FINDING FEASIBLE PARAMETER SETS FOR SHAPE FROM SILHOUETTES WITH UNKNOWN POSITION OF THE VIEWPOINTS

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ABSTRACT

Reconstructing 3D shapes from 2D silhouettes is a common technique in computer vision. It requires knowing the position of the viewpoints with respect to the object. But what can we say when this information is not available? This paper provides a first insight into the problem, introducing the problem of understanding 3D shapes from silhouettes when the relative positions of the viewpoints are unknown. In particular, the case of orthographic silhouettes with viewing directions parallel to the same plane is thoroughly discussed. Also we introduce sets of inequalities, which describe all the possible solution sets and a paving technique to calculate the feasible solution space of each set.

Keywords

Shape-from-silhouette; Volume intersection; Visual hull; Interval Analysis

1. INTRODUCTION

Understanding the shape of 3D objects from image features is a central problem in computer vision. Many algorithms have been presented in literature, based on occluding contours or silhouettes (see for instance [Ast89a], [Zhe94a]). Let $\mathbf{C}_i$ be the solid regions obtained by back-projecting a silhouette S_i from the corresponding viewpoint. The volume **R** shared by the regions $\mathbf{C}_i$ (Fig. 1) summarizes the information provided by a set of silhouettes and viewpoints. Finding this volume is a popular reconstruction technique called *Volume Intersection* (VI) (see [Ahu89a], [Pot87a]). This approach requires knowing the 3D positions of silhouettes and viewpoints in order to position the cones produced by back-projecting each silhouette from its viewpoint and to intersect them. But what can we say when this information is not available?

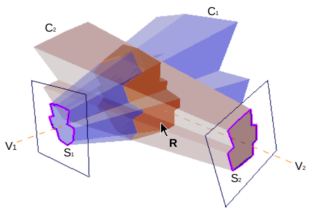


Fig. 1 - 3D shape reconstruction from silhouettes

Before entering the problem, we briefly review some relevant definitions. First, the concept of *visual hull* of an object [Lau95a] which is the object that can be obtained by VI using all the viewpoints that belong to a viewing region completely enclosing the original object without entering its convex hull. It is also the largest object that produces the same silhouettes as the given object. The definition of visual hull allows to state that an object can be reconstructed from its silhouettes iff it is coincident with its visual hull. Another tool for the shape-from-silhouette approach is the concept of *hard point* [Lau95a]. A point of the surface of the reconstructed object **R** is hard if it belongs to any object that produces the same silhouettes from these viewpoints. In the following, for brevity, we will use the expression "set of silhouettes" to specify a set of silhouettes together with the position of the corresponding viewpoint with respect to each silhouette. These data allow constructing a solid cone for each silhouette, but not positioning the cones in the 3D space. The main question that will be considered is therefore: given a set of silhouettes, does an object exist able to produce them? We will call *compatible* a set of silhouettes if the same object can generate them. An object able to produce a compatible set of silhouettes will be said to be *compatible* with the set. Clearly, the main practical issue is to find one or more compatible objects given a compatible set of silhouettes, as that produced by a real object. Let us recall that VI at most allows constructing the visual hull of an object. Infinite objects can have the same visual hull. Although we have not been able to find answers to the previous questions in the general case, we will present a set of results able to provide a first insight into the problem.

2. COMPATIBILITY OF ORTHOGRAPHIC SILHOUETTES

In the rest of this paper we will restrict ourselves to consider simply connected 3D objects and their orthographic projections. This approximates the practical case of objects small with respect to their distance from the camera.

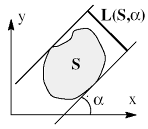


Fig. 2 - The 1D silhouette L(S,α) of a 2D silhouette

First, we will investigate the compatibility of two silhouettes. Let S be a 2D orthographic silhouette of a 3D object, and let us project orthographically S along a direction making in the plane of S the angle α with the *x* axis of a reference system fixed with respect to S (Fig. 2). Let $L(S,\alpha)$ be the length of the 1D silhouette of S. By rotating the projection direction from 0 to π we obtain all possible values of $L(S,\alpha)$. The following statement holds (see [Bot02a] for a proof).

