Optimal control tuning of a redundant robot

Luc Jaulin[*]

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1 Introduction

A robot can generally be described by a vector first-order differential equation, named state equations. A robot is said to be redundant if it has more actuators than necessary. In this case, the number of inputs is higher than the number of outputs (variables to be controlled) and there exists many different ways to achieve the control requirements. We can thus take advantage of the extra number of freedom degrees in order to optimize some performance criterion (involving energy, security, longevity or speed). The resulting problem can be formalized into an parametric optimization problem with equality constraints where the free variables (or the parameters) of the optimization problem correspond to the outputs. Due the non-convexity of the optimization problem, the paper proposes to use an interval approach for the resolution. The approach is illustrated on the optimal sail tuning of a sailboat robot.

2 Formalism

Consider a mobile robot described by the following state equations

$$\left\{ \begin{array}{lll} \dot{\mathrm{x}} & = & \mathrm{f}(\mathrm{x},\mathrm{u}) \\ \mathrm{y} & = & \mathrm{g}(\mathrm{x}) \end{array} \right.$$

where $\mathrm{u} \in \mathbb{R}^m$ is the vector of inputs (or actuators) and $\mathrm{x} \in \mathbb{R}^n$ the state vector. The vector $\mathrm{y} \in \mathbb{R}^p$ is the vector of variables we want to control accurately. If $m = p$ the robot is said to be well actuated and can controlled used nonlinear control techniques. If $m < p$ the robot is underactuated and the control problem is not solvable. If $m > p$ the robot is overactuated and we will have many different way to solve the control problem. In such a case, we may take advantage of this redundancies by maximizing a performance criterion $h(\mathrm{x})$. This criterion may correspond to the power delivered by the batteries (that we want to minimize) or to the speed of a boat (to be maximized), ... We shall consider the case $m > p$ and we assume that the robot is stable, i.e., in an operating mode the signals $\mathrm{x}, \mathrm{u}, \mathrm{y}$ will converge toward the constant values $\bar{\mathrm{x}}, \bar{\mathrm{u}}, \bar{\mathrm{y}}$. Any $(\bar{\mathrm{u}}, \bar{\mathrm{x}}, \bar{\mathrm{y}})$ such that the operating conditions are satisfied is called an operating point. The optimization problem we have to solve is thus defined by

$$\hat{h}(\bar{\mathrm{y}}) = \max_{\bar{\mathrm{u}} \in \mathbb{R}^m, \bar{\mathrm{x}} \in \mathbb{R}^n} h(\bar{\mathrm{x}}) \qquad \text{s.t.} \left\{ \begin{array}{lll} 0 & = & \mathrm{f}(\bar{\mathrm{x}},\bar{\mathrm{u}}) \\ \bar{\mathrm{y}} & = & \mathrm{g}(\bar{\mathrm{x}}). \end{array} \right. \tag{1}$$

[*]OSM, LabSTICC, ENSTA-Bretagne

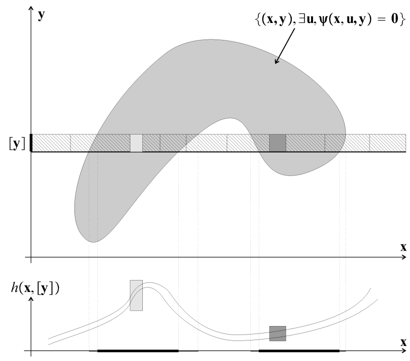

FIG. 1 – Principle of the optimization algorithm

3 Sailboat

The robot to be considered has two inputs : the sail angle u_1 and the rudder angle u_2. The motion the robot can be described by state equations [2]. The state variables are the heading θ, its derivative ω and the velocity v. The output (or the variable to be controlled) is θ and the variable to be maximized is v. The optimization problem (1) can thus be written by

$$\hat{v}(\theta) = \max_{\mathrm{u}\in\mathbb{R}^2, v\in\mathbb{R}} \quad v \quad \text{s.t.} \left\{ \begin{array}{lcl} 0 & = & \sin u_1 (\cos(\theta + u_1) - v \sin u_1) - v \sin^2 u_2 - v \\ 0 & = & (1 - \cos u_1)(\cos(\theta + u_1) - v \sin u_1) - v \frac{\sin 2u_2}{2}. \end{array} \right.$$

4 Resolution

Our optimization problem (1) is a special case of parametrized optimization problem with equality constraints which can be expressed into the following form :

$$\hat{h}(\mathrm{y}) = \max_{\mathrm{u}\in\mathbb{R}^m, \mathrm{x}\in\mathbb{R}^n} h(\mathrm{x},\mathrm{u},\mathrm{y}) \quad \text{s.t. } \psi(\mathrm{x},\mathrm{u},\mathrm{y}) = 0$$

where dim ψ = dim u. The principle of the algorithm is described by Figure 1.

First, we bisect the set y space into boxes and we thus have to solve the optimization problem on each slice of the (x,y) space. Then, we bisect the slice into boxes following the x-space. For each box $[\mathrm{x}] \times [\mathrm{y}]$ of the slice we check [1] if

$$[\mathrm{x}] \times [\mathrm{y}] \subset \underbrace{\{(\mathrm{x},\mathrm{y}), \exists \mathrm{u}, \psi(\mathrm{x},\mathrm{u},\mathrm{y}) = 0\}}_{\mathbb{S}_{\mathrm{xy}}}.$$

Références

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