Outer approximation of the occupancy set left by a mobile robot

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Introduction

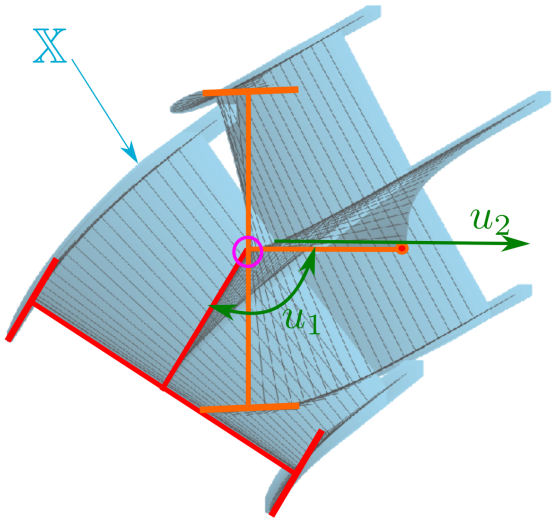
We consider a multi-body mobile robot described by a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$, where the input $\mathbf{u} \in [\mathbf{u}]$ is uncertain. The robot has a given shape. For a given trajectory $\mathbf{x}(t)$, we define the *occupancy set* $\mathbb{X}$ as the set of all points $\mathbf{a}$ of a world that has been occupied by the robot at least once during the mission. It is defined by

$$\mathbb{X} = \left\{ \mathbf{a} \in \mathbb{R}^2 \,|\, \exists t \in [0, \ldots, t_{\max}], \exists i \text{ s.t. } h_i(\mathbf{x}(t), \mathbf{a}) \leq 0 \right\}$$

where h_i is the shape function of the ith body.

Main results

We propose a new interval-based method to enclose $\mathbb{X}$. This is illustrated by the figure for a car-trailer. Here, the robot has two bodies (red and orange) with two inputs u_1 (rotation rate) and u_2 (the acceleration).



It is described by the following state equation

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{pmatrix} = \begin{pmatrix} x_5 \cos x_3 \\ x_5 \sin x_3 \\ u_1 + x_5 \sin(x_3 - x_4) \\ x_5 \sin(x_3 - x_4) \\ u_2 \end{pmatrix}$$

where x_1, x_2 are the coordinates of the center, x_3 is the heading of the first body, x_4 is the internal angle and x_5 is the speed.

The main contribution of this work is to show how to find a diffeomorphism on the state space to rewrite the system into a causal chain. The interval integration of the causal chain can then easily be done even in case of uncertainty in $\mathbf{u}$. In a second step, we show how to characterize $\mathbb{X}$ using an interval-based projection algorithm.