Reference Governors for Linear Systems with Uncertain Polynomial Constraints and Disturbances

Ensta Bretagne
March 10, 2023

by

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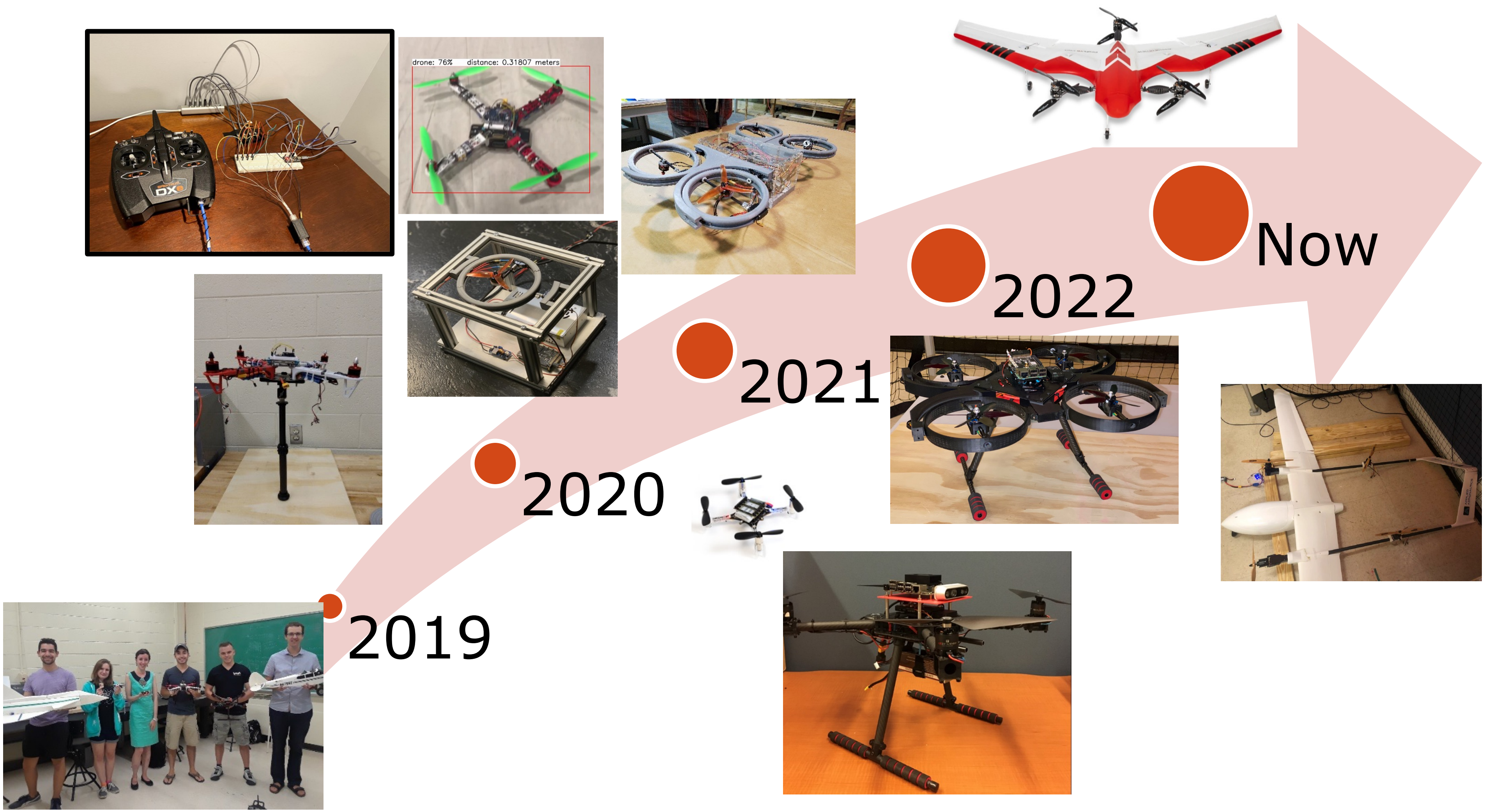
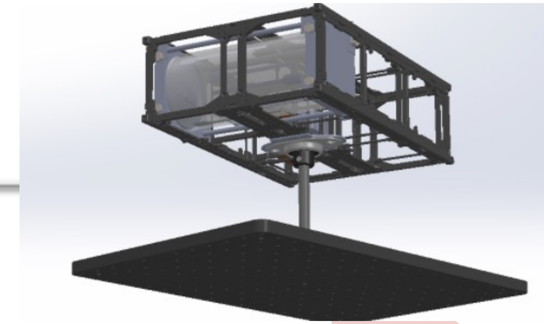
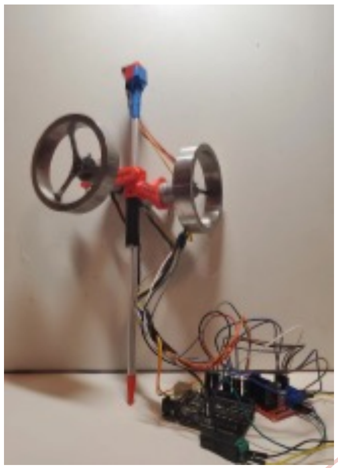
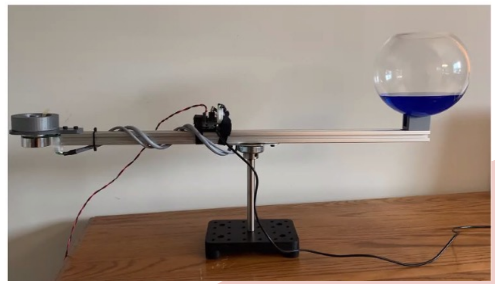
My Laboratory (Drones)



My Laboratory (nanosatellites)







2019

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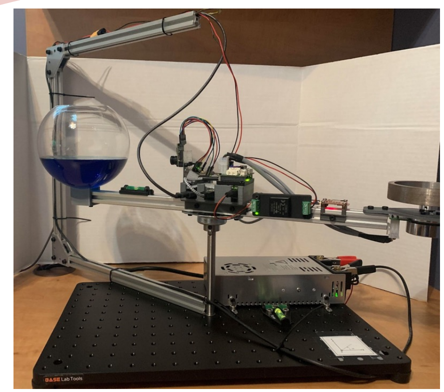

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1. Introduction and Motivation

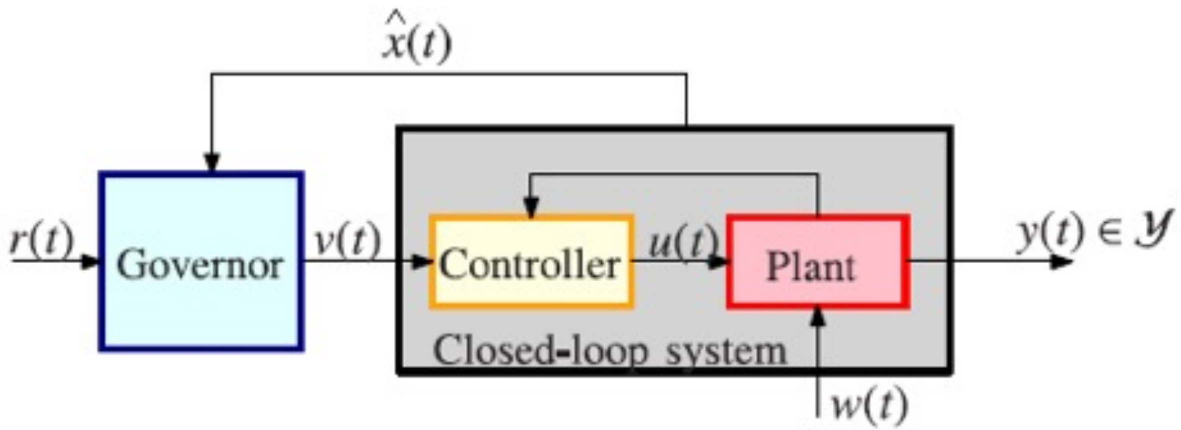
The capabilities of an aircraft in terms of speed, load, and altitude are referred to as its flight envelope.

Ensuring that an aircraft remains within its flight envelope is essential to prevent loss of control (LoC).

Envelope protection currently entails preventing specific constraint violations.

The present work is devoted to methods that prevent constraint violations based on reference governors.

Preliminaries on conventional Reference Governors



E. Garone et al. / Automatica 75 (2017)

1. Augment rather than replace an existing nominal controller.
2. Inactive if no danger of constraint violation.
3. ‘Easy’ to implement / Fast online computations.
4. Special properties.

