Proving the stability of navigation cycles

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When navigating in an unstructured environment a robot may not be able to geolocalize due to the absence of marks or to the fact that it has no access to the GPS. Now, using ancestral navigation methods, it is possible to move without getting lost. The principle is to find a discrete sequence of control such that the robot converges to a limit cycle [2]. The transition from one discrete state to another is triggered by an event such as a timer has reached a given time value or the robot has crossed an isobath line. Consider for instance the robot described by the state equations

$$\begin{cases} \dot{x}_1 &= \cos x_3 \\ \dot{x}_2 &= \sin x_3 \\ \dot{x}_3 &= u, \end{cases}$$

where (x_1, x_2) is the position of the robot and x_3 is its heading. The heading control has the form $u = \sin(\bar{\psi} - x_3)$, where the desired heading $\bar{\psi}$ obeys to the automaton (or Petri net) of Figure 1. The variable $q \in \{0, 1, 2\}$ is discrete and c is a continuous clock initialized to 0 each time q changes.

Using a simulation, we observe (see Figure 2) that the state $\mathbf{x}(t)$ converges to a stable limit cycle. The left figure is a simulation for $t \in [0, 18]$, and $\mathbf{x}(0) = (-3, 1, -1)$. The figure in the center shows the same simulation with

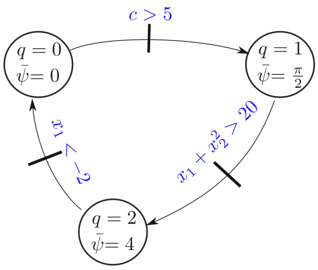

Figure 1: Automaton deciding the desired heading

$t \in [0, \infty]$. In the right, the robot is now represented by points in order to see the cycle. The colors blue, red, greed are associated to $q = 0, 1, 2$, respectively.

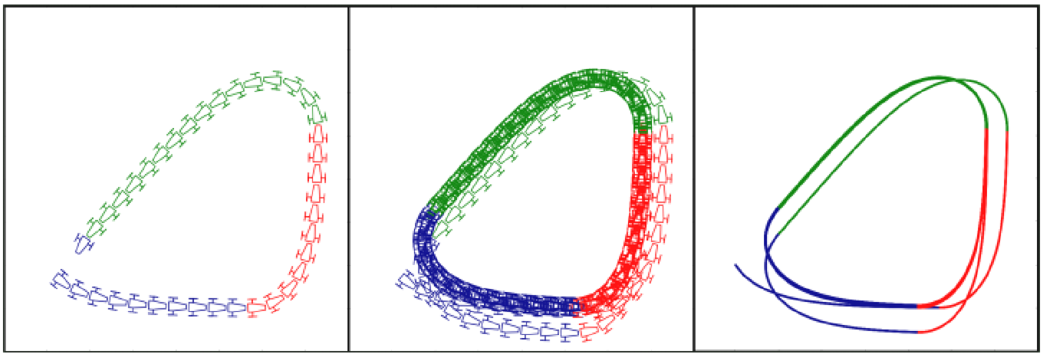

Figure 2: The robot converges to a stable limit cycle in the (x_1, x_2, x_3) space. The frame box is $[-4, 4] \times [-1, 7]$

The goal of this paper is to provide a rigorous method to show that such a cycle is stable. It will combine interval analysis [1] and Poincaré section concepts [3].

References

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