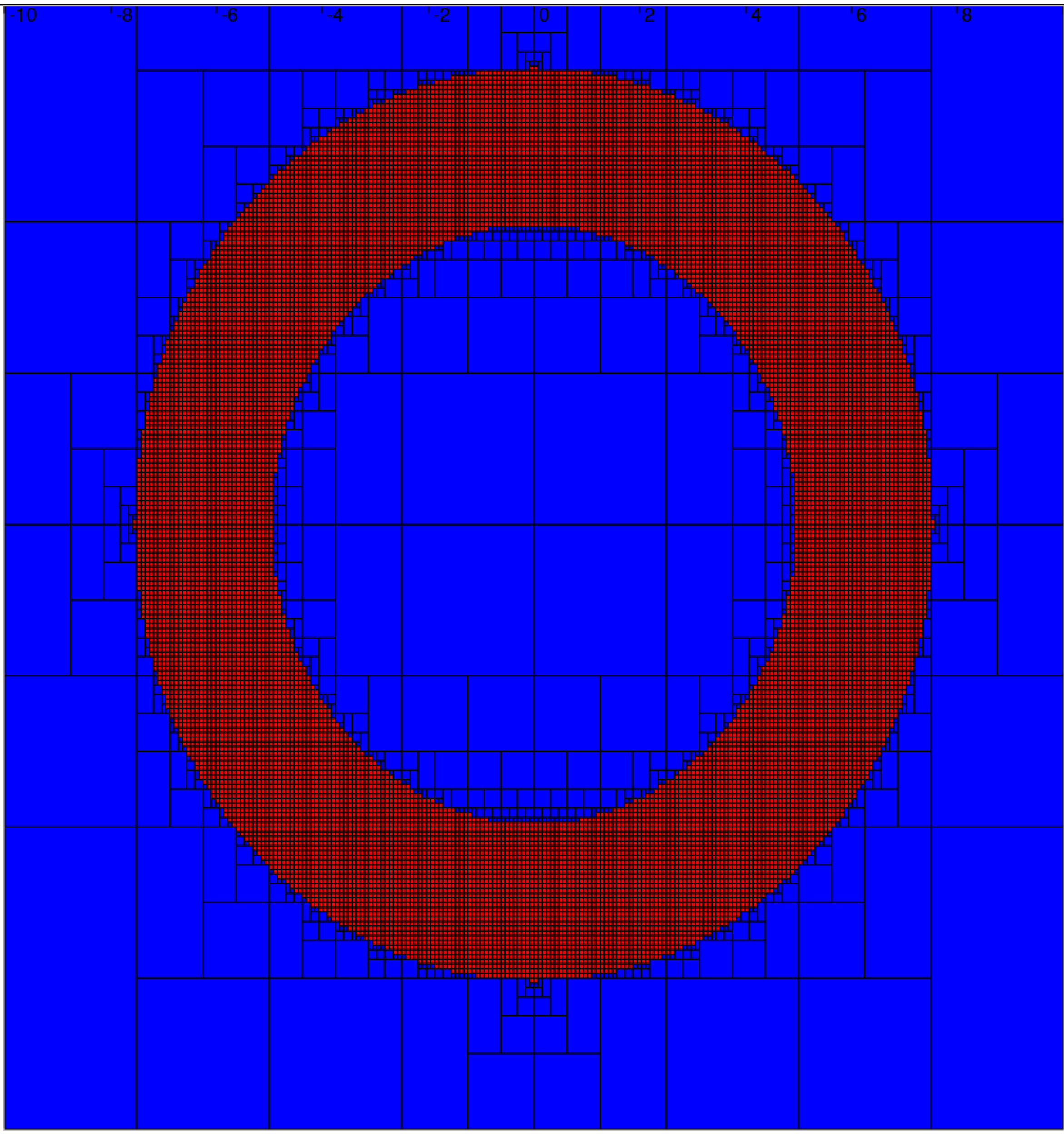


# CI – 2019 SPID/Rob

# Application of the V and G stability as a linearization method for multi agent system


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# Sommaire

## Abstract

The multi-agent system requires a linear model for robots. The main objective of this rapport is to study the possible usage of the V and G stability as a linearization method.



