Robust Multi-Objective TEAM 22 Problem: A Case Study of Uncertainties in Design Optimization

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This paper describes a robust version to the TEAM 22 benchmark optimization problem and presents the methodology WCSA (worst case scenario approximation) to solve this problem and other similar cases. The robust multi-objective TEAM 22 model was built from its classical configuration by assuming the imprecisions in the design space. General and specific robust optimization formulas were developed to elaborate WCSA approach. WCSA adds an uncertainty parameter in the objective and constraint functions to perform the role of the system's imprecisions. A multi-objective genetic algorithm approach was chosen to deal with the robust formulation and to find out the set of robust minimizers that matches with the problem requirements. The behavior of the robust Pareto front is also examined.

***Index Terms*—Genetic algorithms, magnetostatics, robust multi-objective optimization, robust Pareto front, TEAM 22.**

I. Introduction

In OPTIMIZATION systems, the designer may face undesirable interference of uncertainties that can lead to the non-usefulness of the obtained optima, if this optima was achieved by the non-robust optimization framework. In fact, the uncertainty is present in several situations, for example, when data are missing or corrupted, when the laws describing the phenomena are not completely known or when the environment affects the system. Even the constructive limitations may restrict the implementation of a given solution. Most of the time, in optimization papers, the uncertainties are left out. Perhaps the imprecision produced would not bring on serious damage to the system or the designer would have chosen a sensitivity analysis tool to help him find the most suitable solution. Independent of the reason, if the uncertainties may injury the optimal results then it seems natural to insert them in the objective and constraint functions producing a robust optimization formulation. Robust methodologies have been fully developed or even adapted from the non-robust ones with the main goal to cope with the uncertainties generating the *robust solutions* that agree with the problem statement. In this paper, we decided to develop an adaptation of some MOGAs (Multi-Objective Genetic Algorithms) for the following reasons: a) the MOGAs are known for their ability in exploitation and exploration of the optima regions; b) the MOGAs are the most popular class of the evolutionary algorithms and this validates their efficiency; c) the MOGAs are suitable to new formulations, purposes and environments; d) the MOGAs evolve a population of points therefore the robust Pareto front can be found at once; and e) the rules to evolve the population are flexible allowing the designer to adjust them in conformity with his needs.

The SMES (Superconducting Magnetic Energy Storage System) is a device designed to store energy in magnetic fields which were produced by current densities in their superconducting coil system. The *TEAM workshop problem 22* [1],

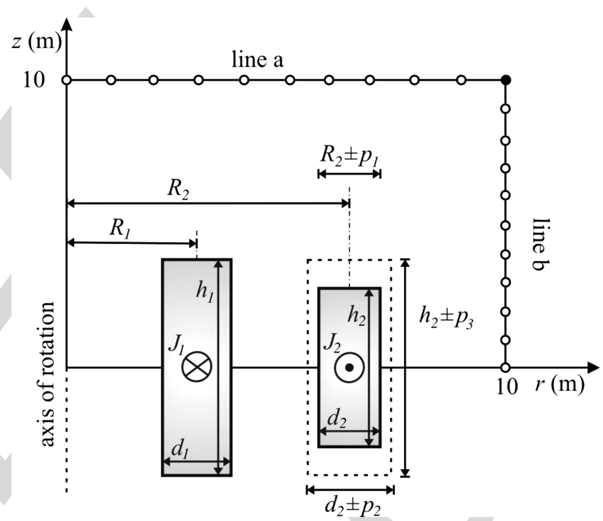

Fig. 1. Robust configuration of the TEAM 22 problem. The parameters R_1, d_1 and h_1 are known, the design parameters $\mathbf{x} = \{R_2, d_2, h_2\}$ have to be optimized by considering the additive uncertainty parameter $\mathbf{p} = \{p_1, p_2, p_3\}$ that represents the rounding or measurement errors, for instance.

henceforth will be known as TEAM 22, it is an optimization case of the SMES that has been used as a benchmark problem in magnetostatics. The TEAM 22 system is composed of two coils with opposite current densities. The first coil is charged to store the energy and the second should be designed to diminish the high magnetic stray caused by the first coil. The classical configuration of the SMES device can be obtained from the Fig. 1 by considering the parameter $\mathbf{p}$ igual to zero. The TEAM 22 has been commonly expressed as one, two and three criteria problem as in [2]–[4]. In all these papers, the researchers have not taken the uncertainty presence into account. There is, however, an increasing interest in handling the uncertainty during the optimization design to produce a set of solutions which are considered the best appropriated to work under the imprecisions inherent to the system. This has motivated us to developed a robust counterpart to the multi-objective TEAM 22 problem and a methodology to solve it.

