Modeling and control of an hoverboard

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Abstract

In this paper, we explain how to control an hoverboard in a two dimensional world. This means that the hoverboard can only move forward and backward on a line. An extension to a three dimensional hoverboard with a heading control is not treated here. Our planar hoverboard has only one actuator (a motor in the axle) and one inertial unit (for the acceleration and the gyrometer). Since the hoverboard has no idea of the position, the weight, and the size of the human rider, a model free approach is considered.

1 Introduction

The hoverboard represented on the left side of Figure 1 is a vehicle with two wheels and a single axle. This vehicle is stable since it is controlled by a micro-controller inside the board. In the modeling step, we have to assume that the engine is not controlled. Its open loop behavior is very close to that of the planar unicycle represented in Figure 1, right. In this figure, u represents the exerted momentum between the board (red) and the wheel. The angle β (called *talocrural*) between the body (blue) and the board is assumed to be constant. This angle β is supposed to be controlled by the human rider.

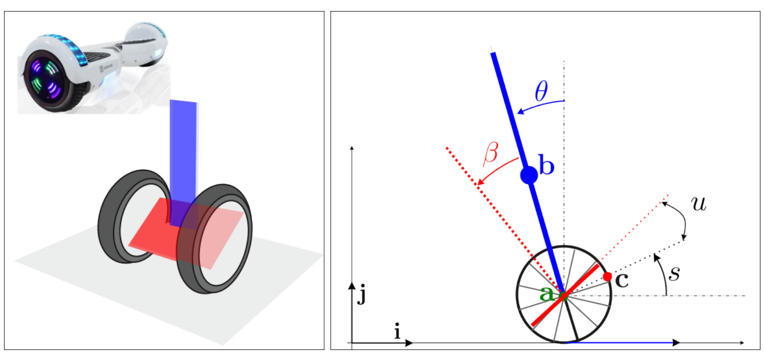

Fig. 1: Hoverboard

The way the rider controls the hoverboard is illustrated by Figure 2. By tilting the board with respect to its own body with his feet, the rider can control indirectly the speed. Note that the rider does control its pitch θ or equivalently, the rider do not decide if he wants to bend forward of backward. The rider can only control the talocrural angle β between its body and its feet.

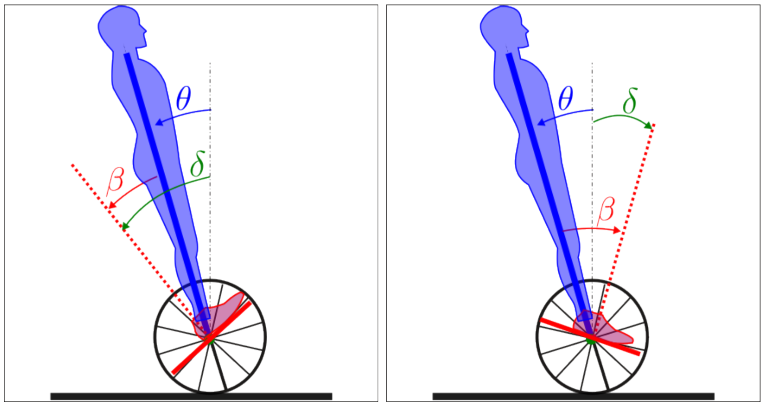

Fig. 2: The rider (blue) controls the hoverboard only through the angle β

In Figure 3, we have the same situation as for Figure 2. But we replaced the rider by a box with different offsets. If the box is centered with respect to the board, then $\beta = 0$. From the point of view of the hoverboard, the two situations (human or box) are similar. In Figure 2 the angle β is constant if the rider do not move his feet with respect to his body. In Figure 3 the angle β is always constant.

