Internship Report
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Algorithms of Inertial Measurement Units based on Kalman Filter

Internship performed at Texys.FR
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Abstract

Precisely estimating the Euler angles of vehicles have an important role on increasing its safety and maneuverability. Based on the Euler angles, professional motorcycles teams have been trying to improve their motorcycle's performance by creating traction control algorithms that convert the throttle demanded by the pilot into a throttle that will avoid the motorcycle from losing friction to the ground. However, these algorithms lay on having precisely estimation of the motorcycle angles, which they haven't. This document is the report of the internship activities of Philipe Miranda de Moura at Texys.FR, where algorithms to estimate Euler angles using gyroscopes and accelerometers were developed, based on Kalman Filter state-observer.

Key words: Kalman Filter, localization, IMU

Résumé

Estimer précisément les angles d'Euler d'un véhicule a un important rôle en ce que concerne augmenter sa sécurité et sa maniabilité. Basés sur les angles d'Euler, des écuries professionnelles des motocyclettes essayent d'améliorer la performance de leurs motocyclettes en créant des algorithmes de contrôle de traction qui convertissent l'accélération demandée par le pilote en une accélération qui évite que la motocyclette perde l'adhérence. Cependant, ces algorithmes reposent sur une estimation précise des angles de la motocyclette, ce qui n'est pas le cas. Ce document est le rapport des activités de stage de Philipe Miranda de Moura à Texys.FR, où des algorithmes pour estimer les angles d'Euler à l'aide de gyroscopes et d'accéléromètres ont été développés, basés sur l'observateur d'état Filtre de Kalman.

Mots-clés: Filtre de Kalman, localisation, Centrale inertielle

Contents

1 Company presentation

Texys is a company that designs, manufactures and distributes several types of sensors for racing, automotive and industrial applications:

- Infrared temperature sensors (for tire and brake disc)
- Thermocouple amplifiers (patented)
- Accelerometers and gyroscopes
- Strain gauge bonding + amplifiers (push rods, gear lever...)
- Pitot sensors and differential pressure sensors

Founded by Etienne Deméocq, former head of Ligier Formula 1 (F1) team's instrumentation department, on 1999 the company profited of both his technical knowledge and experience and his network within Formula 1's teams. Texys's sensors receive Texense brand name, and is renowned for product reliability and accuracy. Nowadays, Texys develops new sensors on demand, adapts existing ones to custom applications and enhances others sensors reliability and robustness.

The moment Ligier moved from Magny-Cours to Paris, Etienne decided quitting his job – so that he could continue living in Burgundy region – and founded Texys, acting at the very beginning as owner, salesman, hardware and software developer and manufacturer. After delivering the first orders to Mercedes F1, the company rapidly grew to its present almost 30 contributors.

As Texys continued growing in recent years, an annex building is being constructed next to its headquarter and a subsidiary was created on United States, mainly to comply with the needs of Nascar's clients. In Asia and Oceania, a partner company keeps in charge of distributing Texys products.

2 Project presentation

Besides its physical expansion, Texys wants to expand its sensors variety. Since its beginning the company has been producing sensors that do not require much computation. In recent years, however, Texys decided developing the so called *smart sensors* – sensors that mix and filter their inputs before outputting – and that is the exact context where the sensor I worked on (the RAD6-M) is inserted in.

RAD6-M (or simply RAD6) stands for *Roll Angle Device for motorcycle with integrated 6-axis inertial box.* Is consists on a 3-axis accelerometer, 3 single axis gyroscopes, the micro-controller for computing the output and a temperature sensor for compensating accelerometers and gyroscopes readings.

Since 2012 Texys has been trying to develop this specific sensor and there were already working prototypes by the time I applied to the internship. Nonetheless nor the company nor potential customers were satisfied with the results. Therefore, it was foreseen on the internship agreement (appendix B) that I would work on the following lines:

- Improve review or validate the existing mechanical kinematics model;
- Determine the inputs required precision to obtain the desired precision outputs;
- Improve or redefine the existing algorithm;
- Simulate on Scilab or Excel the new algorithm;
- Estimate the required computational power;
- Code the algorithm on an embedded system using C language.

Months earlier, a customer called Texys telling they had a proprietary cumbersome wired sensor for measuring a vehicle's steering wheel angle and they would like to replace it by a lightweight easy to install wireless IMU. At the time, Texys had not developed IMUs for this application and the customer ordered some GYRP analog gyroscopes to make essays and verifying whether it was viable or not to simply integrate the angle rates given by gyroscopes for obtaining the steering wheel position.

Seen that the sensor is supposed to work for a moving vehicle, the customer's idea was installing one gyroscope on the steering wheel, with its axis collinear to the steering shaft, and another one on the dashboard, parallel to the first one. That way, they imagined they would be able to isolate the steering wheel's movement from vehicle's movements. They were quite satisfied with preliminary results and ordered Texys developing a new product so that they could improve their result by getting rid of the main issue concerning gyroscopes: the drift.

The product is the SWAD *(Steering Wheel Angle Device)*, consisting on a single axis gyroscope and a 2-axis accelerometer, with its axis orthogonal to the gyroscope axis. With that setup the customer expected using information from accelerometers to quantify the gyroscope drift, what was not the case. By that time, I was stuck on the subject originally designated for me. My tutor and I thought it would be a good idea I worked on this project.

This customer had ordered Texys a particular IMU: a 2-axis accelerometer and a single axis gyroscope. The customer's goal was replacing its actual bulky potentiometer sensor attached to the steering wheel shaft by a wireless practical IMU. This sensor is used for measuring the steering wheel angle of a moving car, by attaching one IMU on the steering wheel's center and another one on the dashboard, parallel to the steering wheel. For that purpose, they had bought some analog single axis gyroscopes and they were quite satisfied with the results they had by only integrating gyroscopes readings. They would use the accelerometer to eliminate gyroscope drift and improve the results.

In sum, my job was to develop two IMUs algorithms on Scilab: The first one to estimate the steering wheel angle of a car and the second one to estimate the roll and the pitch angles of a racing motorcycle.

3 Description of the internship activities and results

3.1 Adopted notation

The following chapters present a dense mathematic, involving probability and linear algebra. The present chapter has the single goal to clarify possible confusion that will be faced from now on. Please note that there is not a consensus on the most appropriate notation among control engineers. I will do my best to state and follow the ones I am most used with.

- Matrices: uppercase, as H;
- Vectors (column matrix): lowercase, as f;
- Vectors (multiple physical dimensions): lowercase with arrow, as $\vec{a}$;
- Identity matrix: bold I, with its dimension subscript, as $\boldsymbol{I}_n$;
- Rotation matrix: bold uppercase R, as $\boldsymbol{R}$;
- Expanded matrix: uppercase with hat, as $\hat{G}$
- Kalman gain: uppercase, as K;
- Instants: lowercase, as k;
- Bayes' rule: letter with events on its subscripts separated by $|$, as $G_{a|b}$;
- Exception: X is a column matrix, to avoid confusion with the direction x

Unless it is stated the opposite, all the noises will be considered white and therefore will be characterized by its mean – equals to zero – and its covariance. *Big* (*small*) values of covariance may be called *large* (*thin*) or *short* (*tall*), due to its plots. This concept will be extended to covariance matrices, meaning a *large* matrix has bigger elements than a *short* one.

Also, "states" refers to the hidden states. The o observed states are called measurements. Finally, words in italic generally are used for qualitative explanation, when the idea is more important then defined values.

3.2 The Kalman Filter

There are several books written about the Kalman Filter presenting the whole theory along with demonstrations, like [1], but the intention here is to be succinct and to present the filter application in a practical way.

3.2.1 An overview

The paragraphs below were strongly inspired by the work done in [2] by an engineering that used to avoid facing the Kalman Filter.

Sensors are noisy. Period. They all depend on many external variables to give their outputs. A GPS sensor precision depends on the weather; a mechanical rotary encoder, on the carbon's brush wear; a MEMS accelerometer, on temperature; and so on. Also, all of them depend on its fabrication and calibration: a 3-axis gyroscope may perceive a slight angular velocity around x when its turned around y only, for instance, due to a little misalignment between axes.

The world is also noisy. A cyclist's prediction on the estimated arrival time may vary due to unexpected head or tail wind, a flat tire or a road deviation. The Kalman Filter was created for taking this uncertainties into account. Both two parts (prediction and measurement) have associated noises, supposed to be Gaussian with mean equal to zero, which means it is as likely to underestimate an actual quantity as it is to overestimate it.

The filter's job is to compute the *most likely output*, based on the confidence we have on the parts. For the Kalman Filter to work, the three following hypothesis must be satisfied, according to [3]:

- The noise is white (i.e. Gaussian centered at zero);
- The process is a Markov's chain;
- The system is linear and time invariant.

The noise's whiteness is required to turn true the mathematical equations used in Kalman's standard algorithm. However, depending on the tunning, the filter can get away with non-white noises.

A Markov chain may be roughly described as a memoryless stochastic process, which means the next states depend only on the present ones. This is needed because of the recursiveness of the Kalman Filter.

The system's linearity and time invariance allows us to use the linear algebra described on Kalman's equations.

There are some alternatives to the Kalman Filter if at least one of the parts doesn't meet these requirements. For a *nearly* white noise or *nearly* linear time invariant system, the algorithm still performs well. For extreme cases, others alternatives must be used:

- Extended Kalman Filter (EKF), that linearizes the system around an operation point
- Unscented Kalman Filter, that stochastically infers the operation point
- Interval Analysis, that presents a totally different problem approach as detailed on [4]
- Particle Filter, that makes no assumptions on the system's model

Each of them has different drawbacks that go beyond the scope of this work. Even though, to clarify this problem, it must be said the reasons why the Extended Kalman Filter was chosen. Compared to other methods, it is not computationally demanding and can get away with the non-linearities presented on the case studies.

