



# Improvement in Interval Analysis based Localization through Neural Networks

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# Localization through Interval Computation

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The correct localization of mobile robots is necessary, even with the existence of imprecise sensors which may record geospatial data with a certain degree of uncertainty.

These mobile robots may follow heavily non-linear and not easily represented relations.

Interval Computation is a numerical tool that allows us to solve these non-linear problems in a reliable way, guaranteeing correct results and avoiding false negatives.

# Issue with Interval Computation

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An issue with Interval Computation is the time it takes to compute results, making real-time observations computationally expensive.

Average computation time using Codac is around  $\sim 0.15$ s to create an image.

Thus, we propose using neural networks, which could supplant the role of Interval Computation and potentially reduce the time to localize mobile robots, especially in cases where a robot may follow a not easily represented relation.

Processor	2 GHz Quad-Core Intel Core i5
Graphics	Intel Iris Plus Graphics 1536 MB
Memory	16 GB 3733 MHz LPDDR4X

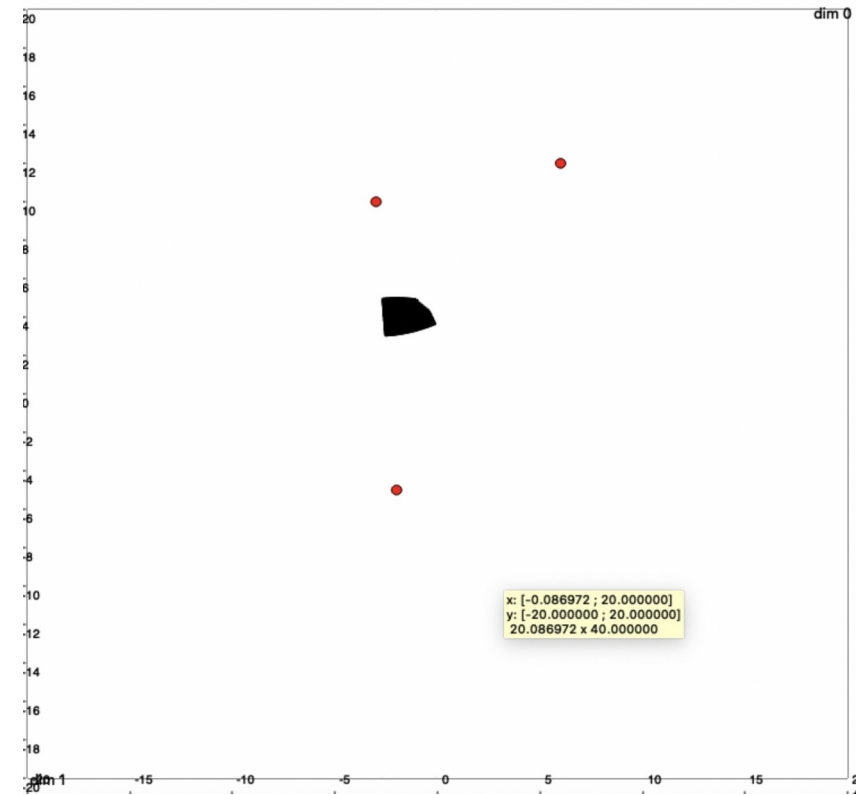
*computer used to run tests*

# Problem Definition

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Given coordinates of landmarks and interval distances between landmarks and robot, predict the area where the robot is localized.

Objective - Replicate outputs produced by interval computation methods, using neural networks.



# First Approach

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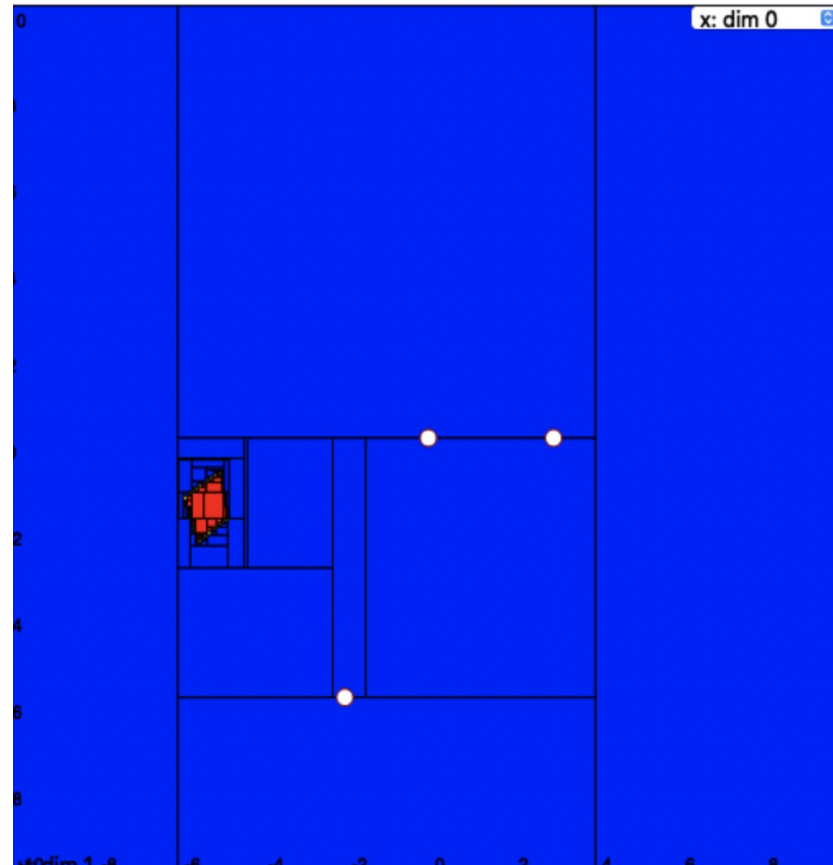
Train neural network with the input being 12 integers representing the coordinates and the interval distances.

We use sigmoid activation and train during 50 epochs with 90,000 images generated with interval computation.

Test different layers widths to compare accuracy, recall and model size.