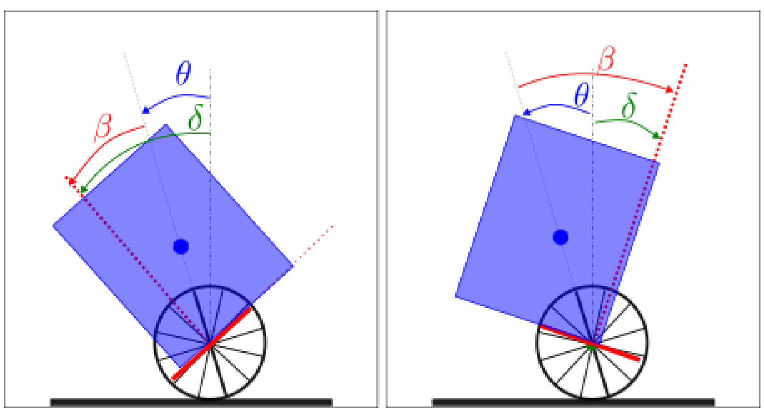

Fig. 3: A box (blue) lies on the board (red) with an offset which generates the talocrural angle β

As explained in the hoverboard manual or in some tutorials (see *e.g.*, https://youtu.be/fDzTtkS_3g8), the principle of the hoverboard is the following. If the hoverboard has a tilt angle $\delta = \theta + \beta$ positive, then the wheels turn forward, if $\delta < 0$, then the wheels turn backward. It is not clear if such a simple strategy will be sufficient to keep the rider standing. Moreover, we need to understand how δ can be estimated from the inertial unit which is only able to measure the accelerations and the rotation speed.

This paper is organized as follows. Section 2 proposes a model that will be used for the simulation and for proving the stability only. This model will not be known by the controller presented in Section 3. The controller can thus be qualified as *model-free* [2]. Section 4 provides an observer for the tilt of the board. This observer is purely inertial and does not use the model of the hoverboard. Section 5 gives a simulation to illustrate the stability of the controller. Section 6 concludes the paper.

2 Model

Modeling an hoverboard can be done using a Lagrangian approach [4][3]. Here, we propose to use a Newton-Euler approach, as for the Segway [1].

Denote by $\mathbf{b}$ the center of gravity of the rider and by $\mathbf{a}$ is the wheel's center. Take one point $\mathbf{c}$ on the wheel. Let us denote by s the angle between the vector $\overrightarrow{\mathbf{ac}}$ and the horizontal axis and by θ the pitch angle. This system has two degrees of freedom s and θ. The state of our system is given by the vector $\mathbf{x} = (s, \theta, v, \omega)^{\mathrm{T}}$, where $v = \dot{s}, \omega = \dot{\theta}$. The parameters are:

- for the wheel: its mass M, its radius ρ, its moment of inertia $J_M = \frac{1}{2} M\rho^2$;
- for the rider: its mass m, its moment of inertia supposed to be $J_p = m\ell^2$ where the distance ℓ between its center of gravity and the center of the wheel.

In order to find the state equations, we apply the fundamental principle of dynamics on each subsystem, more precisely the wheel and the body. We have:

$$\left\{ \begin{array}{ccll} -r_x + f_x & = & -M\rho\ddot{s} & \text{(wheel in translation)} \\ f_x\rho + u & = & \frac{1}{2}M\rho^2\ddot{s} & \text{(wheel in rotation)} \\ r_x\mathbf{i} + r_y\mathbf{j} - mg_0\mathbf{j} & = & m\ddot{\mathbf{b}} & \text{(body in translation)} \\ r_x\ell\cos\theta + r_y\ell\sin\theta - u & = & m\ell^2\ddot{\theta} & \text{(body in rotation)} \end{array} \right. \tag{1}$$

where f_x, r_x, r_y are the reaction forces and $g_0 = 9.81 ms^{-2}$ is the gravity. Since

$$\mathbf{b} = (-\rho s - \ell\sin\theta)\,\mathbf{i} + (\ell\cos\theta + \rho)\,\mathbf{j}, \tag{2}$$

we have

$$\begin{array}{rcl} \dot{\mathbf{b}} & = & \left(-\rho\dot{s} - \ell\dot{\theta}\cos\theta\right)\mathbf{i} - \ell\dot{\theta}\sin\theta\mathbf{j} \\ \ddot{\mathbf{b}} & = & \left(-\rho\ddot{s} - \ell\ddot{\theta}\cos\theta + \ell\dot{\theta}^2\sin\theta\right)\mathbf{i} - \left(\ell\ddot{\theta}\sin\theta + \ell\dot{\theta}^2\cos\theta\right)\mathbf{j} \end{array} \tag{3}$$

