Relaxed intersection of thick sets

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**Relaxed Intersection**: The $q$-relaxed intersection $m$ sets $X_1, \ldots, X_m$ of $\mathbb{R}^n$, is the set of all $x \in \mathbb{R}^n$ which belong to all $X_i$'s except $q$ at most. This notion is important in the context of robust bounded error estimation [1].

**Thick set**. Denote by $\mathcal{P}(\mathbb{R}^n, \subset)$, the powerset of $\mathbb{R}^n$ equipped with the inclusion $\subset$ as an order relation. A **thick set** $[X]$ of $\mathbb{R}^n$ is an interval of $\mathcal{P}(\mathbb{R}^n, \subset)$. If $[X]$ is a thick set of $\mathbb{R}^n$, there exist [2] two subsets of $\mathbb{R}^n$, called the subset bound and the supset bound such that

$$[X] = [X^\subset, X^\supset] = \{X \in \mathcal{P}(\mathbb{R}^n) | X^\subset \subset X \subset X^\supset\}. \quad (1)$$

**Thick test**. Given a thick set $[X]$, the corresponding thick test $[t] : \mathbb{R}^n \to \{0, ?, 1\}$ associates to any $x$ the value 0 if $x \notin X^\supset$, 1 if $x \in X^\subset$ and ? otherwise.

**Algorithm**. Using a paver and an arithmetic on thick tests we show that we can easily obtain a guaranteed approximation of the relaxed intersection of $m$ thick sets.

**TestCase**: Let us take the following system of interval linear equations [3]:

$$\begin{pmatrix}
[2, 4] & [-2, -1] \\
[0, 2] & [-3, -1] \\
[-6, -4] & [-1, 0] \\
[-3, -2] & [-3, -2] \\
[-1, 1] & [-5, -4]
\end{pmatrix} \cdot x = \begin{pmatrix}
[-1, 1] \\
[-5, 0] \\
[-6, 0] \\
[2, 7] \\
[-9, -3]
\end{pmatrix}. \quad (2)$$
This system corresponds to six interval equations, each line of which corresponds to a thick set $[X_i]$ in $\mathbb{R}^2$. Performing a relaxed intersection of the thick sets $[X_i]$, we obtain the characterization relaxed intersection $[X(q)]$ of the $[X_i]$ for $q \in \{1, 2, 3\}$ as given by Figure 1. For $q = 0$, $[X(q)]$ is empty.

Figure 1: $[X^{(1)}]$ (Right), $[X^{(2)}]$ (middle) and $[X^{(3)}]$ (Right). Red boxes correspond to the subset bound $X^{(q)c}$ of $[X^{(q)}]$, blue ones are outside the supset bound $X^{(q)\supset}$ and orange boxes belong to $X^{(q)\supset \setminus X^{(q)c}}$.

References:

