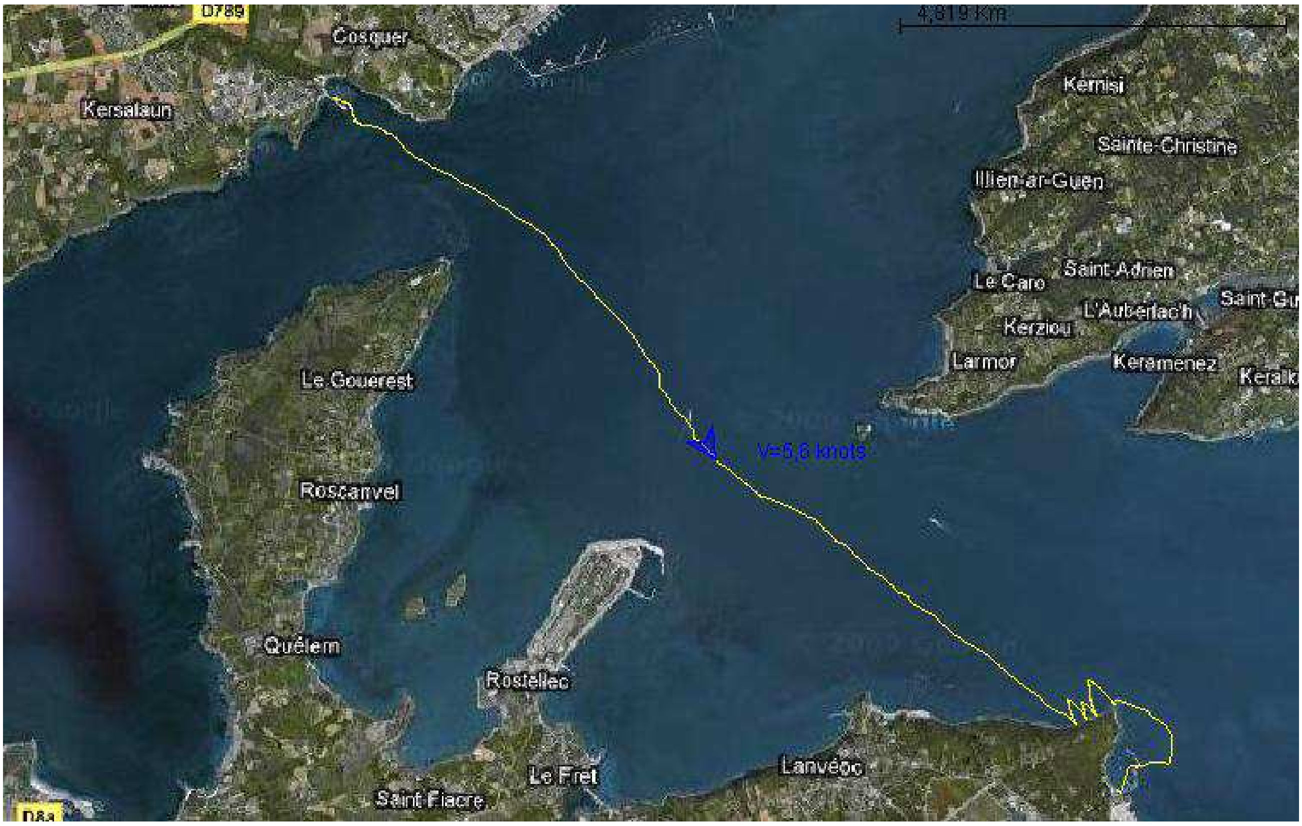
Reliable control using interval analysis. Application to sailboat robotics