# Localization using Codac

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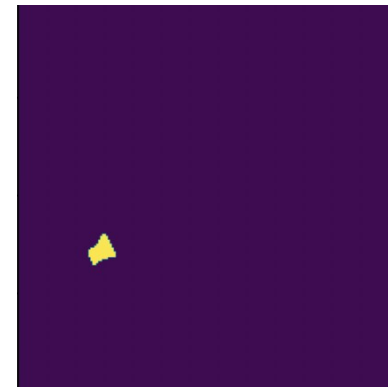
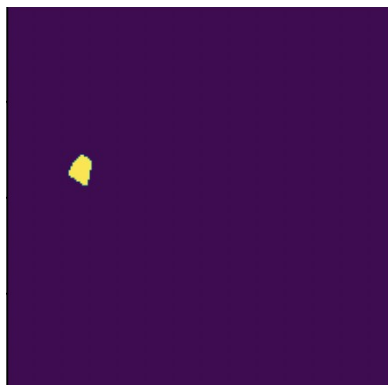
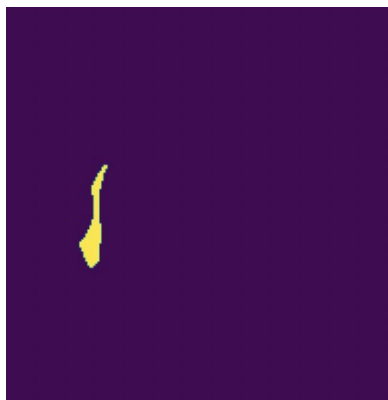


# First Approach

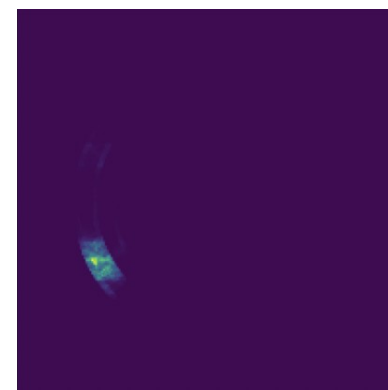
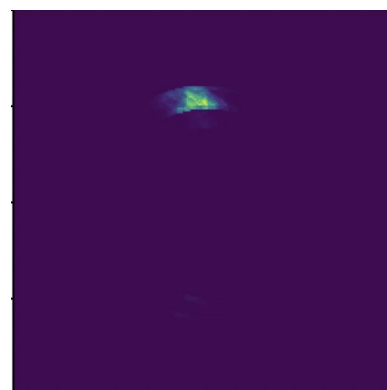
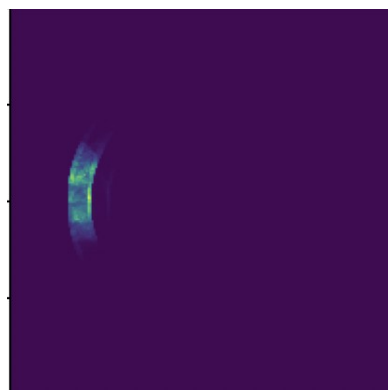
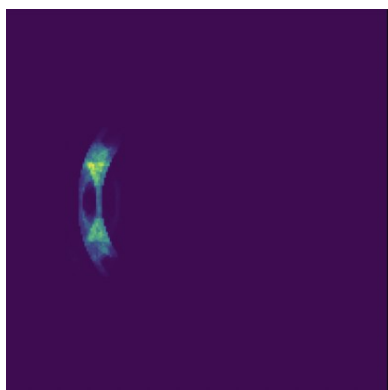
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Model	4 x 256 units	4 x 128 units	4 x 64 units	4 x 32 units	4 x 16 units
Test Accuracy	99.87%	99.81%	99.77%	99.71%	99.67%
Test Precision	82.46%	75.09%	68.85%	63.18%	44.69%
Test Recall	<b>79.38%</b>	<b>62.64%</b>	<b>52.61%</b>	<b>28.42%</b>	<b>08.13%</b>
Model size	125 MG	64 MG	32 MG	17 MG	8 MG
Time of prediction	~0.015s				

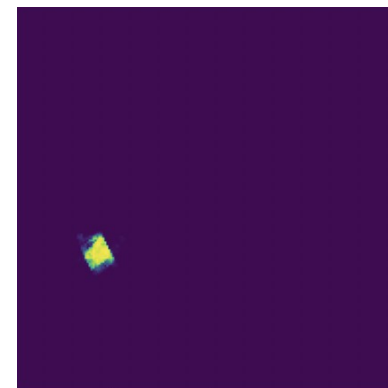
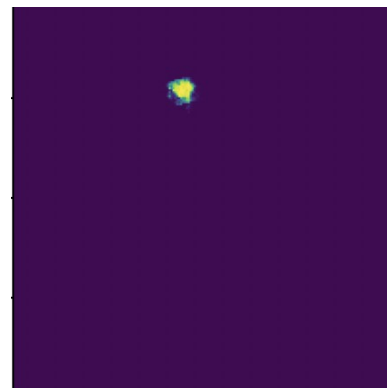
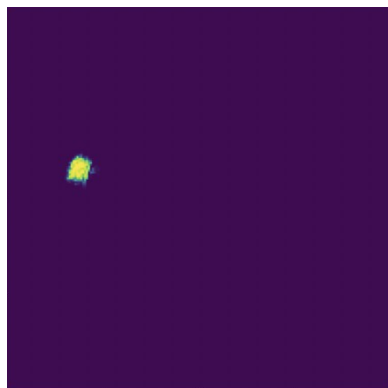
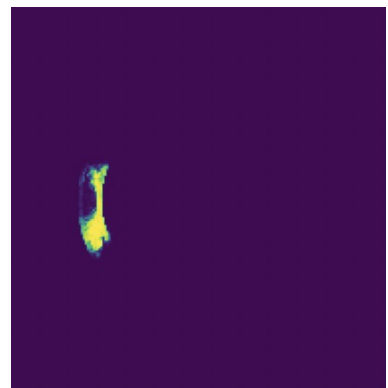
Expected  
output