Preliminaries on conventional Reference Governors

$$x(k+1) = Ax(k) + Bv(k)$$
$$y(k) = C\,x(k) + D\,v(\mathrm{k})$$

Prediction

$$\hat{y}(k|v,x) = C\,A^k x(k) + C\,(I-A)^{-1}(I-A^k)Bv + D\,v$$

Maximum Output Admissible Set (MOAS)

$$\tilde{O}_\infty = \{(v,x)|\hat{y}(k|v,x) \in Y, k = 0 \ldots k^*\} \cap O^\epsilon$$

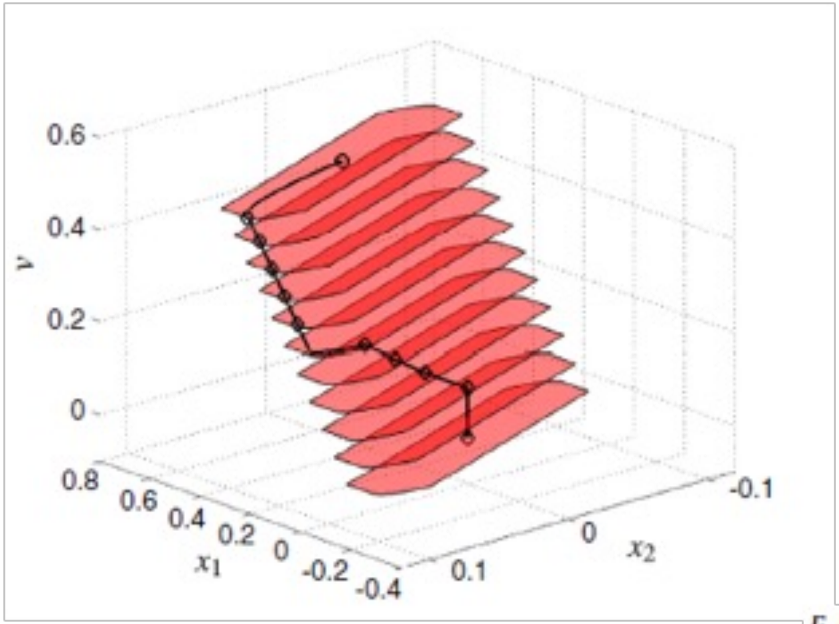
where

$$O^\epsilon \;= \{(v,x)|\bar{y}_v \in (1-\epsilon)Y\}$$

Preliminaries on conventional Reference Governors

Reference governor computation

$$\begin{aligned}\kappa(t) = \max_{\kappa\in[0,1]} \quad & \kappa \\ \text{s.t.} \quad & v = v(t-1) + \kappa(r(t) - v(t-1)), \\ & (v, x(t)) \in \tilde{O}_\infty,\end{aligned}$$



E. Garone et al. / Automatica 75 (2017)

Preliminaries on conventional Reference Governors

$\tilde{O}_\infty$ computation is based on two ingredients:

- Every time we compute the next value of the output, we add and therefore stack some linear inequality constraints,

- Every time we add these new linear inequality constraints, we check if they are redundant with the previous ones. In case, they are all redundant, we stop the algorithm.

Limitations of the Conventional Reference Governors

Nonlinear constraints

It is ‘hard’ to eliminate redundant constraints

Parametric uncertainties

It is ‘hard’ to propagate the constraints through the uncertain dynamics

Constraints elimination and nonlinearities

$$x(k+1) = Ax(k) + Bv$$

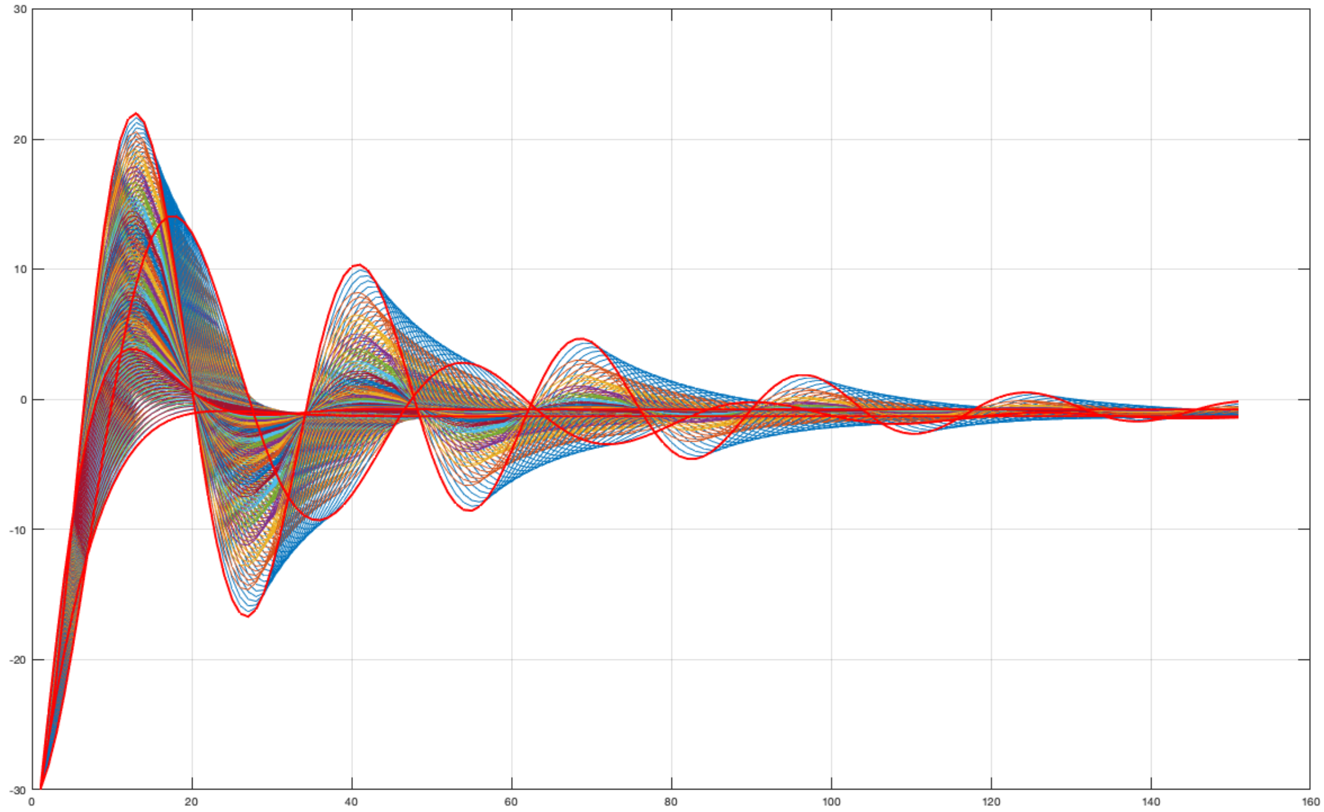
$$y(k) = \sum_{i=1}^{p} C_i \begin{bmatrix} x(k) \\ v \end{bmatrix}^i$$

Given a set defined by some polynomial inequality constraints, how do you efficiently determine that a new polynomial inequality constraint is redundant or not?

A Cotorruelo, I Kolmanovsky, E Garone, "A sum-of-squares-based procedure to approximate the Pontryagin difference of basic semi-algebraic sets", Automatica 135, 109783, 2022.

Constraints propagation through uncertain dynamics

$$x(k+1) = A(U)x(k) + B(U)v$$
$$y(k) = C\,x(k) + D\,v$$



The Quadrotor Model

Translation equations of motion involving aerodynamic effects:

$$\dot{v} = -ge_3 + cRe_3 - RDR^T v - RC\Omega$$

Rotational equations of motion:

$$J\dot{\Omega} = -\Omega \times J\Omega - \tau_G + \tau - AR^T v - B\Omega$$

Assumptions:

- the propellers are rigid (i.e., C = 0),
- the collective thrust remains equal to the commanded collective thrust,
- the yaw angle ψ and its derivatives remain equal to 0,
- the drone flies at a constant altitude z,
- the pitch and roll angles are small all the time, and
- the inertia matrix is diagonal and $J_{xx} = J_{yy}$.

Trajectory Tracking

Introduce change of coordinates to decouple the x and y dynamics:

$$u_1 = \tau_1 - a_{11}v_x - b_{12}q - a_{13}(\theta v_x - \phi v_y),$$
$$u_2 = \tau_2 - a_{22}v_y - b_{21}p - a_{23}(\theta v_x - \phi v_y).$$

Which results in the longitudinal and lateral motions:

Longitudinal

$$\dot{x} = v_x$$
$$\dot{v}_x = g\theta - d_x v_x$$
$$\dot{\theta} = q$$
$$J_{yy}\dot{q} = u_2 - a_{21}v_x - b_{22}q$$

Lateral

$$\dot{y} = v_y$$
$$\dot{v}_y = -g\phi - d_y v_y$$
$$\dot{\phi} = p$$
$$J_{xx}\dot{p} = u_1 - a_{12}v_y - b_{11}p$$

Longitudinal Tracking

u_2 can be specified such that the v_x dynamics enjoy suitable asymptotic tracking properties

$$u_2 = -K_{lon}X_{lon} + \frac{J_{yy}}{g}k_1 v_{lon}.$$

Defining $X_{lon} = [v_x \quad \theta \quad q]^T$, the pre-stabilized system can be rewritten as

$$\dot{X}_{lon} = A_{lon}X_{lon} + B_{lon}v_{lon}.$$

u_2 is designed such that when v_{lon} is constant, then v_x asymptotically tracks the constant v_{lon}, but when $k_1 v_{lon} = r_2(t)$, then x asymptotically tracks x_c.

$x_c(t)$ is the desired reference trajectory for $x(t)$ to track.

Lateral Tracking

Equivalent measures can be taken in the lateral direction where it can be found that

$$u_1 \coloneqq -K_{lat}X_{lat} - \frac{J_{xx}}{g}k_1\nu_{lat}$$

where $X_{lat} \coloneqq [v_y \quad \phi \quad p]^T$

and the pre-stabilized system can be rewritten as

$$\dot{X}_{lat} = A_{lat}X_{lat} + B_{lat}\nu_{lat}.$$

Propeller Thrust Limitations

The collective thrust and torques are given by:

$$\begin{bmatrix} mc \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = M_{eff} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

where the control effectiveness matrix M_{eff} is invertible and is one of a quadrotor operating in the cross configuration.