3.2.2 The Standard Kalman Filter algorithm

The *most likely output* is the output that minimizes the quadratic weighted error between the measurements and the states, as the algorithm's equations 1 presents. Note that $\hat{x}$ represents an estimation on x, $\tilde{y}$ represent the approximated error between the measurement and the state, the subscript $_k$ represents the instant k and $G_{a|b}$ is the Bayes' rule notation.

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k \tag{1a}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q \tag{1b}$$

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1} \tag{1c}$$

$$S_k = H_k P_{k|k-1} H_k^T + R \tag{1d}$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1} \tag{1e}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \tag{1f}$$

$$P_{k|k} = (\boldsymbol{I} - K_k H_k) P_{k|k-1} \tag{1g}$$

At first glance, it is not trivial at all to infer the role of each equation. [2], however, did a great job writing an interactive book with plenty of examples and intuitive thoughts about the Kalman filter, including the equations for the one-dimensional case. I will adapt his work and present the equations for the simplest possible case, where $F_k = 1$, $H_k = 1$ and $B = 0$. Remark that scalars are nothing but 1×1 matrices. As consequence, a matrix inversion turns into a division, while a transposition is the scalar itself. That said, the equations become:

$$\hat{x}_{k|k-1} = \hat{x}_{k-1|k-1} \tag{2a}$$

$$P_{k|k-1} = P_{k-1|k-1} + Q \tag{2b}$$

$$\tilde{y}_k = z_k - \hat{x}_{k|k-1} \tag{2c}$$

$$S_k = P_{k|k-1} + R \tag{2d}$$

$$K_k = P_{k|k-1}/S_k \tag{2e}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\tilde{y}_k \tag{2f}$$

$$P_{k|k} = (1 - K_k)P_{k|k-1} \tag{2g}$$

The equations above could be true for tracking the position x_k of an oil platform that measures directly its position ($z_k = x_{k|k-1}$, for $\tilde{y} = 0$) and is supposed static ($x_{k|k-1} = x_{k-1|k-1}$).

- 2a is the predicted state estimation
- 2b is the predicted state covariance
- 2c is the error between measurement and prediction
- 2d is the error covariance
- 2e is the optimal Kalman gain
- 2f is the updated state covariance
- 2g is the updated state covariance

To better understanding, let's rewrite 2e, as:

$$K_k = \frac{P_{k-1|k-1} + Q}{P_{k-1|k-1} + Q + R} \tag{3}$$

Analyzing equation 3, it is possible to verify that $0 < K_k < 1$. It is next to 0 (resp 1) if $R \gg Q$ (resp $R \gg Q$). Imagine, now, the platform is on furious waters and it is equipped with a high precision sensor. Therefore, Q would be *large* (or *big*) and R would be *narrow* (or *small*). So, $K \approx 1$, means the updated position $\hat{x}_{k|k} \approx \hat{x}_{k|k-1} + \tilde{y}_k$. Its retained from 3 that modifying Q and R plays a lot on the filters behavior, leading to fast convergence or catastrophic divergences.

3.2.3 The Extended Kalman Filter algorithm

The Extended Kalman Filter equations simply are the Standard Kalman Filter ones linearized around the operation point $\hat{x}_k$, thanks to the Jacobian matrices 4b and 4e. It is also possible to modify Q and R over time.

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}) \tag{4a}$$

$$F_{k-1} = \frac{\partial f}{\partial x}\bigg|_{\hat{x}_{k-1|k-1}, u_{k-1}} \tag{4b}$$

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q_{k-1} \tag{4c}$$

$$\tilde{y}_k = z_k - h(\hat{x}_{k|k-1}) \tag{4d}$$

$$H_k = \frac{\partial h}{\partial x}\bigg|_{\hat{x}_{k|k-1}} \tag{4e}$$

$$S_k = H_k P_{k|k-1} H_k^T + R_k \tag{4f}$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1} \tag{4g}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \tag{4h}$$

$$P_{k|k} = (\boldsymbol{I} - K_k H_k) P_{k|k-1} \tag{4i}$$

As it will be shown in case studies (sections 3.3 and 3.4), there are two main issues concerning the use of this filter. They are defining the state vector X and tunning the covariance matrices Q and R.

3.2.4 Some words on tuning Q and R

As stated on 3.2.2, the Kalman Filter algorithm minimizes the weighted quadratic error between the measurements and the states. Therefore, to ensure its proper functioning, one may determine *the right values* for the covariance matrices of weighting.

As a rule of thumb, it is said the covariance matrix also is a diagonal matrix, what means there is no **linear** correlation among the states. This tip has limitations for very specific applications, like on estimating the position and velocity of a constant speed particle, since its position is linear related to its velocity. For a free fall particle, both states (height and velocity) would be non-linear correlated and the covariance matrix should be diagonal.

R is covariance matrix of the observation noise. On the first moment, one could think it is synonym for "sensor noise". In fact, the sensor noise is only a tiny part of the observation noise. R is also supposed to take into account all sort of phenomena that perturbs the measuring. For a pressure sensor, turbulence could be a phenomena of this nature, for instance.

Q is the covariance matrix of the process noise. Even though it is rarely used in the literature, I prefer calling it *system noise*. In other words, this matrix tells us the confidence we have on each state, *imagining the others are true*. That means we can be highly confident on the position, even though we are quite uncertain on the velocity, for instance.

Setting a *small* Q means we are confident about the prediction model. If we set a *large* R along with a small Q, it means the estimation will prioritize respecting the prediction rather than the measurements. The consequence is that in case the an unexpected phenomena perturbs the system, the estimation will slowly be modified, since the filter does not believe the measurements compared to the prediction.

If we choose decreasing the delay (or the inertia) of the estimation, we can set a *smaller* R, by the cost of increasing noise. The main issue on setting these matrices is that there is no technique to calculate their values. In addition, each of their elements can be set individually and it is not trivial to understand how the filter is affected, especially due to the equation 4g.

3.2.5 Smoothers

As it can be inferred from the Kalman Filter equations, it consists on a causal filter, since none of the iterations depends on future measurements to be computed. Smoother simply is a algorithm for taking future measurements into account and, thus, improving the results. Since it uses future measurements, it is not causal and cannot be used on strict real-time applications. Its principle is to run an algorithm similar to the Kalman one, but in the backward direction. There are two main types of smoothers:

- Fixed-Interval
- Fixed-Lag

The Fixed-Interval waits the entire data to be collected before refining the estimation by running the backward algorithm. The Fixed-Lag is a alternative to the following cases:

- Endless data
- Limited memory capacity
- Non-strict real time applications

In all the cases above, the estimation is improved at the cost of being lagged, as the name suggests. That means if we set a lag of l steps, the estimation of the instant n will be available in $n+l$ only. The Fixed-Lag smoother is the particular case of the fixed-interval when the backward processing occurs after having a fixed amount of future points.

One of the most used smoother algorithms is the Rauch–Tung–Striebel (RTS) one. It is easy to implement and provides significantly improvements on results, as it will be shown in the following chapters. It is equated as:

$$C_k = P_{k|k} F_{k+1}^T P_{k+1|k}^{-1} \tag{5a}$$

$$\hat{x}_{k|n} = \hat{x}_{k|k} + C_k(\hat{x}_{k+1|n} - \hat{x}_{k+1|k}) \tag{5b}$$

$$P_{k|n} = P_{k|k} + C_k(P_{k+1|n} - P_{k+1|k})C_k^T \tag{5c}$$

It is important to note that this name is somehow ambiguous, since a low-pass filter also smooths a data. A smoother is able not only to smooth the estimation, but also to improve it.

3.3 Estimating the steer angle of a steering wheel

3.3.1 Problem statement

Nowadays there are companies interested on measuring a vehicle's steering wheel steer angle for several purposes. One tries to relate the steer angle with the tire wear, while another uses the steering wheel to develop a stability control. In case a car is conceived for measuring the steering wheel turning angle, one of the cheapest and most reliable alternatives is to use of a rotary encoder, which could be a potentiometer connected to the steering wheel axle. However, for an existing car, this solution may be difficult to implement, compared to an IMU.

This case study consists on attaching a IMU on the steering wheel's center and another on the dashboard panel. The only constraint of model is that both IMU must be parallel.

Both IMUs are equipped with a 2-axis accelerometer α^x and α^y and a gyroscope ω^z, where the superscripts are not exponents but the axis. Subscripts are reserved for denoting the sensor; $_f$ for the fixed one (i.e. the one on the dashboard), and $_v$ for the one on the steering wheel. The idea here is to mix both sensors data in such way the vehicle's movements are despised. To do so, a hypothesis is posed: the acceleration seen by $\vec{\alpha_v} = \boldsymbol{R} \times \vec{\alpha_f}$, which is only true if $\vec{\alpha_v}$ is positioned right on the center of the steering wheel, as it was stated it should be.

The customer opted for acquiring the IMU raw data and post-treat it, what allowed us to make use of the RTS smoother to improve the results.

3.3.2 First approach

When we first approached this problem, we made it in the simplest way we though: simulating the angle for each one of the sensors separately and then subtracting one from the other to get the final result, which proved to give a unsatisfactory estimation. This occurs mainly because both data are not mixed during the process. Anyway, served for better understanding how to set Q and R matrices and now may help the reader to understand how to design a Kalman Filter.