This paper is organized as follows. Section II covers some basic robust concepts and the robust multi-objective formulation. Section III expounds the robust TEAM 22 problem and

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presents the specific robust optimization formula. Section IV describes the genetic algorithm approach used. Section V presents and discusses the numerical experiments. Finally, the paper's conclusions are in Section VI.

II. Robust Optimization

Several papers pointed Taguchi [5] as the precursor of Robust Optimization. He used statistical methods to cope with the uncertainties in the parameter design with final objective of improving the quality of the manufactured products. Taguchi's methodology has been applied in various different areas mainly in engineering. His method consists of three stages: a) system design, b) parameter design, and c) tolerance design. In few words, in a) the objective functions are defined in terms of quality, in b) the parameters are optimized, and in c) the fine-tuning is used to choose the solutions. In the optimization stage, beyond the usual control parameters $\mathbf{x}$, there are noise factors ξ entering in the objective functions. These factors represent the environmental variation during the product's usage, manufacturing variation, component deterioration, etc. There are other ways to interpret the robust optimization. For example, in the *worst case design approach* Parkinson *et al.* [6] classified as the best solutions those that had the best performance after considering the full uncertainty interference. In similar direction, Kouvelis and Yu [7] have used *scenarios* to represent the uncertainties in their formulation and they defined the robust optimization problem as a min max approach: minimization of maximal deviation from individual criterion best values. On the other hand, Deb and Gupta [8] have not considered the uncertainty as a separate parameter in their robust formal expression. They have employed a probabilistic framework to define two types of robust solutions. Basically, the solutions' performance are quantified using a particular technique to compute the mean of the samples which were based on the image's value of the perturbations in the vicinity of the solutions.

We mixed the previous robust methodologies to create the WCSA (worst case scenario approximation) methodology to solve the robust problem proposed as well as other similar cases. The *general* WCSA formulation is as follows: consider the design variables $\mathbf{x} \in \mathbf{X} \subseteq \mathbb{R}^{n_x}$ and the uncertainty parameter $\mathbf{p} \in \mathbf{P} \subseteq \mathbb{R}^{n_p}$. The vectors of the objective and constraint functions are defined by $\mathbf{f}(\mathbf{x},\mathbf{p}) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_f}$ and by $\mathbf{g}(\mathbf{x},\mathbf{p}) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_g}$, respectively. Then, the robust constrained minimization problem can be written as

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbf{X}} \quad \max_{\mathbf{p}\in\mathbf{P}} \mathbf{f}(\mathbf{x},\mathbf{p}), \\ &s.t. \quad \mathbf{g}(\mathbf{x},\mathbf{p}) \leq \mathbf{0}. \end{aligned} \tag{1}$$

Solving (1) consists in finding a set of *robust minimizers* $\mathbf{X}^*$

$$\begin{aligned} \mathbf{X}^* = \{\mathbf{x}^* \in \mathbf{X} |\ \nexists \mathbf{x} \in \mathbf{X}, \forall(\mathbf{p},\mathbf{p}^\circ) \in \mathbf{P}, \\ \mathbf{f}(\mathbf{x},\mathbf{p}) \preceq \mathbf{f}(\mathbf{x}^*,\mathbf{p}^\circ) \wedge \mathbf{g}(\mathbf{x},\mathbf{p}) \leq 0\} \end{aligned} \tag{2}$$

where the symbol $\preceq$ means $\mathbf{f}(\mathbf{x},\mathbf{p}) \leq \mathbf{f}(\mathbf{x}^*,\mathbf{p}^\circ)$ and $\mathbf{f}(\mathbf{x},\mathbf{p}) \neq \mathbf{f}(\mathbf{x}^*,\mathbf{p}^\circ)$. The minimization process in (1) is performed by a MOGA whose mechanism is explained later. The maximization procedure computes an approximation of the worst case scenario. This procedure is the *specific* part of WCSA formulation and it is explained as follows. Considering a particular sample $\mathbf{x}_0 \in \mathbf{X}$ and a discrete uncertainty set $\mathbf{P}' \subset \mathbf{P}$, the worst case objective function $\mathbf{f_{wc}}$ of $\mathbf{x}_0$ is computed as