The following constraints are then enforced:

$$M_{eff}^{-1} \begin{bmatrix} mc \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = M_{eff}^{-1} \begin{bmatrix} mg - md_x\theta v_x + md_y\phi v_y \\ -K_{latt}X_{lat} - \frac{J_{xx}}{g}k_1 v_{lat} + a_{11}v_x + b_{12}q + a_{13}(\theta v_x - \phi v_y) \\ -K_{lon}X_{lon} - \frac{J_{yy}}{g}k_1 v_{lon} + a_{22}v_y + b_{21}p + a_{23}(\theta v_x - \phi v_y) \\ a_{31}v_x + a_{32}v_y + a_{33}(\theta v_x - \phi v_y) \end{bmatrix} \epsilon [0, T_{max}]^4$$

2. Proposed solution: a Reference Governor for Polynomial Constraints

Considering the previously established dynamics

$$\dot{X}_{lon} = A_{lon}X_{lon} + B_{lon}v_{lon}$$

$$\dot{X}_{lat} = A_{lat}X_{lat} + B_{lat}v_{lat}.$$

Additionally, the reference dynamics are defined by

$$\dot{v}_{lon} = -\beta v_{lon} \qquad \dot{v}_{lat} = -\beta v_{lat}.$$

Discretizing the system and augmenting the state such that

$$Z := [\ [X_{lon}, X_{lat}, v_{lon}, v_{lat}], [X_{lon}, X_{lat}, v_{lon}, v_{lat}]^2\ ]$$

the system now takes the form

$$Z(k+1) = \Phi Z(k)$$

where the polynomial constraints are now expressed as linear constraints.

Algorithms

MOAS computation

1-Compute $\widetilde{O}_{\infty,Z_1}$

2-Compute $\widetilde{O}_{\infty,Z}$

Reference Governor updates

1- Initialization by solving a nonlinear optimization problem

2- Bisection algorithm

Numerical Results

MOAS computations:

First Step:

$\mathrm{Dim}(Z_1) = 8$

$\widetilde{\mathrm{O}}_{\infty,Z_1}$ is finitely determined in 84 iterations and is defined by 252 linear inequality constraints.

Second Step:

$\mathrm{Dim}(\mathrm{Z}) = 36$

$\widetilde{\mathrm{O}}_{\infty,Z}$ is finitely determined in 142 iterations and is defined by 390 linear inequality constraints.

Numerical Results

Consider the following cases:

Case 1: Starting close to the reference trajectory and following it at moderate speed.

$$[x(0), y(0)] = [x_c(0), y_c(0)] + [0.5, 0.5]\text{m}$$
$$\omega = 1\, rad/s$$

Case 2: Starting far from the reference trajectory and following it at moderate speed.

$$[x(0), y(0)] = [x_c(0), y_c(0)] + [2.0, 2.0]\text{m}$$
$$\omega = 1\, rad/s$$

Case 3: Starting close to the reference trajectory and following it at high speed

$$[x(0), y(0)] = [x_c(0), y_c(0)] + [0.5, 0.5]\text{m}$$
$$\omega = 2\, rad/s$$

Numerical Results

Same cases while following a more complex trajectory (Figure ‘8’)

Case 1b: Starting close to the reference trajectory and following it at moderate speed.

$$[x(0), y(0)] = [\mathrm{x_c}(0), y_c(0)] + [0.5, 0.5]\mathrm{m}$$
$$\omega = 1\, rad/s$$

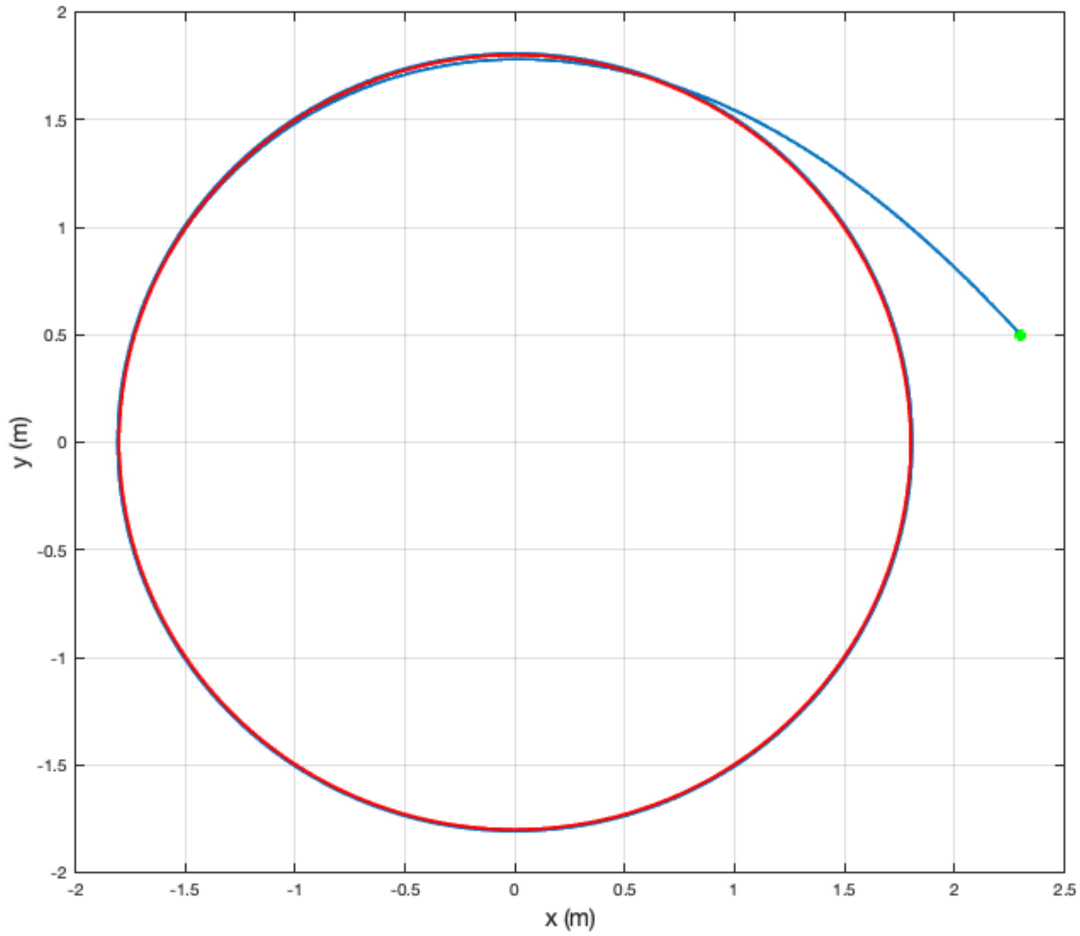
Case 2b: Starting far from the reference trajectory and following it at moderate speed.

$$[x(0), y(0)] = [\mathrm{x_c}(0), y_c(0)] + [2.0, 2.0]\mathrm{m}$$
$$\omega = 1\, rad/s$$

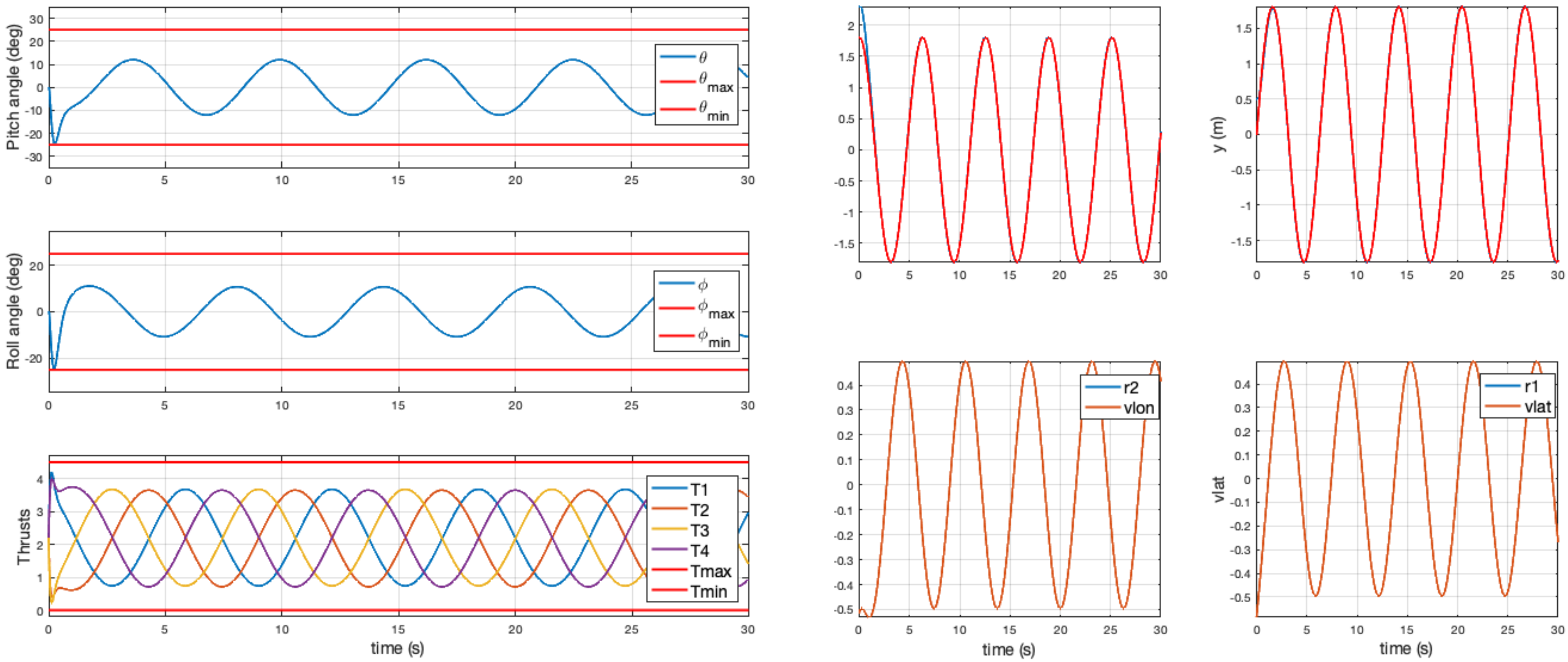
Case 3b: Starting close to the reference trajectory and following it at high speed

$$[x(0), y(0)] = [\mathrm{x_c}(0), y_c(0)] + [0.5, 0.5]\mathrm{m}$$
$$\omega = 2\, rad/s$$

