Computation of attractors of an hybrid system

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Abstract. In this paper, we propose to use an interval approach to compute an outer approximation of the attractor a continuous-time hybrid dynamical system. We show the importance of Poincaré maps to build an event-based discretization. The latter will be combined with an interval integration to compute a guaranteed enclosure for the attractor.

1 Introduction

A large class of dynamical system can be modeled by an hybrid system which converges to a stable periodic orbit. This orbit is set $\mathbb{A}$ is an attractor of the system. A characterization of $\mathbb{A}$ is important for security reasons (for instance, $\mathbb{A}$ should not intersect a forbidden zone), for default detection (if the system leaves its attractor, we suspect a failure) or for control specification (we want the system remains inside a desired region). José Ragot and his team have shown the importance of using set membership method is such context, see e.g., [1] [2].

This work presents how tools such has Poincaré maps, interval analysis and guaranteed integration can be used to characterize an attractor of an hybrid system. In the context of hybrid systems, the use of guaranteed integration has been used for characterizing reachability sets [3], and but to our knowledge these tools have not yet been used to characterize attractors of hybrid systems.

2 Motivation

Consider a Dubins car [4]:

$$\begin{cases} \dot{x} &= \cos\theta \\ \dot{y} &= \sin\theta \\ \dot{\theta} &= u \end{cases} \tag{1}$$

where (x, y) is the position of the robot, θ is its heading and u is the input. The robot has no possibility to measure its state, neither its position nor its heading. It is only able to measure the sign of y. We want that the robot moves along the *wanted* line $y = 0$. For this, we suggest to use the Trinity pattern proposed in [5] which yields a *rolling* behavior for the motion. The stability of the resulting navigation has been shown experimentally in [5] with an autonomous plane turning around a cloud with an unknown shape. A theoretical analysis has been provided in [6].
The principle of the rolling navigation is as illustrated by Figure 1. The controller alternates between a small circle (red) of radius $\rho_0 = 2$ when $y < 0$ (hatched area) and a large circle (blue) of radius $\rho_1 = 1$ when $y > 0$. When a small circle is started, it continues until the heading has changed of π. When a large circle is started, it continues as long as $y > 0$.

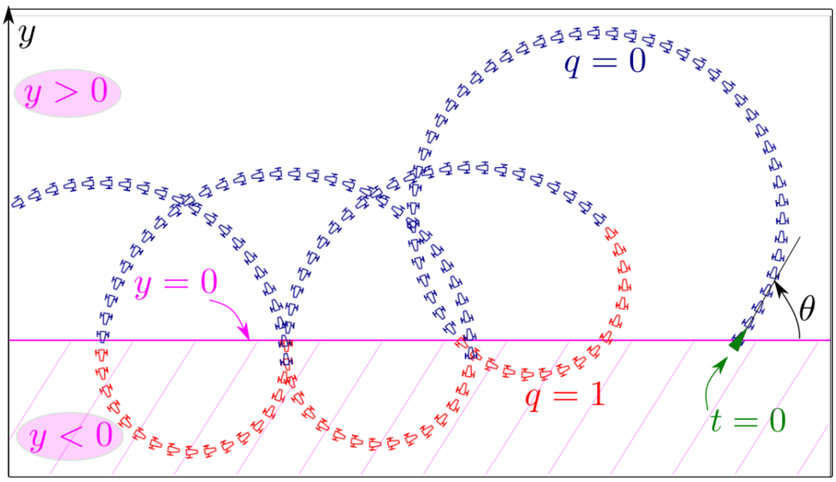

Figure 1: Stable behavior of the rolling motion

The behavior of the motion can be modeled by the hybrid system of Figure 2. The state variable x which is not of interest has been removed. Instead, we have added a clock variable c needed by our controller to know that half of the small red circle has been covered. In Figure 1, we took for the initial state vector of the robot $\mathbf{x} = (0, 0, 1)$ and for the controller $q = 0$, $c = 0$.

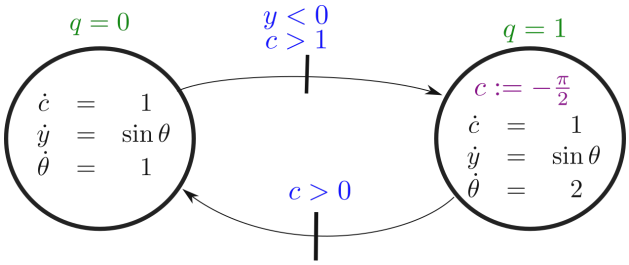

Figure 2: The attractor for this system is a stable periodic orbit if we start around $(q, c, y, \theta) = (0, 0, 0, \frac{\pi}{2})$

In this paper, we want to enclose the set $\mathbb{A}$ of all pairs (y, θ) to which the controlled system will converge.

3 Method

The hybrid system we consider has a discrete state $q \in \{0, 1\}$ and a continuous state $\mathbf{x})(c, y, \theta) \in \mathbb{R}^3$. The continuous state $\mathbf{x}$ follows a state equation of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, q).$$

A transition occurs when some equality conditions are satisfied for $\mathbf{x}$, say $g(\mathbf{x}, q) = 0$. When a transition is crossed, q jumps $1 - q$. The state vector $\mathbf{x}$ jumps also. Since $q \in \{0, 1\}$, we have two surfaces $\mathcal{S}_1 : g(\mathbf{x}, 0) = 0$ and $\mathcal{S}_2 : g(\mathbf{x}, 1) = 0$, called the *Poincaré sections*. The latter can be used to show the stability. Figure 3 represents a situation with the two sections $\mathcal{S}_1$, $\mathcal{S}_2$. The partial Poincaré map is $\mathbf{p} : \mathcal{S}_1 \mapsto \mathcal{S}_2$ is defined from the dynamics, as represented in the Figure. The composition of two Poincaré maps will allow us to check the stability using linear criterion such as those used for linear systems. The procedure may be guaranteed if we use interval methods [7] [8][9] .

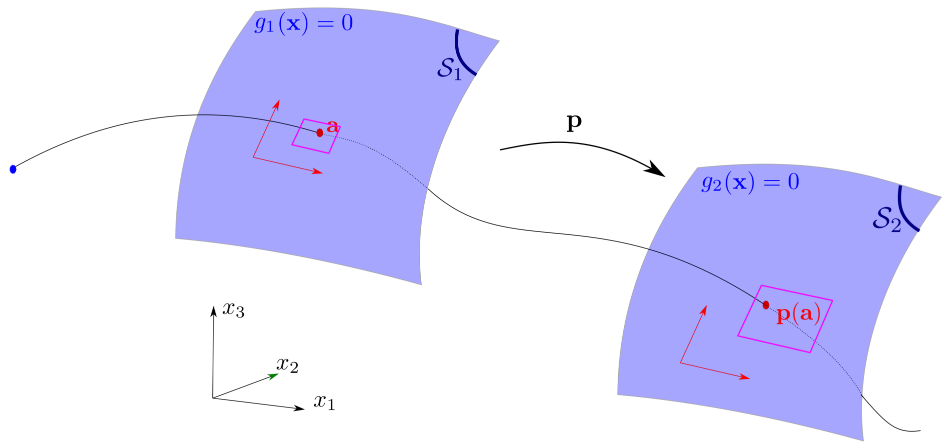

Figure 3: Partial Poincaré map to go from one section to another

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