The observations are:

$$Z = \begin{pmatrix} \vec{\alpha} \\ \omega^z \end{pmatrix} = \begin{pmatrix} \alpha^x \\ \alpha^y \\ \omega^z \end{pmatrix} \tag{6}$$

Since we desire to estimate the angle ϕ it should be part of the process model, along all other states that affect ϕ. In this case, we have no model predicting the system behavior, meaning the driver can turn the steering wheel the way he desires and it has no *significant* dynamic associated. The only equation we can state for ϕ is $\phi = \phi + dt\dot{\phi}$, leading to the process (or predicting) model:

$$X = \begin{pmatrix} \phi \\ \dot{\phi} \end{pmatrix} \tag{7}$$

$$f = X_{k+1} = \begin{pmatrix} \phi_k + dt\dot{\phi}_k \\ \dot{\phi}_k \end{pmatrix} \tag{8}$$

Being h the equation that related the hidden states (equation 7) with the observed states (equation 6) and assuming the steering wheel is vertical, we obtain:

$$h = \begin{pmatrix} g\cos(\phi) \\ g\sin(\phi) \\ \dot{\phi} \end{pmatrix} \tag{9}$$

As explained on section 3.2.3, the Extended Kalman Filter linearizes the equations around the operation point with the Jacobian observation matrix 4e, as well as the Jacobian process matrix 4b:

$$H = \begin{bmatrix} -g\sin(\phi) & 0 \\ g\cos(\phi) & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

$$F = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix} \tag{11}$$

I hope this section has illustrated how to design a Kalman Filter: First you list the observed states and the hidden ones. Then, you relate both and compute the Jacobian matrices.

3.3.3 Filter design

The available measures Z were defined on 3.3.1. The job now is to relate Z with the states (equation 16).

$$Z = \begin{pmatrix} \vec{\alpha_v} \\ \omega_v^z \\ \vec{\alpha_f} \\ \omega_f^z \end{pmatrix} = \begin{pmatrix} \alpha_v^x \\ \alpha_v^y \\ \omega_v^z \\ \alpha_f^x \\ \alpha_f^y \\ \omega_f^z \end{pmatrix} \tag{12}$$

For the moment, we also know some of our states, from chapter 3.3.2:

$$X = \begin{pmatrix} \phi \\ \dot{\phi} \end{pmatrix} \tag{13}$$

It is time, then, to define h. The problem is we cannot equate it with the current states, since there is no relation between $\vec{\alpha_v}$ and the states, for instance. The trick is to equate h anyway and add the missing variables to X:

$$h = \begin{pmatrix} \vec{\alpha_v} \\ \omega_v^z \\ \vec{\alpha_f} \\ \omega_f^z \end{pmatrix} = \begin{pmatrix} \boldsymbol{R} \times \vec{a_f} \\ \dot{\phi} + w_f^z \\ \vec{a_f} \\ w_f^z \end{pmatrix} = \begin{pmatrix} a_f^x cos\phi + a_f^y sin\phi \\ -a_f^x sin\phi + a_f^y cos\phi \\ \dot{\phi} + w_f^z \\ a_f^x \\ a_f^y \\ w_f^z \end{pmatrix} \tag{14}$$

$\omega_v^z = \dot{\phi} + w_f^z$ means that the angular speed seen by the steering wheel sensor is the steering wheel angular speed plus the vehicle angular speed (in the concerning axis). Similarly, $\vec{\alpha_v} = \boldsymbol{R} \times \vec{a_f}$ means the acceleration seen by the steering wheel sensor is the acceleration seen by the vehicle sensor, rotated by the steering wheel angle. The others states only exists to allow computing h.

$$\boldsymbol{R} = \begin{bmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{bmatrix} \tag{15}$$

As stated above, $\vec{\alpha_v} = \boldsymbol{R} \times \vec{\alpha_f}$. $\omega_v^z = \dot{\phi} + w_f^z$, $\vec{\alpha_f} = \vec{a_f}$ and $\omega_f^z = w_f^z$. Remark the difference between α and a, as well as between ω and w. The Greek letters denote the observations, while the Roman ones represent the states.

The states turn into:

$$X = \begin{pmatrix} \phi \\ \dot{\phi} \\ a_f^x \\ a_f^y \\ w_f^z \end{pmatrix} \tag{16}$$

$$f = X_{k+1} = \begin{pmatrix} \phi_k + dt\dot{\phi}_k \\ \dot{\phi}_k \\ a_{fk}^x \\ a_{fk}^y \\ w_{fk}^z \end{pmatrix} \tag{17}$$

Now we have X, Z, h and f, we compute the Jacobian matrices H and F.

$$H_{i,j} = \frac{d_{h_i}}{d_{X_j}} \tag{18}$$

$$H = \begin{bmatrix} -a_f^x sin\phi + a_f^y cos\phi & 0 & cos\phi & sin\phi & 0 \\ -a_f^x cos\phi + a_f^y sin\phi & 0 & -sin\phi & cos\phi & 0 \\ & 1 & 0 & 0 & 1 \\ & & 1 & 0 & 0 \\ \text{\Huge 0} & & & 1 & 0 \\ & & & & 1 \end{bmatrix} \tag{19}$$

$$F_{k-1} = \frac{\partial f}{\partial x}\bigg|_{\hat{x}_{k-1|k-1}, u_{k-1}} \tag{20}$$

$$F = \begin{bmatrix} 1 & dt & & & \\ & 1 & & \text{\Huge 0} & \\ & & 1 & & \\ & \text{\Huge 0} & & 1 & \\ & & & & 1 \end{bmatrix} \tag{21}$$

Note that even though the state transition model is linear ($\dot{X} = FX$), it was chosen to present it as if it wasn't, to make it easier for the reader to follow the most commonly used Extended Kalman Filter literature.

3.3.4 Results

In the appendix E it is presented how I set the initial conditions and how I tuned the covariance matrices Q and R.

The following plots show the performance of the designed Kalman Filter, its error with respect to the reference and the error after having applied the RTS algorithm (equation 5). Zooming the figure 1, it is possible to perceive the spikes seen on figures 2 and 3 are mainly due to a small delay of the filter to react to changes, as explained on the section 3.2.4.

The customer sent us the a file containing the reference steering angle – obtained using a potentiometer attached to the steering wheel –, as well as Z. It was asked them to place both IMUs parallels, with the gyroscope axis collinear to the steering wheel axis. We have no photos or other details on how they mounted the sensors.

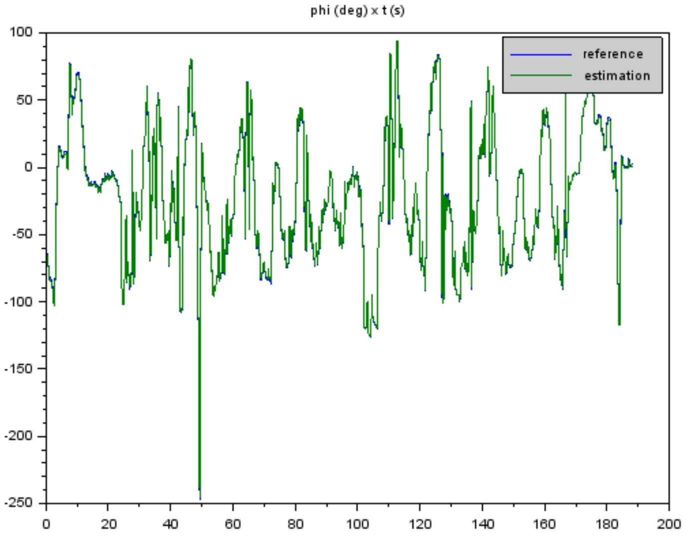

Figure 1: Reference and estimated angles over time.

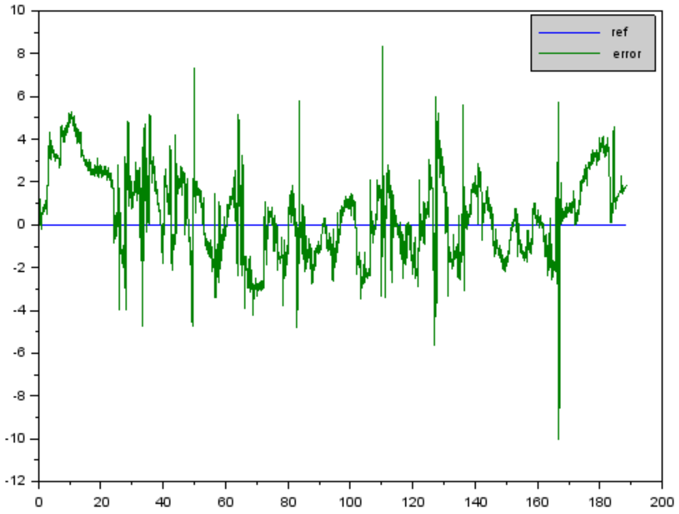

Figure 2: Error between reference and estimation without smoothing.

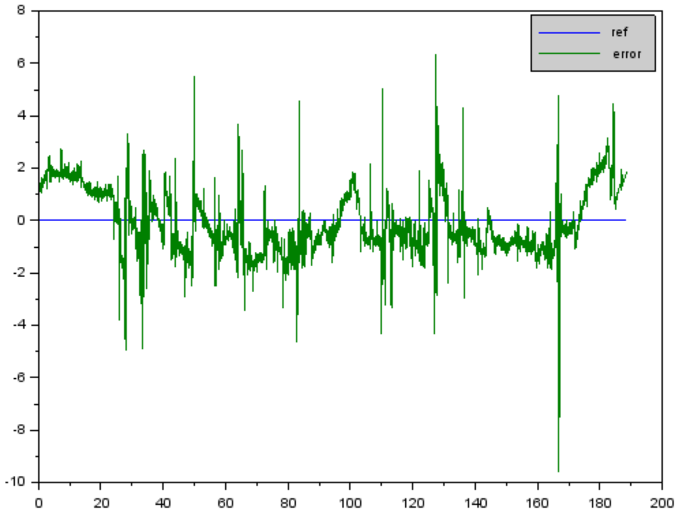

Figure 3: Error between reference and estimation smoothing.

3.4 Estimating attitude angles of a motorcycle

3.4.1 Problem statement

Moto GP teams have been using IMU's outputs for traction control, i.e. to avoid sliding the motorcycle sideway by limiting the throttle depending on roll and pitch, regardless the pilot's action.

For this purpose Texys has developed an IMU (RAD6-M) containing a 3-axis gyroscope, a 3-axis accelerometer, a temperature sensor and a micro-controller. The temperature sensor is used for correcting gyroscopes and accelerometers output as a function of the temperature.

RAD6-M may also receives the motorcycle's speed via CAN in order to better estimate roll and pitch.

3.4.2 Filter design

Previous chapters served to understand the process of designing a Kalman Filter. Therefore, this section will prioritize presenting the equations and explaining some tricks used rather than detailing the development of the equations.

In addition, the observation model was deeply inspired on the work done by Ivo Boniolo on [5]. On his book, he defines the rotation matrices to switch between the inertial and the motorcycle coordinate systems. Also, he develops what he calls Inertial Output Model (IOM). It uses the 3-axis gyroscope and the 3-axis accelerometer outputs to estimate roll, pitch, speed and its derivatives, as well as the yaw derivative, as follows:

$$Z = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \tag{22}$$

$$h = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -cos\phi sin\theta g + sin\phi sin\theta\dot{\psi}v_x + cos\theta\dot{v}_x \\ sin\phi g + cos\phi\dot{\psi}v_x \\ cos\phi cos\theta g - sin\phi cos\theta\dot{\psi}v_x + sin\theta\dot{v}_x \\ cos\theta\dot{\phi} - sin\theta cos\phi\dot{\psi} \\ \dot{\theta} + sin\phi\dot{\psi} \\ sin\theta\dot{\phi} + cos\phi cos\theta\dot{\psi} \end{pmatrix} \tag{23}$$

$$X = \begin{pmatrix} \phi \\ \theta \\ v_x \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{v_x} \end{pmatrix} \tag{24}$$

$$f = X_{k+1} = \begin{pmatrix} \phi + dt\dot{\phi} \\ \theta + dt\dot{\theta} \\ v_x + dt\dot{v_x} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{v_x} \end{pmatrix} \tag{25}$$

The Jacobian matrices F and H are, therefore:

$$F = \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 & 0 \\ & 1 & 0 & 0 & dt & 0 & 0 \\ & & 1 & 0 & 0 & 0 & dt \\ & & & 1 & 0 & 0 & 0 \\ & \text{\Huge 0} & & & 1 & 0 & 0 \\ & & & & & 1 & 0 \\ & & & & & & 1 \end{bmatrix} \tag{26}$$

$$H = \begin{bmatrix} s_\phi s_\theta g + c_\phi \dot{\psi} v_x s_\theta & -c_\theta c_\phi g - s_\theta d v_x + s_\phi c_\theta \dot{\psi} v_x & s_\phi \dot{\psi} s_\theta & 0 & 0 & s_\phi v_x s_\theta & c_\theta \\ c_\phi g - s_\phi \dot{\psi} v_x & 0 & c_\phi \dot{\psi} & 0 & 0 & c_\phi v_x & 0 \\ -s_\phi c_\theta g - c_\phi c_\theta \dot{\psi} v_x & -c_\phi s_\theta g + s_\phi s_\theta \dot{\psi} v_x + c_\theta \dot{v_x} & -s_\phi c_\theta \dot{\psi} & 0 & 0 & -s_\phi c_\theta v_x & s_\theta \\ s_\theta s_\phi \dot{\psi} & -s_\theta \dot{\phi} - c_\theta c_\phi \dot{\psi} & 0 & c_\theta & 0 & -s_\theta c_\phi & 0 \\ c_\phi \dot{\psi} & 0 & 0 & 0 & 1 & s_\phi & 0 \\ -s_\phi c_\theta \dot{\psi} & c_\theta \dot{\phi} - c_\phi s_\theta \dot{\psi} & 0 & s_\theta & 0 & c_\phi c_\theta & 0 \end{bmatrix} \tag{27}$$

To improve the observation model proposed by Boniolo, it were added to Z two new observed variables thanks to the speed sensor. They are v_{ox} and $\dot{v}_{ox}$, $_o$ standing for observed. This modification leads to the following equations:

$$Z = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \omega_x \\ \omega_y \\ \omega_z \\ v_{ox} \\ \dot{v}_{ox} \end{pmatrix} \tag{28}$$

$$h = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ \omega_x \\ \omega_y \\ \omega_z \\ v_{ox} \\ \dot{v}_{ox} \end{pmatrix} = \begin{pmatrix} -cos\phi sin\theta g + sin\phi sin\theta\dot{\psi}v_x + cos\theta\dot{v}_x \\ sin\phi g + cos\phi\dot{\psi}v_x \\ cos\phi cos\theta g - sin\phi cos\theta\dot{\psi}v_x + sin\theta\dot{v}_x \\ cos\theta\dot{\phi} - sin\theta cos\phi\dot{\psi} \\ \dot{\theta} + sin\phi\dot{\psi} \\ sin\theta\dot{\phi} + cos\phi cos\theta\dot{\psi} \\ v_x \\ \dot{v}_x \end{pmatrix} \tag{29}$$

$$H = \begin{bmatrix} s_\phi s_\theta g + c_\phi\dot{\psi}v_x s_\theta & -c_\theta c_\phi g - s_\theta dv_x + s_\phi c_\theta\dot{\psi}v_x & s_\phi\dot{\psi}s_\theta & 0 & 0 & s_\phi v_x s_\theta & c_\theta \\ c_\phi g - s_\phi\dot{\psi}v_x & 0 & c_\phi\dot{\psi} & 0 & 0 & c_\phi v_x & 0 \\ -s_\phi c_\theta g - c_\phi c_\theta\dot{\psi}v_x & -c_\phi s_\theta g + s_\phi s_\theta\dot{\psi}v_x + c_\theta\dot{v}_x & -s_\phi c_\theta\dot{\psi} & 0 & 0 & -s_\phi c_\theta v_x & s_\theta \\ s_\theta s_\phi\dot{\psi} & -s_\theta\dot{\phi} - c_\theta c_\phi\dot{\psi} & 0 & c_\theta & 0 & -s_\theta c_\phi & 0 \\ c_\phi\dot{\psi} & 0 & 0 & 0 & 1 & s_\phi & 0 \\ -s_\phi c_\theta\dot{\psi} & c_\theta\dot{\phi} - c_\phi s_\theta\dot{\psi} & 0 & s_\theta & 0 & c_\phi c_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

Initially, only the v_{ox} was included, what improved the filter performance. Even so, we noticed that the $\dot{v}_x$ estimation drifted from the reality and that it prevented the filter from working even better. So, we decided computing the indirect observed state $\dot{v}_{ox}$. While v_{ox} comes via CAN bus directly from the speed sensor mounted on the front wheel, $\dot{v}_{ox}$ is computed by numerical differentiation, using the equation

$$\dot{v}_{ox}(k) = \frac{v_{ox}(k) - v_{ox}(k-1)}{dt} \tag{31}$$

The speed sensor runs at $100Hz$, while the RAD6-M runs at $200Hz$, meaning RAD6-M had no new speed information on half of its iterations, approximatively and, therefore, the indirect observed $\dot{v}_{ox}$ was 0. To solve this issue, the equation 31 only is called when RAD6-M has receives a different v_{ox} from the speed sensor. For all other instants, RAD6-M uses the last computed value.

We also implemented other tricks to face boundaries problems. If the a posteriori $v_x < 0 \Rightarrow v_x = 0$, since motorcycles move forward only. Similarly, if the a posteriori $(\dot{v}_o x < 0) \wedge (v_o x < 0) \Rightarrow \dot{v}_o x = 0$, meaning motorcycles cannot accelerate backward when they are stopped.

3.4.3 Methodology

In order to verify the filter's performance, both roll and pitch were computed using other sensors. 4 laser distance sensors were fixed to the motorcycle with its beams pointing vertical. The left pair is symmetric to the right one. Photos and further details are available in the D.

The sensors are noisy and, thus, a first order IIR filter was used to get rid of the high frequency noise.

The roll is determined by a lookup table that relates the difference between the left and the right sensor to the roll angle. This lookup table is created during a calibration session and then used on the real test. For calibrating, we make sure the motorcycle is on a horizontal smooth floor and we slowly lean it to the left as much as possible and then to the right, trying to not pumping much the shock absorbers, nor turning the handlebar, to minimize external effects.

It is important to do this process at least within the range $[-40, 40]$ degrees to have an accurate extrapolation during real tests, where the roll reaches up to 55 degrees. It is equally important to do it slowly, seen that during the calibration the roll angle is obtained by the accelerometers:

$$\phi = \sin^{-1}\left(\frac{\alpha_y}{\alpha_z}\right) \tag{32}$$

The pitch angle is computed using a simplified geometric model:

$$\theta = -\sin^{-1}\left(\frac{(front - k1) - (rear - k2)}{k3}\right) \tag{33}$$

Where $k1$ ($k2$) is the front (rear) distance when the pilot is on the motorcycle, while $k3$ is the distance between the front and the rear sensors. Note that $front$ ($rear$) is the mean of both the $front$ ($rear$) measurements.

