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IFAC PapersOnLine 51-29 (2018) 14–19

Position keeping control of an autonomous sailboat

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Abstract

This paper addresses the problem of reaching and keeping a target position for a sailboat. A method to control sail angle is proposed, using an adaptive adjustment of the sail to regulate the acceleration of the sailboat. A tacking strategy is defined to navigate into the wind, arrive upwind to slow down the boat and then to stay close to the target point. Simulation results show the effectiveness of the proposed approach. The stability of the sailboat using the proposed control has been proven.

Key words: Position keeping, autonomous sailboat, non-linear control.

1 Introduction

Reaching and maintaining a specific position has many applications for sailboats: collecting undersea data, getting back a buoy, meeting another vehicle to exchange materials at sea, or following an underwater vehicle to transmit its information to the surface. However, numerous factors such as wind orientation and velocity, presence of obstacles and complex dynamics of a sailboat make it a challenging problem compared to motor boats. Many low-level and high-level control systems design can be found in literature [3, 5, 6, 7, 8, 9, 10, 11, 13] for various kinds of tasks. The problem caused by the complex dynamics of a sailboat is exposed in [6, 7, 10, 13, 14], while trying to obtain an accurate prediction of the sailboat behaviour. However, the major inconvenience of these methods is that the control scheme can only be made with a perfect knowledge of the dynamic parameters of the sailboat, which are not always feasible in practice. In works like [3, 5, 9, 12], a line following control has been developed, and a zigzag trajectory, named tack strategy, is proposed to move upwind while staying in a defined corridor. Two reference points or attractive areas (potential field method) are defined to orientate the boat. Problems to reaching a target point and/or path planning are studied in [1, 2, 4, 5, 8, 11]. However, these techniques are developed to reach a position and move to a new objective rather than maintain a specific position during a long time. Indeed, keeping a position with a sailboat is not simple due to wind and waves pushing against the sailboat while its velocity is low.

In this paper, we aim to reach a static target position and to stay therein. A high-level control method is proposed to provide a coupled heading and speed of the sailboat to converge to a target position and to maintain its position afterwards. In a second part, a low-level control of the sail is exposed to respect the desired speed and heading of the high-level control. Moreover, we desire techniques developed in this paper does not require any prior knowledge of dynamic parameters/model of the sailboat to be controlled.

The outline of the paper is as follows. Problem statement is described in Section 2.1 and model equation of the sailboat is given in Section 2.2. The method to choose rudder angle is defined Section 3.1. The method to determine sail angle using an adaptive approach, is described in Section 3.2. The problem of reaching a target position is described in Section 4. An approach to evaluate target acceleration and orientation of the sailboat is proposed in Section 4.2 and 4.3. Section 5 presents some simulation results and Section 6 concludes the paper.

2 Problem description

2.1 Problem statement and parameter

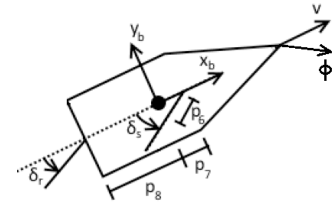

Fig. 1. [6] Fixed distance parameters p_6, p_7, p_8.

The following notation will be used in this paper:
- θ : orientation of the sailboat,

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.09.462

- v : velocity of the sailboat,
- ω : rotation speed,
- ϕ : course angle,
- δ_r : angle of rudder, $\delta_r \leq \delta_{r,\max}$ with $\delta_{r,\max} = \frac{\pi}{4}$.
- δ_s : angle of sail, $|\delta_s| \leq \delta_{s,\max}$ with $\delta_{s,\max} = \frac{\pi}{2}$
- ψ_{tw}, a_{tw}: orientation and speed of the true wind,
- δ: hauled angle which define the dead area. $\delta = \frac{\pi}{4}$ here.

