Gascogne project

1 Problem

This project is motivated by the detection of submarine intruders inside the Bay of Biscay (golfe de Gascogne). For simplicity, in this project, we consider a 2D environment, and the robots are equipped with a GPS. The zone to be secured is the Bay of Biscay. The objective of the project GASCOGNE is to demonstrate how the secute zone can be obtained in a reliable way. This has to be done using mathematics, simulations and experiments.

2 Mathematical morphology

The Minkowski sum and the Minkowski difference of two sets $\mathbb{A}$ and $\mathbb{B}$ are defined by

$$\begin{array}{rcl} \mathbb{A}+\mathbb{B} & = & \{\mathbf{a}+\mathbf{b}\in\mathbb{R}^n|\mathbf{a}\in\mathbb{A} \text{ and } \mathbf{b}\in\mathbb{B}\} \\ \mathbb{A}-\mathbb{B} & = & \{\mathbf{a}-\mathbf{b}\in\mathbb{R}^n|\mathbf{a}\in\mathbb{A} \text{ and } \mathbf{b}\in\mathbb{B}\} \end{array} \tag{1}$$

These operations correspond to the addition and difference used in interval computation. When $\mathbb{A}$ or $\mathbb{B}$ are singletons, we get these rules yield:

$$\begin{array}{rclc} \mathbb{A}+\mathbf{b} & = & \{\mathbf{a}+\mathbf{b}\in\mathbb{R}^n|\mathbf{a}\in\mathbb{A}\} & \text{(translation by } \mathbf{b}) \\ \mathbb{A}-\mathbf{b} & = & \{\mathbf{a}-\mathbf{b}\in\mathbb{R}^n|\mathbf{a}\in\mathbb{A}\} & \text{(translation by } -\mathbf{b}) \\ -\mathbb{A} & = & \{-\mathbf{a}\in\mathbb{R}^n|\mathbf{a}\in\mathbb{A}\}. & \text{(symmetry)} \end{array} \tag{2}$$

Dilatation. The *dilatation* of a set $\mathbb{A}$ by the *structuring element* $\mathbb{B}$ is defined by $\mathbb{A}+\mathbb{B}$.

Erosion. The *erosion* of $\mathbb{A}$ by $\mathbb{B}$ is defined by the three following equivalent relations:

$$\begin{array}{rcl} \mathbb{A}\ominus\mathbb{B} & = & \{\mathbf{z}\in\mathbb{R}^n|\mathbb{B}+\mathbf{z}\subseteq\mathbb{A}\} \\ & = & \bigcap_{\mathbf{b}\in\mathbb{B}} \mathbb{A}-\mathbf{b} \\ & = & \overline{\overline{\mathbb{A}}-\mathbb{B}}. \end{array} \tag{3}$$

where $\overline{\mathbb{X}}$ denotes the complement of $\mathbb{X}$ relative to $\mathbb{R}^n$.

Opening. The *opening* of $\mathbb{A}$ by $\mathbb{B}$ is obtained by the erosion of $\mathbb{A}$ by $\mathbb{B}$, followed by dilation by $\mathbb{B}$:

$$\mathbb{A}\circ\mathbb{B}=(\mathbb{A}\ominus\mathbb{B})+\mathbb{B} \tag{4}$$

Closing. The *closing* of $\mathbb{A}$ by $\mathbb{B}$ is obtained by the dilation of $\mathbb{A}$ by $\mathbb{B}$, followed by erosion by $\mathbb{B}$:

$$\mathbb{A} \bullet \mathbb{B} = (\mathbb{A} + \mathbb{B}) \ominus \mathbb{B} \tag{5}$$

Properties of the basic operators. Here are some properties of the basic binary morphological operators :