3.4.4 Results and further works

When analyzing the graphs, we cannot forget some of the disparities causes between the reference and the estimation. First, there are several spikes on roll and pitch angles computed using the lasers. Secondly, the RAD6-M computes the angles with respect to the vertical, while the lasers cannot distinguish a tilted motorcycle from a tilted road.

Finally, we faced the problem of defining the origin of the pitch angle. Unlike the roll angle – where zero degrees means the motorcycle is symmetric to the vertical plane –, it is not clear where the pitch angle is null. We tried to mount the RAD6-M parallel to the horizontal and to the vertical planes (details on appendix D) when the motorcycle had the pilot seated on, but given that the pitch angle range is significantly narrower than the roll angle one, even small deviations on the mounting imply on huge relative errors to the pitch computed using the lasers.

The following plots are the results achieved on an essay using the Honda CBR1000RR racing motorcycle used by Tecmas team, drove by a professional pilot at Val de Vienne circuit located at Le Vigeant, France. For this motorcycle, there was a problem on the front lasers mounting and, thus, the pitch angle is not available.

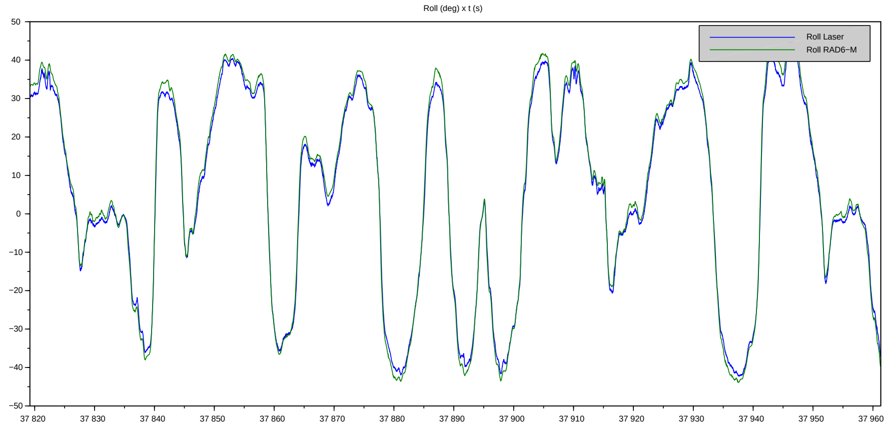

Figure 4: Reference and estimated roll angles over time.

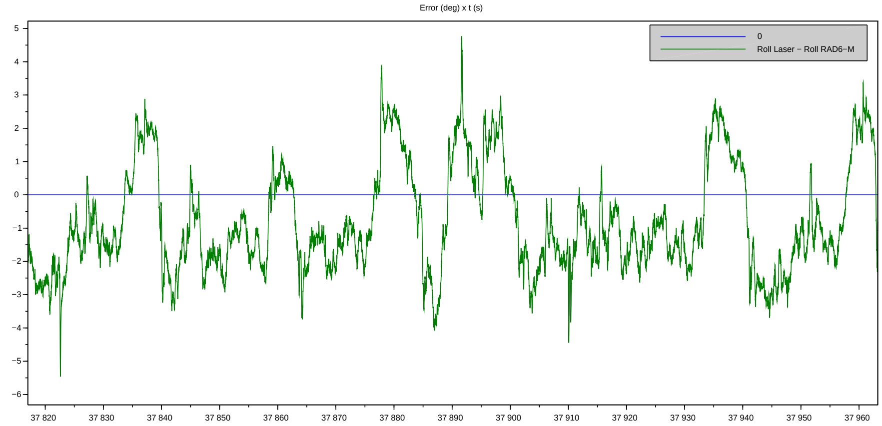

Figure 5: Roll angle error between reference and estimation.

The following plots are the results achieved on an essay using a BMW 1200GS motorcycle at a *mostly flat horizontal* parking lot. Note that during this essay, the maximum speed reached was 82km/h and the maximum leaning angle was 30 degrees, much less than the typical 300km/h and 55 degrees of leaning on racing situation. It is equally important to observe the particular suspension geometry of the BMW 1200GS, detailed on appendix D.

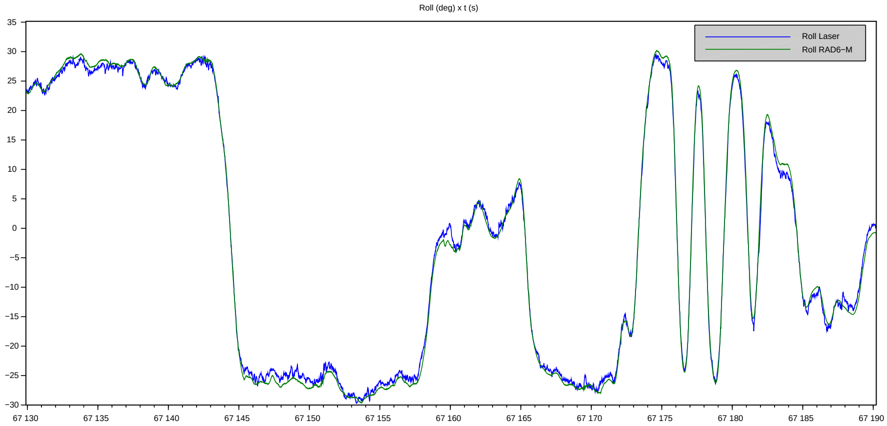

Figure 6: Reference and estimated roll angles over time.

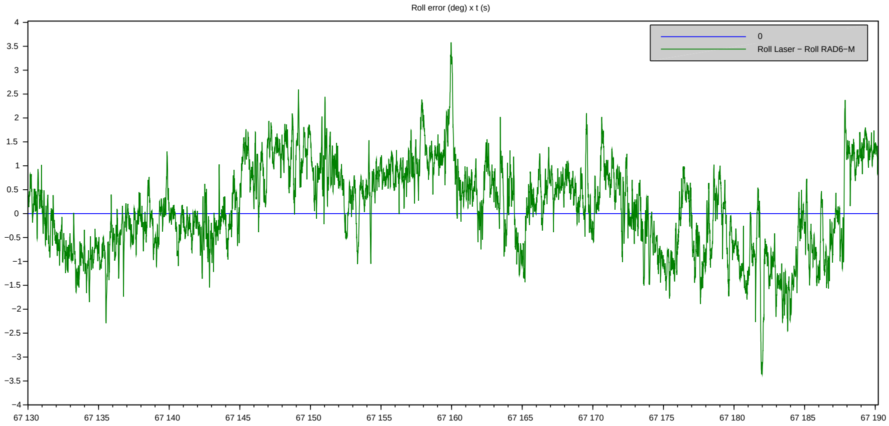

Figure 7: Roll angle error between reference and estimation.

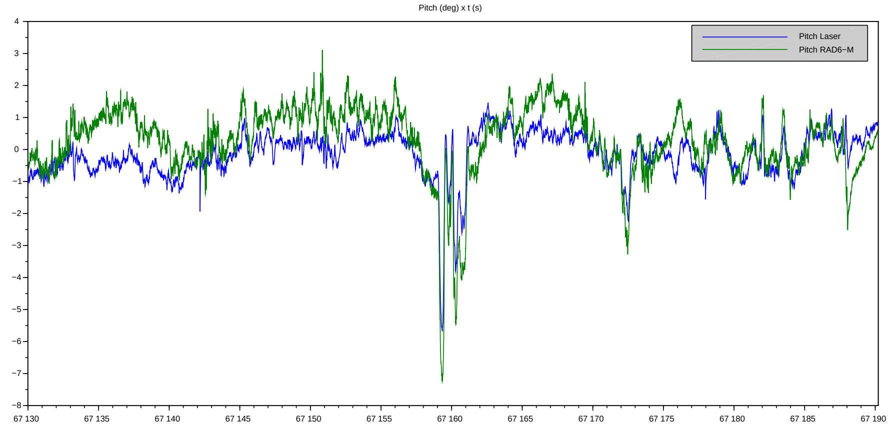

Figure 8: Reference and estimated pitch angles over time.

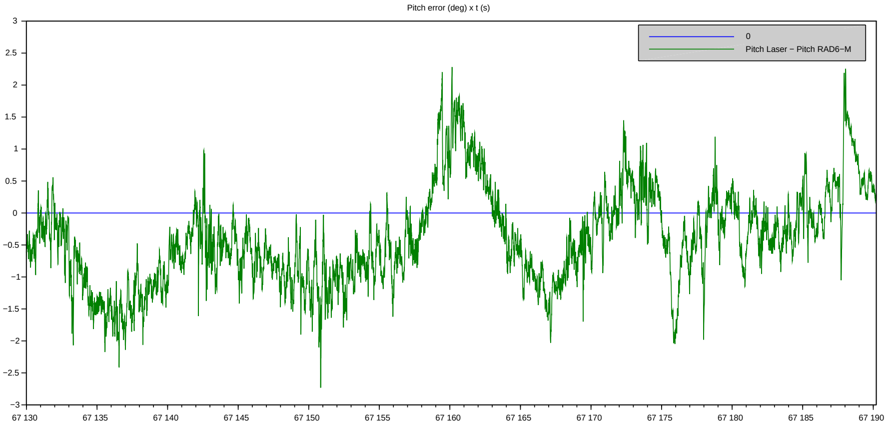

Figure 9: Pitch angle error between reference and estimation.

What concerns the observation model h and tuning the covariance matrices Q and R, I believe almost no improvements can be made. I reckon the main constraint for the moment is the process model. For instance, it is extremely difficult to set the covariance associated with $\dot{\psi}$, since we have no reference to verify its values.