4x16  
layer  
output



4x256  
layer  
output

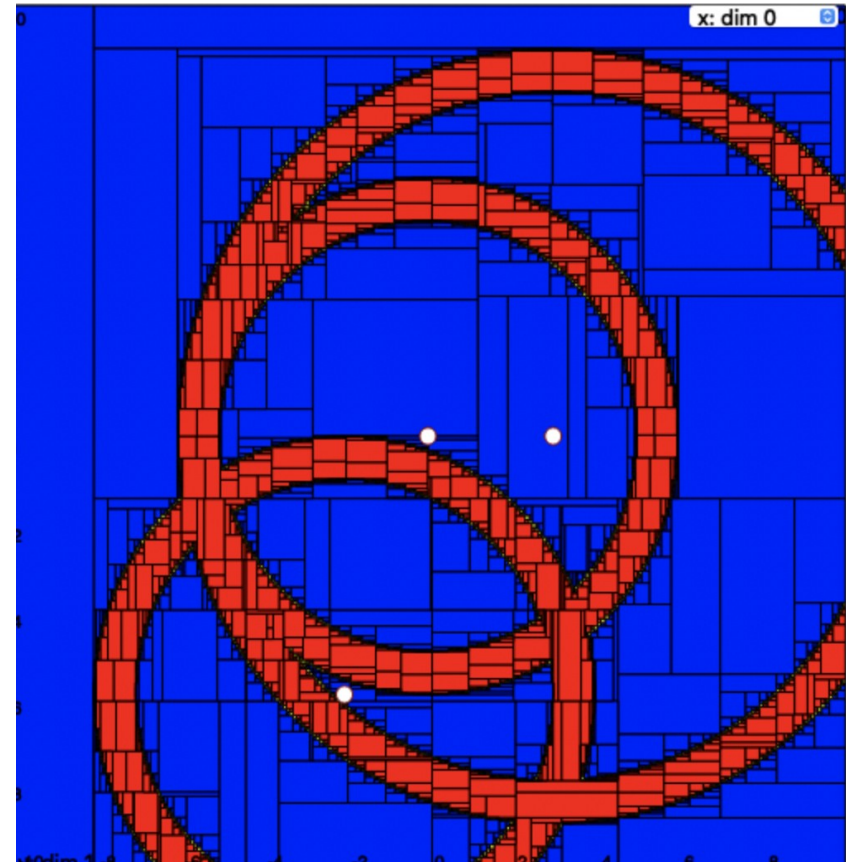




# Second Approach

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Train neural network for individual landmarks and intersect results.



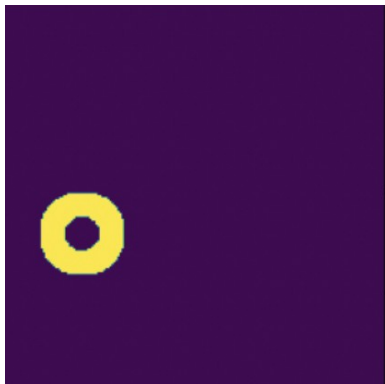
# Second Approach

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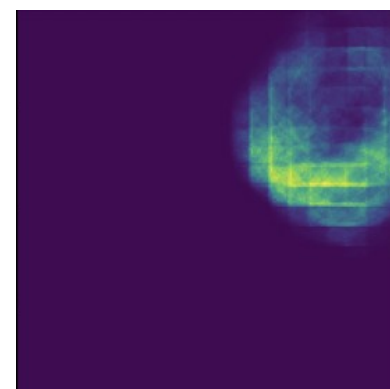
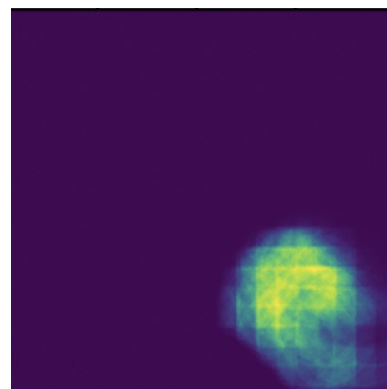
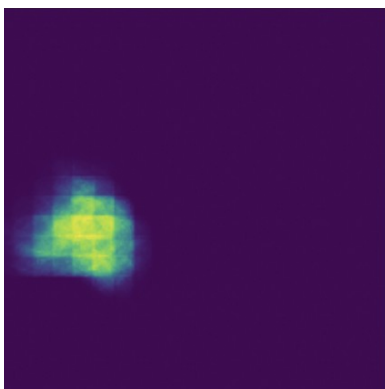
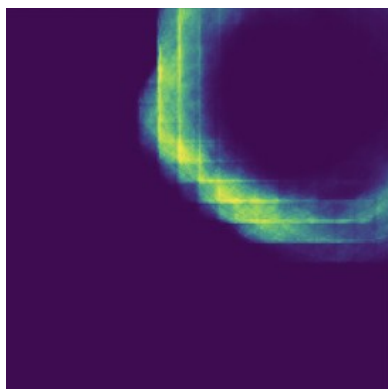
Model	4 x 256 units	4 x 128 units	4 x 64 units	4 x 32 units	4 x 16 units
Test Accuracy	99.99%	99.52%	98.24%	96.84%	94.68%
Test Precision	99.96%	95.82%	85.49%	76.09%	62.06%
Test Recall	<b>99.95%</b>	<b>96.65%</b>	<b>86.81%</b>	<b>72.67%</b>	<b>39.57%</b>
Model size	125 MG	64 MG	32 MG	17 MG	8 MG
Time of prediction	~0.015s				

*test results for individual outputs, not final image*

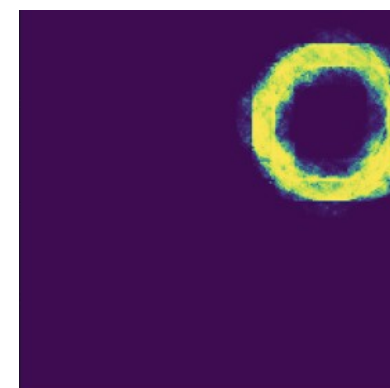
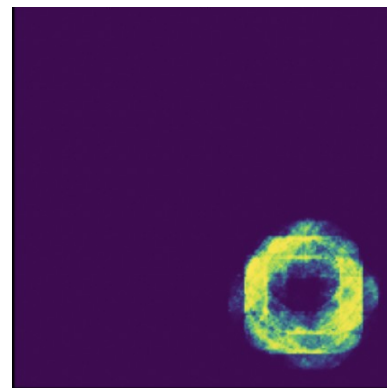
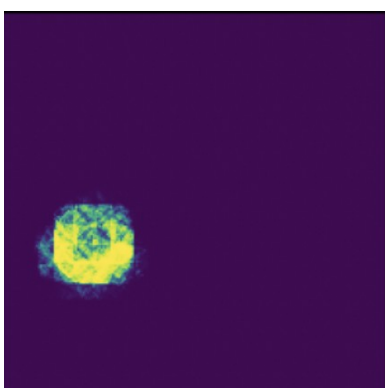
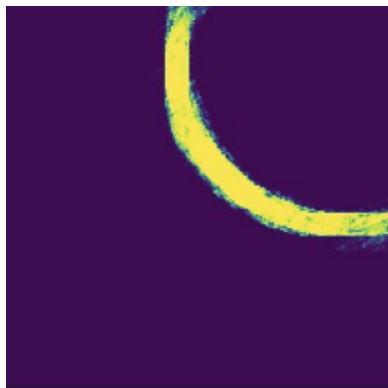
Expected  
output