L. Jaulin, F. Lebars, J. Sliwka, O. Ménage, T. Terre, . . .
ENSTA-Bretagne, IFREMER, Brest.

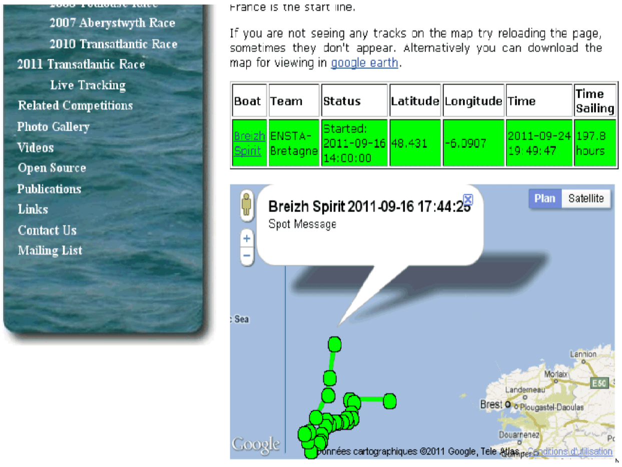
http://www.ensta-bretagne.fr/jaulin/

1 Sailboat robotics


BREIZH
ENSIETA
SPIRIT
ENSIETA



4,819 Km
D789
Cosquer
Kersalaun
Kernisi
Sainte-Christine
Illien-ar-Guen
Saint-Adrien
Le Caro
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
V=5,6 knots
Roscanvel
Quélern
Rostellec
Le Fret
Lanvéoc
Saint-Fiacre


2007 Aberystwyth Race
2010 Transatlantic Race
2011 Transatlantic Race
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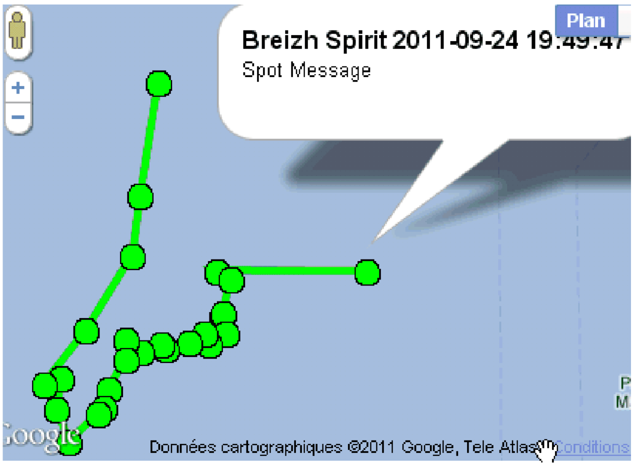
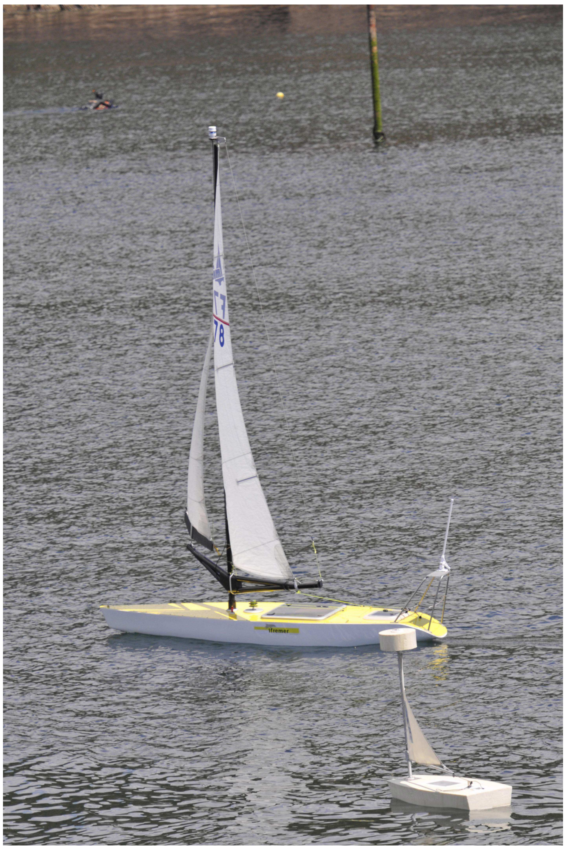
France is the start line.