Moreover, the motorcycle mechanical model is not took into account. There are some studies on this area, as [6] and [7], but they require setting up to 70 constants that are drastically different from one motorcycle to another.

Even if the mechanical model is fully described for a particular motorcycle, there is still a variable playing a major role: the pilot's behavior. The position of Center Of Gravity (COG) plays a lot on cornering, for instance. It is, however, possible to estimate the pilot's attitude, using the technique explained on [8].

4 Company's economical analysis

Before economically analyzing Texys, it must be told the company works differently from others, mainly concerning its clients budget, needs and loyalty. As stated in chapter 1, the company is renowned for product reliability and accuracy. What makes Texys so special is that its clients are primarily professional racing teams, who usually need fast and customized solutions for its specific harsh applications, with a reliable after sales service. Knowing these client's needs, Texys positioned itself to meet these demands. This positioning assures the company always have new orders. Hence, its products are sold with all these facilities costs embedded in its prices, what guarantees clients' satisfaction and Texys' profit margins to be large on every service and sensor.

That said, to the economical analysis. These large profit margins allow Texys to not care much about investing money with R&D. Given that the company does not really depend on the R&D department's results to obtain its profit, the engineers are under pressure due to deadlines only and, therefore, often achieve better results than they would if they worked under full stress. As consequence, the standard products are constantly improved and new products are developed. That is the case of the RAD6 sensor I worked on during the internship, which needs to fit the Moto GP specs, as well as to beat Magneti Marelli's sensor performance.

Profit margins are about 50% for the sensors and services, what, among with a growth in selling volume, provoke Texys' turnover to dramatically increase in the last decade. It jumped from 0.8 million€ in 2009 to about 4.5 million€ in 2016.

The Enterprises' Bank File – or *Fichier bancaire des entreprises*, in French – (FIBEN) is a database managed by the Bank of France – Banque de France – that attributes a quote that provides evidence of their ability to honor their financial commitments. In 2016, Texys has obtained the highest grade from FIBEN.

As the RAD6 puts Texys into the *glamorous* list of smart sensors manufacturers, the budget available for developing the product is not limited to a well defined threshold. Even so, it is expected to allocate around 12000€ during 2017, the biggest portion being labor cost. 50 units of RAD6 are estimated to be sold a year for 2000€ each, representing a turnover of 100,000€, from which 38,000€ are profit (subtracting development costs). The company deliberately does not take into account development costs of unsold versions. It is their choice facing failures in development outside the sum, rather than embedding it to product's final cost. In the longterm it is all about being able to develop smart sensors.

The numbers for the steering angle are also great. It is predicted to sell about 100 pieces a year, for around 800€ each. This contributes with 80,000€ in the company's turnover, from which almost 40,000€ is profit.

5 Analysis of the contribution of the internship

The internship brought me several experiences and skills, mainly concerning technical and management.

In the technical domain, both problems of angle estimation required the use of multiple tools to obtain proper results. Day after day I became more capable to figure out how to prioritize the sequence of tasks I had to accomplish to improve the results. This ability of prioritizing tasks came as direct consequence of learning to analyze the results I had. In my experience, analyzing data is not a skill you can really learn by reading books, nor a skill you can self-teach. During the internship, Gregory Servaud – my tutor –, and Benoit Buteau – the engineer who works on the project – sit next to me uncountable times to help me understanding all the *messy* data displayed on my screen. I learned to find patterns, isolate data, correlate different data, and even *guessing* the causes of unexpected phenomena.

Still on the technical domain, I learned to use Scilab – an open source alternative to MATLAB –, as well as Finite and Infinite Impulse Response filters (FIR and IIR). The topic I am satisfied the most on the technical aspect – apart from having learned to analyze data – is that I deeply understood the Kalman Filter, from defining the observation model and system's states, to tuning the filter's parameters and adding constraints from the real world.

Concerning the management domain, I could experience company's day to day and understand how the departments work together smoothly. I believe that an engineer must understand how his job's environment is like, so that he can work to optimize what is expected from him in terms of deliverables. The R&D department I worked on has weekly meetings, which serves not only as a moment to presenting to its members what was done during the previous week, but also to brainstorm alternatives to projects issues and to defining priorities for the following week. These experiences gave me clear ideas on proper running a project.

Before the internship I had no clue what I would like to do as an engineer, i.e. I had no professional project at all. The internship opened my mind and now I am working on multiple lines. Definitely it was a breakthrough discovering how fast and pleasant is learning techniques on the day to day of an innovative company's R&D department. Now I feel I am more **capable** of working on projects that seem to be really tough. Nevertheless, for some reason I cannot clear understand, I am not sure of willing to work with R&D until I retire. One thing I can state: it was really enjoyable learning the French culture – especially French puns and idiomatic expressions – and that pushes abroad for my next internship.

6 Conclusion

During the internship, I have been confronted some problematics entirely new to me. Even though I am living in France for the past year, the experience of working was totally different from the one I had studying: the people I worked with are older than I, meaning I was not used to their vocabulary, especially specific technique words and slangs.

Also, working on an assistant engineer, I faced real-world projects, which solutions are not known. In the beginning, it was quite scary to be in charge of developing new technology, but the situation soon showed to be challenge (rather than scary) and, thus, exciting.

Finally, working in real-world projects taught me to handle measurement noises, uncertainties, constraints, pressure and deadlines, as well as to trade-off between crucial project's aspects, like precision and power consumption.

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A RAD6-M data sheet

RAD6-M

Roll Angle Device for Motorcycle with integrated 6 axis inertial box.

Ref: RAD6-M

SN: B######## Software version: v#.##

Texys sensors are designed for data recording. If the user wants to include this sensor in a close loop system or active control, he must assume all responsibility.

Mechanical specification		
Dimensions	32x42.5x20	mm
Material	Aluminum	
Weight	60	g

CAN parameters	
Configurable parameters	CAN type, baudrate, emission frequency, identifiers.
CAN type	2.0A or 2.0B
CAN termination resistor	Switchable, 120Ω
Baud rate	125k to 1Mbps
Output Frequency	200Hz

Operating conditions		
Max Supply Current (with 12V supply)	80	mA
Supply Voltage	6 to 16	V
Protection	IP66	
Vibration test	20Gpp 5'	
Operating Temp	-20 to +100	°C
Storage Temp	-40 to +125	°C

Accelerometer specification		
Technology	GAS	
Range	±5	G
Bandwidth 3dB	DC to 20 ±15%	Hz
Max offset error (20 to 80°C)	±0.5	%FS
Max sensitivity error (20 to 80°C)	±1	%
Max error (offset, repeatability, sensitivity, linearity)	±1.5	%FS
Max cross axis sensitivity	±2	%
Gyroscope specification		
Range	±300	°/s
Cut off frequency 1st order	30	Hz
Max offset error (20 to 80°C)	±0.5	%FS
Max sensitivity error (20 to 80°C)	±0.5	%
Max error (offset, repeatability, sensitivity, linearity)	±1	%FS
Max cross axis sensitivity	±2	%

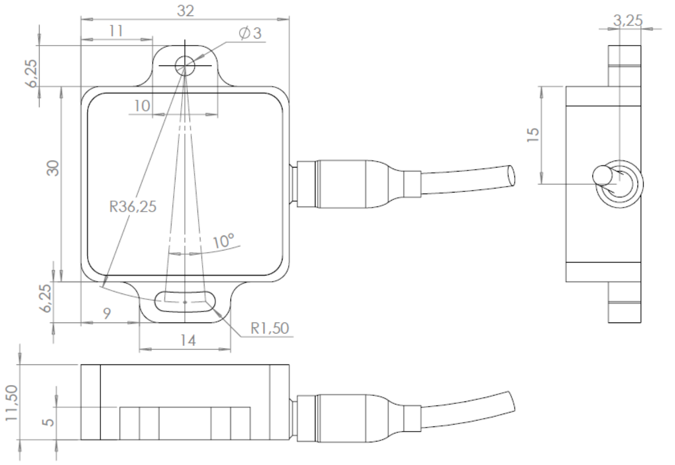
Accelerometer Sensor Readings			
	X	Y	Z
Signal @ -1G			
Signal @ 0G			
Signal @ +1G			
Cross axis (%)			
Gyroscope Sensor Readings			
	X	Y	Z
@ 0°/s			
@ 300°/s			
Cross axis (%)			

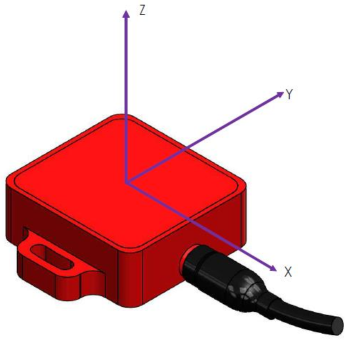
Setup parameters		
CAN	2.0A ~~2.0B~~	-
CAN Termination Resistor	☐yes ☒no	-
Baudrate	1000000	bps
Tx1 ID	0x3B0	Hex
Tx2 ID	0x3B4	Hex
Tx3 ID	0x3B8	Hex
Tx4 ID	0x3BC	Hex
Rx1 ID	0x300	Hex
Permutation axis	1 2 3	-

Cable: 5x26AWG FEP tinned copper braided (250V 200°C)

Length: 1000 mm

Function	Description	Wire color
Supply	Supply (6 to 16 V)	Red
	GND	Black
CAN	CAN HIGH	Green
	CAN LOW	White
IHM	One-Wire	Yellow





Data output

Tx Frame #1 (output frequency: 200Hz)