2.2 Model equations

Inspired from [6], the sailboat dynamics is described by the following non-linear differential equations

$$\dot{x} = v\cos(\theta) - u\sin(\theta) \quad (1)$$
$$\dot{y} = v\sin(\theta) + u\cos(\theta) \quad (2)$$
$$\dot{\theta} = \omega \quad (3)$$
$$p_9\dot{v} = g_s\sin(\delta_s) - g_{rv}p_{11}\sin(\delta_r) - p_2 v|v| + p_1 a_{tw}^2\cos(\psi_{tw} - \theta) \quad (4)$$
$$p_9\dot{u} = -g_{ru}p_{11}\cos(\delta_r) - p_2 u|u| + p_1 a_{tw}^2\sin(\psi_{tw} - \theta) \quad (5)$$
$$p_{10}\dot{\omega} = g_s(p_6 - p_7\cos(\delta_s)) - g_{rv}p_8\cos(\delta_r) - p_3\omega v \quad (6)$$

where all parameters p_i can be found in Table 1, g_s and g_r are respective forces on the sail and the rudder, given by

$$g_{rv} = p_5 v^2\sin(\delta_r) \quad (7)$$
$$g_{ru} = p_5 u|u|\cos(\delta_r) \quad (8)$$
$$g_s = p_4 a_{aw}\sin(\delta_s - \psi_{aw}) \quad (9)$$

where $W_{p,aw} = [a_{aw}, \psi_{aw}]$ is the apparent wind and $W_{p,tw} = [a_{tw}, \psi_{tw}]$ is the true wind, exposed in section 2.3. Terms $p_1 a_{tw}^2$ represents the wind force on the hull. We suppose here that the action of the wind on the sail is compensated by the keel on u. Terms $p_2 v^2$ and $p_3\omega v$ represent the tangential and the angular friction force.

Remark 1 *Since parameters $p_1, \ldots, p_{11}$ are difficult to obtain, control strategies proposed will not require them and are independent of these system parameters. Thus others systems with different parameters are also applicable.*

2.3 True and apparent wind

Wind can be described in two different referential spaces. True wind (tw) is the velocity a_{tw} and the direction of the wind ψ_{tw} measured in a fixed global referential. Apparent wind (aw) is the velocity a_{aw} and direction of the wind ψ_{aw} measured from the ship by the weather vane. As exposed in [6], apparent wind can be evaluated from true wind in Cartesian coordinate by

$$W_{c,aw} = \left[\, a_{tw}\cos(\psi_{tw} - \theta) - |v|\,,\; a_{tw}\sin(\psi_{tw} - \theta)\,\right]$$

and in corresponding polar coordinate as

$$W_{p,aw} = \left[\, a_{aw},\, \psi_{aw}\,\right] = \left[\, |W_{c,aw}|\,,\; \mathrm{atan2}(W_{c,aw})\,\right]$$

where atan2 is the arctangent function returning an angle in the correct quadrant. Same transformation can be made from apparent wind to true wind.

3 Low level control

In this section, low level controls of the sail and rudder angle are exposed to follow a desired heading. Evaluation of the desired direction will be proposed in Section 4.

3.1 Rudder angle

In classic works like [3, 5], rudder angle is evaluated using the heading θ. However, due to the sideway forces of the wind, the course angle ϕ and heading angle θ are not necessarily equal, see [11]. This discrepancy is mainly observed when sailing close-hauled, drifting the sailboat from a line it follows. Thus, combining ideas from [11] and [5], we propose the following rudder control which uses course angle to compensate the perturbations:

$$\delta_r = \delta_{r,\max}\sin\left(\Theta - \bar{\theta}\right) \quad \text{if } \cos\left(\Theta - \bar{\theta}\right) \geq 0 \quad (10)$$
$$\delta_r = \delta_{r,\max}\mathrm{sign}\left(\sin\left(\Theta - \bar{\theta}\right)\right) \quad \text{else.} \quad (11)$$

where $\bar{\theta}$ is the desired orientation, $\Theta = \phi$ if $\cos(\theta - \phi) - \cos(\epsilon_\theta) \geq 0$ with $\epsilon_\theta \in \left[0,\, \frac{\pi}{2}\right[$ as design parameter angle, $\Theta = \theta$ else.

3.2 Sail angle

In this section, a method to choose the sail angle is proposed. It allows control of the acceleration of the sailboat by choosing the sail angle. First notice that since the sail cannot hold against the wind, the angle of the sail δ_s cannot exceed a limit angle defined by the apparent wind. This condition can be expressed like in [6] as

$$\delta_s \in -\mathrm{sign}(\psi_{aw}) * [0,\, \delta_{s,M}] \quad (12)$$

where $\delta_{s,M} = \min(|\pi - |\psi_{aw}||\,, \delta_{s,\max})$.