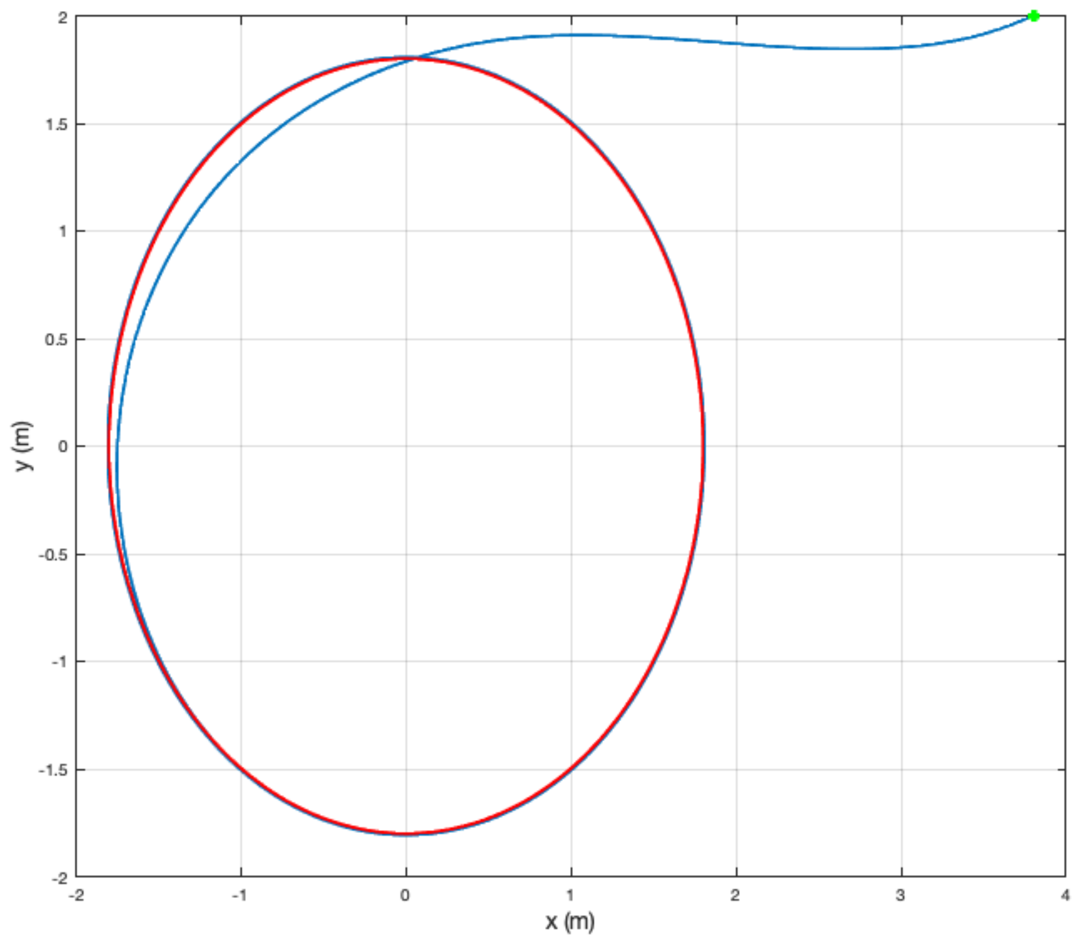
Numerical Results: Case 1



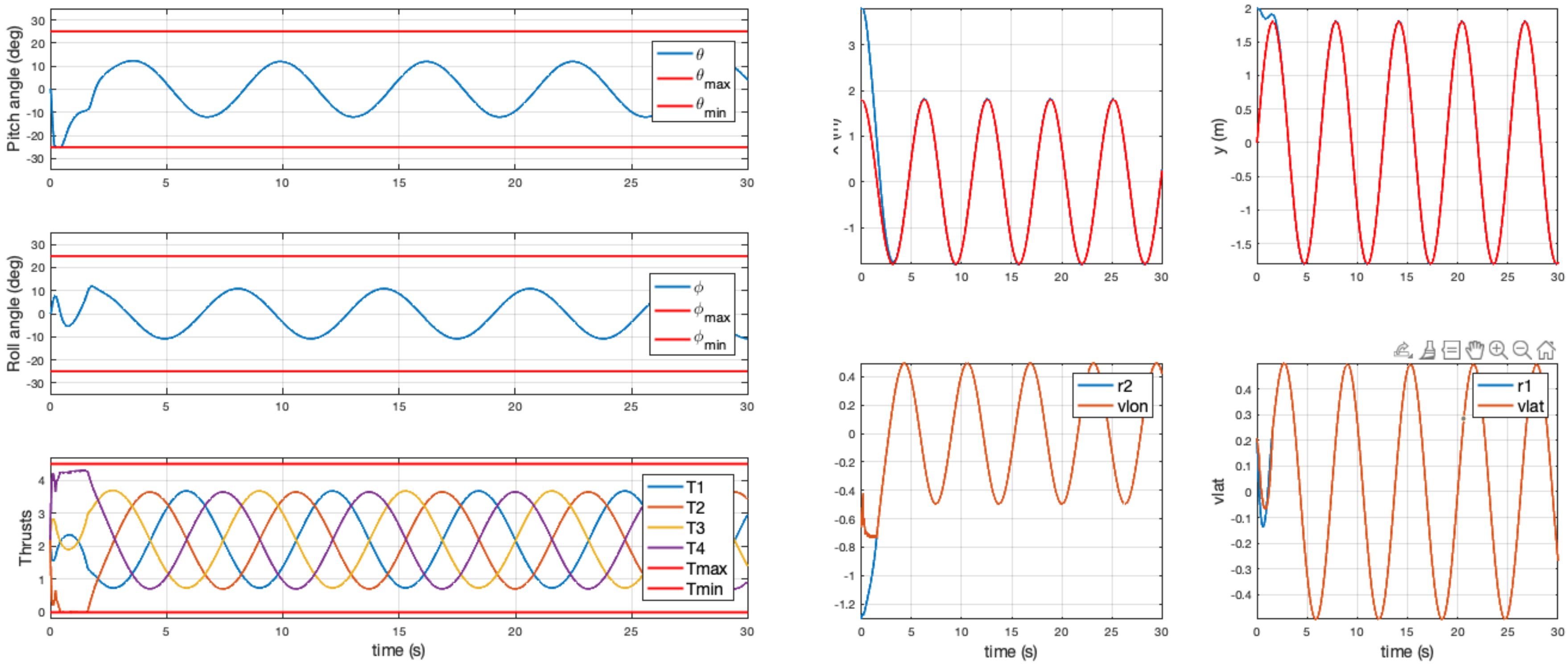
Numerical Results: Case 1



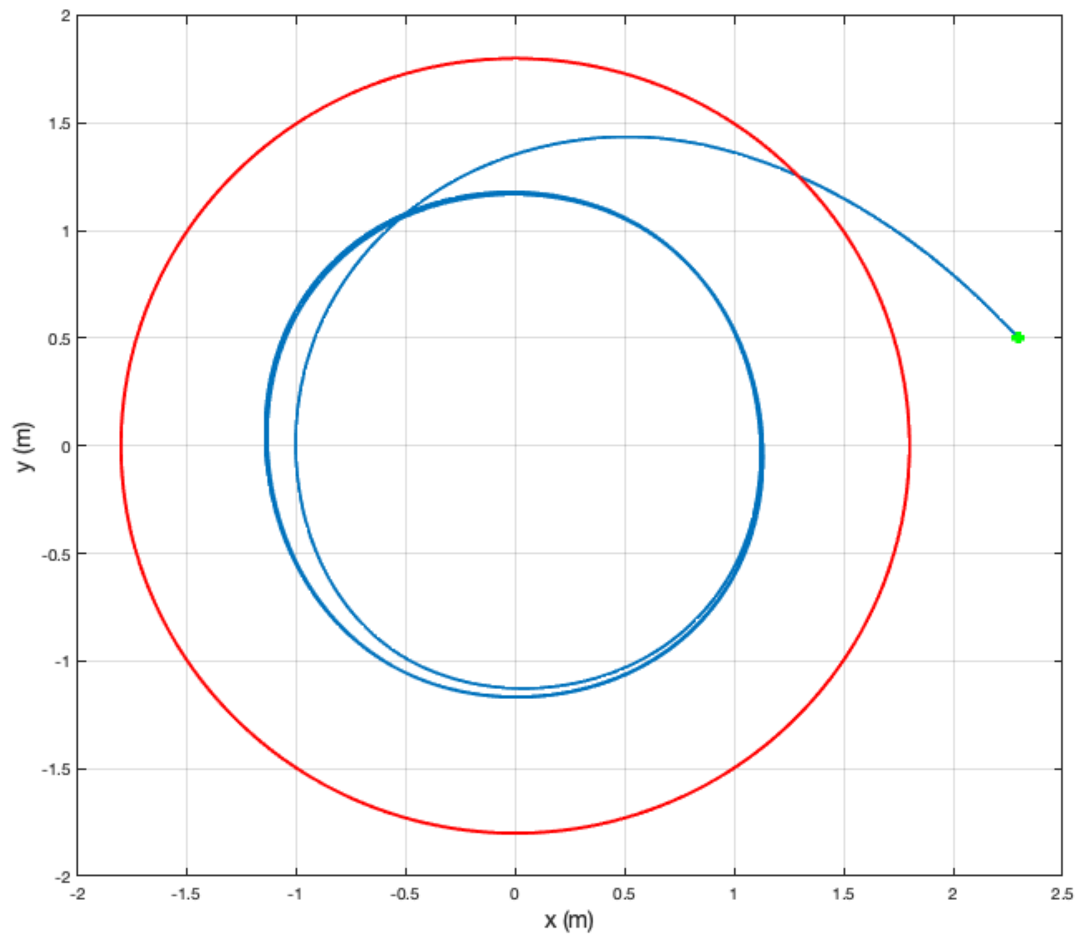
Numerical Results: Case 2



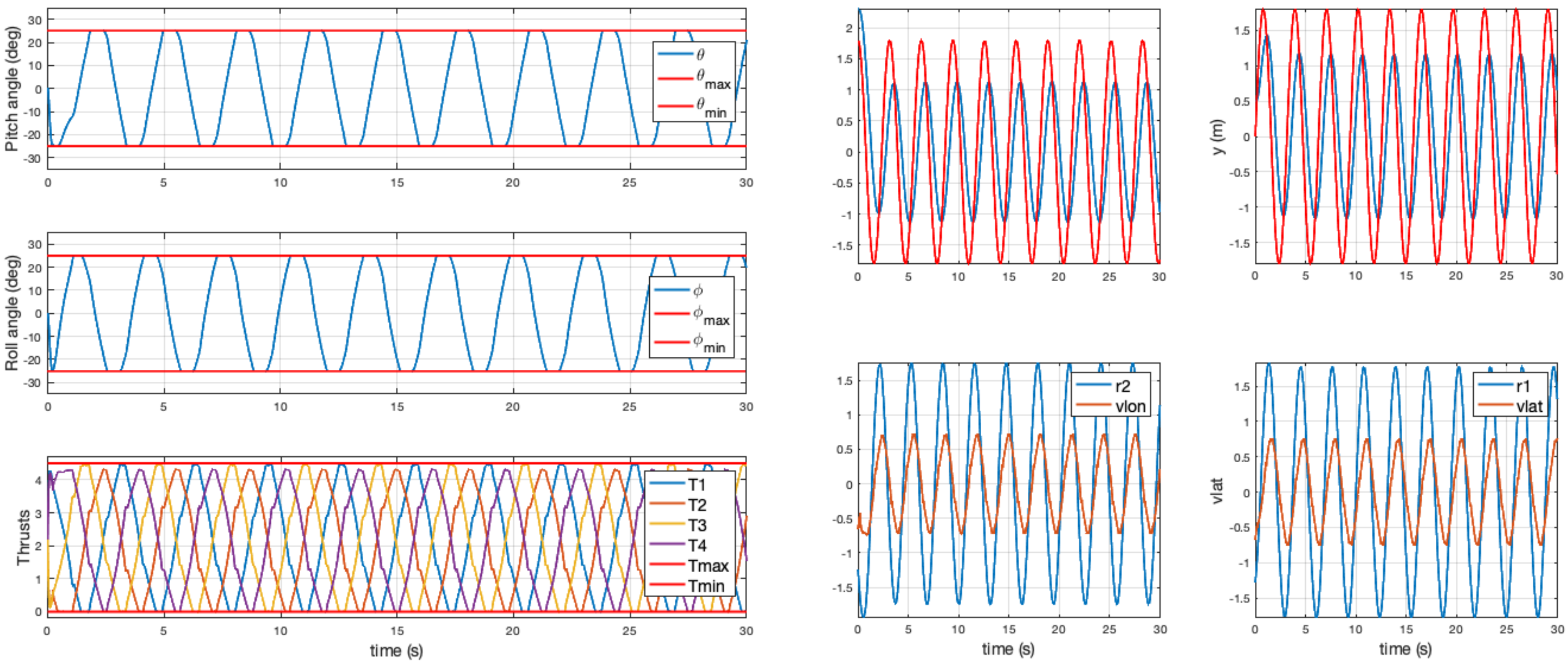
Numerical Results: Case 2



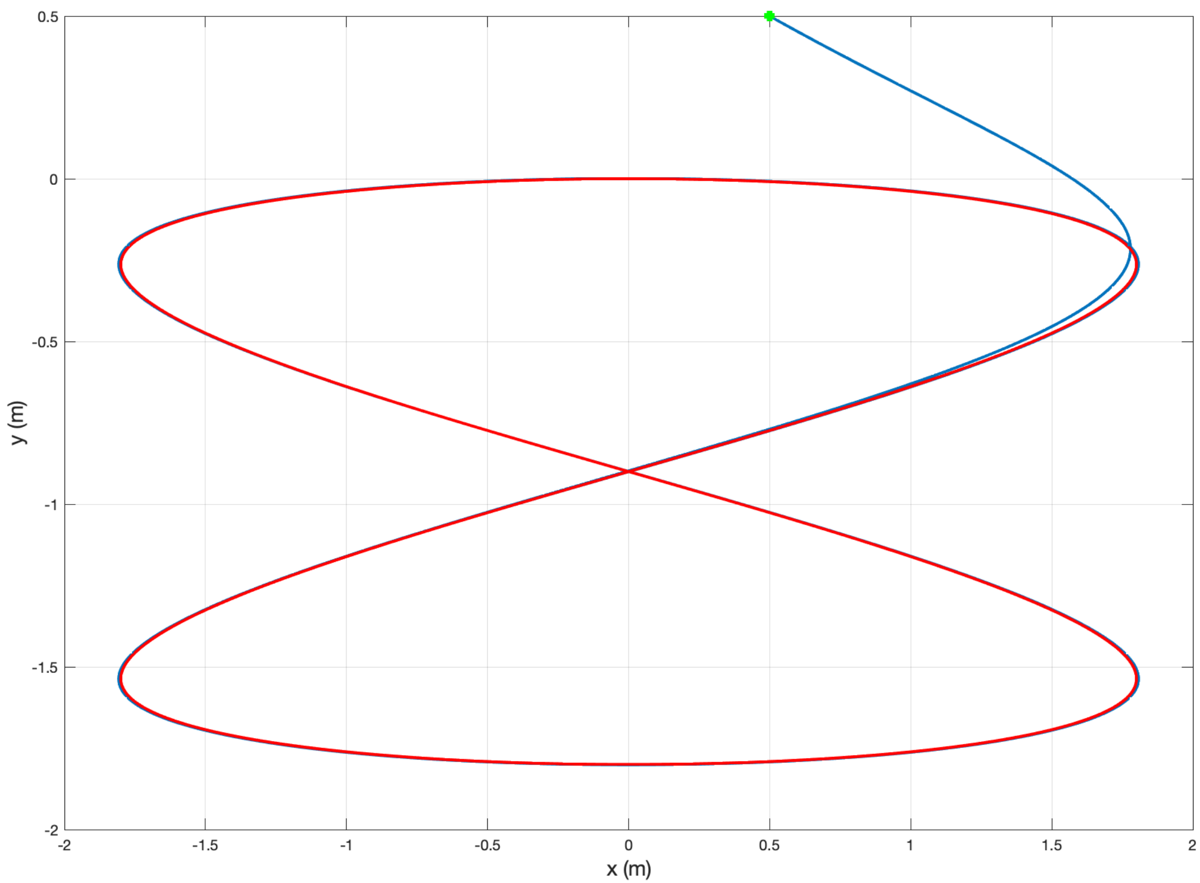
Numerical Results: Case 3



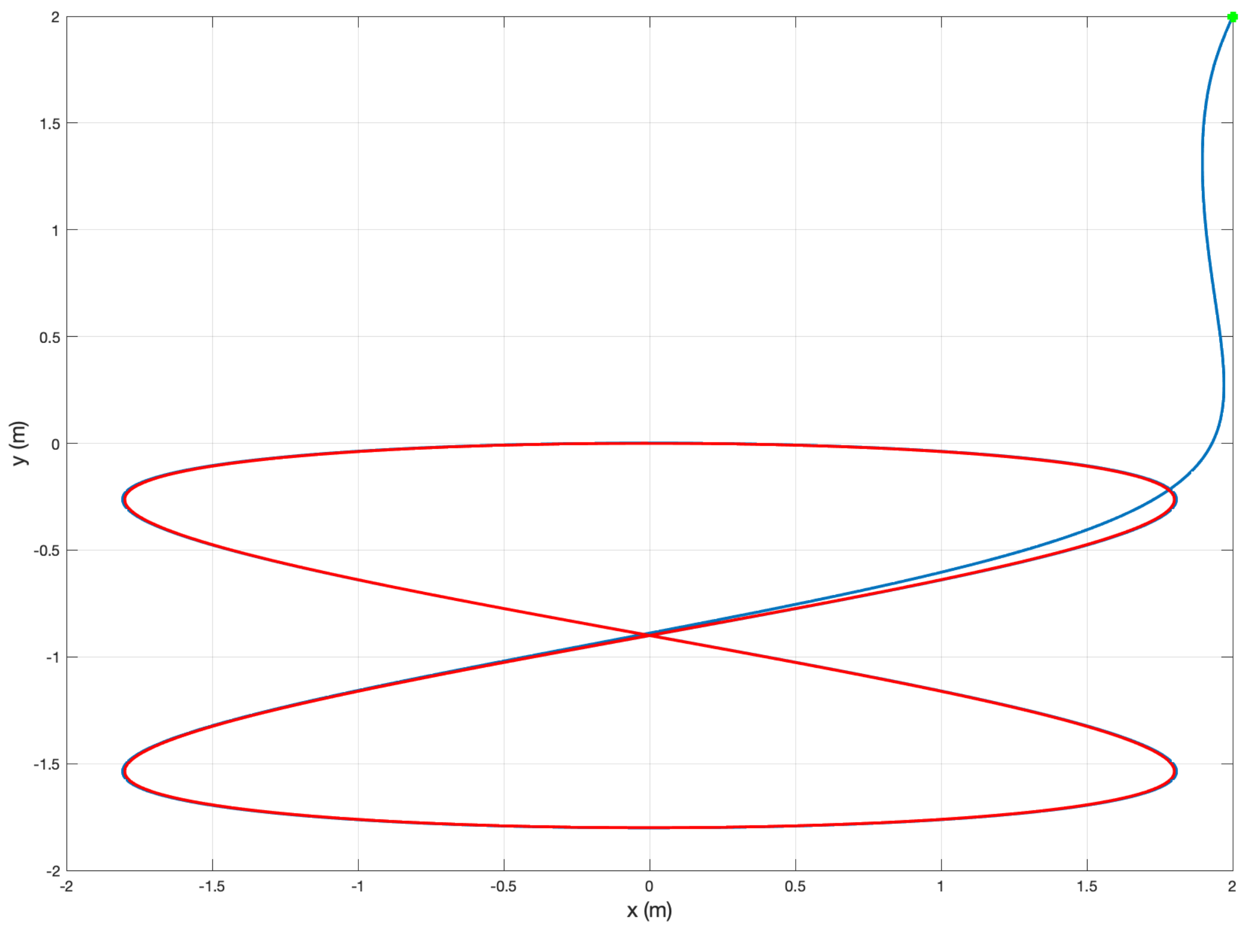
Numerical Results: Case 3



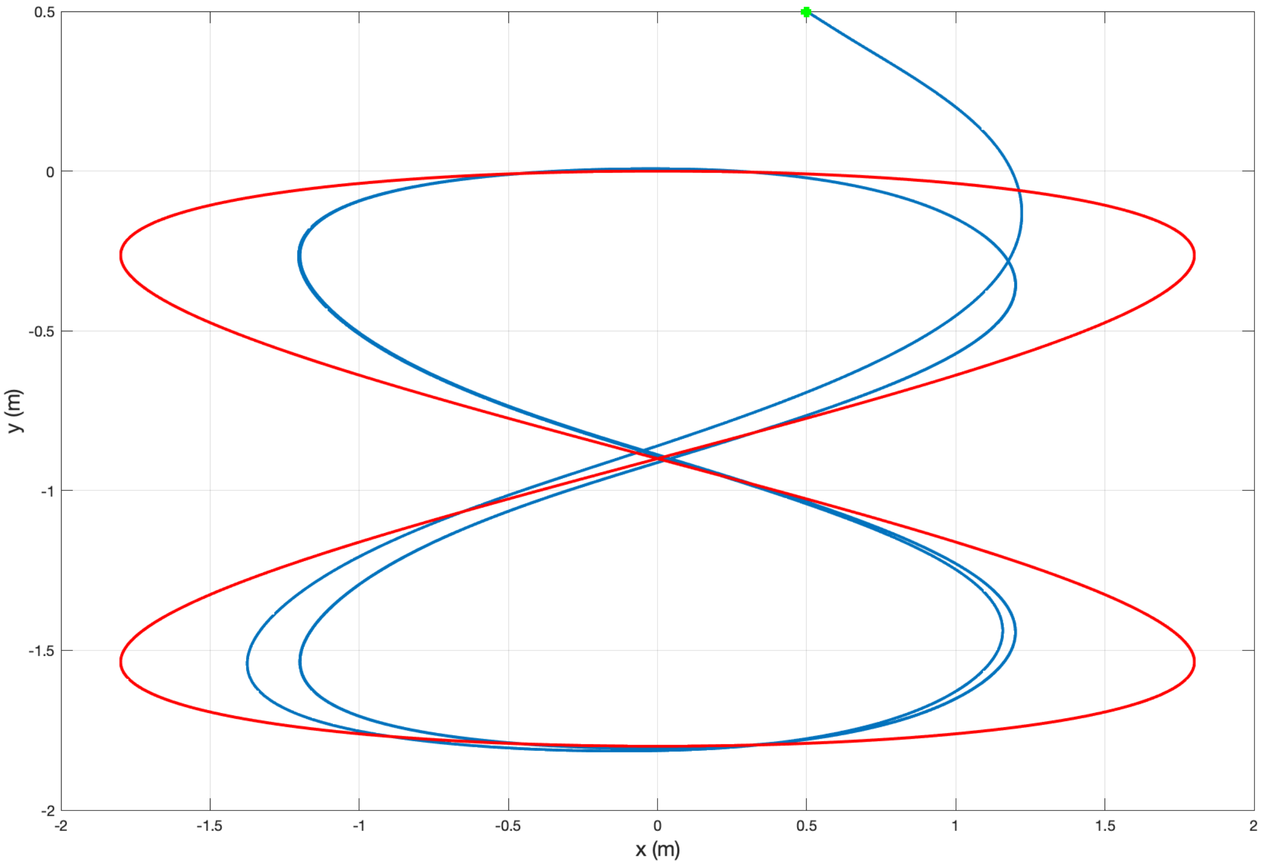
Numerical results: Case 1b



Numerical results: Case 2b



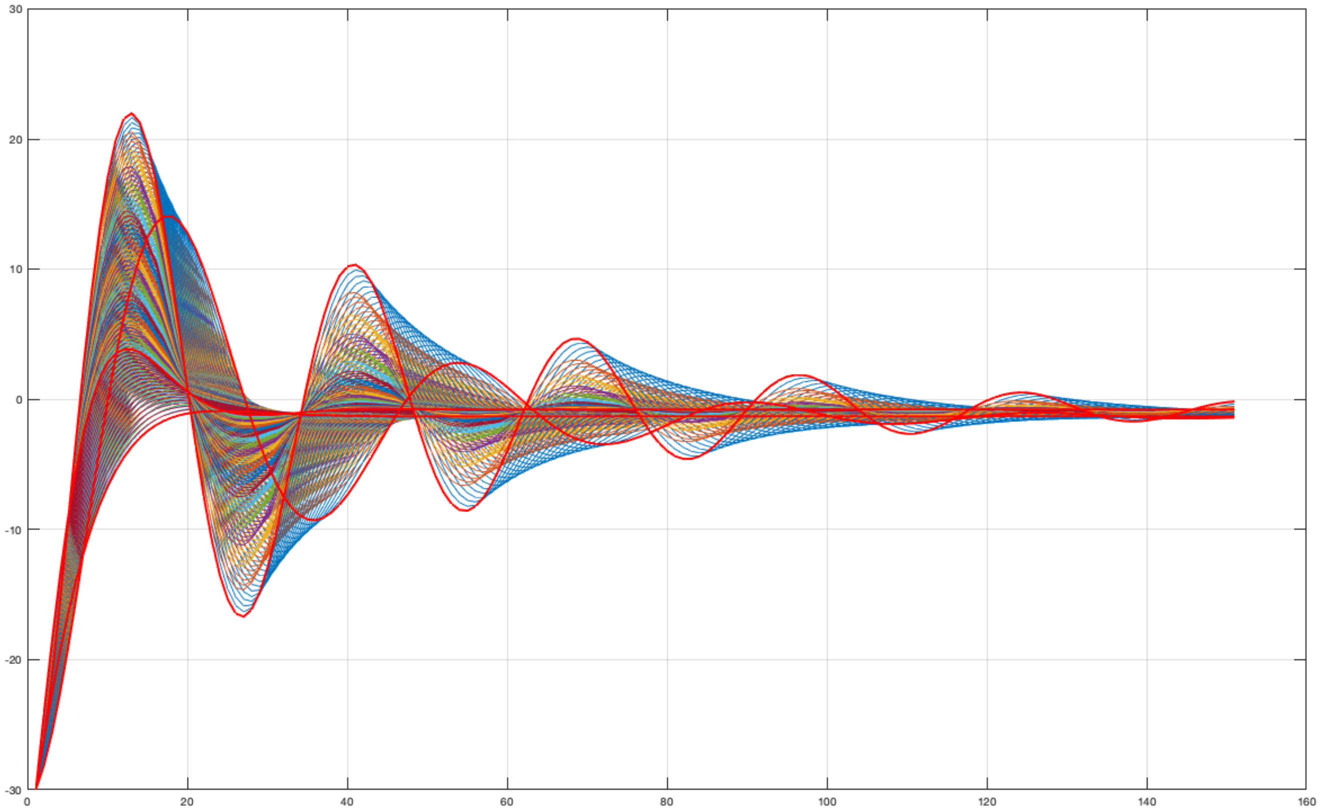
Numerical results: Case 3b



3. Extension: a Reference Governor for Uncertain Polynomial Constraints

Constraints propagation through uncertain dynamics

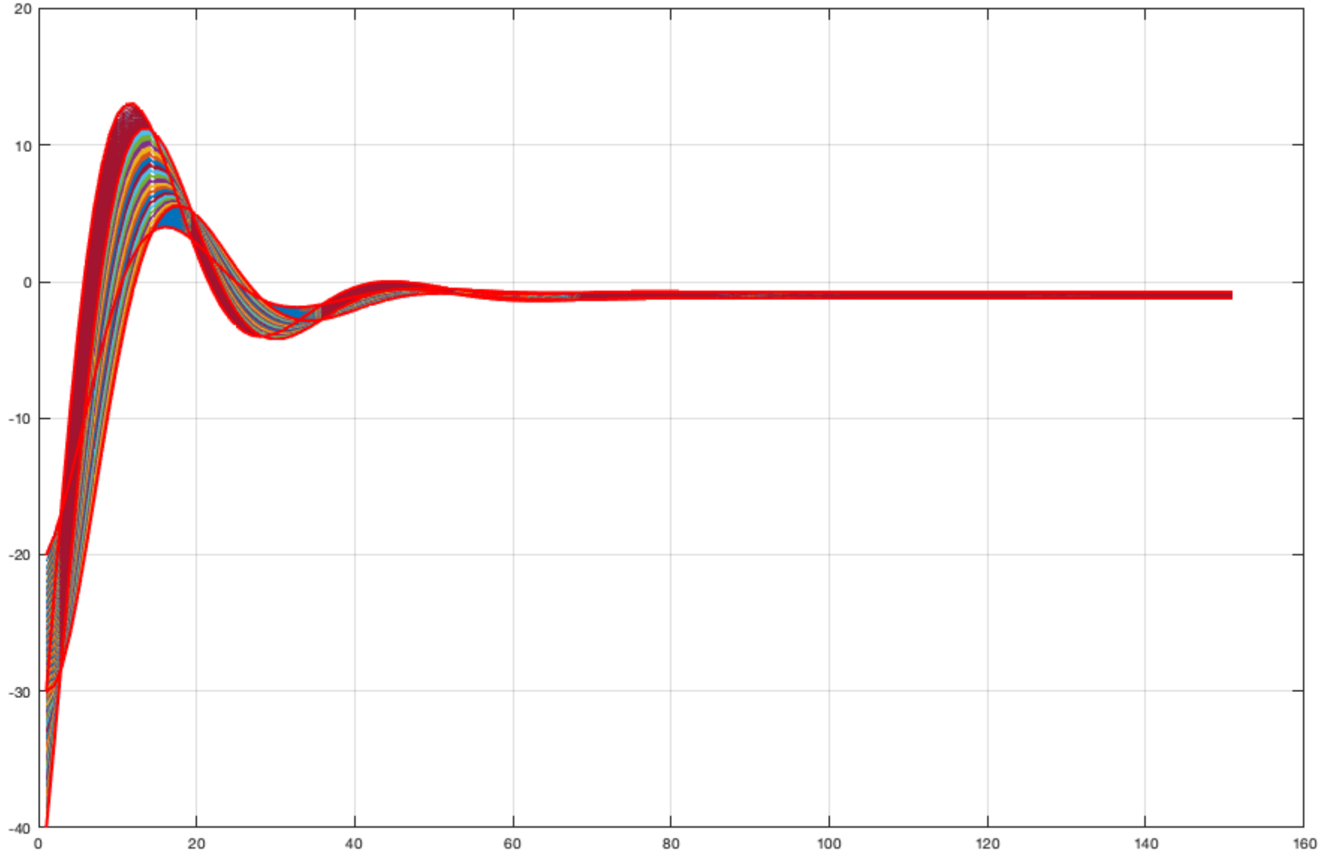
$$x(k+1) = A(U)x(k) + B(U)v$$
$$y(k) = Cx(k) + Dv$$



Constraints propagation through uncertain dynamics

$$x(k+1) = Ax(k) + Bv$$

$$y(k) = C(U)x(k) + D(U)v$$



Model-free Control

- Summary of key results in [1]:

$$\mathbb{F}(z, \dot{z}, \ldots, z^{(a)}, u, \ldots, u^{(b)}) = 0$$