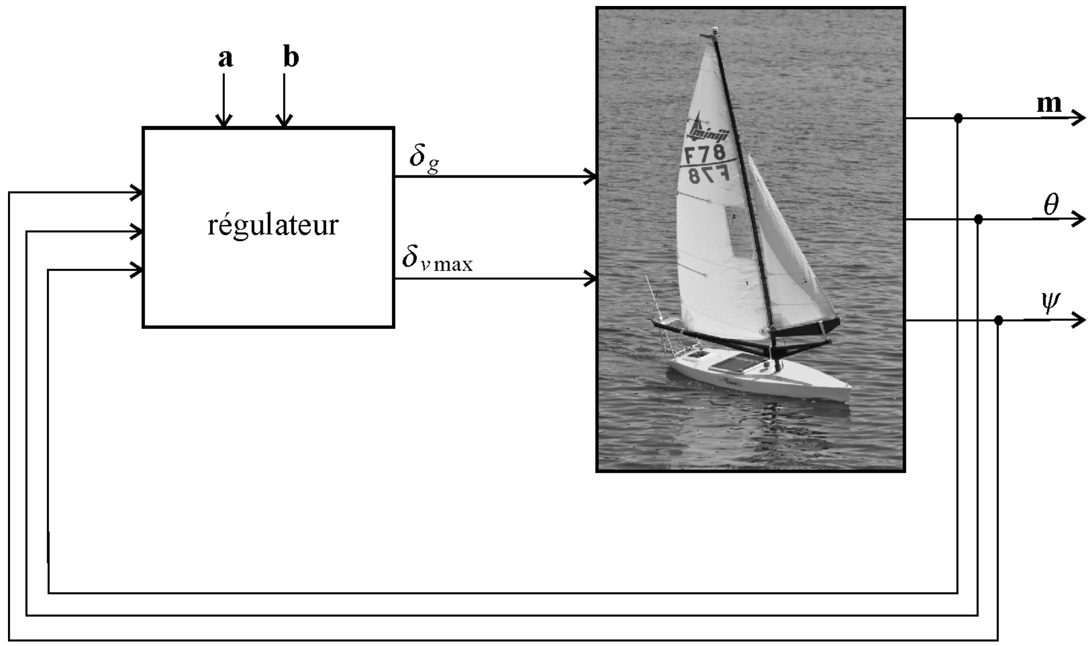
If you are not seeing any tracks on the map try reloading the page, sometimes they don't appear. Alternatively you can download the map for viewing in google earth.

Boat	Team	Status	Latitude	Longitude	Time	Time Sailing
Breizh Spirit	ENSTA-Bretagne	Started: 2011-09-16 14:00:00	48.431	-6.0907	2011-09-24 19:49:47	197.8 hours




F78




a
b
régulateur
δ_g
$\delta_{v\max}$
F78
m
θ
ψ

2 Brest-Douarnenez

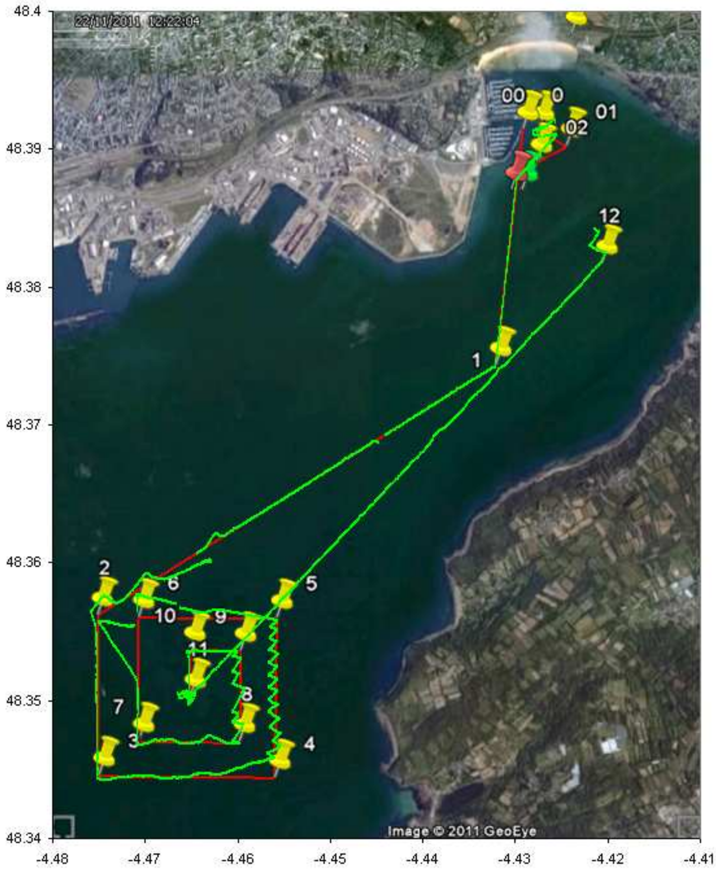
Départ le 17 janvier 2012 à 8h.