In some cases, we desire to control the acceleration of the boat $\dot{v}$ using the sail adjustment δ_s. Due to the non-linearity of (4), linearized outputs provokes singularities as shown in [2]. Instead of that, the sail adjustment δ_s can be chosen to be as close as possible of the desire acceleration $\dot{\bar{v}}$. However, the main inconvenience of such methods like in [2] is the knowledge of dynamics parameters of the sailboat, which are difficult to obtain. Thus, a control of the δ_s without using dynamics parameters and allowing to control sailboat acceleration is proposed.

The main idea is to take δ_s close to the limit angle $\delta_{s,M}$ when we desire to be slow, *i.e.* reduce sail lift, and to take δ_s close to the optimal sail angle when we desire to speed up. Define the control of the sail with the following steps:

- Let define the angular acceleration such

$$\dot{\delta}_s^* = -k_s(\dot{v}^* - \dot{v})\,\mathrm{sign}(\delta_s) \quad (13)$$

with $k_s > 0$ a design parameter and $\dot{v}^*$ the desired acceleration of the sailboat, which will be exposed in Section 4.2. Using a discrete step dt, the angle obtain using (13) is expressed as

$$\delta_s^*(t) = \max\left(|\delta_s(t)| + \dot{\delta}_s^* dt,\, 0\right). \tag{14}$$

- If $\dot{\delta}_s^* < 0$, we desire to speed up. As exposed in [5, 3], the optimal sail angle is

$$\delta_s^{opt} = \frac{\pi}{2}\left(\frac{\cos(\psi_{tw} - \bar{\theta}) + 1}{2}\right) \tag{15}$$

Thus, one takes

$$\delta_s = -\text{sign}(\psi_{aw}) \max\left(\delta_s^*,\, \min\left[\left|\delta_s^{opt}\right|,\, \delta_s^{\lim}\right]\right) \tag{16}$$

where $\delta_s^{\lim} = \max(\delta_{s,M} - \epsilon_{\delta_s},\, 0)$ and ϵ_{δ_s} design parameter to avoid sail and apparent wind are aligned.

- If $\dot{\delta}_s^* > 0$, we desire to slow down. The limit angle is defined by the apparent wind $\delta_{s,M}$, one takes

$$\delta_s = -\text{sign}(\psi_{aw}) \min(\delta_s^*,\, \delta_{s,M}) \tag{17}$$

The main advantage of this technique is the control of the sailboat acceleration without knowledge of parameters $p_1, \ldots, p_{11}$. Moreover, this technique is simple to implement with a smaller calculation time.

4 Reach target position

In this section, strategies are proposed to reach a static target and to stay close to it. To achieve that, different cases are studied with regard to the distance and orientation between sailboat and its target. Problem statement is exposed Section 4.1. A control of acceleration is exposed in Section 4.2 to reach the target position with a small velocity. Orientation of the sailboat is presented in Section 4.3 to arrive upwind.

4.1 Problem statement

Reaching a target position might not be a difficult problem. However, keeping this position could be more challenging because sailboat position is not simple to control with wind and waves pushing the sailboat far to its target. It requires the sailboat to arrive with a low velocity and to perform small corrections to compensate action of the wind and wave.

To help the sailboat to slow down when it is coming close to the target, it is useful to arrive upwind, and keep its orientation when the boat has reached its target to move as little as possible. We define an area where the sailboat will try to stay upwind called the target area, and an area where the sailboat will maneuver to arrive upwind called pre-arrival area.

Put $p_T = [x_T,\, y_T]^T$ is the target coordinate and $p_S = [x,\, y]^T$ the sailboat coordinate. $\mathcal{C}(p, d)$ is the circle of center p and radius d. Let define the following notations:

- $\mathcal{C}(p_T, d_T)$ with $d_T \geq 2r_t$ is the target area, where $r_t = \frac{p_8 + p_7}{\sin(\delta_{r,\max})}$ is the turning radius, a non-holonomic constraints of the sailboat. Consider the sailboat has reached the target when $\|p_S - p_T\| < d_T$.
- Area between $\mathcal{C}(p_T, d_T)$ and $\mathcal{C}(p_T, d_M)$ with $d_M \geq d_T + 2r_t$ is the pre-arrival area. Manoeuvre to arrive upwind while target starts inside.