Proposition 1. *A necessary and sufficient condition for two orthographic silhouettes* S_1 *and* S_2 *to be compatible is that two angles* α_1 *and* α_2 *exist such that* $L(S_1,\alpha_1)=L(S_2,\alpha_2)$.

When we have more silhouettes, clearly, we have that:

Proposition 2. *A necessary condition for a set of silhouettes to be compatible is that all pairs of silhouettes of the set are compatible.*

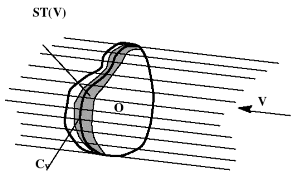


Fig. 3 - The annular strip ST(V) and the curve C_v

However, in general, to be compatible in pairs is not sufficient for a set of silhouettes to be compatible (see [Bot02a]). To derive a necessary and sufficient condition for the compatibility of more than two silhouettes we can exploit one property of the reconstructed object **R**. Let us consider one of the silhouettes involved in the process, the corresponding viewing direction V and the cylinder circumscribed to the object **O**, which is made of lines parallel to this direction (Fig. 3). Each line of this cylindrical surface must share with the surface of **O** at least one point. These points form a curve C_V belonging to a closed annular surface, a strip ST(V) of variable width (measured along a line of the cylinder), which is what is left of the original circumscribed cylinder after the various intersections. During the reconstruction process, this annular strip cannot be interrupted: at most it can reduce to a curve with zero width. In this case, the curve consists of hard points. Therefore, we can formulate the following condition for the VI algorithm to be feasible:

Proposition 3. *A necessary and sufficient condition for a set of silhouettes to be compatible is that it be possible to find viewpoints such that no annular strip of the reconstructed object is interrupted.*

In the next sections this condition will be used for constructing algorithms both to verify the compatibility of a set of silhouettes and to reconstruct compatible 3D objects.

3. SILHOUETTES WITH VIEWING DIRECTIONS PARALLEL TO A PLANE

In this section we deal with a particular case of the general problem, where all viewing directions are parallel to the same plane (Fig. 4). This case idealizes some practical situations, as observing a vehicle on a planar surface. Clearly, all silhouettes have the same height and the same plane must support all cylinders obtained by back-projection (see [Bot02a]).

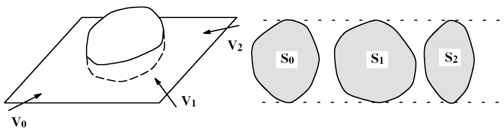


Fig. 4 - Viewing directions parallel to the same plane

Let's start by considering the compatibility of three silhouettes (S_0, S_1 and S_2). Let us introduce some notations (see Fig. 5). Each planar silhouette S_i is defined, for $0\leq y\leq y_{max}$ by two curves $S_{il}(y)$ and $S_{ir}(y)$. For simplicity, let us consider mono-valued functions. Also let $S_i(y)=S_{ir}(y)-S_{il}(y)$. Given a set of silhouettes, we will apply the condition of Proposition 3 and derive various sets of inequalities.

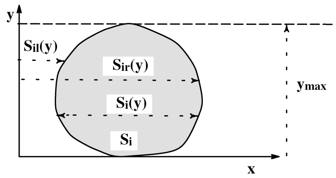


Fig. 5 - Notations used for a silhouette.

Let us consider a horizontal plane corresponding to a value of y between 0 and y_{max}, and its intersection with the three cylinders obtained by back-projecting the silhouettes.