## Résumé

La théorie des système multi robots nécessitent que ces derniers aient un modèle linéaire. L'objectif principal de ce rapport est d'étudier la possibilité d'utiliser les théories de la V et G stabilité comme méthode de linéarisation.

# Introduction

The presence of several robots collaborated which one another is more frequent. One of the theories used is the multi-agent theory based on communication between robots to insure keep a relative position. However, the multi-agent theory uses linear based model for robots except that most robot are not simulated by a linear model. To overcome this requirement, several methods of linearization exist. This rapport will focus on the V-stability and G-stability has a new linearization method. The V and G-stability will be used with ground robots with the objectives of using this theory for the multi-agent theory. The V and G stability insures the robot to stay in a precise zone and simplify the collisions avoidance between robots.

The V stability and G stability need to solve complex and non-linear system. The interval method will be used to help solving this problem.

# 1. Presentation of the different theory and model

## 1.1. The multi-agent system theory

The multi-agent system theory [1] is based on a controller that allows robots to keep a formation by knowing the state of the other robots. For the multi-agent system, robots are organised with a graph. With two robots I and J, an edge (I, J) means that J have access to the distance between I and J.

The controller for the robot I is:

$$\dot{X_\iota} = LX_i$$

With L the adjacency matrix.

## 1.2. The V and G stability

The V-stability theory is based on the Lyapunov stability theorem.
Considered a differential function $V: \mathbb{R}^n \rightarrow \mathbb{R}$ . The system is V-stable if there exist ε >0 such that:

$If\ V(x) \geq 0 \Rightarrow V(\dot{x}) \leq$ -ε<0

For a dynamic system $\dot{x} = f(x)$ given:

$$\begin{cases} \frac{\partial V}{\partial x}(x).f(x) \geq 0 \\ V(x) \geq 0 \end{cases} \Leftrightarrow \dot{x} = f(x)\ is\ V\ stable$$ [X]

The V stability guarantee [2] the stability for a system with time-invariant differential equation.
For time-dependent system the V-stability isn't enough but the G-stability work for this case.

Consider a capture tube G(t) = {x,g(x,t)<=0} where g:RnxR->Rm

If the system:

$$\begin{cases} \text{(i)}\ \frac{\partial g_i}{\partial \mathbf{x}}(\mathbf{x},t).\mathbf{f}(\mathbf{x}) + \frac{\partial g_i}{\partial t}(\mathbf{x},t) \geq 0 \\ \text{(ii)} \qquad g_i(\mathbf{x},t) = 0 \\ \text{(iii)} \qquad \mathbf{g}(\mathbf{x},t) \leq 0 \end{cases}$$

Is inconsistent for all x, all t>0 and all i = 1,..,m the G(t) is a capture tube for the time dependent system[2].

## 1.3.Model of the ground robot

For the test of the theory, two model of ground robot will be used:

$$X = \begin{pmatrix} x \\ y \end{pmatrix}, \dot{X} = \begin{pmatrix} u_2 * \cos(u_1) \\ u_2 * \sin(u_1) \end{pmatrix}$$

$$Controller:$$

$$\begin{cases} u_1 = \operatorname{arctan2}\big((ay \times t - y), (ax \times t - x)\big) \\ u_2 = k1 \times \sqrt{(x - bx \times t)^2 + (y - by \times t)^2} + k2 \times \sqrt{bx^2 + by^2} \end{cases}$$

$$X = \begin{pmatrix} x \\ y \\ \theta \\ v \end{pmatrix}, \dot{X} = \begin{pmatrix} v \times \cos(\theta) \\ v \times \sin(\theta) \\ u_1 \\ u_2 \end{pmatrix}$$

$$Controller:$$

$$\begin{cases} e = (\bar{x} - x, \bar{y} - y) \\ u_1 = k3 \times (angle(e) - \theta) - k4 \times \dot{\theta} \\ u_2 = k1 \times norm(e) + k2 \times \left(\sqrt{bx^2 + by^2} - v\right) \end{cases}$$

The first authorize the robot to have a precise angle at every time and allow an instant U-turn. The second one set a rotation speed and is a more realistic model for a char ground robot.