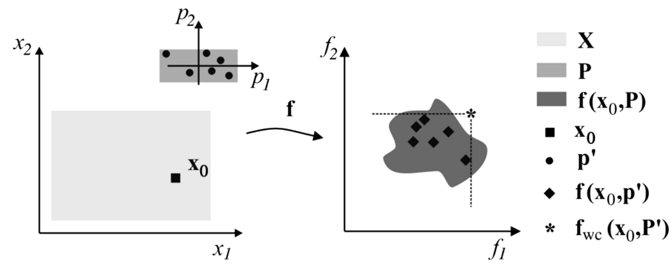

Fig. 2. Worst case scenario approximation of a particular solution $\mathbf{x}_0$. The function $\mathbf{f}$ maps $\mathbf{x}_0 \in \mathbf{X}$ and a set of samples $\mathbf{P}' \subset \mathbf{P}$ to objective space. The full image of $\mathbf{x}_0$ is the region in the objective space marked in dark gray. Note that the worst case is an approximation because it depends on the number of the samples n_s and their spreading quality; if $n_s \rightarrow \infty$, $\mathbf{P}' \rightarrow \mathbf{P}$.

$$f_{\mathbf{wc}i}(\mathbf{x}_0,\mathbf{P}') = \max_{\mathbf{p}'\in\mathbf{P}'} f_i(\mathbf{x}_0,\mathbf{p}'), \quad i = \{1,\ldots,n_f\}. \tag{3}$$

Fig. 2 illustrates the computation of $\mathbf{f_{wc}}(\mathbf{x}_0,\mathbf{P}')$. The vector of constraint functions can be rewritten as

$$\mathbf{g_{wc}}(\mathbf{x}_0,\mathbf{P}') = \mathbf{g}(\mathbf{x}_0,\mathbf{p}'), \quad \mathbf{p}' \in \mathbf{P}'. \tag{4}$$

Regarding (3) and (4) and assuming $\mathbf{P}'$ composed by a finite number of samples n_s, we point two directions to go: a) $\mathbf{P}'$ is considered random at the beginning and unchangeable during the optimization process; b) $\mathbf{P}'$ is always considered random. Option (a) seems more suitable to the interpretation of the worst case scenario approach because the solutions are compared using the same samples of the uncertainty space; however it is quite a deterministic mechanism. Whereas, option (b) is stochastic, and we found more coherent to work with genetic algorithms.

Remark: The worst case point shown in Fig. 2 works like an ideal point for maximization, thus it may be unreachable or be outside of the objective space. However, this does not affect the WCSA approach since the point is used only as a reference point to guide the search to the non-dominated solution set.

III. Problem Description

The configuration of the robust TEAM 22 is presented in Fig. 1. The arrangement of parts is identical to that found in the classical problem; the formulation, however, is quite different. The parameters of coil 1 are fixed and known. They establish the amount of current density J_1 yielded and consequently the energy stored in magnetic fields. The parameters of coil 2 must be adjusted to establish the current density J_2 which is responsible for decreasing the stray field caused by J_1. At this point, the main difference between the robust and the classical TEAM 22 is evidenced. It is assumed that the adjustable parameters in coil 2 suffer undesirable and unavoidable presence of the uncertainties, denoted by $\mathbf{p}$, that can cause damage in the optimization system and its results. The uncertainties are computed by the worst case approximation function $\mathbf{f_{wc}}$ as showed in (3). The goals of the robust approach, nonetheless, remain the same of the classical problem, that is, to find the best configuration in the SMES device to maintain the stored energy $E_0 = 180$ MJ diminishing the stray field to the minimal amount possible. The