The values of $S_0(y)$, $S_1(y)$, $S_2(y)$ and their relative position in Fig. 6(a) satisfy in this plane the condition of Proposition 3. It is not difficult to see that this condition requires that the two lines that project the endpoints of $S_2(y)$ along the direction V_2 must lie inside the two areas highlighted in Fig. 6(a), otherwise a silhouette smaller that $S_0(y)$ would be obtained from V_0. For the whole silhouettes to be compatible, this condition must hold for all y. Fig. 6(b) shows the orthogonal projection for all y of the vertices of the parallelogram marked as **1**, **2**, **3** and **4** in Fig. 6(a) onto the plane of S_2. For the reconstruction to be feasible, $S_{2l}(y)$ must lie between the two curves projections of the vertices **3** and **4**, and $S_{2r}(y)$ must lie between the two curves projections of the vertices **1** and **2**.

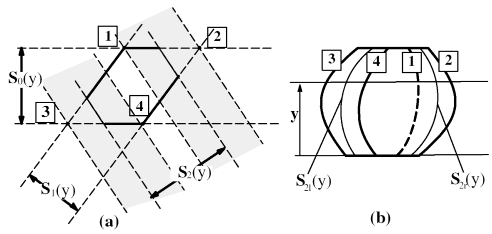


Fig. 6 - The condition for the compatibility of $S_2(y)$

The set of inequalities that define feasible intersection parameters can be derived inspecting in more detail the intersection in a horizontal plane Fig. 7.

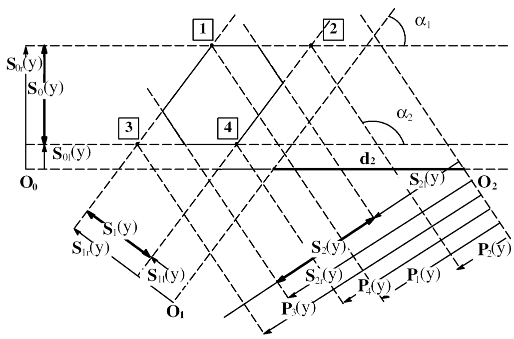


Fig. 7 - The intersections in a horizontal plane

Let $\mathbf{O_0}$, $\mathbf{O_1}$, and **O2** be the intersections of the axis y of each silhouette with this plane. Intersecting $S_0(y)$ and $S_1(y)$ requires fixing an angle, let it be α_1. Intersecting also $S_2(y)$ requires choosing two more parameters: the angle α_2 and the distance d_2 between the projection of $\mathbf{O_1}$ along V_1 and of $\mathbf{O_2}$ along V_2 on the line projecting $\mathbf{O_0}$ along the direction V_0 (see Fig. 7). Thus, to find feasible solutions we must search the 3-dimensional space $[\alpha_1,\alpha_2,d_2]$. Now, let $P_1(y)$, $P_2(y)$, $P_3(y)$ and $P_4(y)$ be the distances from $\mathbf{O_2}$ of the orthographic projections of the vertices of the parallelogram onto the line supporting

$S_2(y)$. The compatibility condition for the three silhouettes is expressed by the following inequalities, which can be worked out from the figure.

$$S_{2r}(y) \geq P_4(y),\quad S_{2r}(y) \leq P_3(y),\quad S_{2l}(y) \geq P_2(y) \qquad (1)$$
$$S_{2l}(y) \leq P_1(y),\quad P_4(y) \geq P_1(y)$$

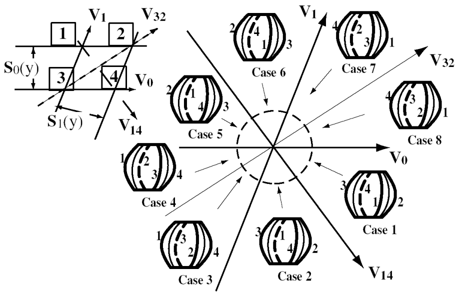


Fig. 8 - The four intersection cases

In (1), the purpose of the fifth inequality is to characterize the case just analyzed, let it be Case 1. Seven other cases are possible, each producing different sets of inequalities, as shown in Fig. 8. For each case, a possible orthographic projection onto the plane of S_2 of the edges of the object produced by the first intersection is shown with thick lines. The boundaries of S_2 are the thin lines.