Challenger

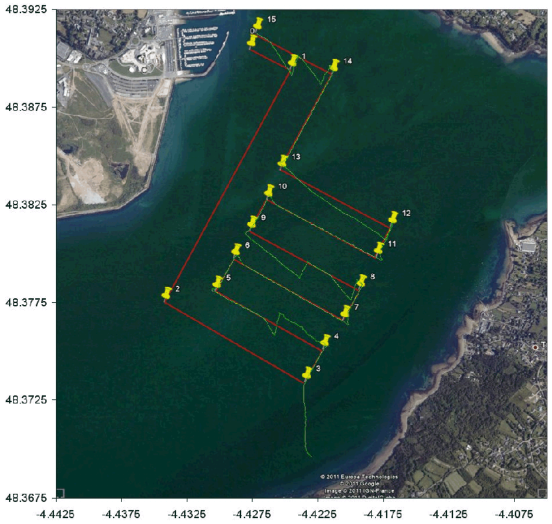

Brest
PRO
BR D98 505
YAMAHA

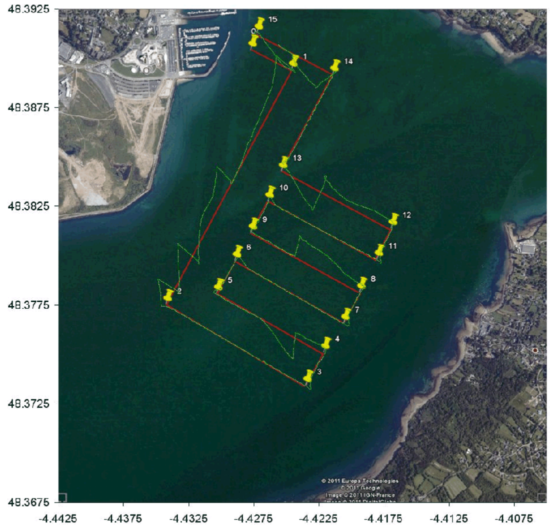

miniji
F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Plougastel-Daoulas
Le Conquet
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
lat 48.230580° long -4.564526° élév. 6 m
Altitude 53.47 km
©2010 Google
Start
vaimos.txt -...
Logs
sftp://root...
Command Pr...
Command Pr...
data
Interface po...
VCtrl (Runni...
VAIMOS aut...
Google Earth
FR
98%
01:42



48.4
48.39
48.38
48.37
48.36
48.35
48.34
-4.48
-4.47
-4.46
-4.45
-4.44
-4.43
-4.42
-4.41
22/11/2011 12:22:04
00
0
01
02
12
1
2
6
5
10
9
11
8
7
3
4
Image © 2011 GeoEye


48.3925
48.3875
48.3825
48.3775
48.3725
48.3675
-4.4425
-4.4375
-4.4325
-4.4275
-4.4225
-4.4175
-4.4125
-4.4075
15
1
14
13
10
12
9
11
6
5
8
2
7
4
3