They are increasing, that is, if

$$\begin{array}{ccc} \mathbb{A} \subset \mathbb{C} & \Rightarrow & \mathbb{A} + \mathbb{B} \subset \mathbb{C} + \mathbb{B} \\ \mathbb{A} \subset \mathbb{C} & \Rightarrow & \mathbb{A} \ominus \mathbb{B} \subset \mathbb{C} \ominus \mathbb{B} \\ \mathbf{0} \in \mathbb{B} & \Rightarrow & \mathbb{A} \ominus \mathbb{B} \subseteq \mathbb{A} \circ \mathbb{B} \subseteq \mathbb{A} \subseteq \mathbb{A} \bullet \mathbb{B} \subseteq \mathbb{A} + \mathbb{B} \\ \mathbb{A} \subseteq (\mathbb{C} \ominus \mathbb{B}) & \Leftrightarrow & (\mathbb{A} \oplus \mathbb{B}) \subseteq \mathbb{C} \end{array} \tag{6}$$

Moreover

$$\begin{array}{ccc} \mathbb{A} + \mathbb{B} & = & \mathbb{B} + \mathbb{A} \\ (\mathbb{A} + \mathbb{B}) + \mathbb{C} & = & \mathbb{A} + (\mathbb{B} + \mathbb{C}) \\ (\mathbb{A} \ominus \mathbb{B}) \ominus \mathbb{C} & = & \mathbb{A} \ominus (\mathbb{B} + \mathbb{C}) \\ \mathbb{A} + \mathbb{B} & = & \overline{\overline{\mathbb{A}} \ominus -\mathbb{B}} \\ \mathbb{A} \bullet \mathbb{B} & = & \overline{\overline{\mathbb{A}} \circ -\mathbb{B}} \end{array} \tag{7}$$

3 Secure zone

Several robots $\mathcal{R}_1, \ldots, \mathcal{R}_n$ at positions $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are moving in a 2D world and collaborate to guarantee that they is no moving intruder inside a zone $\mathbb{G}$ (G for *Gascogne*). For this, we assume that there exists a virtual intruder with a speed less than 5m/sec with an unknown position $\mathbf{x}$, i.e., $\|\dot{\mathbf{x}}\| \leq \bar{v}_{\mathbf{x}} = 5$.

Each robot $\mathcal{R}_i$ is able to detect the intruder if the distance is less than a scope distance d_i, (for instance 10m). Since we assume that $\mathcal{R}_i$ detects nothing, we have $\|\mathbf{x} - \mathbf{a}_i\| > d_i$.

Secure zone. We define the secure zone as the set of all points for which we can guarantee that there is no intruder, assuming that the robots do not detect anything. If we define the set $\mathbb{X}$ of all positions where the virtual intruder could be, then the secure zone corresponds to the complementary of $\mathbb{X}$. For the initialization, we assume that $\mathbb{X}(0) \subset \mathbb{G} \subset [\mathbf{x}](0)$ where $[\mathbf{x}](0)$ is a box. Estimating $\mathbb{X}$, corresponds to a state estimation problem in a set-membership context.

Evolution. We assume that the motion of the virtual intruder obeys to the following state equation:

$$\left\{ \begin{array}{lcl} \dot{x}_1 & = & v \cos \psi \\ \dot{x}_2 & = & v \sin \psi \end{array} \right. \tag{8}$$

where $v \leq \bar{v}_{\mathbf{x}} = 5$. Equivalently, we can represent this evolution under the form of a differential inclusion

$$\dot{\mathbf{x}}(t) \in \mathbb{F}(\mathbf{x}). \tag{9}$$

For our specific application, $\mathbb{F}(\mathbf{x})$ corresponds to a disk $\mathbb{D}(\mathbf{0}, \bar{v}_{\mathbf{x}})$ of center $\mathbf{0}$ and radius $\bar{v}_{\mathbf{x}}$ and does not depend on $\mathbf{x}$. For a small sampling time δ, we can write

$$\mathbf{x}(t) \in \mathbb{F}_{\delta}(\mathbf{x}(t-\delta)) \tag{10}$$

where

$$\mathbb{F}_{\delta}(\mathbf{x}) = \mathbf{x} + \delta \cdot \mathbb{D}(\mathbf{0}, \bar{v}_{\mathbf{x}}). \tag{11}$$

Note that

$$\begin{array}{rcl} \mathbb{F}_{\delta}(\mathbf{x}) & = & \mathbf{x} \\ \mathbb{F}_{-\delta}(\mathbf{x}) & = & \mathbb{F}_{\delta}(\mathbf{x}) \\ \mathbb{F}_{\delta_1+\delta_2}(\mathbf{x}) & = & \mathbb{F}_{\delta_1} \circ \mathbb{F}_{\delta_2}(\mathbf{x}) \quad , \quad \text{if } \delta_1 \geq 0 \text{ and } \delta_2 \geq 0 \end{array} \tag{12}$$