The inequalities for more than three silhouettes

Let us consider any of the four cases described in the previous section, for instance Case 1, and let us add a fourth silhouette S_3. In each horizontal plane $S_0(y)$, $S_1(y)$ and $S_2(y)$ produce a polygon with six vertices and three pairs of parallel edges. The new volume intersection is defined by two more parameters, the angle α_3 between V_0 and V_3 and the distance d_3, measured, as d_2, along the line that projects $\mathbf{O_0}$. Satisfying the condition of Proposition 3 requires, in each horizontal plane, to cut away two opposite vertices, for instance vertices **7** and **5** in Fig. 9, without eliminating completely the edges that meet at these vertices. By orthographically projecting the six vertices onto the plane of S_3 we obtain six curves. For the new intersection to be feasible, the boundaries $S_{3l}(y)$ and $S_{3r}(y)$ of S_3 must lie in the areas bounded by the two leftmost and the two rightmost curves respectively. Various sets of inequalities result. First, let us distinguish two cases (case (**a**) and (**b**) of Fig. 9) related to which are the leftmost and rightmost vertices (respectively, **5** and **7** for case (**a**) and **7** and **5** for case (**b**)). For each case, eight distinct sub-cases result (see Fig. 10). The inequalities corresponding to each sub-case are easily written. For instance, for the first sub-case of case (**a**) it is:

$$P_5(y) \leq S_{3l}(y) \quad S_{3l}(y) \leq P_4(y) \quad P_1(y) \leq S_{3r}(y) \quad S_{3r}(y) \leq P_7(y)$$
$$P_4(y) \leq P_6(y) \quad P_6(y) \leq P_8(y) \quad P_8(y) \leq P_1(y)$$

where the $P_i(y)$ are defined on the line supporting S_3. Summarizing, each set of inequalities contains 12 inequalities, the five of Section 4 and seven new also referring to S_3. As for the number of sets of inequalities, we have 8 cases for three silhouettes, 3 pairs of opposite vertices and 16 cases for each pair, and thus 384 sets each containing 12 inequalities.

The previous discussion holds for any further silhouette. In fact, we must always cut a pair of opposite vertices without deleting completely the edges converging at these edges. It follows that each new silhouette adds two parameters, seven inequalities for each case and 16 sub-cases for each pair of opposite vertices. For the n^{th} silhouette, the pairs of vertices are n-1. Let $N_c(n)$ be the number of sets of inequalities for n silhouettes. For $n>3$ it is: $N_c(n)=16(n-1)N_c(n-1)$.

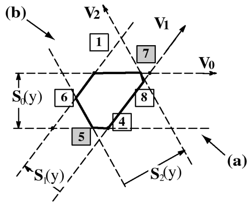


Fig. 9

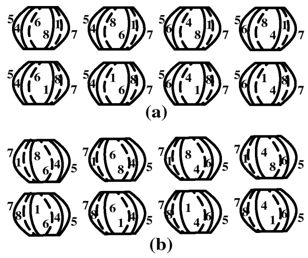


Fig. 10 – Cases (a**) and (**b**) and the 16 sub-cases**

4. SOLVING THE SETS OF INEQUALITIES

We have developed an algorithm to write automatically the sets of inequalities. The axes of the reference system are aligned with the axis of the projection of S_0 on the plane. Let's assume, without loss of generality, that V_0 is parallel to the y axis of the reference system and the line supporting S_0 is parallel to the x axis. The origin of the reference system corresponds with the intersection of the projections of $\mathbf{O_0}$ along V_0 and $\mathbf{O_1}$ along V_1 on the plane. The position of the i^{th} silhouette is determined by two parameters, d_i and α_i, previously defined. In particular, we assume that α_i (the angle between V_i and V_0) is positive if $V_0 \times V_i$ has the same verse of $x \times y$; it is also $V_i = (sen\alpha_i, -cos\alpha_i)$. Let C_j be the vertices of the polygon; the equations of the first 4 vertices, as in Fig. 11, are:

