Inner approximation of a capture basin of a dynamical system

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Introduction

Consider a dynamic system $\mathcal{S}$ defined by the following state equation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector and $\mathbf{f} : \mathbb{R}^n \mapsto \mathbb{R}^n$ is the evolution function of $\mathcal{S}$. Interval analysis has been used for many years to deal with dynamical systems. It is for instance possible to perform guaranteed integration as shown in [1,3,5,6] which is used in control theory [4] or robotics [2]. More and more tools for interval arithmetic exist as it is the case for the new package for Octave [3]. Our objective is to compute capture basin that is now defined. Let $\boldsymbol{\varphi}$ be the flow map of $\mathcal{S}$, *i.e.*, with the initial condition $\mathbf{x}_0 = \mathbf{x}(0)$, the system $\mathcal{S}$ reaches the state $\boldsymbol{\varphi}(t, \mathbf{x}_0)$ at time t. The *capture basin* of the *target* $\mathbb{T} \subset \mathbb{R}^n$ is the set $Capt(\mathbb{T})$ of initial states $\mathbf{x}$ from which at least one evolution of $\mathcal{S}$ reaches the target $\mathbb{T}$ in finite time:

$$Capt(\mathbb{T}) = \{\mathbf{x}_0 \mid \exists t \geq 0, \boldsymbol{\varphi}(t, \mathbf{x}_0) \in \mathbb{T}\}\,. \tag{2}$$

Note that $\mathbb{T} \subset Capt(\mathbb{T})$. We propose here a new method to compute an inner approximation of $Capt(\mathbb{T})$.

Dead path

A *trajectory* is a smooth function $\mathbf{x}(\cdot)$ from $\mathbb{R}^+$ to $\mathbb{R}^n$. The *path* associated with a trajectory $\mathbf{x}(\cdot)$ is the set of all $\mathbf{x}(t) \in \mathbb{R}^n$ and an orientation with respect to t. A path which satisfies (1) is said to be *feasible*. A path is *elected* if at least one of its points is inside $\mathbb{T}$. Otherwise it is a *dead path*. A state $\mathbf{x}$ which belongs to a dead path is outside $Capt(\mathbb{T})$. Figure provides five paths. All of them satisfy (1) except (v) which makes a loops and this cannot satisfy the state equation. The path (ii) enters in $\mathbb{T}$ and converges to an equilibrium point. The path (iii) is elected since it enters in $\mathbb{T}$, but since it leave it later, it contains some subpaths that are dead. The path (iv) corresponds to a limit cycle which is dead since it does not enter in $\mathbb{T}$.

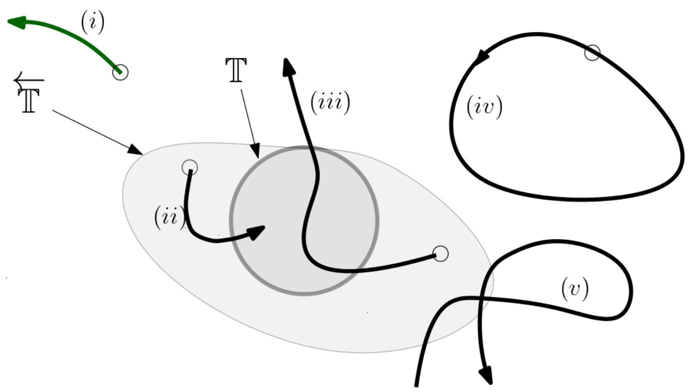

Figure 1: The paths (i),(iv) are dead, (v) does not satisfy (1) since it loops, and paths (ii), (iii) are elected

Methods

To compute an inner approximation of $Capt(\mathbb{T})$ we search for a dead path using a contractor-based approach. Note that the fact that the set of dead paths has a dimension equal to infinity is not a problem for our method. Then, we derive a finite dimensional polygonal contractor for $Capt(\mathbb{T})$. Using a paver in $\mathbb{R}^n$, we will show that we are able to obtain an inner approximation of $Capt(\mathbb{T})$ without any interval integration.

References

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