Correction. If a robot located at position $\mathbf{a}$ detects no intruder within a scope distance of d, we have

$$g_{\mathbf{a}}(\mathbf{x}) = \|\mathbf{x} - \mathbf{a}\| \notin [0, d]. \tag{13}$$

Equivalently

$$\mathbf{x} \in g_{\mathbf{a}}^{-1}([d, \infty]). \tag{14}$$

Causal observer. As a consequence, for a small sampling time δ, we have

$$\mathbb{X}(t) = \mathbb{G} \cap \mathbb{F}_{\delta}(\mathbb{X}(t-\delta)) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty]. \tag{15}$$

The secure set corresponds to the complementary of $\mathbb{X}(t)$.

Non causal observer. In a non-recursive formulation, the causal observer is

$$\mathbb{X}(t) = \mathbb{G} \cap \bigcap_{t_1 \in \mathbb{T}} \mathbb{F}_{t-t_1}(\bigcap_i g_{\mathbf{a}_i(t_1)}^{-1}([d_i(t_1), \infty]). \tag{16}$$

where $\mathbb{T} = [0, t]$ for a causal estimator, $\mathbb{T} = [t, \infty]$ for an anticausal state estimator and $\mathbb{T} = [0, \infty]$ for a state observer without any causality.

$$\mathbb{X}(t) = \mathbb{G} \cap \mathbb{F}_{\delta}(\mathbb{X}(t-\delta)) \cap \bigcap_{t_1 \geq t} \mathbb{F}_{t-t_1}(\bigcap_i g_{\mathbf{a}_i(t_1)}^{-1}([d_i(t_1), \infty]) \tag{17}$$

With uncertainty. We have the following uncertainties

$$\begin{array}{c} d_i(t) \in [d_i(t)] \\ \mathbf{a}_i(t) \in [\mathbf{a}_i]\,(t) \end{array}$$

and thus the thick set for causal observer:

$$[\![\mathbb{X}]\!]\,(t) = \mathbb{G} \cap \mathbb{F}_{\delta}([\![\mathbb{X}]\!]\,(t-\delta)) \cap \bigcap_i g_{[\mathbf{a}_i](t)}^{-1}([\![[d_i(t)], \infty]\!]) \tag{18}$$

and for the thick state observer without any causality:

$$[\![\mathbb{X}]\!](t) = \mathbb{G} \cap \mathbb{F}_{\delta}([\![\mathbb{X}]\!](t-\delta)) \cap \bigcap_{t_1 \geq t} \mathbb{F}_{t-t_1}(\bigcap_i g_{[\mathbf{a}_i](t)}^{-1}([\![[d_i(t)], \infty]\!])). \tag{19}$$

4 Control

Explain the controllers that are implementedin the project: Feedback linearization, potential field, control of the group on a moving ellipsoid.

5 Implementation

Explain the principle of the integral image, the SIVIA. How the communication is done.

6 Robots

7 Experiments

8 A faire

1) For each robot, generate the control low so that it follows a trajectory given by a perodic equation.

2) Each robot broadcast its position which is reveived by the centralized computer.

3) Draw the corresponding secure zone.

4) Build the subpaving from the image in order to improve the efficiency of the state observer.

5) Implement the causal state observer.

6) Management. Find a manager who (1) detects the inconsistencies in the project, (2) helps the teams to communicate, (3) is aware of the state of the project and (4) starts to write the report with Lyx.

9 Experiment

1) Five robots are moving on the football field. They are have two modes: teleoperated or automous. For each sampling time t_k, they broadcast a box $[\mathbf{a}_i](t_k)$ with they position, (possibly their future positions with uncertainty $[\mathbf{a}_i](t_k+\delta)$, $[\mathbf{a}_i](t_k+2\delta)$, $\dots$) and the scope distance interval $[d_i](t_k)$. A centralized computer reveives all these data and draw the secure zone. The virtual intruder is only allowed to enter through the boundary of the secured zone.