Thus

$$\begin{cases} -r_x + f_x & = & -M\rho\ddot{s} \\ f_x\rho + u & = & \frac{1}{2}M\rho^2\ddot{s} \\ r_x & = & m(-\rho\ddot{s} - \ell\ddot{\theta}\cos\theta + \ell\dot{\theta}^2\sin\theta) \\ r_y - mg_0 & = & -m(\ell\ddot{\theta}\sin\theta + \ell\dot{\theta}^2\cos\theta) \\ r_x\ell\cos\theta + r_y\ell\sin\theta - u & = & m\ell^2\ddot{\theta} \end{cases} \tag{4}$$

Isolating $\ddot{s}$ and $\ddot{\theta}$ and eliminating the internal forces (here r_x, r_y and f_x) yields

$$\begin{cases} \ddot{s} = \dfrac{\mu_3\left(\mu_2\dot{\theta}^2 - \mu_g\cos\theta\right)\sin\theta + (\mu_2 + \mu_3\cos\theta)\,u}{\mu_4 + \mu_3^2\sin^2\theta} \\ \ddot{\theta} = \dfrac{\left(\mu_1\mu_g - \mu_3^2\dot{\theta}^2\cos\theta\right)\sin\theta - (\mu_1 + \mu_3\cos\theta)\,u}{\mu_4 + \mu_3^2\sin^2\theta} \end{cases} \tag{5}$$

where

$$\begin{array}{lcc} \mu_1 & = & \frac{3}{2}M\rho^2 + m\rho^2 \\ \mu_2 & = & 2m\ell^2 \\ \mu_3 & = & \rho m\ell \\ \mu_4 & = & \rho^2 m\ell^2\,(3M + m) \\ \mu_g & = & g_0\ell m \end{array} \tag{6}$$

The state equations are thus:

$$\begin{pmatrix} \dot{s} \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \\ \frac{\mu_3\left(\mu_2\omega^2 - \mu_g\cos\theta\right)\sin\theta + (\mu_2 + \mu_3\cos\theta)u}{\mu_4 + \mu_3^2\sin^2\theta} \\ \frac{\left(\mu_1\mu_g - \mu_3^2\omega^2\cos\theta\right)\sin\theta - (\mu_1 + \mu_3\cos\theta)u}{\mu_4 + \mu_3^2\sin^2\theta} \end{pmatrix} \tag{7}$$

All parameters $\mu_1, \mu_2, \mu_3, \mu_4, \mu_g$ are positive.

3 Controller

All sensors are inertial (two accelerometers and a gyrometer) and are located at the center of the board, *i.e.*, at the wheel's center. The tilt, *i.e.*, angle of the board, is $\delta = \theta + \beta$. Since there is no external sensors such as a GPS, camera or a pitch sensor to get the angle θ of the rider, we guess that we have to find a controller which uses the inertial unit only. Assume first that this inertial sensor returns only the tilt δ (see next section) and its derivative ω. This leads us to the proportional and derivative control given by

$$u = k_1\delta + k_2\omega. \tag{8}$$

This controller considers δ as an error that has to be canceled. Equivalently, the controller tries to maintain the board flat.

Proposition 1. *If k_1 is large enough and if $k_2 > 0$ the controller (8) stabilizes the pitch θ.*

Proof. We study the stability of the subsystem

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{\left(\mu_1\mu_g - \mu_3^2\omega^2\cos\theta\right)\sin\theta - (\mu_1+\mu_3\cos\theta)u}{\mu_4+\mu_3^2\sin^2\theta} \end{pmatrix} \tag{9}$$

With the controller

$$u = k_1(\theta+\beta) + k_2\omega, \tag{10}$$

the closed loop system is

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{\left(\mu_1\mu_g - \mu_3^2\omega^2\cos\theta\right)\sin\theta - (\mu_1+\mu_3\cos\theta)(k_1(\theta+\beta)+k_2\omega)}{\mu_4+\mu_3^2\sin^2\theta} \end{pmatrix} \tag{11}$$

Near $(\theta,\omega) = (0,0)$, we have

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{\mu_1\mu_g\theta - k_1(\mu_1+\mu_3)(\theta+\beta) - k_2(\mu_1+\mu_3)\omega}{\mu_4} \end{pmatrix} \tag{12}$$

i.e.