- Ultra Local model:

$$z_m^{(\nu)} = \lambda u + F_z$$

- If ν =1, then

$$\hat{F}_z = \frac{6}{T^3} \int_{t-T}^{t} \{[T - 2(t-\xi)] z_m(\xi) - \lambda [T - (t-\xi)](t-\xi) u(\xi)\} d\xi$$

- If ν =2, then

$$\hat{F}_z = \frac{5!}{2T^5} \int_{t-T}^{t} \{[T^2 - 6T(t-\xi) + 6(t-\xi)^2] z_m(\xi) - \frac{\lambda}{2} [T - (t-\xi)]^2 (t-\xi)^2 u(\xi)\} d\xi$$

- Controller:

$$u = \frac{1}{\lambda}(-k_p e - k_d \dot{e} + \ddot{z}_d - \hat{F}_z)$$

z: output

z_m: measurement of z

u: input

F_z: unknown dynamics

e: error

[1]:Fliess, M. and Join, C. (2013). Model-free control. International Journal of Control, 86(12), 2228–2252.

Uncertain quadrotor dynamics

We have the following longitudinal and lateral motions where d_x and d_y are uncertain.

<u>Longitudinal</u>

$$\dot{x} = v_x$$
$$\dot{v}_x = g\theta - d_x v_x$$
$$\dot{\theta} = q$$
$$J_{yy}\dot{q} = u_2 - a_{21}v_x - b_{22}q$$

<u>Lateral</u>

$$\dot{y} = v_y$$
$$\dot{v}_y = -g\phi - d_y v_y$$
$$\dot{\phi} = p$$
