BE 1 Inclusion tests and set inversion

Download the QT project `http://www.ensta-bretagne.fr/jaulin/mer_sivia_simple`, compile it and run it.

1) Propose a recursive version of SIVIA in order to avoid using any list.

2) A robot located at (x, y) is able to measure its distance to $n = 5$ landmarks. The landmark positions and the corresponding distances are reported below:

x_i	3	7	-3	5	-3
y_i	4	3	7	-2	-6
d_i	3	6	6	6	9

The errors on the distances in ± 0.5. Using a set inversion algorithm, find all feasible locations for the robot.

3) In `interval.h` add a new sum operator $+$ between an interval $[x]$ and a boolean interval $[b]$ such that

$$\begin{aligned}
\left[x^-, x^+\right] + [b] &= \left[x^-, x^+\right] \text{ if } [b] = [0,0] \\
&= \left[x^-, x^+ + 1\right] \text{ if } [b] = [0,1] \\
&= \left[x^- + 1, x^+ + 1\right] \text{ if } [b] = [1,1] \,.
\end{aligned}$$

4) In the data set, we insert two outliers: $d_1 = 9$ and $d_2 = 2$ but the robot ignores this and only knows that outliers could occur. Using a set inversion robust to two outliers (and using also the previous operator $+$) adapt the inclusion test in order to find all feasible locations of the robot.

5) Adapt SIVIA in order characterize a set defined as a projection draw an inner and an outer. As an application, characterize the set

$$\mathbb{X} = \left\{ (x_1, x_2) \mid \exists y,\ x_1^2 + x_2^2 + y_1^2 \in [4, 9] \right\}.$$

6) We assume that we do not measure the distances, but time flight given by

$$\delta_i = c\sqrt{(x - x_i)^2 + (y - y_i)^2},$$

where $c \in [0, 5]$ is unknown. For the same experimental conditions, we get the data set

$\delta_i =$	10	18	17	18	28

with an error of ± 1. Compute the set of all feasible locations given by

$$\mathbb{X} = \left\{ (x, y) \mid \exists c \in [0, 5], \forall i,\ c\sqrt{(x - x_i)^2 + (y - y_i)^2} \in [\delta_i - 1, \delta_i + 1] \right\}$$