Remark 2 *Since r_t corresponds to the turning radius, $\mathcal{C}(p_T, r_t)$ is the smaller circle boat can perform.*

4.2 Control of velocity

The sailboat needs to reach the target position p_T with the smallest velocity possible. A proportional control is used to solve this problem, allowing a reduced sailboat velocity when it is close to the target. The sail opening δ_s is evaluated using Section 3.2 with

$$\dot{v}^* = -k_v(v - v_0) + k_p d_{ST} \tag{18}$$

where $v_0 = \max\left(0,\, u\tan\left(\theta - \tilde{\theta}\right)\right)$, $d_{ST} = \|p_S - p_T\|$, $k_p > 0$, and $k_v > 2\sqrt{k_p}$. Note since the sailboat cannot go backward easily, than why k_v and k_p must be chosen as to obtain a behavior without overshoot. Proof of control stability is exposed in Appendix A.

4.3 Orientation control

To reach the target point p_T, the fastest way is to choose the direct angle between the sailboat and its target. However, some inconveniences like wind orientation can force to choose another path. Moreover, to help the sailboat to slow down when it is coming close to the target and maintain its position, we can imitate the behavior of an experienced human sailor by arriving upwind, and keeping its orientation when the boat has reached its target. Then, strategy to define the desired orientation $\bar{\theta}$ proposed here is define such that respect the different areas exposed in Section (4.1):

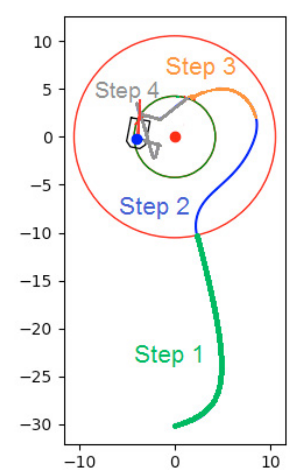

Fig. 2: Steps of orientation control.

- Step 1: Direct path. Let define the shorter path to the target $\theta^* = \text{phase}(x_T - x + 1i(y_T - y))$.
- Step 2: Go round target. Since the sailboat is far to this target, *i.e.* $\|p_S - p_T\| > d_M$, it will follow this angle to reduce this distance with it, like exposed in Step 1. When $\|p_S - p_T\| \leq d_M$, it arrives in the pre-arrival area. Thus, if θ^* does not induce to arrive easily upwind, a maneuver is made to go round the target area:

$$\text{If } (\|p_S - p_T\| \in [d_T,\, d_M])\,\&\,(\cos(\theta^* - \psi_{tw}) > 0)\,,$$
$$\text{If } \cos\left(\alpha - \left(\psi_{tw} + \frac{\pi}{2}\right)\right) > \cos\left(\alpha - \left(\psi_{tw} - \frac{\pi}{2}\right)\right)$$
$$\theta^* = \theta^* + \frac{\pi}{2}$$
$$\text{Else } \theta^* = \theta^* - \frac{\pi}{2}$$
$$\text{Else } \theta^* = \theta^*$$

with $\alpha = \text{phase}\,(x - x_T + 1i\,(y - y_T))$.

- Step 3: Tack strategy. To move upwind, we make one tack. Variable q is the tack variable and update since θ^* is not inside the dead area. If θ^* is inside the dead area, q is used to select and keep the closer tack:

$$\text{If } \cos(\psi_{tw} - \theta^*) + \cos(\delta) > 0,$$
$$\text{Put } \bar{\theta} = \theta^*$$
$$\text{If } \cos(\theta^* - (\psi_{tw} + \pi - \delta)) > \cos(\theta^* - (\psi_{tw} + \pi + \delta))$$
$$q = 1$$
$$\text{Else, } q = -1$$
$$\text{Else } \bar{\theta} = \psi_{tw} + \pi + q\delta$$