4x16  
layer  
output



4x256  
layer  
output



# Comparison of Approaches

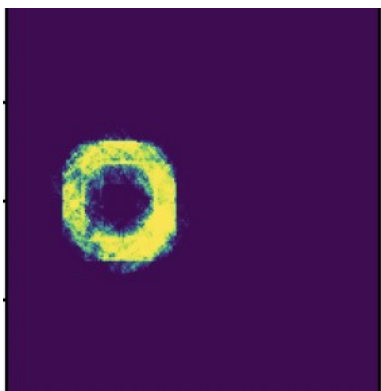
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Model	4 x 256 units	4 x 128 units	4 x 64 units	4 x 32 units	4 x 16 units
Test recall of 1st approach	79.38%	62.64%	52.61%	28.42%	08.13%
Test recall of 2nd approach	97.02%	88.40%	63.64%	43.32%	11.89%
Model size	125 Mo	64 MG	32 MG	17 MG	8 MG
Time of prediction	~0.015s vs ~0.045s				

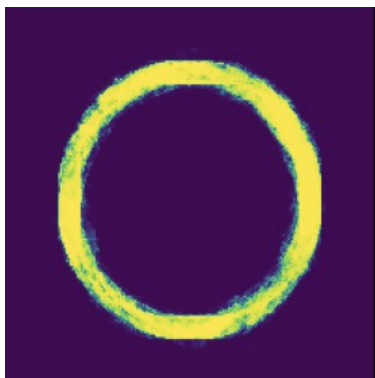
Expected  
output

Second  
approach

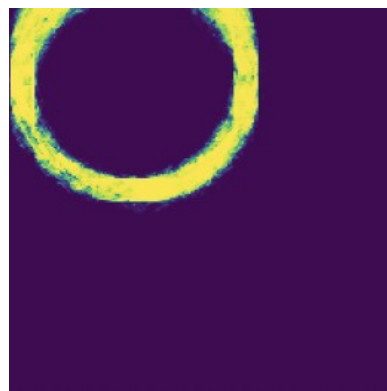
First  
approach



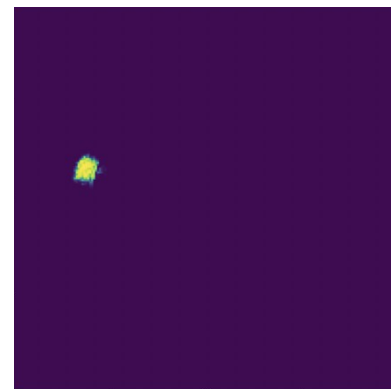
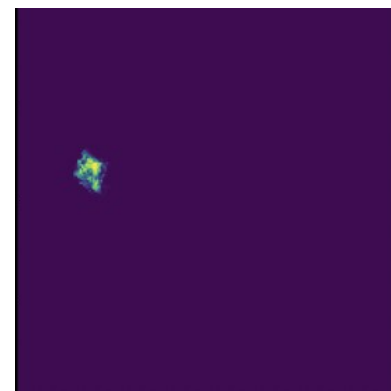
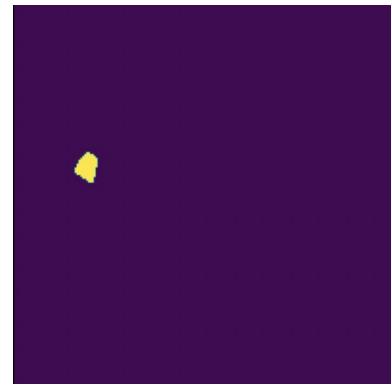
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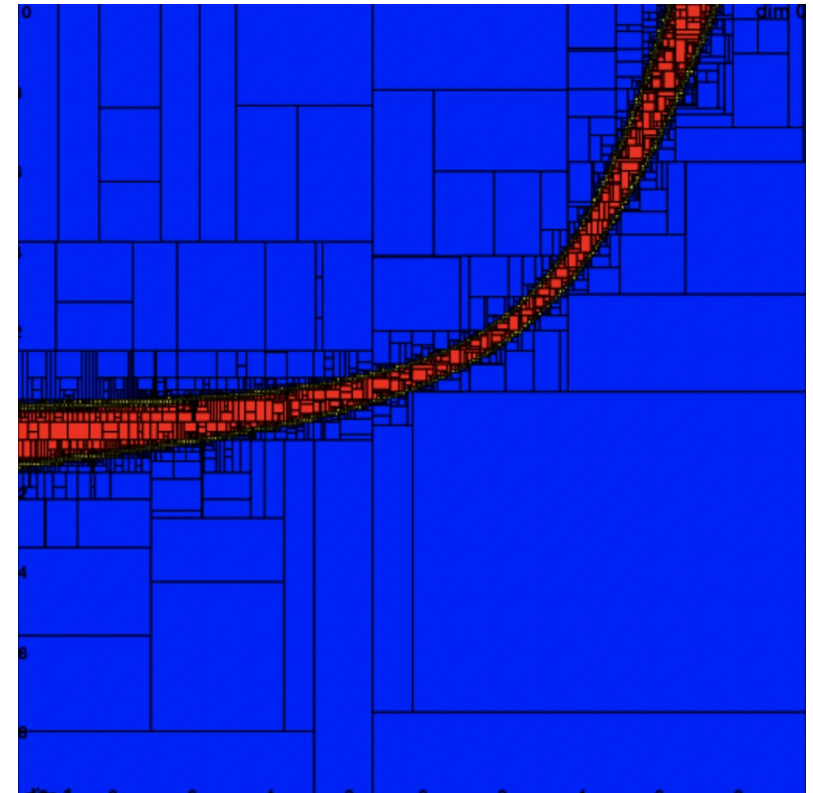


# 2<sup>nd</sup> Problem Definition: TDoA

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Time Difference of Arrival - Given coordinates of landmarks and interval difference of distances between any two landmarks and robot, predict the area where the robot is localized.