ID	Byte 0	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6	Byte 7
0x3B0 (default)	AccX_user		AccY_user		AccZ_user		Acc. Diag	Acc. Temp
	MSB	LSB	MSB	LSB	MSB	LSB	4 bits	12 bits
	signed integer 16 bits		signed integer 16 bits		signed integer 16 bits			signed integer
	1mG/bit		1mG/bit		1mG/bit			0.1°C/bit

Tx Frame #2 (output frequency: 200Hz)

ID	Byte 0	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6	Byte 7
0x3B4 (default)	GyrX_user		GyrY_user		GyrZ_user		Gyr. Diag	Gyr. Temp
	MSB	LSB	MSB	LSB	MSB	LSB	4 bits	12 bits
	signed integer 16 bits		signed integer 16 bits		signed integer 16 bits			signed integer
	0.01deg/s/bit		0.01deg/s/bit		0.01deg/s/bit			0.1°C/bit

Tx Frame #3 (output frequency: 200Hz)

ID	Byte 0	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6	Byte 7
0x3B8 (default)	Roll angle		Pitch angle		Not used	Not used	Not used	Not used
	MSB	LSB	MSB	LSB				
	signed integer 16 bits		signed integer 16 bits					
	0.1deg/bit		0.1deg/bit					

Tx Frame #4 (output frequency: 50Hz)

ID	Byte 0	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6	Byte 7
0x3BC (default)	Not used	Not used	Not used	Not used	Not used	Not used	Base Diag	
							16 bits	

Data input

Rx Frame #1 (input frequency: 50Hz)

ID	Byte 0	Byte 1	Byte 2	Byte 3	Byte 4	Byte 5	Byte 6	Byte 7
0x300 (default)	Vehicle speed		Vehicle speed diag		Not used	Not used	Not used	Not used
	MSB	LSB	MSB	LSB				
	signed integer 16 bits		16 bits					
	1mG/bit							

B Assessment report

RAPPORT D'EVALUATION
ASSESSMENT REPORT

Merci de retourner ce rapport en fin du stage à :
Please return this report at the end of the internship to :

ENSTA Bretagne – *Bureau des stages - 2 rue François Verny - 29806 BREST cedex 9 – FRANCE*
☎ 00.33 (0) 2.98.34.87.70 - Fax 00.33 (0) 2.98.38.87.90 - stages@ensta-bretagne.fr

I - ORGANISME / *HOST ORGANISATION*

NOM / *Name* TEXYS FR

Adresse / *Address* 16 rue Edouard Branly
58640 Varennes Vauzelles

Tél / *Phone (including country and area code)* +33 3 86 21 27 18

Fax / *Fax (including country and area code)* +33 3 86 21 24 49

Nom du superviseur / *Name of placement supervisor*
SERVAUD Grégory

Fonction / *Function* Responsable R&D

Adresse e-mail / *E-mail address* g.servaud@texense.com

Nom du stagiaire accueilli / *Name of trainee* Philipe Miranda de Moura

II - EVALUATION / *ASSESSMENT*

Veuillez attribuer une note, en encerclant la lettre appropriée, pour chacune des caractéristiques suivantes. Cette note devra se situer entre **A (très bien)** et **F (très faible)**
*Please attribute a mark from **A (very good)** to **F (very weak).***

MISSION / *TASK*

- La mission de départ a-t-elle été remplie ? A [✓ B] C D E F
  Was the initial contract carried out to your satisfaction?

- Manquait-il au stagiaire des connaissances ? ☐ oui/*yes* ☒ non/*no*
  Was the trainee lacking skills?

  Si oui, lesquelles ? / *If so, which skills?*

ESPRIT D'EQUIPE / *TEAM SPIRIT*

- Le stagiaire s'est-il bien intégré dans l'organisme d'accueil (disponible, sérieux, s'est adapté au travail en groupe) / *Did the trainee easily integrate the host organisation? (flexible, conscientious, adapted to team work)*

  [✓ A] B C D E F

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here*

COMPORTEMENT AU TRAVAIL / *BEHAVIOUR TOWARDS WORK*

Le comportement du stagiaire était-il conforme à vos attentes (Ponctuel, ordonné, respectueux, soucieux de participer et d'acquérir de nouvelles connaissances) ?
Did the trainee live up to expectations? (Punctual, methodical, responsive to management instructions, attentive to quality, concerned with acquiring new skills)?

[✓ A] B C D E F

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* ______

INITIATIVE – AUTONOMIE / *INITIATIVE – AUTONOMY*

Le stagiaire s'est –il rapidement adapté à de nouvelles situations ? A [✓ B] C D E F
(Proposition de solutions aux problèmes rencontrés, autonomie dans le travail, etc.)

Did the trainee adapt well to new situations? A B C D E F
(eg. suggested solutions to problems encountered, demonstrated autonomy in his/her job, etc.)

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* ______

CULTUREL – COMMUNICATION / *CULTURAL – COMMUNICATION*

Le stagiaire était-il ouvert, d'une manière générale, à la communication ? [✓ A] B C D E F
Was the trainee open to listening and expressing himself /herself?

Souhaitez-vous nous faire part d'observations ou suggestions ? / *If you wish to comment or make a suggestion, please do so here* ______

OPINION GLOBALE / *OVERALL ASSESSMENT*

- La valeur technique du stagiaire était : [✓ A] B C D E F
  Evaluate the technical skills of the trainee:

III - PARTENARIAT FUTUR / *FUTURE PARTNERSHIP*

- Etes-vous prêt à accueillir un autre stagiaire l'an prochain ?
  Would you be willing to host another trainee next year? ☒ oui/*yes* ☐ non/*no*

Fait à Varennes Vauzelles, le 4/09/17
In ______, *on* ______

Signature Entreprise ______ Signature stagiaire
Company stamp ______ *Trainee's signature*

TEXYS FR
Sarl au capital de 552 000€
ZA des Chamonds 16 Rue E.Branly
58640 Varennes-Vauzelles -FR
Siret : 490 935 756 000 26

Merci pour votre coopération
We thank you very much for your cooperation

C BMW 1200GS suspension

Figure 10: BMW 1200GS front suspension.

Figure 11: BMW 1200GS rear suspension.

The figure 10 reveals the particular fork design of this motorcycle that allows it to rotate with respect to the frame, so that the pitch angle is reduced during abrupt breaking periods.

Similarly, the figure 11 zooms in the swingarm, that is also very unique. It possesses multiple pivot point to soften the pitch angle during hard accelerations. Plus, this motorcycle is equipped with a Cardin axle – rather than a chain – to serve as transmission.

All the factors explained right above, in addition with its upright pilot positioning (and, thus, higher COG) may partially explain why the RAD6-M computed data does not match better the data computed using the laser sensors.

D Sensors locations and setups

Figure 12: BMW 1200GS used for testing.

Figure 13: Rear laser in detail.

Figure 14: Front laser in detail.

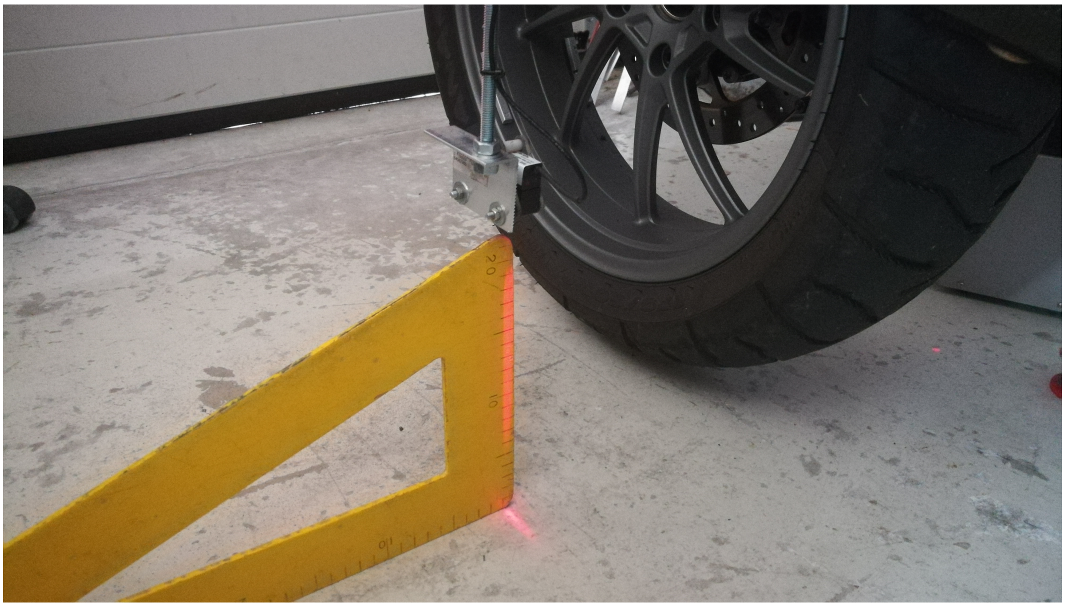


Figure 15: Vertical laser beam.