# 2. Application

## 2.1. G-stability

The first objective of the stage was to apply the G stability with the acceleration model and to find condition for the size and the maximal speed for the bubble.
Below there are the equations for the G-stability with acceleration model.

$$\begin{cases} 2\left[v\cos(\theta)\left(2x - b_x t\right) + v\sin(\theta)\left(2y - b_y t\right)\right] - 2\left[b_x(2x - b_x t) + b_y\left(2y - b_y t\right)\right] \geq 0 \\ (x - b_x t)^2 + (y - b_y t)^2 - r^2 \leq 0 \end{cases}$$

For this theory, the exact trajectory must be known however in the multi agent system, the exact trajectory can't be known so the g stability can't be used to insure the non-collision of robots.
To use this theory for multi agent system, the variable time must be removing so the V-stability will be use and not the G-stability.
The system of equation will have solutions due to the controller can't be included in the G-stability because the controller appears on the second derivate and the g-stability only derivate once and the same problem appear for the V-stability. So, the next tentative will be with the V-stability and the velocity-based model.

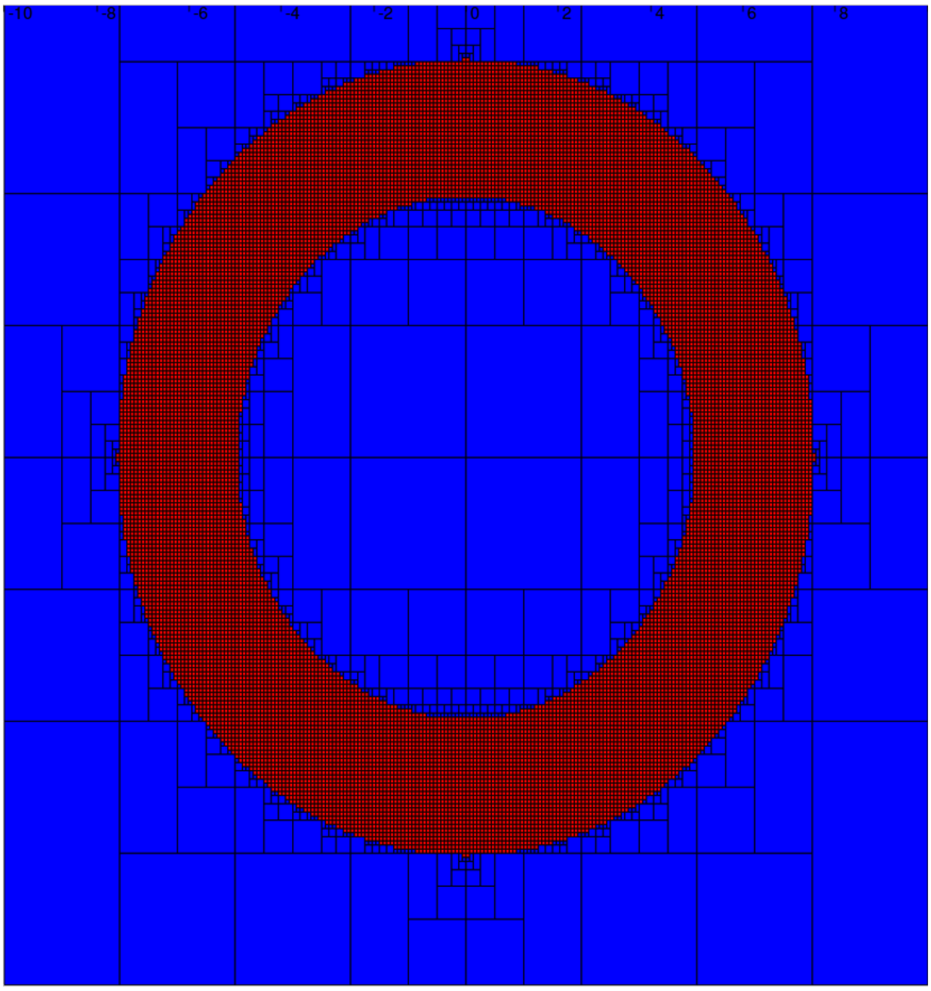
## 2.2.V stability and velocity-based model