- Step 4: Upwind strategy. When the sailboat is inside the target area, we desire the sailboat moves as little as possible. Thus, objective is to orient boat front the wind.

$$\text{If } (\|p_S - p_T\| < d_T)\,\&\,(v > 0)\,,$$
$$\bar{\theta} = \psi_{tw} + \pi \tag{19}$$
$$\delta_s = -\text{sign}\,(\psi_{aw})\,|\pi - |\psi_{aw}|| \tag{20}$$
$$\text{If } (\|p_S - p_T\| < d_T)\,\&\,(v \le 0)\,,$$
$$\bar{\theta} = \psi_{tw} + \pi - \delta\text{sign}\,(u) \tag{21}$$
$$\delta_s = -\text{sign}\,(\psi_{aw})\min\left[\left|\delta_s^{opt}\right|,\, \delta_s^{\lim}\right] \tag{22}$$

To be more reactive, this strategy is maintained only if $v \geq 0$ so as to counter-balance the wind effect.

Proof for the stability of the sailboat using the proposed strategy is provided in Appendix A. Steps of previous strategy are illustrated in Figure 2.

5 Simulation

5.1 Parameter

The performance of the proposed method is evaluated considering six simulations with a duration of $T = 200$ s. Parameters of the simulated sailboat are expressed in Table 1. The radius of different areas are $d_M = 14$ and $d_T = 7$. We choose $\epsilon_{\delta_s} = \frac{\pi}{36}$, $k_s = 5$ with $dt = 0.01$, $k_p = 1$ and $k_v = 8$. Target position is $p_T = [0,\, 0]^T$, for example a buoy. The wind orientation and strength are as follows:

- case 1: $a_{tw} = 10$, $\psi_{tw} = \frac{\pi}{2}$
- case 2: $a_{tw} = 5$, $\psi_{tw} = \frac{\pi}{2}$
- case 3: $a_{tw} = 5\sin(2\pi t) + 10$, $\psi_{tw} = \frac{\pi}{2}$
- case 4: $a_{tw} = 10$, $\psi_{tw} = \frac{\pi}{12}\sin(2\pi t) + \frac{\pi}{2}$

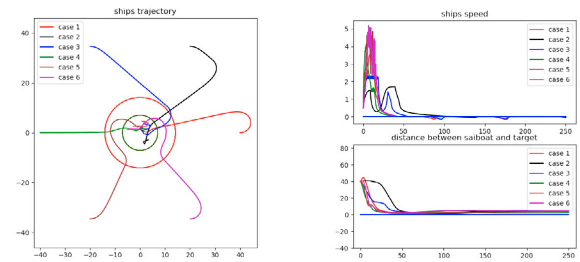

Fig. 3. Sailboat reaching the target point p_T. Red arrow shows wind direction. Pre-arrival area is represented by the two red circles, target area is the green circle.

- case 5: $a_{tw} = 5\sin(2\pi t) + 10$, $\psi_{tw} = \frac{\pi}{12}\sin(2\pi t) + \frac{\pi}{2}$
- case 6: $a_{tw} = 5\sin(2\pi t) + 10$, $\psi_{tw} = \frac{\pi}{6}\sin(2\pi t) + \frac{\pi}{2}$

5.2 Results

Figure 3 illustrates the performance of the method. In all cases, sailboat reaches the target area, stops inside, and comes back to it when the wind puts the sailboat out. Figure 3 also shows the distance between sailboat and target remains lower than 6.4m after sailboat has entered in the target area. Notice that the sailboat velocity is kept close to zero, but never equal to it so as to stay reactive. A case of the sailboat going round the target point to arrive upwind is also observed. An animation is available on https://youtu.be/mv-qqFQczs4.

6 Conclusion

In this paper, a method to reach a target point and to keep the position for a sailboat has been defined. A controller for rudder and angular sail is proposed, and a desired acceleration of the sailboat to stop at the target point is exposed. A method of regulating the sail angle is proposed and it is used to follow the desired acceleration. This method does not require knowledge of dynamic parameters to be implemented. Proof for the stability of the sailboat using the proposed strategy has been given. Finally, strategy to control sailboat orientation have been defined to arrive upwind, making the immobilization of the sailboat easier. Simulation results shows the effectiveness of the approach.

