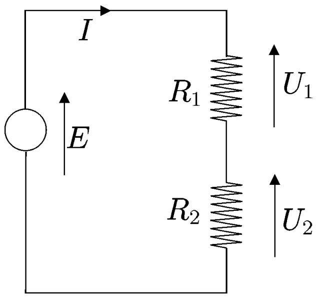
BE3 Contractors with Ibex

Exercise 1. Consider the electronic circuit consisting of one battery and two resistors represented below



Assume that measuments have been collected on this circuit, leading to the following relations:

$$E \in [23V, 26V],\ I \in [4A, 8A],\ U_1 \in [10V, 11V], U_2 \in [14V, 17V],\ P \in [124W, 130W],$$

where P is the power delivered by the battery. Nothing is known about the values of the resistors except that they are positive. Using an interval constraint propagation algorithm, compute the feasible values for R_1 and R_2. Contract also the domains for all other variables E, I, U_1, U_2, P.

Exercise 2. Consider the three following sets

$$\begin{array}{rcl} \mathbb{X}_1 & = & \left\{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \in [4, 9]\right\} \\ \mathbb{X}_2 & = & \left\{\mathbf{x} \in \mathbb{R}^2 \mid \sin(x_1 + 2x_2) \in [0, 1]\right\} \\ \mathbb{X}_3 & = & \left\{\mathbf{x} \in \mathbb{R}^2 \mid x_1 + x_2^2 \in [1, 2]\right\}. \end{array}$$

Draw the sets defined by

$$\mathbb{X}_i, \overline{\mathbb{X}}_i, \left(\mathbb{X}_1 \cap \overline{\mathbb{X}}_2\right) \cup \left(\mathbb{X}_3 \cap \mathbb{X}_2\right),\ \bigcap_i \mathbb{X}_i,\ \bigcup_i \mathbb{X}_i,\ \bigcap_i \overline{\mathbb{X}}_i,\ \bigcup_i \overline{\mathbb{X}}_i, \overset{\{1\}}{\bigcap_i} \mathbb{X}_i,\ \overset{\{2\}}{\bigcup_i} \mathbb{X}_i.$$

Exercise 3. Consider 5 robots $\mathcal{R}_1, \dots, \mathcal{R}_5$ sleeping under the water since a very long time. Assume that all robots wake up and that all clocks are not synchronized anymore. The speed of the sound is assumed to be $c = 1$. Denote by τ_i the clock of robot $\mathcal{R}_i$. At time $\tau_i = 10 \cdot i$, (in the clock frame of $\mathcal{R}_i$), the robot $\mathcal{R}_i$ emits a sonar ping which is received at time τ_{ij} by the robots $\mathcal{R}_j$, $j \neq i$ measured with their own clock. The corresponding matrix is

$$\mathbf{T} = \begin{pmatrix} 10 & 14 & 16 & 17 & 21 \\ 22 & 20 & 23 & 26 & 32 \\ 32 & 31 & 30 & 34 & 40 \\ 41 & 42 & 42 & 40 & 46 \\ 53 & 56 & 56 & 54 & 50 \end{pmatrix}.$$

The error is ± 0.5. Define the distance matrix $\mathbf{D}$ with entries

$$d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

where (x_i, y_i) is the coordinate vector of $\mathcal{R}_i$. Define also the time shift matrix $\mathbf{S}$ with entries

$$s_{ij} = s_i - s_j,$$

where s_i is the shift of the clock (i.e., when $s_i = -2$, it means that the clock has a delay of 2 sec with respect to the absolute time). Using IBEX find interval matrices $[\mathbf{D}]$ and $[\mathbf{S}]$ enclosing $\mathbf{D}$ and $\mathbf{S}$. Deduce a picture which provides an approximation of the configuration of the swarm. Explain why the variables x_i, y_i, s_i are not identifiable.

Exercise 4. Consider n robots $\mathcal{R}_1, \dots, \mathcal{R}_n$ each of them moving inside a bubble $\mathcal{B}_i$ with radius 1 and center

$$\begin{cases} x_i &= 10.\cos(t + p_1.i) + 3\cos(p_2(t + p_1.i)) \\ y_i &= 15.\cos(t + p_1.i) + 3\cos(p_2(t + p_1.i)). \end{cases}$$

1) For $n = 1, 2, 3, \dots$, draw the set of all $\mathbf{p}$ such that no collision occur.

2) What is the maximal number of robots n for which we can guarantee that no collision could occur?

3) We now consider that if we have less than q robots in the same bubble, a basic anticollision system can be considered as reliable. Draw, for some arbitrary n, q, the set of all $\mathbf{p}$ such that we never have strictly more that q robots is the same bubble.