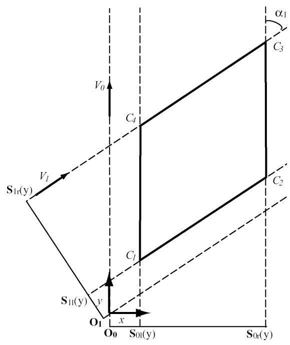


Fig. 11

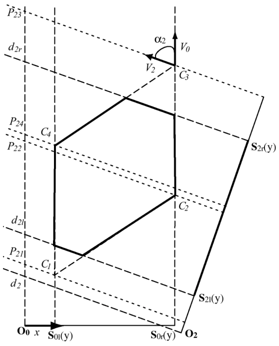


Fig. 12

$$C_1 = (S_{0l}, S_{1l}/sen\alpha_1 - S_{0l}/\tan\alpha_1), C_2 = (S_{0r}, S_{1l}/sen\alpha_1 - S_{0r}/\tan\alpha_1)$$
$$C_3 = (S_{0r}, S_{1r}/sen\alpha_1 - S_{0r}/\tan\alpha_1), C_4 = (S_{0l}, S_{1r}/sen\alpha_1 - S_{0l}/\tan\alpha_1)$$

The sets of inequalities previously introduced can be rewritten in terms of the distances from the origin along the y axis of the projections of the vertices of the parallelogram and of S_{il} and S_{ir} along the viewing direction of the i^{th} silhouette. For each projection, the lines passing through the vertices of the polygons have equations $C_j + V_i t$ and their intersections (P_{ij}) with the y axis of the reference system are:

$$P_{ij} = c_{yj} - c_{xj}\frac{sen(\alpha_i + \pi/2)}{\cos(\alpha_i + \pi/2)} = c_{yj} + c_{xj}/\tan\alpha_i$$

Now, let d_{il}, d_{is} be the projections on the y axis of S_{il} and S_{ir}. It follows that:

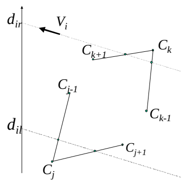
$$d_{il} = d_i + S_{il}(y)/\mathrm{sen}(\alpha_i)$$
$$d_{ir} = d_i + S_{ir}(y)/\mathrm{sen}(\alpha_i)$$

Projecting the vertices and S_i onto the y axis, the verse of the inequalities also depends on the value of α_i. For instance, in the example shown in Fig. 12, we have:

$$P_{21} \le d_{2l} \le P_{22} \le P_{24} \le d_{2r} \le P_{23}, \quad 0 < \alpha_2 < \pi$$
$$P_{21} \ge d_{2l} \ge P_{22} \ge P_{24} \ge d_{2r} \ge P_{23}, \quad \pi < \alpha_2 < 2\pi$$

In order to be able to write the inequalities in an automatic way, the general form of the inequalities can be rewritten multiplying each term by $sin(\alpha_i)$.

We also have to express the co-ordinates of the vertices C_j, j>4, as function of the VI parameters (see Fig. 13, where C_j and C_k are the couple of opposite vertices affected by the i[th] silhouette). Each new vertex is the intersection of the line every edge lies on and the specific projection line relative to V_i. All these lines are projection lines, and can be written as:



$$D_{il} + V_i r \ or \ D_{ir} + V_i r, where:$$
$$D_{il} = (S_{0l}, 0), i = 0; (0, d_{il}), i > 0$$
$$D_{ir} = (S_{0r}, 0), i = 0; (0, d_{ir}), i > 0$$

Fig. 13

Writing the equations as $A + Br = D + V_i t$ and given a vector W=(cosα_i,senα_i) perpendicular to V_i, the co-ordinates of the new vertex can be found as:

$$(A - D) + Br = V_i t \Rightarrow (A - D + Br)\cdot W = 0 \Rightarrow r = (D - A)\cdot W / B\cdot W$$
$$C_{new} = A + B(D - A)\cdot W / B\cdot W$$