In future works, this method will be implemented on a real autonomous sailboat robot to test its effectiveness. Case of a moving target will also be considered, inducing a different strategy to stay close to the target. Moreover, path planning must be added to find a path in case of obstacles or long trips, strategies described here are only focused on the problem of maintaining the target position rather than planning a path.

A Appendix: Proof of stability

In this section, proof for the stability of the proposed approach has been shown. This one is divided in two parts: the convergence to the target area and the stability in the target area. Two Lyapunov functions are expressed to represent the different challenges of theses cases.

	parameter	value		parameter	value		parameter	value
p_1	drift coefficient	0.03	$p_5[\text{kgs}^{-1}]$	rudder lift	1500	$p_9[\text{kg}]$	mass of boat	300
$p_2[\text{kgs}^{-1}]$	tangential friction	40	$p_6[\text{m}]$	distance to sail	0.5	$p_{10}[\text{kgm}^2]$	moment of inertia	400
$p_3[\text{kgm}]$	angular friction	6000	$p_7[\text{m}]$	distance to mast	0.5	$p_{11}-$	rudder break coefficient	0.2
$p_4[\text{kgs}^{-1}]$	sail lift	200	$p_8[\text{m}]$	distance to rudder	2			

Table 1
Model parameters value, from [6]

A.1 Notations

Let define the target coordinate $p_T = [x_T,\, y_T] = [0,\, 0]$. Note all problems can be transformed such as obtain $p_T = [0,\, 0]$. Let d be the distance between the target and the sailboat such $d = \sqrt{x^2+y^2}$, and $\tilde{\theta} = atan(\frac{y}{x})$. Using $\tilde{\theta}$, one can write $x = d\cos\left(\tilde{\theta}\right)$ and $y = d\sin\left(\tilde{\theta}\right)$. Remark $\tilde{\theta} = \theta^* + \pi$.

A.2 Convergence to the target area: $d > d_T$

If $d > d_T$, we desire to converge to the target area, *i.e.* $d^2 - d_T^2 \le 0$. Let define the candidate Lyapunov function $V = \frac{1}{2}\left(d^2 - d_T^2\right)$ and show V is stable since $d > d_T$. The derivative if V is

$$\dot{V} = d\dot{d}. \tag{A.1}$$

Using (1)-(2), let study $\dot{d}$

$$\begin{aligned}\dot{d} &= \frac{2\,(x\dot{x} + y\dot{y})}{2\sqrt{x^2+y^2}} = \frac{d\cos\left(\tilde{\theta}\right)(v\cos(\theta) - u\sin(\theta))}{d} \\ &+ \frac{d\sin\left(\tilde{\theta}\right)(v\sin(\theta) + u\cos(\theta))}{d} \\ &= v\left(\cos(\theta)\cos\left(\tilde{\theta}\right) + \sin(\theta)\sin\left(\tilde{\theta}\right)\right) \\ &+ u\left(\sin\left(\tilde{\theta}\right)\cos(\theta) - \cos\left(\tilde{\theta}\right)\sin(\theta)\right) \\ &= v\cos\left(\theta - \tilde{\theta}\right) - u\sin\left(\theta - \tilde{\theta}\right).\end{aligned} \tag{A.2}$$

Injecting (A.2) in (A.1), one has

$$\dot{V} = d\left(v\cos\left(\theta - \tilde{\theta}\right) - u\sin\left(\theta - \tilde{\theta}\right)\right) \tag{A.3}$$

It can be shown $v > v_0 \ge 0$ if $d > d_T$ (see Appendix A.2.1). Let now consider the following cases. If $\theta = \tilde{\theta} + \pi$, one has $\dot{V} = -dv \le 0$. Else, we have to show

$$dv\cos\left(\theta - \tilde{\theta}\right) - du\sin\left(\theta - \tilde{\theta}\right) \le 0. \tag{A.4}$$