Objective - Replicate outputs produced by interval analysis methods, using neural networks.



# Results

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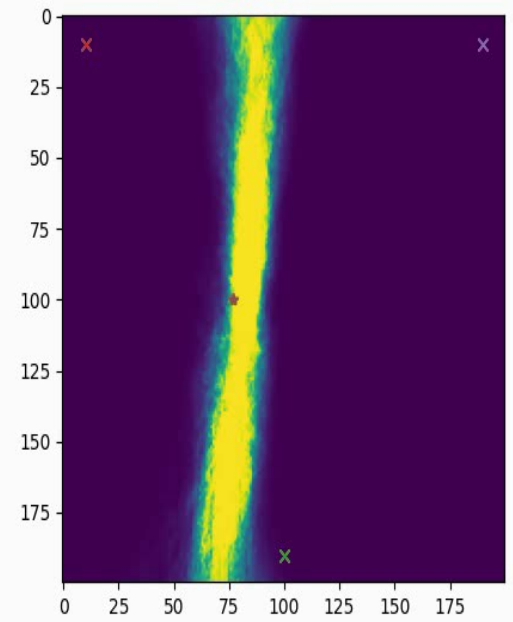
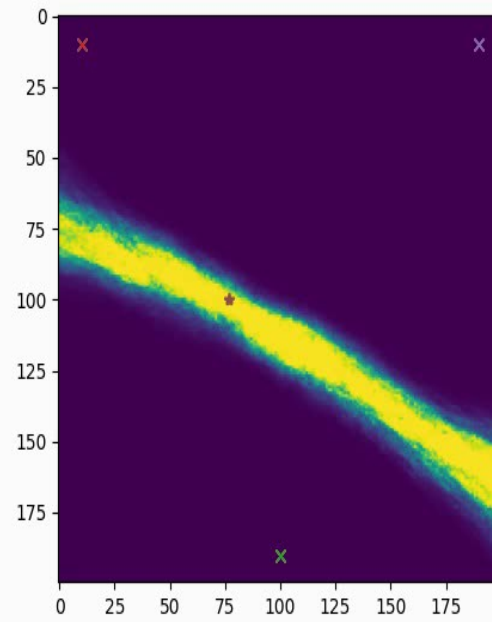
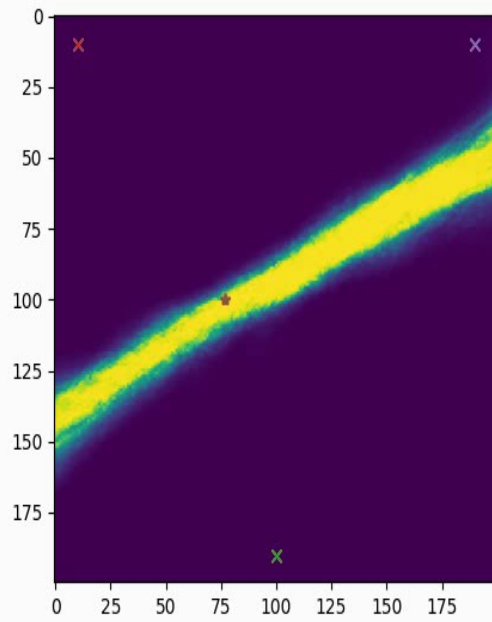
Model	4 x 256 units	4 x 128 units	4 x 64 units	4 x 32 units	4 x 16 units
Test Accuracy	98.84%	98.46%	97.54%	96.22%	94.43%
Test Precision	90.14%	87.65%	81.49%	71.90%	61.91%
Test Recall	92.23%	88.66%	80.07%	68.14%	35.72%
Test Recall for Final Output	90.06%	77.43%	54.11%	36.41%	4.26%

# Circular Path Test

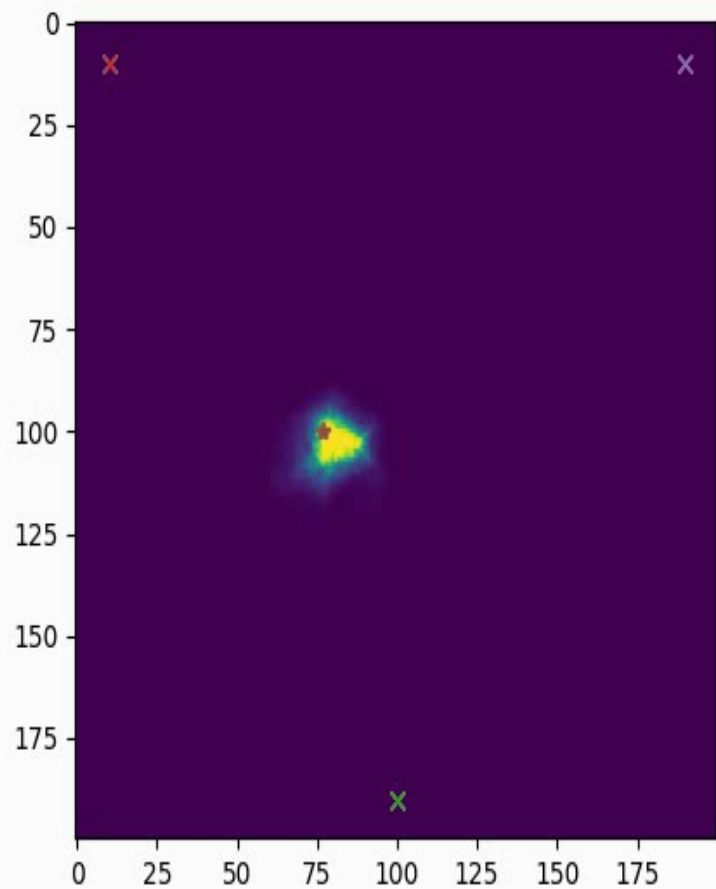
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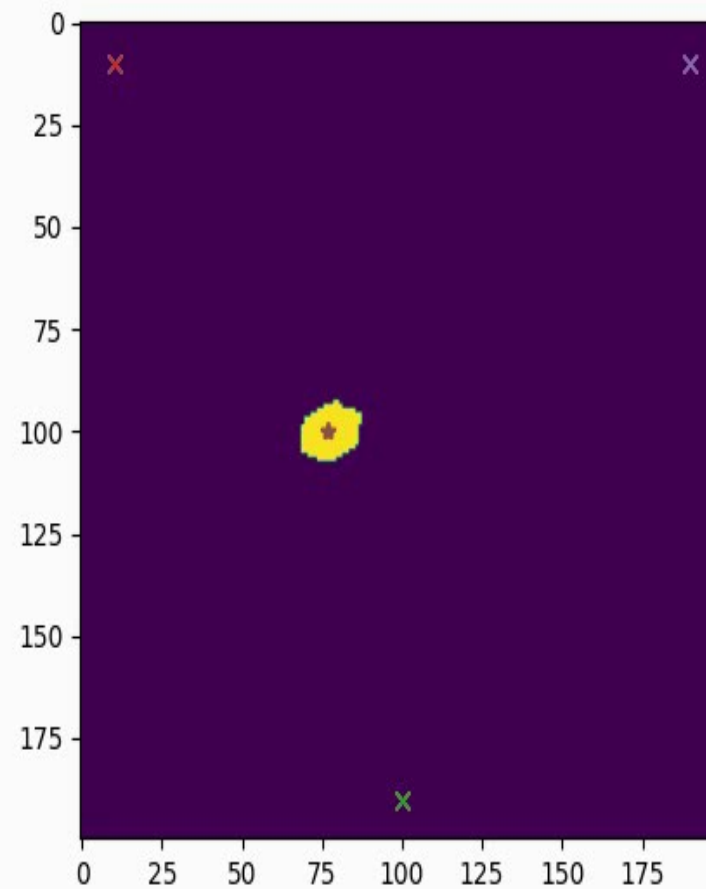
## Per pair of landmarks