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{\mu_1\mu_g - k_1(\mu_1+\mu_3)}{\mu_4} & \frac{-k_2(\mu_1+\mu_3)}{\mu_4} \end{pmatrix} \begin{pmatrix} \theta \\ \omega \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-k_1(\mu_1+\mu_3)}{\mu_4} \end{pmatrix} \beta \tag{13}$$

The characteristic polynomial is

$$\det \begin{pmatrix} s & -1 \\ \frac{-\mu_1\mu_g + k_1(\mu_1+\mu_3)}{\mu_4} & s + \frac{k_2(\mu_1+\mu_3)}{\mu_4} \end{pmatrix} = s^2 + \frac{k_2\left(\mu_1+\mu_3\right)}{\mu_4} s + \frac{-\mu_1\mu_g + k_1\left(\mu_1+\mu_3\right)}{\mu_4} \tag{14}$$

Now, the polynomial $P(s) = s^2 + p_1 s + p_0$ is Hurwitz iff p_0 and p_1 are positive. Since μ_1, μ_3, μ_g are positive, we get that the parameters of the controller should satisfy

$$\begin{array}{rcl} k_1 & > & \frac{\mu_1\mu_g}{\mu_1+\mu_3} \\ k_2 & > & 0 \end{array} \tag{15}$$

to have the stability. □

4 Observer

To implement the controller, we need two quantities:

- the tilt δ which can be deduced from an accelerometer (see below).
- the rotation rate ω which can be directly measured from a gyrometer (since β is constant).

We now show how we can get the tilt δ. As illustrated by Figure 4, the measured acceleration $\mathbf{a}^m$, expressed in the board frame, satisfies

$$\mathbf{a}^m = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \dot{v} \\ g_0 \end{pmatrix} \tag{16}$$

where $\dot{v}$ is the horizontal acceleration and $g_0 = 9.81$ corresponds to the gravity.

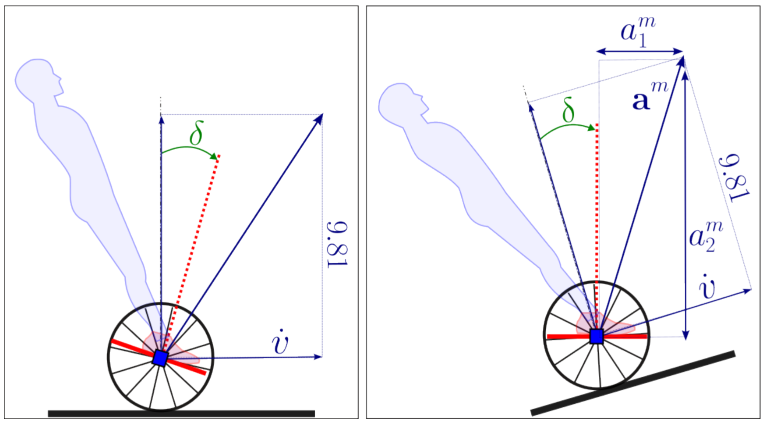

Fig. 4: Left: acceleration in the world frame; Right: in the board frame

We have

$$\|\mathbf{a}^m\|^2 = \dot{v}^2 + g_0^2. \tag{17}$$

Thus, the acceleration of the hoverboard satisfies

$$\dot{v} = \sigma \cdot \sqrt{|\|\mathbf{a}^m\|^2 - g_0^2|} \tag{18}$$

where

$$\sigma = \text{sign } \dot{v}. \tag{19}$$

The existence of two solutions is illustrated by Figure 5. In (a), $\dot{v} < 0$ (i.e., $\sigma = -1$) and in (b) $\dot{v} < 0$ (i.e., $\sigma = 1$). In both situations, the inertial unit feels a vertical acceleration greeter than g_0. The inertial unit can guess that it accelerates, but cannot deduce in which direction σ the vehicle goes.

