A contractor is minimal for narrow boxes

Luc Jaulin

Robex, Lab-STICC, ENSTA-Bretagne, Brest, France lucjaulin@gmail.com

Keywords: Interval analysis, Contractors, Centered form

Introduction

Interval analysis is an efficient tool used for solving rigorously complex nonlinear problems involving bounded uncertainties [1] [2] [3]. Many interval algorithms are based on the notion of *contractor* [5] which is an operator which shrinks an axis-aligned box $[\mathbf{x}]$ of $\mathbb{R}^n$ without removing any point of the solution set $\mathbb{X}$. Combined with a paver which bisects boxes, the contractor builds an outer approximation of the set $\mathbb{X}$. The resulting methodology can be applied in several domains of engineering such as localization [7], SLAM [8] [9], reachability [10], etc.

This paper proposes a new interval-based contractor for nonlinear equations which is minimal for narrow boxes. The method is based on the centered form combined with a Gauss Jordan band diagonalization preconditioning.

1 Illustration

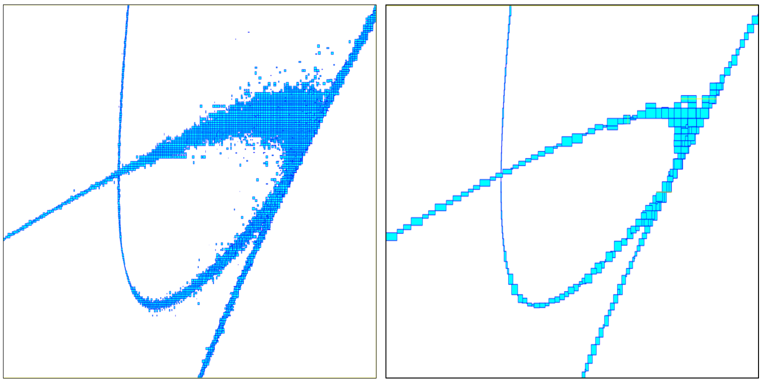
Consider the set

$$\mathcal{P} = \{\mathbf{p} \mid \exists \omega > 0,\, \mathbf{f}(\mathbf{p}, j\omega) = 0\}. \tag{1}$$

where

$$\mathbf{f}(p_1, p_2, j\omega) = \begin{pmatrix} -\omega^2 + 2\omega\sin(\omega p_1) + \cos(\omega p_2) \\ 2\omega\cos(\omega p_1) - \sin(\omega p_2) \end{pmatrix} \tag{2}$$

Take $[p_1] = [0, 2.5]$, $[p_2] = [1, 4]$,$[\omega] = [0, 10]$ and let us characterize the set $\mathcal{P}$. Using a branch and prune algorithm with a accuracy of $\varepsilon = 2^{-8}$ with an HC4 algorithm [1][12] (the state of the art), we get the paving of Figure 1, left. The number of boxes of the approximation is 43173. Similar results where obtained were obtained on the same example in [13]. With an accuracy of $\varepsilon = 2^{-4}$, our centered contractor yields Figure 1, right. The number of boxes of the approximation is 282 (instead of 43173), for a more accurate approximation.



A video illustrating the asymptotic minimality of the contractor is given at:

`https://youtu.be/nM9rR4jDj74`

References

[1] M. Cébério and L. Granvilliers, "Solving nonlinear systems by constraint inversion and interval arithmetic," in *Artificial Intelligence and Symbolic Computation*, vol. 1930, LNCS 5202, 2001, pp. 127–141.

[2] V. Kreinovich, A. Lakeyev, J. Rohn, and P. Kahl, *Computational Complexity and Feasibility of Data Processing and Interval Computations.* Springer, 1997.

[3] A. Rauh and E. Auer, *Modeling, Design, and Simulation of Systems with Uncertainties.* Springer, 2011.

[4] G. Chabert and L. Jaulin, "Contractor Programming," *Artificial Intelligence*, vol. 173, pp. 1079–1100, 2009.

[5] R. Guyonneau, S. Lagrange, and L. Hardouin, "A visibility information for multi-robot localization," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2013.

[6] M. Mustafa, A. Stancu, N. Delanoue, and E. Codres, "Guaranteed SLAM; An Interval Approach," *Robotics and Autonomous Systems*, vol. 100, pp. 160–170, 2018.

[7] S. Rohou, L. Jaulin, L. Mihaylova, F. L. Bars, and S. Veres, *Reliable Robot Localization.* Wiley, dec 2019. [Online]. Available: https://doi.org/10.1002%2F9781119680970

[8] E. Goubault and S. Putot, "Robust under-approximations and application to reachability of non-linear control systems with disturbances," *IEEE Control. Syst. Lett.*, vol. 4, no. 4, pp. 928–933, 2020. [Online]. Available: https://doi.org/10.1109/LCSYS.2020.2997261

[9] F. Benhamou, F. Goualard, L. Granvilliers, and J. F. Puget, "Revising hull and box consistency," in *Proceedings of the International Conference on Logic Programming*, Las Cruces, NM, 1999, pp. 230–244.

[10] R. Malti, M. Rapaić, and V. Turkulov, "A unified framework for robust stability analysis of linear irrational systems in the parametric space," *Automatica*, 2022, second version, under review (see also https://hal.archives-ouvertes.fr/hal-03646956). [Online]. Available: https://hal.archives-ouvertes.fr/hal-03646956/