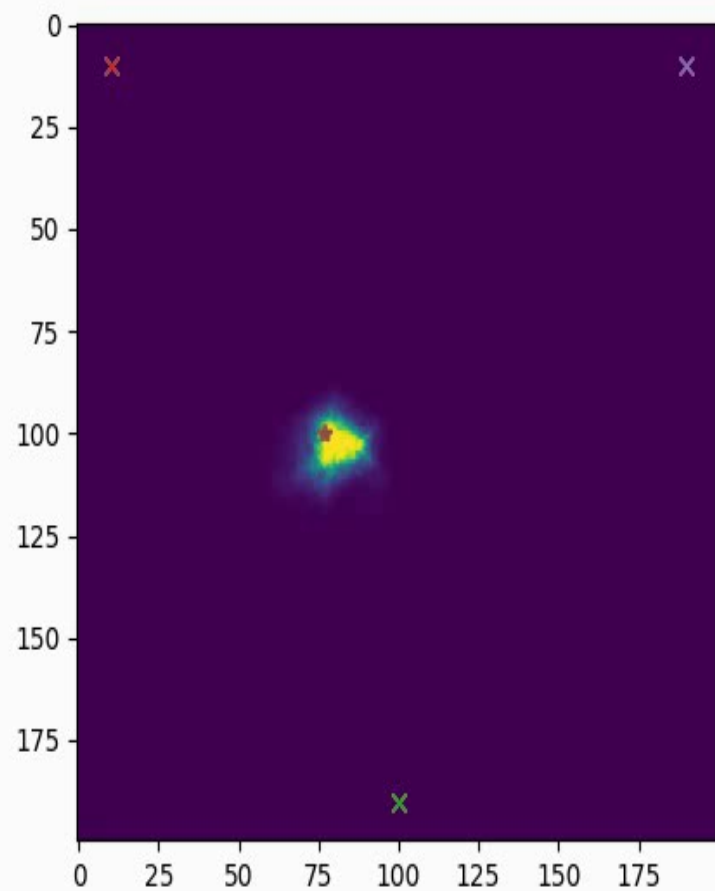
# Neural Network



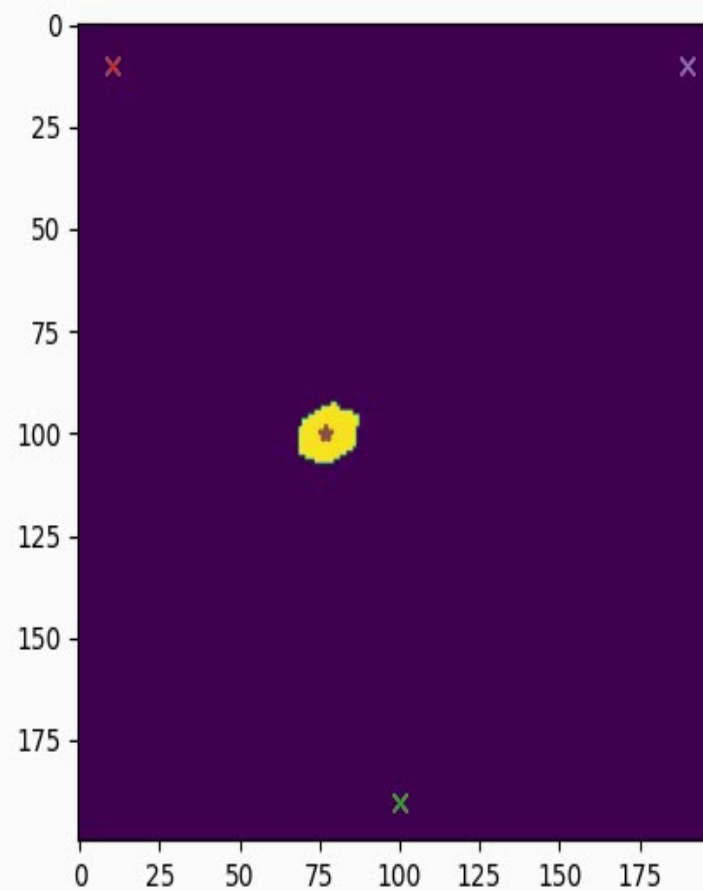
# Interval Computation



# Neural Network



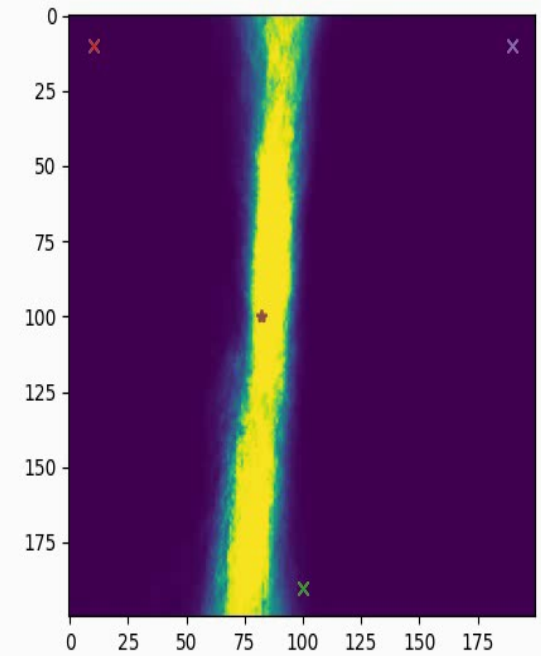
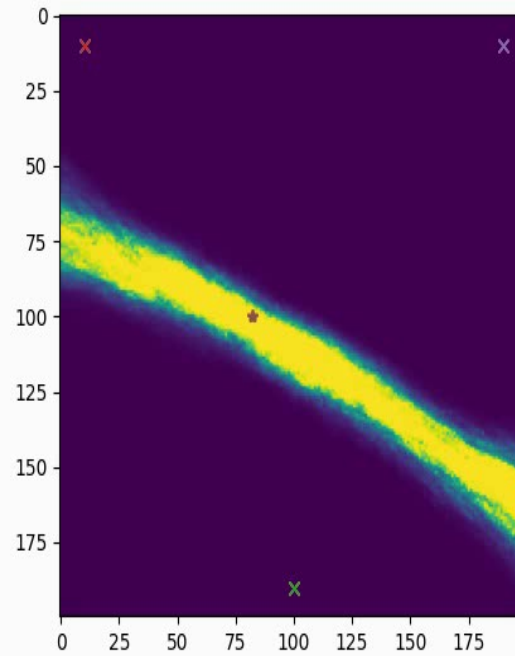
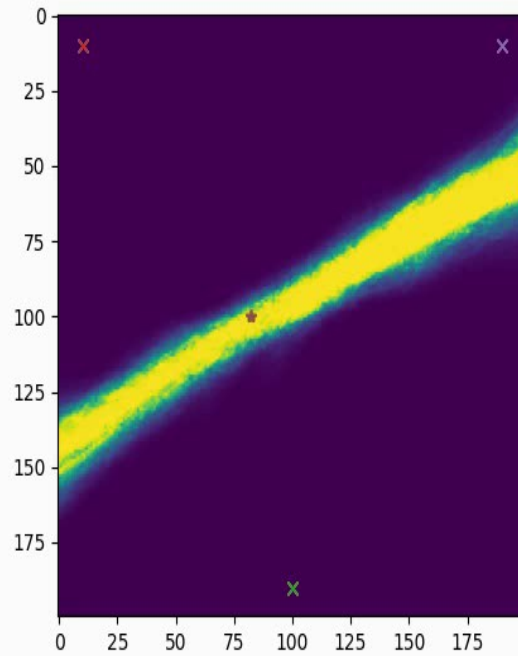
# Interval Computation