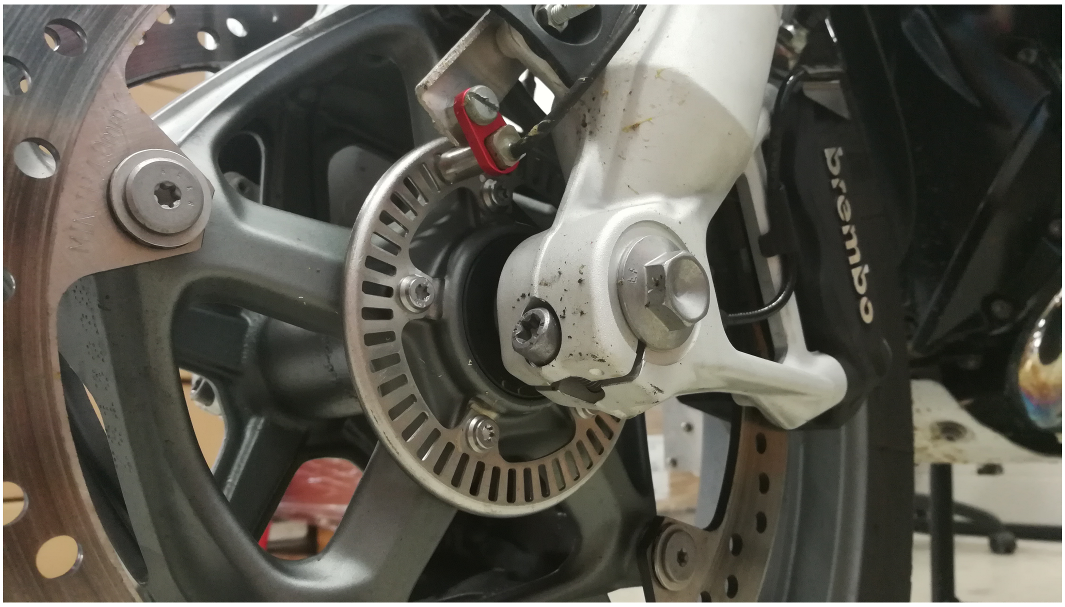


Figure 16: Speed sensor (in red) mounted on the front ABS disk.

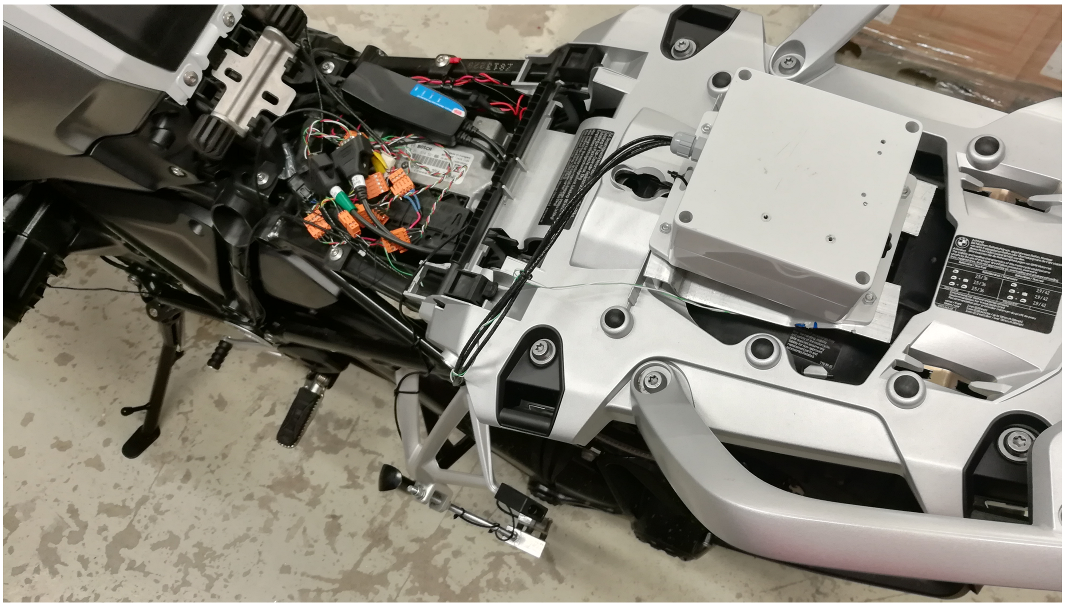


Figure 17: The white box mounted on the rear seat contains the RAD6-M sensor.

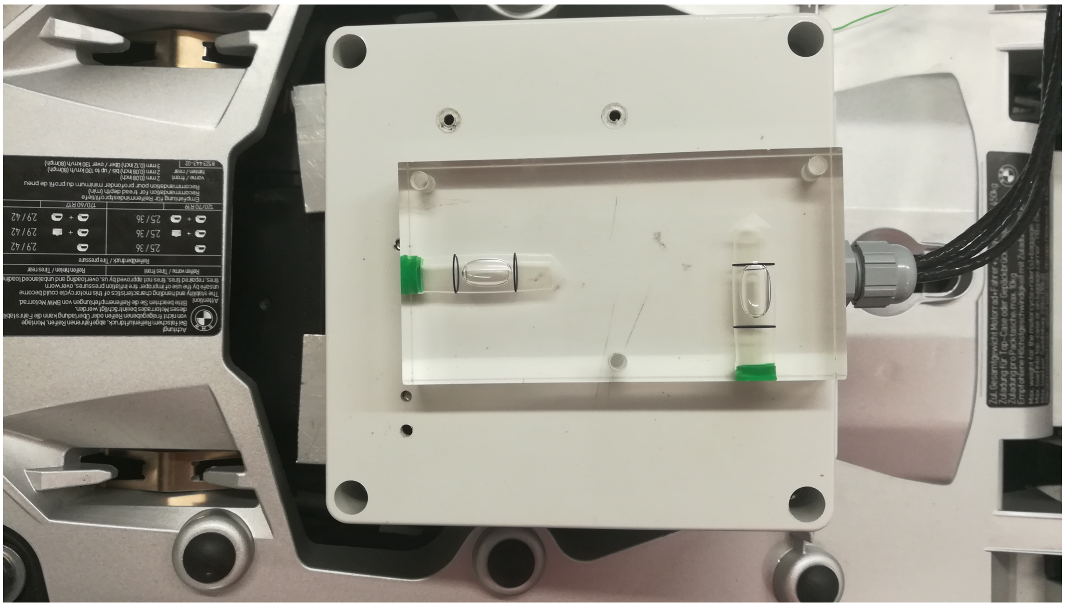


Figure 18: The RAD6-M container is fixed parallel to the ground with the aid of a bubble level.

E Steering wheel's problem Kalman parameters

$$X_0 = \begin{pmatrix} 0 \\ 0 \\ g \\ 0 \\ 0 \end{pmatrix} \tag{34}$$

The states are initialized using the most suitable values, what, in this case means zero everywhere, except from the third one. That means we expect to start the algorithm with the steering wheel being centered, not moving. The sensor mounted on the dashboard is equally expected not to move, what corresponds to reading the gravity on the x axis and zero elsewhere.

$$P_0 = 0.1 \times \boldsymbol{I}_5 \tag{35}$$

Honestly, setting larger or smaller P_0 had not much impact on the filter performance, after the first few seconds. Setting a small P_0 avoids obtaining a noisy estimation, with the drawback of less robustness to the initial condition. An alternative to having these drawbacks is using the first measurements as the initial condition.

$$Q = \boldsymbol{I}_5 \times \begin{pmatrix} 2.031e^{-}14 \\ 0.0126643 \\ 0.005517 \\ 0.009784 \\ 0.010767 \end{pmatrix} \tag{36}$$

$$R = 10^{-4} \times \boldsymbol{I}_6 \times \begin{pmatrix} 2.789 \\ 2.789 \\ 0.002 \\ 2.789 \\ 2.789 \\ 0.002 \end{pmatrix} \tag{37}$$

These Q and R matrices were initially manually tunned following the general directives explained in section 3.2.4. Then, since the reference is not noisy at all and can be considered accurate, a Particle Swarm Optimization (PSO) code was used to optimize the simulation quadratic error with respect to the reference. It is, however, not advisable to use this kind of technique to tune Q and R. It can lead to parameters suitable for noisy processes only, for instance.

F Motorcycle's problem Kalman parameters

$$X_0 = \vec{0}_{7\times1} \tag{38}$$

$$P_0 = \boldsymbol{I}_7 \tag{39}$$

$$Q = 0.5 \times \boldsymbol{I}_7 \times \begin{pmatrix} 10^{-14} \\ 10^{-14} \\ 5\times10^{-12} \\ 10^{-2} \\ 10^{-2} \\ 10^{-2} \\ 5\times10^{-4} \end{pmatrix} \tag{40}$$

$$R = \boldsymbol{I}_6 \times \begin{pmatrix} 5\times10^{-2} \\ 5\times10^{-2} \\ 10^{3} \\ 3\times10^{-5} \\ 3\times10^{-5} \\ 3\times10^{-5} \end{pmatrix} \tag{41}$$

$$\hat{R} = \begin{bmatrix} [R] & 0 \\ 0 & \begin{bmatrix} 5\times10^{-3} & 0 \\ 0 & 10^{-1} \end{bmatrix} \end{bmatrix} \tag{42}$$

These parameters were tuned after having the experience of tuning simpler filters. The process is exact the same, but even so, a few remarks are needed.

Firstly, we can observe a huge difference between the covariance of the first three and the last four elements on Q. Not by chance, the first ones are ϕ, θ and v_x, while the others are their derivatives (and $\dot{\psi}$). In addition, the non-derivative terms have their prediction equations on the form

$$\beta(k+1) = \beta(k) + dt\dot{\beta}(k) \tag{43}$$

, and the derivatives equations states

$$\dot{\beta}(k+1) = \dot{\beta}(k) \tag{44}$$

This difference on the typology explains why the covariances of the first three states are much smaller than the others.

Secondly, the covariance of α_z (the third element of R) is giant, compared to the ones of α_x and α_y. In fact, this is a good example of what "observation noise" means. Although the three accelerometers have the same noise, the observation noise on the vertical direction α_z is larger than on the other directions due to road irregularities, that play mainly on this axis.

Finally, we observe the relatively large covariance on $\dot{v}_{ox}$, that is explained by the indirect way it is obtained.