Relaxed intersection of thick sets

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Relaxed Intersection : The q-relaxed intersection m sets $\mathbb{X}_1, \ldots, \mathbb{X}_m$ of $\mathbb{R}^n$, is the set of all $\mathbf{x} \in \mathbb{R}^n$ which belong to all $\mathbb{X}_i$'s except q at most. This notion is important in the context of robust bounded error estimation [**1**].

Thick set. Denote by $(\mathcal{P}(\mathbb{R}^n), \subset)$, the powerset of $\mathbb{R}^n$ equipped with the inclusion $\subset$ as an order relation. A *thick set* $[\![\mathbb{X}]\!]$ of $\mathbb{R}^n$ is an interval of $(\mathcal{P}(\mathbb{R}^n), \subset)$. If $[\![\mathbb{X}]\!]$ is a thick set of $\mathbb{R}^n$, there exist [**2**] two subsets of $\mathbb{R}^n$, called the subset bound and the supset bound such that

$$[\![\mathbb{X}]\!] = \qquad [\![\mathbb{X}^{\subset}, \mathbb{X}^{\supset}]\!] = \{\mathbb{X} \in \mathcal{P}(\mathbb{R}^n) | \mathbb{X}^{\subset} \subset \mathbb{X} \subset \mathbb{X}^{\supset}\}. \tag{1}$$

Thick test. Given a thick set $[\![\mathbb{X}]\!]$, the corresponding thick test $[\![t]\!] : \mathbb{R}^n \to \{0, ?, 1\}$ associates to any $\mathbf{x}$ the value 0 if $\mathbf{x} \notin \mathbb{X}^{\supset}$, 1 if $\mathbf{x} \in \mathbb{X}^{\subset}$ and ? otherwise.

Algorithm. Using a paver and an arithmetic on thick tests we show that we can easily obtain a guaranteed approximation of the relaxed intersection of m thick sets.

TestCase: Let us take the following system of interval linear equations [**3**]:

$$\begin{pmatrix} [2,4] & [-2,-1] \\ [0,2] & [-3,-1] \\ [-6,-4] & [-1,0] \\ [-3,-2] & [-3,-2] \\ [-1,1] & [-5,-4] \end{pmatrix} \cdot \mathbf{x} = \begin{pmatrix} [-1,1] \\ [-5,0] \\ [-6,0] \\ [2,7] \\ [-9,-3] \end{pmatrix}. \tag{2}$$

This system corresponds to six interval equations, each line of which corresponds to a thick set $[\![\mathbb{X}_i]\!]$ in $\mathbb{R}^2$. Performing a relaxed intersection of the thick sets $[\![\mathbb{X}_i]\!]$, we obtain the characterization relaxed intersection $[\![\mathbb{X}^{\{q\}}]\!]$ of the $[\![\mathbb{X}_i]\!]$ for $q \in \{1, 2, 3\}$ as given by Figure 1. For $q = 0$, $[\![\mathbb{X}^{\{q\}}]\!]$ is empty.

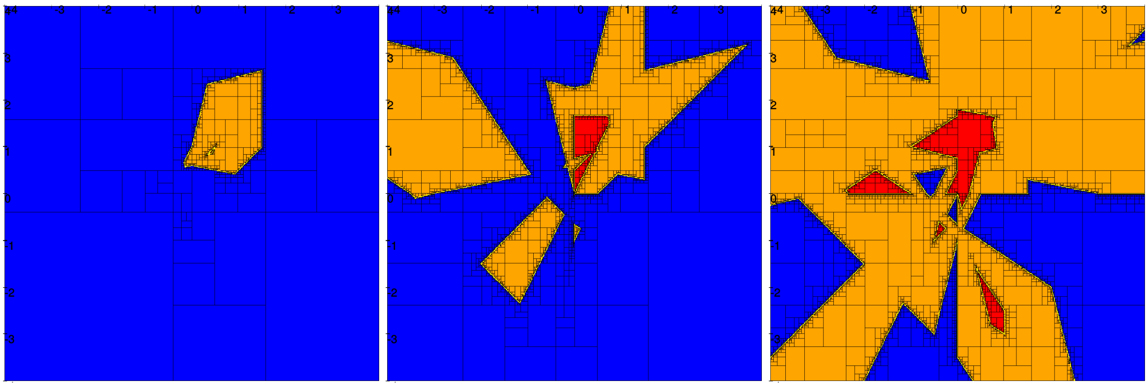

Figure 1: $[\![\mathbb{X}^{\{1\}}]\!]$ (Right), $[\![\mathbb{X}^{\{2\}}]\!]$(middle) and $[\![\mathbb{X}^{\{3\}}]\!]$ (Right). Red boxes correspond to the subset bound $\mathbb{X}^{\{q\}\subset}$ of $[\![\mathbb{X}^{\{q\}}]\!]$, blue ones are outside the supset bound $\mathbb{X}^{\{q\}\supset}$ and orange boxes belong to $\mathbb{X}^{\{q\}\supset} \setminus \mathbb{X}^{\{q\}\subset}$.

References:

[**1**] Q. Brefort, L Jaulin, M. Ceberio, V. Kreinovich, Towards Fast and Reliable Localization of an Underwater Object: An Interval Approach, *Journal of Uncertain Systems*, (2015).

[**2**] B. Desrochers, L. Jaulin, Computing a guaranteed approximation the zone explored by a robot, *IEEE Transaction on Automatic Control*, (2016).

[**3**] V. Kreinovich and S. Shary, Interval Methods for Data Fitting under Uncertainty: A Probabilistic Treatment, *Reliable Computing*, (2016).