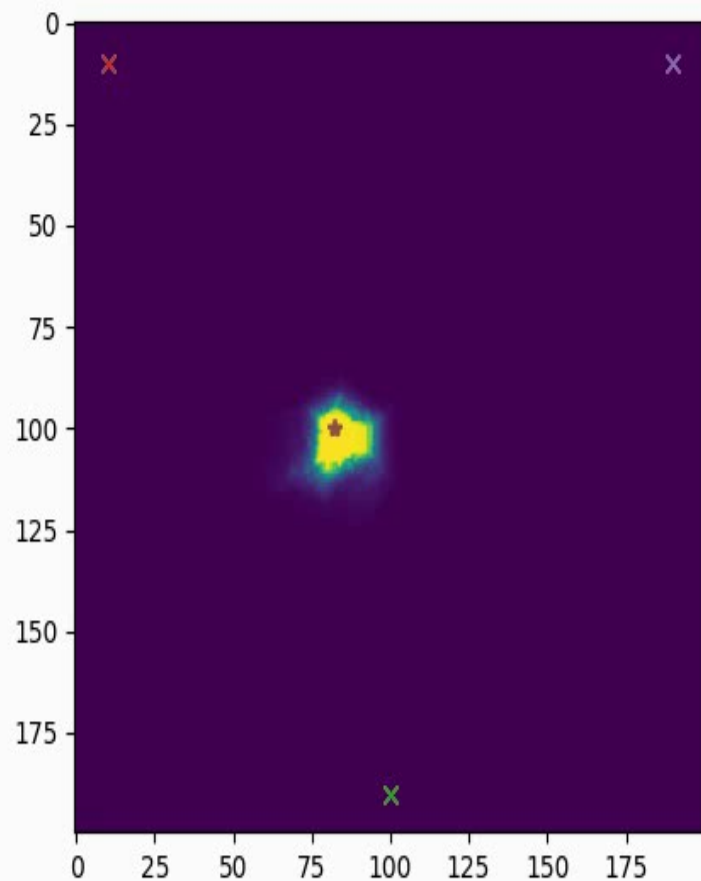
# Straight Path Test

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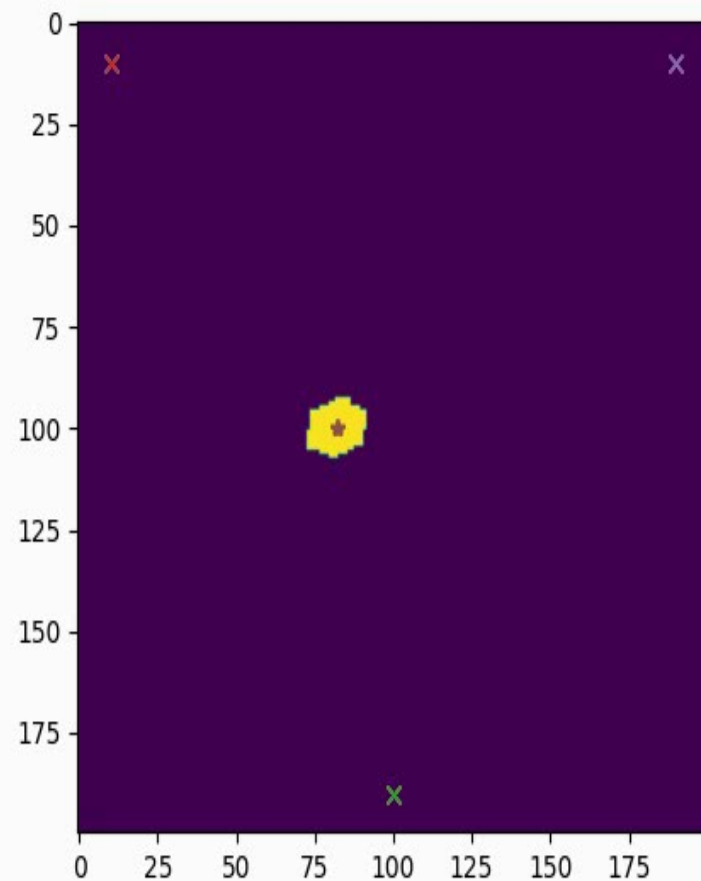
Per pair of landmarks



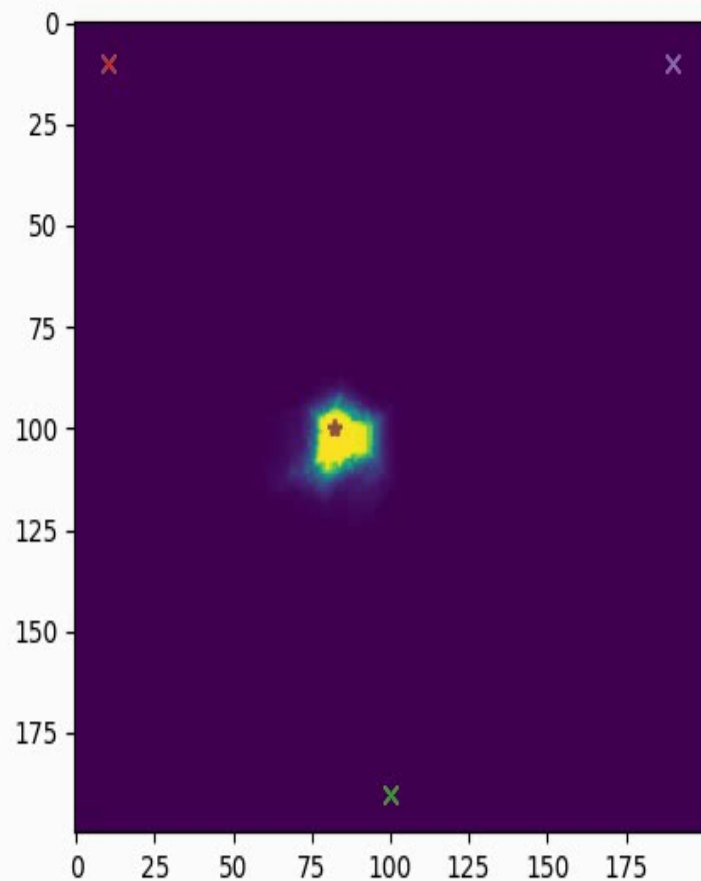
# Neural Network



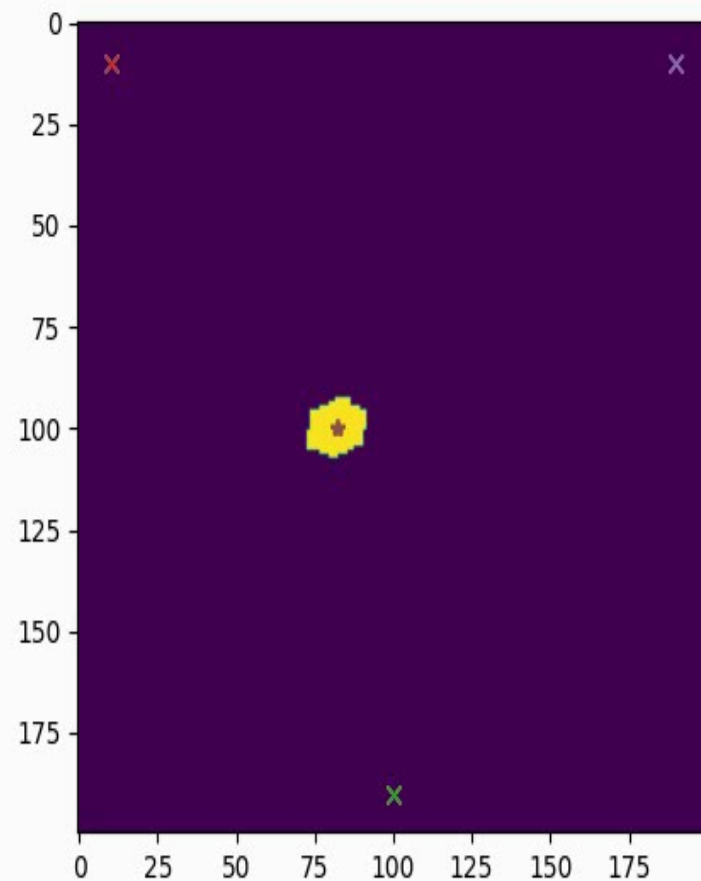
# Interval Computation



# Neural Network



# Interval Computation



# Conclusion

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Neural Networks reduce time to localize objects, although not significantly. In our examples, they reduced computation time by a third approximately.

Neural Networks may be used to represent more complex relations that may be difficult to graph with simple equations.

Neural Networks can offer highly accurate localization although there will still be false negatives present, unlike in interval computation.



# Future Developments

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Add more data where robot is localized near boundaries created by landmarks.

Test different architecture depths.

Train models on fixed landmarks and use geometric transformations to generalize outputs to different landmark locations.

Use pruning methods and TensorFlow Lite to reduce the size of neural networks.

Search and test on more complex relations.

# References

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***Thank you  
for your  
attention!***