Remark $\cos\left(\theta - \tilde{\theta}\right) = \cos(\theta - \theta^* - \pi) = -\cos(\theta - \theta^*)$. By following steps exposed in Section 4.3, one has $\bar{\theta} \in \left[\theta^* - \frac{\pi}{2},\, \theta^* + \frac{\pi}{2}\right]$. Thus, since θ is closed to $\bar{\theta}$, one has $\cos(\theta - \theta^*) \in [0,1]$, so $\cos\left(\theta - \tilde{\theta}\right) \le 0$. Using it in (A.4), one get

$$v \ge u\tan\left(\theta - \tilde{\theta}\right) \tag{A.5}$$

Then, since $v \ge v_0$ with $v_0 = \max\left(0,\, u\tan\left(\theta - \tilde{\theta}\right)\right)$ as shown in Section A.2.1, one has $\dot{V} \le 0$. Using the theorem of Lyapunov, $V \ge 0$ for $d > d_T$ and $\dot{V} \le 0$, prove the system is stable.

A.2.1 Proof $v > 0$ if $d > d_T$

Consider $d > d_T$. v is growing since $\dot{v}^* \ge 0$, *i.e.* (18)

$$\begin{aligned}-k_v\,(v - v_0) + k_p d_{ST} &> 0 \\ v_0 + \frac{k_p}{k_v} d_{ST} &> v\end{aligned} \tag{A.6}$$

Then, v is decreasing/increasing since $v = v_0 + \frac{k_p}{k_v} d_{ST} > 0$ and maintain at this velocity. Thus, v is kept positive.

A.3 Stability in the target area: $d < d_T$

Due to strategy exposed in Section 4.3 and proof in Section A.2, sailboat arrives upwind, *i.e.* $\cos(\psi_{tw} - \theta) < 0$. This condition will be satisfied in all this section.

If $d < d_T$, we desire $v = 0$ and $u = 0$. Let define the candidate Lyapunov function $V_T = \frac{1}{2}\left(v^2 + u^2\right)$. The derivative of V is

$$\begin{aligned}\dot{V} &= v\dot{v} + u\dot{u} \\ &= \frac{p_4}{p_9} v a_{aw}\sin(\delta_s - \psi_{aw})\sin(\delta_s) - \frac{p_5 p_{11}}{p_9} v v^2 \sin(\delta_r)^2 \\ &- \frac{p_2}{p_9} v^2\,|v| + v\frac{p_1}{p_9} a_{tw}^2 \cos(\psi_{tw} - \theta) - \frac{p_5}{p_9} u^2\,|u|\cos(\delta_r)^2 \\ &- \frac{p_2}{p_9} u^2\,|u| + u\frac{p_1}{p_9} a_{tw}^2 \sin(\psi_{tw} - \theta)\end{aligned} \tag{A.7}$$

Note first $-\frac{p_2}{p_9} v^2\,|v| \le 0$, $-\frac{p_5}{p_9} u^2\,|u|\cos(\delta_r)^2 \le 0$ and $-\frac{p_2}{p_9} u^2\,|u| \le 0$. Then

$$\begin{aligned}\dot{V} &\le v\frac{p_4}{p_9} a_{aw}\sin(\delta_s - \psi_{aw})\sin(\delta_s) - \frac{p_5 p_{11}}{p_9} v v^2 \sin(\delta_r)^2 \\ &+ \frac{p_1}{p_9} a_{tw}^2 \left(v\cos(\psi_{tw} - \theta) + u\sin(\psi_{tw} - \theta)\right)\end{aligned} \tag{A.8}$$

We have to show $\dot{V} \le 0$ to prove the stability of the system. Consider now $\theta = \bar{\theta}$ and the two following case: $v > 0$ and $v \le 0$.

A.3.1 Case $v > 0$ and $\theta = \bar{\theta}$:

In this case, the following control described in Section 4.3 is chosen (19)-(20). Moreover, since $\psi_{aw} \in [-\pi, \pi]$, one has $|\pi - |\psi_{aw}|| = \pi - |\psi_{aw}|$. Thus, one gets

$$\begin{aligned}\sin(\delta_s - \psi_{aw}) &= \sin\left(-\text{sign}(\psi_{aw}) * |\pi - |\psi_{aw}|| - \psi_{aw}\right) \\ &= \sin\left(\psi_{aw} - \text{sign}(\psi_{aw})\,\pi - \psi_{aw}\right) = 0\end{aligned} \tag{A.9}$$