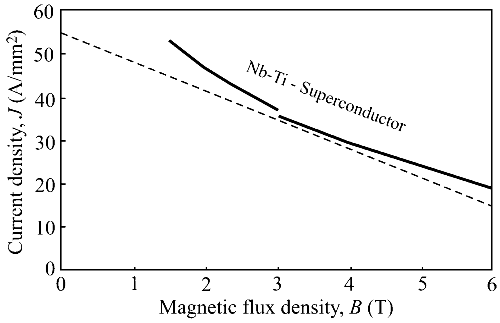

Fig. 3. Critical curve of the superconductor. The true critical curve is in continuous black. The dotted line represents the linear approximation to the previous curve used as quench condition limit in this paper.

stray field is represented by magnetic flux density B_{stray} and it is evaluated in 22 equidistant points marked on lines a and b,

$$B_{stray}^2 = \frac{\sum_{i=1}^{22} \|B_{stray,i}\|^2}{22}. \quad (5)$$

Furthermore, to keep the superconductivity characteristic, the materials of the coils must not exceed the prescribed bounds established by the *quench condition* (see Fig. 3). The dotted curve is given by the equation

$$\|J\| = -6.4\|B\| + 54. \quad (6)$$

Observing the objectives and the restriction above the robust TEAM 22 can be formulated. Consider the design variables $\mathbf{x} \in \mathbf{X} \subseteq \mathbb{R}^3$ and the discrete uncertainty set $\mathbf{P'} \subset \mathbf{P} \subseteq \mathbb{R}^3$. Assume the notations $\mathbf{f_{wc}}(\mathbf{x}, \mathbf{P'}) : \mathbb{R}^2$ and $\mathbf{g_{wc}}(\mathbf{x}, \mathbf{P'}) : \mathbb{R}$ for objective and the constraint functions, as it was defined in (3) and (4). The robust multi-objective TEAM 22 problem can be written as

$$\begin{aligned} \min_{\mathbf{x} \in \mathbf{X}} \quad & f_{\mathbf{wc}1}(\mathbf{x}, \mathbf{P'}) : \frac{B_{stray}^2}{B_{norm}^2} \\ & f_{\mathbf{wc}2}(\mathbf{x}, \mathbf{P'}) : \frac{\|E - E_0\|}{E_0} \\ s.t. \quad & \mathbf{g_{wc}}(\mathbf{x}, \mathbf{P'}) : \|\mathbf{J}\| + 6.4\|\mathbf{B}_{\max}\| - 54 \le 0, \end{aligned} \quad (7)$$

where $B_{norm} = 3.0 \times 10^{-3}$, $\mathbf{B}_{\max}$ is the maximum magnetic flux density, and $\mathbf{g_{wc}}(\mathbf{x}, \mathbf{P'})$ is measured in $\mathrm{A/mm^2}$. The first objective quantifies the stray field. The second objective measures the percentual deviation from E_0 to computed energy E. Finally, the constraint function is addressed to ensure the quench condition. In this context, by considering a fixed n_s for $\mathbf{P'}$, the main goal of this paper is to find the set of minimizers $\mathbf{X}^*$ to (7) and the robust Pareto front associated with $\mathbf{X}^*$.

IV. A Multi-Objective Genetic Algorithm Approach for Robust Optimization

The multi-objective genetic algorithm to solve the TEAM 22 was implemented with the following genetic configuration: a) binary code; b) one-cut-point crossover; c) bit-change mutation; d) roulette selection; e) a simple niche technique; and f) NSGA II elitism [9]. Options (a)–(d) are classical in genetic algorithms. In the niche technique, the objective space is divided in nonoverlapping regions (niches) with the same size. Each niche is stated 'valid' if at least it contains one of the n_{pop} members of the population. The performance of the all n_{niche} individuals of the niche i are defined as $f_{niche}^i = n_{niche}$. In addition, the population is sorted into non-dominated frontiers, in every generation. The performance due to the front f_{front}^i is scored as 1 if the individual i lies on the first front, $f_{front}^i = 2$ for the second front, etc. Then the roulette individual performance function $f_{roulette}^i$ is given by

$$f_{roulette}^i = \frac{1}{f_{niche}^i} + \frac{1}{f_{front}^i}. \quad (8)$$

The roulette selection method uses $f_{roulette}^i / \Sigma_{j=1}^{n_{pop}} f_{roulette}^j$ to compute the probability of each individual i to be chosen. In option (f), the NSGA II elitism strategy is based on the union from the current population with the incoming population. After front classification, the n_{pop} first individuals are chosen from the firsts fronts.