$$J_{xx}\dot{p} = u_1 - a_{12}v_y - b_{11}p$$

Robust Longitudinal Tracking using MFC

$$v_x^{(3)} = \frac{g}{J_{yy}} u_2 + F_2$$

$$u_2 = -K_{lon} X_{lon} + \frac{J_{yy}}{g}(k_1 v_{lon} - \hat{F}_2)$$

Defining $X_{lon} = [v_x \quad \theta \quad q]^T$, the pre-stabilized system can be rewritten as

$$\dot{X}_{lon} = A_{lon} X_{lon} + B_{lon}(v_{lon} + d_2)$$

u_2 is designed such that when v_{lon} is constant, then v_x asymptotically tracks the constant v_{lon}, but when $k_1 v_{lon} = r_2(t)$, then x asymptotically tracks x_c.

$x_c(t)$ is the desired reference trajectory for $x(t)$ to track.

Robust Lateral Tracking using MFC

Equivalent measures can be taken in the lateral direction where it can be found that

$$u_1 \coloneqq -K_{lat}X_{lat} - \frac{J_{xx}}{g}(k_1 v_{lat} + \hat{F}_1)$$

where $X_{lat} \coloneqq [v_y \quad \phi \quad p]^T$

and the pre-stabilized system can be rewritten as

$$\dot{X}_{lat} = A_{lat}X_{lat} + B_{lat}(v_{lat} + d_1)$$

Propeller Thrust Limitations

The following constraints are then enforced:

$$M_{eff}^{-1}\begin{bmatrix} mg - md_x\theta v_x + md_y\phi v_y \\ -K_{latt}X_{lat} - \frac{J_{xx}}{g}(k_1 v_{lat} + \hat{F}_1) + a_{11}v_x + b_{12}q + a_{13}(\theta v_x - \phi v_y) \\ -K_{lon}X_{lon} - \frac{J_{yy}}{g}(k_1 v_{lon} - \hat{F}_2) + a_{22}v_y + b_{21}p + a_{23}(\theta v_x - \phi v_y) \\ a_{31}v_x + a_{32}v_y + a_{33}(\theta v_x - \phi v_y) \end{bmatrix} \epsilon [0, T_{max}]^4$$

which is the same as:

$$M_{eff}^{-1}\begin{bmatrix} mg - md_x\theta v_x + md_y\phi v_y \\ -K_{latt}X_{lat} - \frac{J_{xx}}{g}(k_1(v_{lat} + d_1) + F_1) + a_{11}v_x + b_{12}q + a_{13}(\theta v_x - \phi v_y) \\ -K_{lon}X_{lon} - \frac{J_{yy}}{g}(k_1(v_{lon} + d_2) - F_2) + a_{22}v_y + b_{21}p + a_{23}(\theta v_x - \phi v_y) \\ a_{31}v_x + a_{32}v_y + a_{33}(\theta v_x - \phi v_y) \end{bmatrix} \epsilon [0, T_{max}]^4$$

Numerical Results

MOAS computations:

<u>First Step:</u>

Dim$(Z_1) = 8$

$\widetilde{O}_{\infty,Z_1}$ is finitely determined in 84 iterations and is defined by 252 linear inequality constraints

