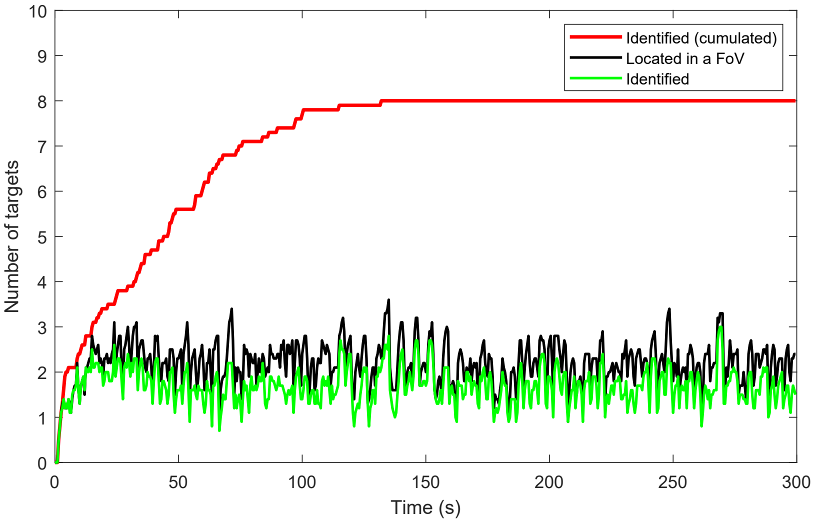
Simulation conditions

Conditions

- $N^{\mathrm{u}} = 4$ UAVs
- $N^{\mathrm{t}} = 8$ targets, speed $v_{\mathrm{max}} = 1$ m/s

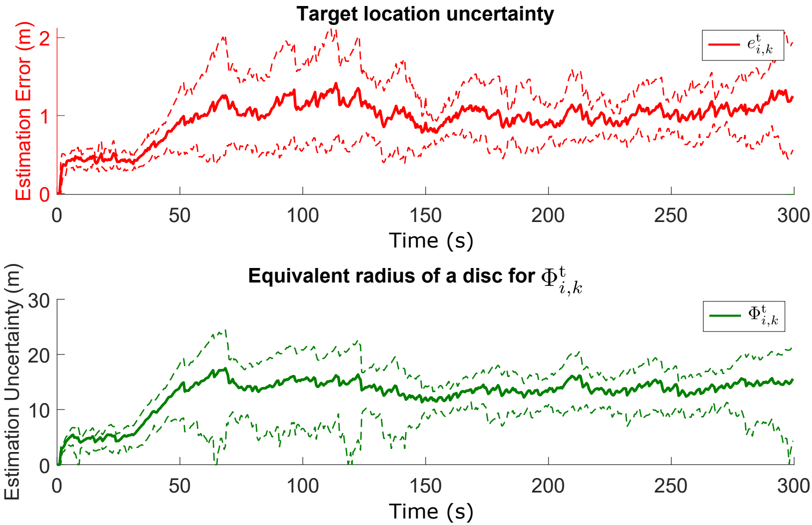


Simulation results



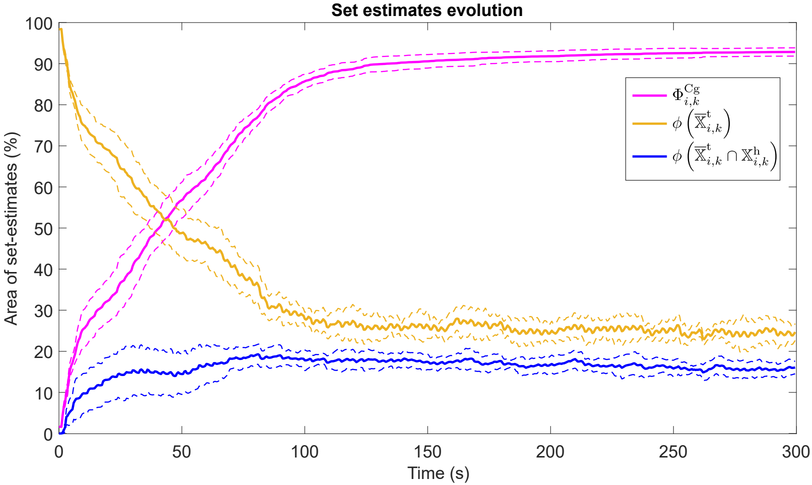
Cumulated number of detected targets

Simulation results



Estimation uncertainty

Simulation results



Exploration efficiency

Agenda

Summary

Development of a set-membership target location estimator that

- exploits **multiple CVS measurements**
- to characterize set estimates containing target location,
- while accounting for **negative information**
- and being **robust to depth-map noise** and **bad target detection**

CVS measurements are also used to

- approximate/predict the hidden portion of the ground
- **2.5D mapping** of the environment

Limitations:

- Unjustified depth-map noise bounds
- no target misidentification (False positive) [8]
- no target non-detection (False negative)

Summary

Development of a set-membership target location estimator that

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