The equations are rewrite for the velocity-based model and presented below.

$$\begin{cases} (x - bx \times t)^2 + (y - by \times t)^2 - r^2 \geq 0 \\ 2 * (x - bx \times t) \times u_2 * \cos(u_1) + 2 \times (y - by \times t) \times u_2 \times \sin(u_1) \geq 0 \\ k1 \times \sqrt{(x - bx \times t)^2 + (y - by \times t)^2} + k2 \times \sqrt{bx^2 + by^2} = u_2 \\ \text{arctan2}\big((by \times t - y), (bx \times t - x)\big) = u_1 \end{cases}$$

For bx ϵ [-25,25], by ϵ [-25,25], Vmax = 30 and R = 5 the below result is obtaining.

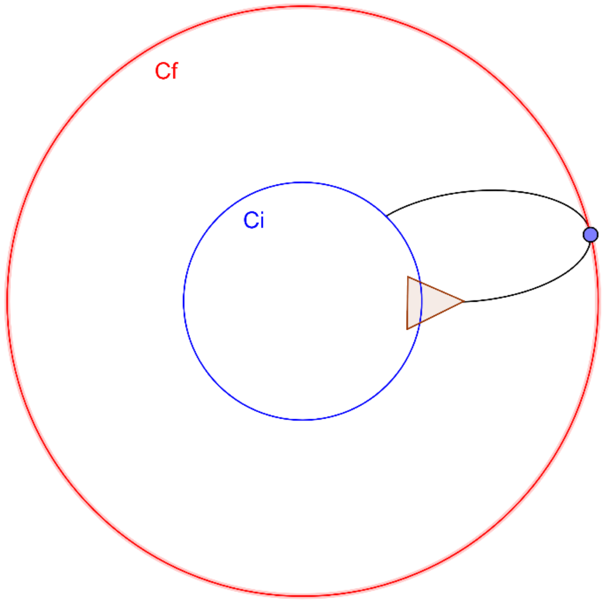
The red boxes are the box unstable end the blue ones are stable. An inner circle is stable however the outer circle is unstable, so the radius of the wanted bubble is R so the radius of the bubble needed to insure the non-collision is greater than R, for example 2R.