As an example, the equations for case 1 of Fig. 8 are:

$$\sin\alpha_2(S_{1l}/\sin\alpha_1 - S_{0l}/\tan\alpha_1 + S_{0l}/\tan\alpha_2 - (d_2 + S_{2l}/\sin\alpha_2)) \le 0$$
$$\sin\alpha_2(d_2 + S_{2l}/\sin\alpha_2 - (S_{1l}/\sin\alpha_1 - S_{0r}/\tan\alpha_1 + S_{0r}/\tan\alpha_2)) \le 0$$
$$\sin\alpha_2((S_{1l} - S_{1r})/\sin\alpha_1 + (S_{0l} - S_{0r})/\tan\alpha_1 + (S_{0r} - S_{0l})/\tan\alpha_2) \le 0$$
$$\sin\alpha_2(S_{1r}/\sin\alpha_1 - S_{0l}/\tan\alpha_1 + S_{0l}/\tan\alpha_2 - (d_2 + S_{2r}/\sin\alpha_2)) \le 0$$
$$\sin\alpha_2(d_2 + S_{2r}/\sin\alpha_2 - (S_{1r}/\sin\alpha_1 - S_{0r}/\tan\alpha_1 + S_{0r}/\tan\alpha_2)) \le 0$$

A set inversion technique ([Jau01a]) has been applied for finding the feasible solution set **S** of the set of non-linear inequalities that characterizes each sub-case. This technique performs a paving of the parameter space with boxes. If the current box [**p**] is proved to be inside **S**, then the box is kept as part of the solution space or discarded if it has an empty intersection with **S** then it is discarded. Otherwise, [**p**] is bisected except if its width is smaller than a defined threshold. Bisection occurs in the middle of the box, perpendicularly to the side of the largest length. The dimensionality of the initial box is equal to the number of variables involved in the set of inequalities, and each dimension has initial size $[0,2\pi]$ for angular variables α_i and $[-\infty,\infty]$ for linear variables d_i. To prove that a given box [**p**] is inside **S**, interval computation ([Moo79a]) has been used. This technique can be used to find feasible parameter sets for one value of y between 0 and y_{max}. If one of the parameter sets is empty, the corresponding group of inequalities can be discarded. Otherwise, we could perform an incremental computation, adding each time a small Δy, related to the shape of the silhouettes, to the previous y. For each group of inequalities, the new feasible parameter set at $y+\Delta y$ must be a subset of the set at y.

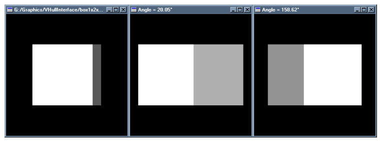


Fig. 14 - The silhouette S_0, S_1 and S_2

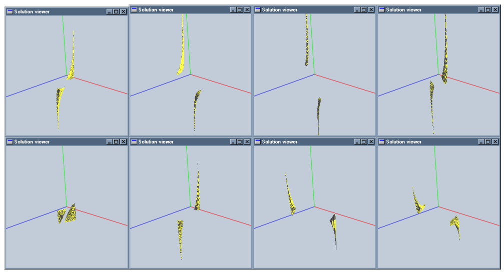


Fig. 15 – The eight solution spaces

In Fig. 14 three silhouettes S_0, S_1 and S_2 of a parallelepipedon are shown. The boxes defining the paving of the solution set of each of the eight sub-cases obtained are depicted in Fig. 15, where the axis of the reference system are α_1 and α_2 on the plane and d_2 as vertical axis.

5. CONCLUSION AND OPEN PROBLEMS

In this paper we have presented an approach to the new problem of understanding the shape of 3D objects from silhouettes when the relative position of the viewpoints is not known. We have presented a compatibility condition, which has been applied to the particular case of orthographic projections with viewing directions parallel to a plane. For this case, we have been able to work out sets of inequalities, involving the volume intersection parameters, which allow computing feasible solution sets, if they exist. An algorithm for automatically writing the inequalities has been developed, and some preliminary results have been presented. Several problems are open. Among them, the case of orthographic projection with unrestricted viewing directions, and the case of perspective projections.

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