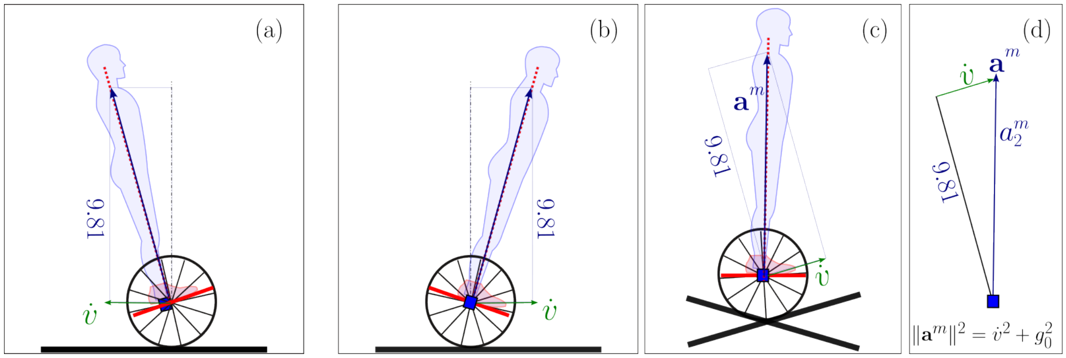

Fig. 5: (a) and (b): two symmetrical situations where the inertial unit collects the same data; (c) In the board frame; (d) acceleration triangle

The sign σ for $\dot{v}$ is difficult to determine without using the model, which is not available since we have no idea of the position and the weight of the rider. To remain model free, we will only assume that δ is continuous in t. To estimate $\sigma = \text{sign}(\dot{v})$ we may use the gyrometer. Indeed, from the previous estimation $\delta^m(t - dt)$ we can predict that as time t, the tilt angle $\delta(t)$ is approximately

$$\delta^p(t) = \delta^m(t - dt) + dt \cdot \omega^m(t) \tag{20}$$

Since

$$\begin{pmatrix} \dot{v} \\ g_0 \end{pmatrix} = \begin{pmatrix} \cos\delta^p & -\sin\delta^p \\ \sin\delta^p & \cos\delta^p \end{pmatrix} \begin{pmatrix} a_1^m \\ a_2^m \end{pmatrix}. \tag{21}$$

an estimation of $\dot{v}$ is

$$\dot{v} \simeq \cos\delta^p \cdot a_1^m - \sin\delta^p \cdot a_2^m. \tag{22}$$

Thus

$$\sigma = \text{sign}(\cos\delta^p \cdot a_1^m - \sin\delta^p \cdot a_2^m). \tag{23}$$

Therefore an approximation $\hat{\delta}$ of δ is

$$\begin{aligned} \hat{\delta} &= \text{angle}\left(\mathbf{a}^m, \begin{pmatrix} \dot{v} \\ g_0 \end{pmatrix}\right) \\ &= \text{angle}\left(\mathbf{a}^m, \begin{pmatrix} \text{sign}(\cos\delta^p \cdot a_1^m - \sin\delta^p \cdot a_2^m) \cdot \sqrt{|\|\mathbf{a}^m\|^2 - g_0^2|} \\ g_0 \end{pmatrix}\right) \end{aligned} \tag{24}$$

where

$$\text{angle}(\mathbf{u}, \mathbf{v}) = \arcsin\left(\frac{u_1 v_2 - u_2 v_1}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|}\right). \tag{25}$$

The expression of the controller is thus

$$u = k_1 \hat{\delta} + k_2 \omega^m. \tag{26}$$

As illustrated by Figure 6, the control is purely inertial. There is no need of any exteroceptive sensor. All the information is in the red box (an IMU) fixed with respect to the board. The hoverboard does not know what is its own speed, neither its position or its heading. The controller tries to cancel δ, which is considered as an error, *i.e.*, it tries to maintain the board horizontal. The human rider influences the closed loop system by tuning the talocrural angle β. This will create a disequilibrium that the controller will have to compensate. As a result, the hoverboard will go forward or backward depending of the sign of β.