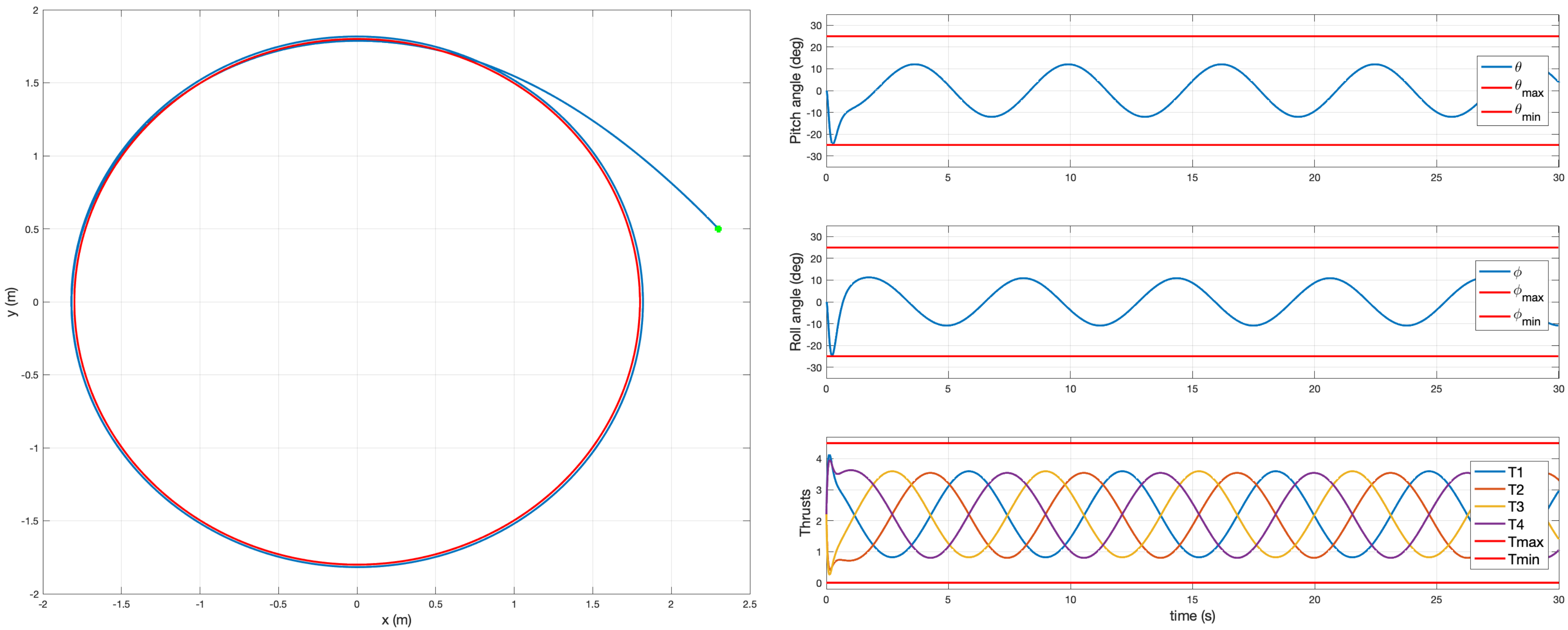
<u>Second Step:</u>

Dim(Z) $= 36$

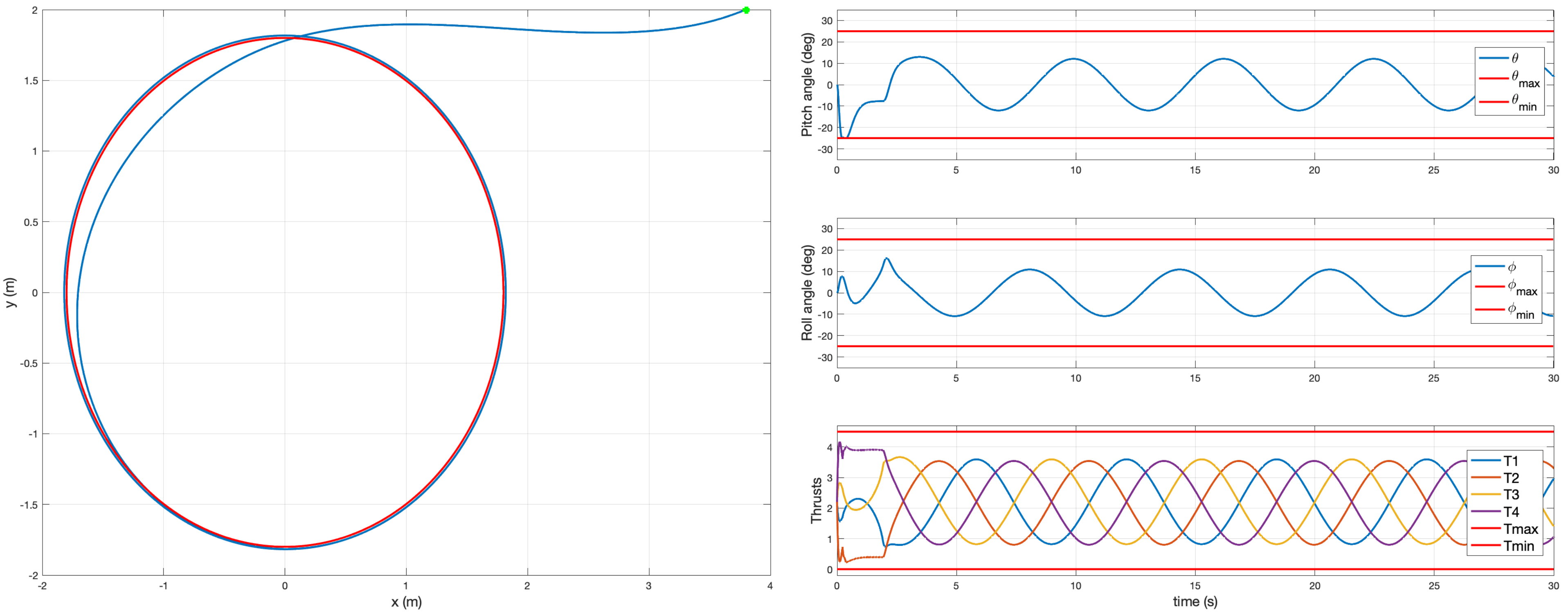
Dim(U) $= 6$

$\widetilde{O}_{\infty,Z}$ is finitely determined in 2 iterations and is defined by 702 linear inequality constraints

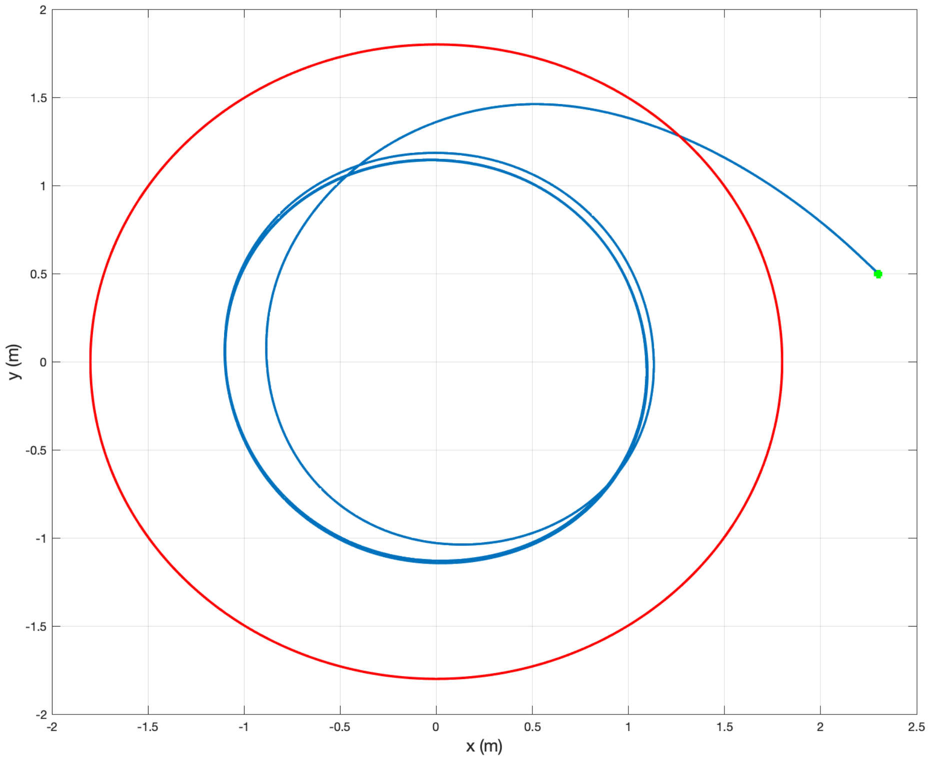
Numerical Results: Uncertain Case I

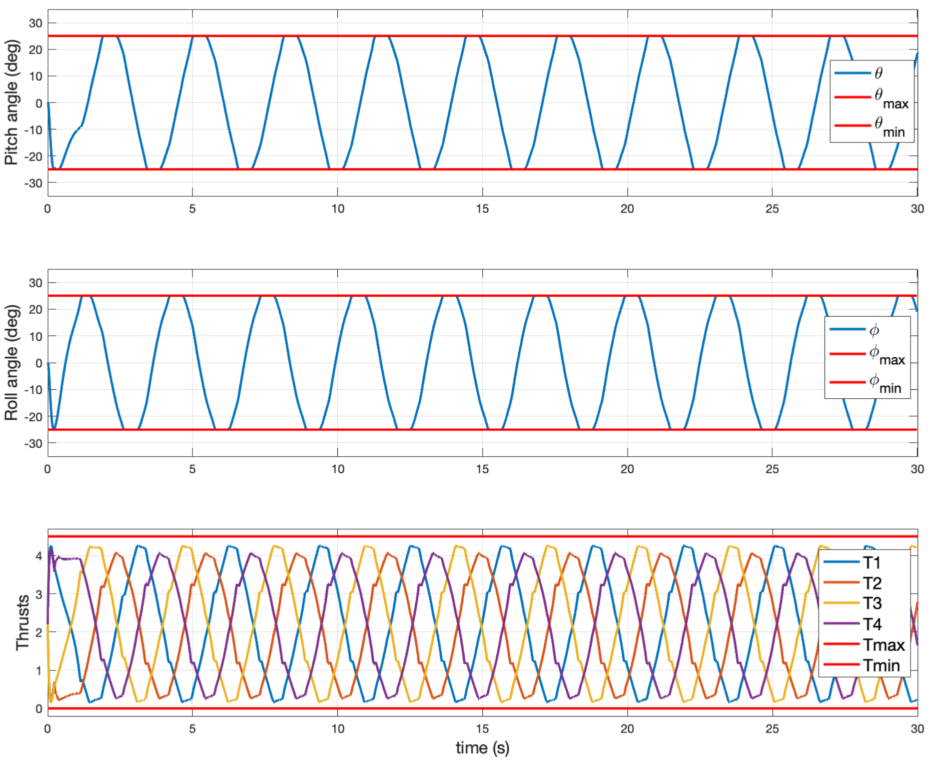


Numerical Results: Uncertain Case 2



Numerical Results: Uncertain Case 3





Conclusions

The methods employed were shown to be capable of ‘robustly’ protecting the flight envelope of a quadrotor following high-speed trajectories.

The method proposed is based on the computation a ‘safe’ forward invariant set in which the state (and reference) of the quadrotor must remain while tracking a given trajectory.

The method was recently extended to account for some parametric uncertainties.

Conclusions

Future work includes expanding to more complex trajectories, including model uncertainties and/ or disturbances, and real-time implementation on a physical drone.

A few references

L. Burlion, R. Schieni and I. Kolmanovsky, “A reference governor for linear systems with polynomial constraints”, Automatica, vol.142, 2022.

R. Schieni, “Reference Governors For Systems With Polynomial Constraints:Theory And Extensions“, PhD Thesis, 12-2022.

R. Schieni, C. Zhao, J. Barreira, M. Malisoff and L. Burlion, “Quadrotor Flight Envelope Protection while Following High-Speed Trajectories: a Reference Governor Approach”, AIAA Scitech forum, January 2023.

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