Using A.9 in $\dot{V}$. When $\bar{\theta} = \theta$ with $\bar{\theta} = \psi_{tw} + \pi$, one has

$$\dot{V} \leq -\frac{p_5 p_{11}}{p_9} v v^2 \sin(\delta_r)^2 + v\frac{p_1}{p_9} a_{tw}^2 \cos(\psi_{tw} - (\psi_{tw} + \pi))$$
$$+ u\frac{p_1}{p_9} a_{tw}^2 \sin(\psi_{tw} - (\psi_{tw} + \pi))$$
$$\leq -\frac{p_5 p_{11}}{p_9} v v^2 \sin(\delta_r)^2 - v\frac{p_1}{p_9} a_{tw}^2 < 0 \quad \text{(A.10)}$$

$\dot{V} < 0$ since $v > 0$, thus the system is stable if $v > 0$.

Case $v \leq 0$ and $\theta = \bar{\theta}$:

In this case, the following control described in Section 4.3 is chosen (21)-(22). Since $\text{sign}(\delta_s) = -\text{sign}(\psi_{aw})$,

$$\sin(\delta_s - \psi_{aw}) \sin(\delta_s) \quad \text{(A.11)}$$
$$= \sin(\delta_s) \sin(\text{sign}(\delta_s)|\delta_s| - \text{sign}(\psi_{aw})|\psi_{aw}|)$$
$$= \text{sign}(\delta_s)^2 \sin(|\delta_s|) \sin(|\delta_s| + |\psi_{aw}|) \quad \text{(A.12)}$$
$$= \sin(|\delta_s|) \sin(|\delta_s| + |\psi_{aw}|) \quad \text{(A.13)}$$

and since $|\psi_{aw}| \leq \pi$, deduce $|\delta_s| \leq \pi - |\psi_{aw}| \Leftrightarrow |\delta_s| + |\psi_{aw}| \leq \pi$. Injecting it in (A.12), one get $\sin(|\delta_s| + |\psi_{aw}|)\sin(|\delta_s|) \geq 0$ so $\sin(\delta_s - \psi_{aw})\sin(\delta_s) \geq 0$. Thus, if $v \leq 0$, one has

$$v\frac{p_4}{p_9} a_{aw} \sin(\delta_s - \psi_{aw}) \sin(\delta_s) \leq 0 \quad \text{(A.14)}$$

Using (15) to evaluate δ_s, it can be shown using interval analysis than

$$v\frac{p_4}{p_9} a_{aw} \sin(\delta_s - \psi_{aw}) \sin(\delta_s) + \frac{p_1}{p_9} a_{tw}^2 v \cos(\psi_{tw} - \theta) \leq 0 \quad \text{(A.15)}$$

$\forall v \in]-\infty, 0]$ and for all $|\delta| \in \left[\frac{\pi}{18}, \frac{\pi}{3}\right]$. Figure A.1 illustrates results obtain using interval analysis. Moreover, one has using (21)

$$u \sin(\psi_{tw} - \theta) = u \sin(\psi_{tw} - (\psi_{tw} - \pi - \delta \text{sign}(u)))$$
$$= u \sin(\pi + \delta \text{sign}(u))$$
$$= -|u| \sin(\delta) \leq 0 \quad \text{(A.16)}$$

Finally, $\frac{p_5 p_{11}}{p_9} v v^2 \sin(\delta_r)^2 = 0$ if $\theta = \bar{\theta}$. Injecting it with (A.15),(A.16) in (A.8), one gets $\dot{V}_T \leq 0$ if $v < 0$. The system is stable and convergent if $v < 0$.

Acknowledgement

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) of the U.K. under Grant EP/R005532/1.

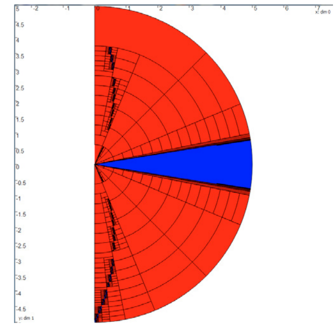

Fig. A.1. Characterization of (A.12) for parameter (v, δ). The red area represented cases (A.12) when is true. Radius correspond to the value of $v \in [-10, 0]$ and angular correspond to the value of $\delta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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