## 2.3. V stability with acceleration-based model

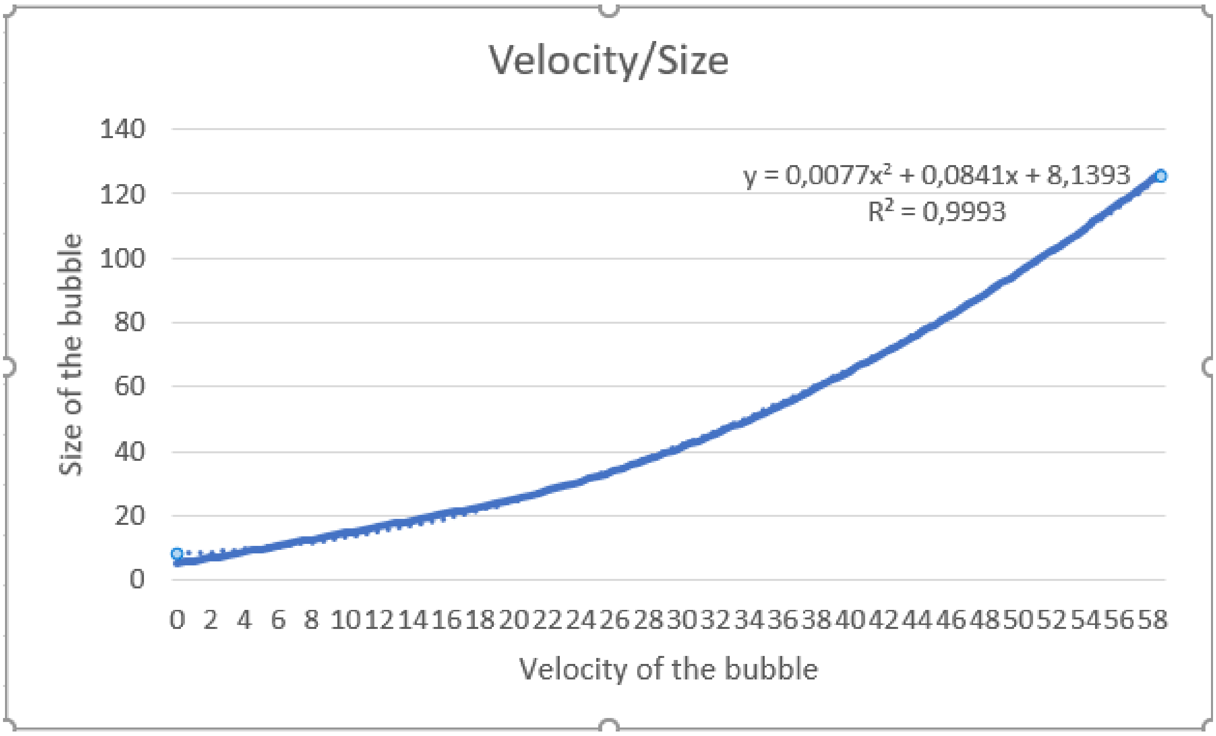
In a second time, below are the equations for the acceleration-based model. Due to the necessity to derivate twice, the previous method can't be used.

The method propose is to choose the radius of the bubble, create boxes at the frontier of the bubble and simulate the evolution of the boxes until all the boxes come back to the inner bubble. The maximum distance between the centre of the bubble and the boxes during the movement give the size of the outer bubble to ensure that the robot will not be escaped.




The robot is symbolized by the red triangle, Ci is the initial circle, in black there is the trajectory of the robot and Cf is the final circle, the circle that allows the robots the go back in the initial circle without leaving it.

The size of the bubble is related to the speed of the bubble, the maximum velocity of the robot, the maximal acceleration and the maximal rotation speed of the robot. The exact influence is currently unknown.




The graph represents the relation of the size of the bubble and the velocity of the bubble with a maximal speed of 60 AU, a maximal acceleration of 15AU and a maximal rotation speed of pi rad/s.

This graphics show that the size of the bubble increase like the square of the velocity of the bubble, however more tests are required to find the exact relation. Due to the fast-increasing size of the bubble link to the velocity, a large area is needed to insure the non-collisions of the bubble and so the non-collisions of the robots.

# 3 The contribution of the internship to the student's professional project

The objective of the stage was difficult to satisfy due to some communication problems with the tutors of the stage. This stage confirms my first idea of choosing the robotics option at ENSTA Bretagne. This stage also convinces me that I prefer to work with the objectives of an application than only doing global research. The main objective of this stage, even if it was complicated to understand due to my necessity to learn completely two important theories and to find a way to link them, could be satisfy without this communication problem.

## Conclusion

The V stability permit to linearize a model and simplify the collision avoidance problematic however for this theory, the robots need to be wide spread due to the size of the bubble. This condition is difficult to be satisfy for ground robot because currently there is no application where a several robots are used with an important clear are. However, this theory can be applying to surface vessels, submarine robots or aerial robots where the large clear area condition is easily satisfied.

# References

[1] *J. Alexander Fax, Member, IEEE, and Richard M. Murray, Member, IEEE*
Information Flow and Cooperative Control of Vehicle Formations 2004

[2] *Alexandru Stancu, Luc Jaulin, and Aymeric Bethencourt*
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