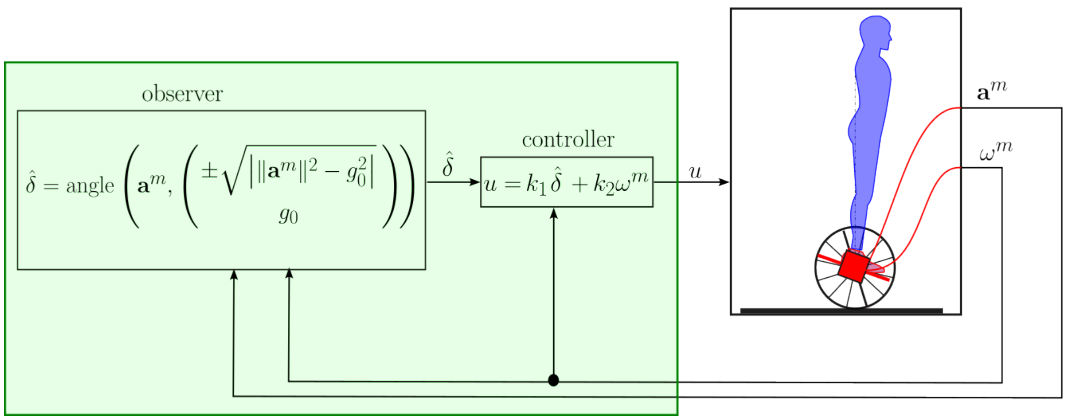

Fig. 6: The inertial control of the hoverboard

5 Simulation

To illustrate the behavior of the control strategy, we emulate the action of the rider by a second controller, named *human controller* (see Figure 7). This controller has two inputs

- The measured speed corresponding to the speed of the vehicle seen by the rider
- The desired speed $\bar{v}$.

The difference $\bar{v} - v^m$ expresses the will of the rider to increase or decrease its speed. At first sight, the rider has no possibility to communicate (via a joystick or any remote control system) with the inertial controller of the hoverboard. The rider can only modify the talocrural angle β with its feet in order to create a bias between its pitch θ and the tilt δ. It is via this bias that the rider can communicate with the hoverboard.

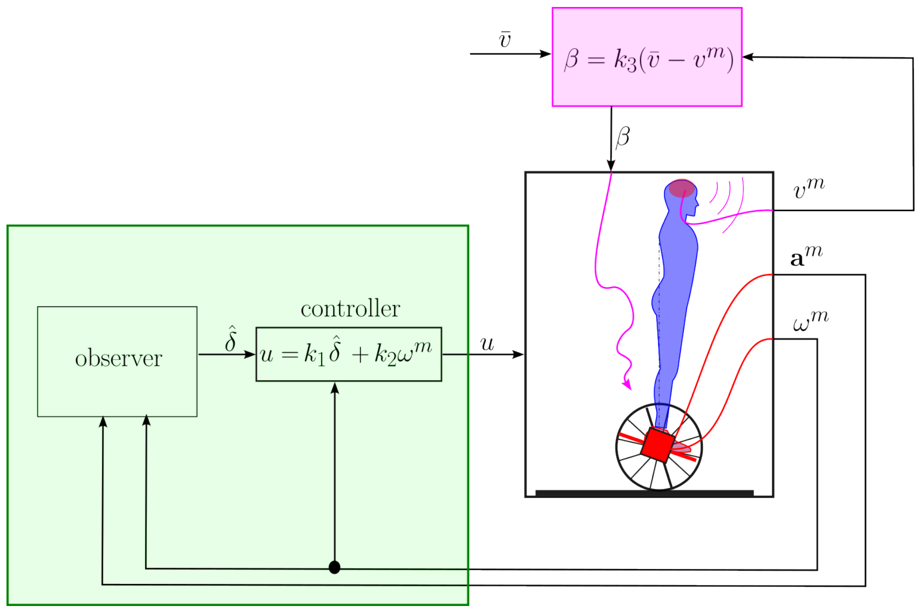

Fig. 7: The inertial control of the hoverboard is supervised by the human controller

Let us propose the proportional control

$$\beta = k_3(\bar{v} - v^m). \tag{27}$$

As illustrated by Figure 8, for $t \in [0, 20s]$, after a few seconds, the hoverboard reaches the desired speed. For the simulation, we have chosen : $m = 10Kg$, $M = 1Kg$, $\ell = 1m$, $g_0 = 9.81ms^{-2}$ and $\rho = 1m$. For the parameters of the controller we have taken $k_1 = 200, k_2 = 100, k_3 = -0.2$.

