## Shepherd project

## 1 Problem

This project is motivated a project proposed by Ifremer to collect autonomously data in the ocean by underwater drifting buoys. To monitor the buoys Ifremer proposed to use 4 saiboat robots.

Sailboat. Each sailboat has a position $\mathbf{a}_{i}, i \in\{1,2,3,4\}$. Each sailboat follows a small triangle with center $\hat{\mathbf{a}}_{i}$. We have

$$
\hat{\mathbf{a}}_{i}=\hat{\mathbf{a}}_{0}+10^{3}\binom{\cos \left(i \frac{\pi}{2}\right)}{\sin \left(i \frac{\pi}{2}\right)},
$$

where $\hat{\mathbf{a}}_{0}=\hat{\mathbf{b}}_{0}+\tilde{\mathbf{a}}_{0}$. The point $\hat{\mathbf{b}}_{0}$ is the barycenter of the buoys and $\tilde{\mathbf{a}}_{0}$ is a vector given by the supervisor to control the position of the group. Define

$$
\mathbf{r}_{i, \ell}=\hat{\mathbf{a}}_{i}+50\binom{\cos \left((\ell+1) \frac{2 \pi}{3}\right)}{\sin \left((\ell+1) \frac{2 \pi}{3}\right)}
$$

The robot $\mathcal{R}_{i}$ follows a line the vertices of which are $\mathbf{r}_{i, \ell}, \mathbf{r}_{i, \ell+1}$ where $\ell$ is the number of edges that have been followed from the beginning of the mission. When

$$
\left\langle\mathbf{r}_{i, \ell+1}-\mathbf{r}_{i, \ell}, \mathbf{r}_{i, \ell+1}-\mathbf{a}_{i}\right\rangle<0
$$

then $\ell$ is incremented by 1 .
Buoys. Each buoy has a position $\mathbf{b}_{j}, j \in\{1, \ldots, 20\}$ and moves up and down with a depth between 0 to 1 km . The duration of a cycle of 1 hour. A buoy can stay longer at a depth which fosters a given detection. A buoy is attracted by $\hat{\mathbf{a}}_{0}$ but is repulsed by nearby buoys. A buoy can move verticaly using its only actuator which is a ballast. Horizontaly, the buoy follows underwater current. We assume that the state equations of a buoy is

$$
\dot{\mathbf{x}}=\left(\begin{array}{c}
\sin \left(10^{-3}\left(x_{2}+0.9 x_{3}\right)\right) \\
-\sin \left(10^{-3}\left(x_{1}+x_{3}\right)\right) \\
u
\end{array}\right)
$$

where $u \in[-1,1]$ is the ballast. The state vector $\mathbf{x}=\left(x_{1}, x_{2}, x_{3}\right)$ corresponds to the horizontal coordinates $\left(x_{1}, x_{2}\right)$ and the depth $x_{3}$.

Supervisor. The supervisor control the vector of the point $\tilde{\mathbf{a}}_{0}$. The supervisor can be a human operator, but a pheromone technique can also be used to avoid staying for too long in a given region of the ocean.

Notation. Hats are used to indicate an estimation. For instance $\hat{\mathbf{a}}_{i}$ corresponds to an estimation of the position $\mathbf{a}_{i}$ of the boats. The zero as a subscript corresponds to a abstraction of the group of boat or the group of buoys to a single abstract boat or a single abstract buoy.


Overview of the group of sailboat and buoy robots

## 2 Formalism