In few words, our MOGA starts with random population. Its members are evaluated using (7). The unfeasible points are replaced by other random feasible ones so they are classified in agreement with the front and the niche they belong. The selection is done and the chosen individuals are submitted to crossover and mutation operations in conformance to the probabilities p_c and p_m respectively. Then elitism strategy is applied. The evaluation-selection-crossover-mutation-elitism mechanism is repeated until a convergence criteria is reached.

A common drawback pointed in the robust optimization approaches is the computational effort CE. In our algorithm, it is measured in terms of objective function as

$$\mathrm{CE} = (n_{gen} + 1) \times \underbrace{(n_s \times n_{pop})}_{by\ generation} + \sum_{i=0}^{n_{gen}} f_{wasted}^i, \quad n_s > 0, \quad (9)$$

where n_{gen} is the number of generations (our convergence criteria) after initial generation and f_{wasted}^i denotes the number of function evaluation wasted with the unfeasible individuals in each generation.

V. Results

The experiments consisted in solving (7) using the MOGA presented with different numbers of the samples n_s in the space of uncertainty. The dependency of the robust Pareto front's location and n_s is shown and discussed. The n_s parameter varied from 0 to 20 (range determined empirically) with intervals of 5 unities; where $n_s = 0$ means non-robust case obtained by considering $\mathbf{p} = \mathbf{0}$ in Fig. 1. The solution set due to a fixed n_s is denoted by $\mathbf{X}_{n_s}^*$. For instance, $\mathbf{X}_{n_s=5}^*$ represents the solution set achieved using 5 samples in $\mathbf{P'}$. Other simulation details are: a) the stray field, B_{stray} which was calculated applying the Biot-Savart law to evaluate $B_{stray,i}$ in (5); b) the energy, E which was computed by finite-element method (triangular elements, first order, $\mathrm{mesh\ factor} = 0.75$); and c) the constraint function was evaluated considering $\|\mathbf{J}\| = 22.5\ \mathrm{A/mm^2}$. The TEAM 22's parameters are summarized in Table I. The MOGA's parameters are: $n_{pop} = 50$; $n_{gen} = 50$; $p_c = 1.0$; and $p_m = 0.001$.

As the first result, Fig. 4 exhibits the robust Pareto front changing its location with the sampling's size $n_s = \{0; 5; 10; 20\}$. By Fig. 2, it should be clear that the worst case induced by the uncertainty parameter can be better

TABLE I
TEAM 22 PARAMETERS (MEASUREMENTS IN METERS)

	R_1	R_2	h_1	h_2	d_1	d_2	p_1	p_2	p_3
min	-	2.6	-	0.408	-	0.1	0	0	0
max	-	3.4	-	2.2	-	0.4	0.03	0.01	0.01
fixed	2.0	-	1.6	-	0.27	-	-	-	-

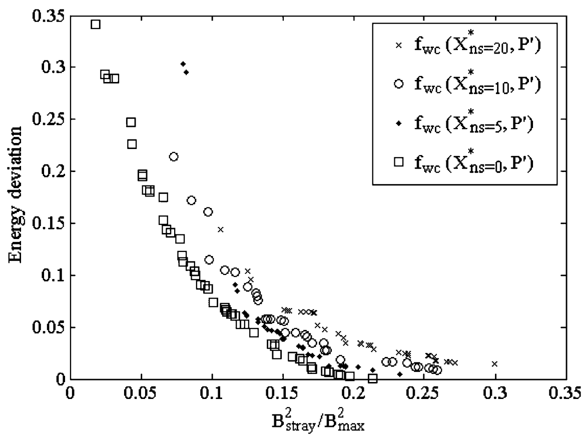

Fig. 4. The dependency of the robust Pareto front on the sample's amount. The smallers n_s caused bad estimation of the individuals' worst case performance, leading their fronts situate quite below of an ideal location.