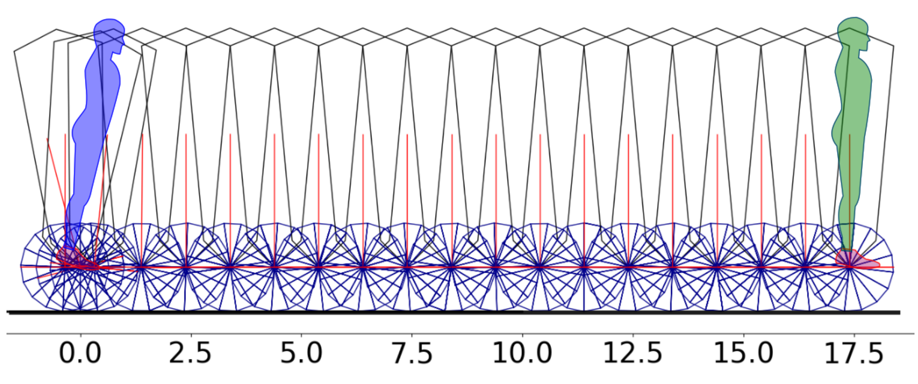

Fig. 8: The rider starts in blue and ends in green at a constant speed $v = 1$

Figure 9 shows that the controller gets a good estimation of the tilt δ and generates a control input u which is smooth.

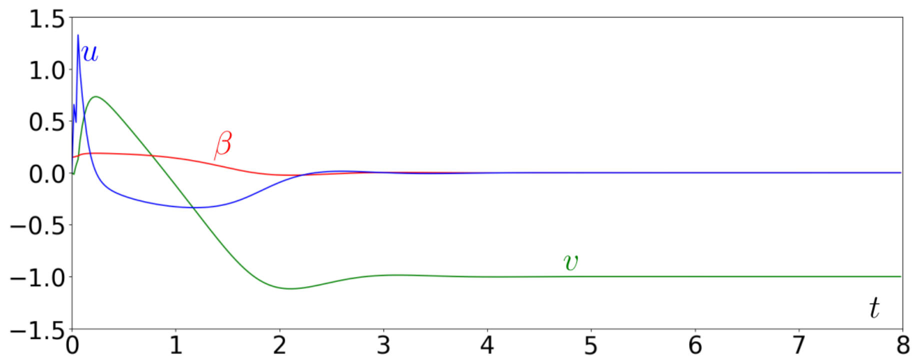

Fig. 9: The hoverboard is stable and the speed converges to the desired value

6 Conclusion

In this paper, we have presented a model of an hoverboard and we have shown how a pure inertial controller can allow us to stabilize the tilt angle δ of the board. The controller is model-free, *i.e.*, it does not used the state equation of the system. There is no possibility with this controller to control the speed. With a human in the loop, an estimation of the speed can be performed in order to get a velocity control of the vehicle. Using odometers (between the board and the wheels) we could probably have a speed control independent from the action of the human, but there is no such a need in practice, except for security reasons: to limit the speed of the vehicle and to avoid the saturation of the motors.

The Python source code associated with the test-case can be found at:

https://www.ensta-bretagne.fr/jaulin/hoverboard.html

References

[1] L. Jaulin. *Automation for Robotics*. ISTE editions, 2015. 2

[2] L. Jaulin. *Mobile Robotics*. ISTE editions, 2015. 1

[3] D. Sangotra, M. Mendhe, S. Kshirsagar, and R. Tamboli. Mathematical modelling of hoverboard. *Journal of Physics: Conference Series*, 1913:012102, 05 2021. 2

[4] D.A. Wells. *Lagrangian Mechanics*. Schaum's outlines, London, 1967. 2