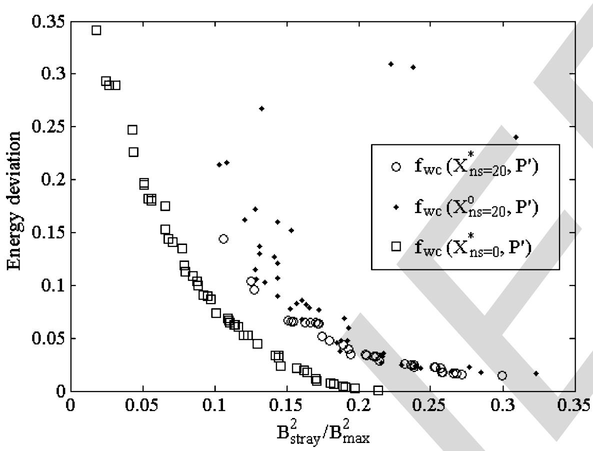

Fig. 5. The interference of uncertainties in the solution sets. Note that when the uncertainty is considered some solutions of $\mathbf{X}^{\circ}_{n_s=20}$ were clearly outperformed, in the objective space, by one or more minimizers of $\mathbf{X}^{*}_{n_s=20}$.

computed with larger n_s. Thus it was already expected that the robust Pareto front $\mathbf{f_{wc}}(\mathbf{X}^*_{n_s=20}, \mathbf{P}')$ represented the best front location of all the simulations planned. Besides, the $\mathbf{f_{wc}}(\mathbf{X}^*_{n_s\to\infty}, \mathbf{P}')$ would result in the true robust Pareto front location position, that is certainly above to $\mathbf{f_{wc}}(\mathbf{X}^*_{n_s=20}, \mathbf{P}')$ curve.

The second result has the purpose of proving that the solutions sets change with the presence of uncertainties. The evidence is in Fig. 5. Assume the notation $\mathbf{X}^{\circ}_{n_s=20}$ to represent the set $\mathbf{X}^*_{n_s=0}$ reevaluated considering $n_s = 20$. Some solutions of $\mathbf{X}^{\circ}_{n_s=20}$ became unfeasible, other preserved the robust quality, and other were surpassed by $\mathbf{f_{wc}}(\mathbf{X}^*_{n_s=20}, \mathbf{P}')$.

Finally, the results from optimization process have to be filtered to point one robust minimizer as the solution to compose the SMES device. As in [10], we discarded the solutions that have $B_{stray} > 3$ mT and we chose the one with the smaller volume of material: $V_{min} = 2\pi R_2 h_2 d_2$. Then using $\mathbf{X}^*_{n_s=20}$, we selected

$$[R_2\ h_2\ d_2] = [3.3790\ 0.4887\ 0.3168] \tag{10}$$

yielding $E = 180.36$ MJ, $B_{stray} = 1.38$ mT and $V_{min} = 3.287$ m^3.

VI. CONCLUSION

The minimizers of the robust multi-objective TEAM 22 problem changed in the presence of uncertainties. Hence, in this case the imprecisions could not be neglected. Therefore, the robust methodologies that define the minimizers such as those solutions less sensitive inside the non-robust solution set, have to be aware of possibility of failure.

The way of adapting the classical TEAM 22's configuration to its robust counterpart (see Fig. 1) was quite natural; the additive uncertainty parameter can be equally inserted in several design optimization cases. The proposed formulations (1)–(4) are simple and they can be adapted for different problems. Finally, our MOGA ran efficiently in finding the robust minimizers. However, it can be replaced by other algorithms because the only change necessary is the computation of the specific WCSA functions (3), (4). Considering the configuration, formulas and algorithm employed in robust TEAM 22 problem, we believe that this paper achieved its purpose of studying a robust optimization case providing yet a flexible methodology to solve it.

Certainly, there are other possible ways to understand and formulate the robust optimization. New methods must be developed as incoming problems urge for solutions. We expect this paper can be a starting point for further robust approaches.

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