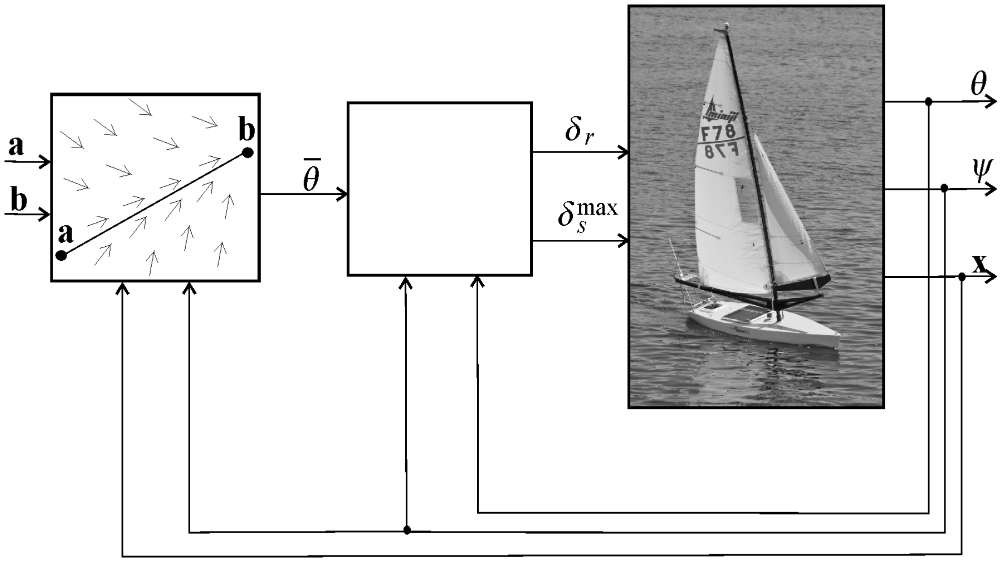

48.3925
48.3875
48.3825
48.3775
48.3725
48.3675
-4.4425
-4.4375
-4.4325
-4.4275
-4.4225
-4.4175
-4.4125
-4.4075
15
1
14
13
10
12
9
11
6
5
8
2
7
4
3

Il est donc possible pour un robot voilier de rester dans sa bande.

Indispensable pour créer des règles de circulation lorsqu'on travaille avec des meutes.

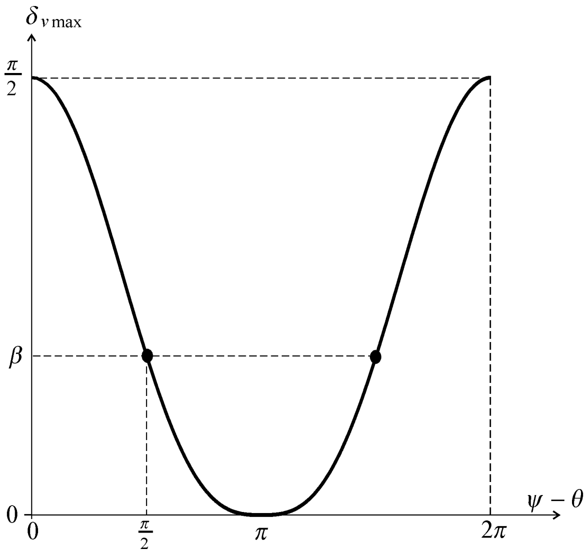
Indispensable pour déterminer les responsabilités en cas d'accident.

3 Line following

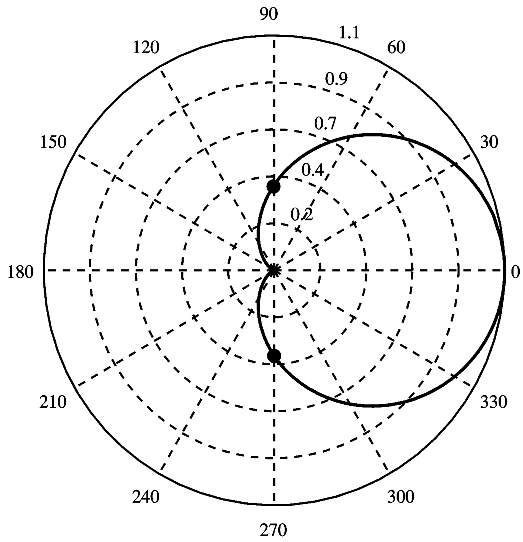


a
b
a
b
$\bar{\theta}$
δ_r
$\delta_s^{\max}$
F78
θ
ψ
x

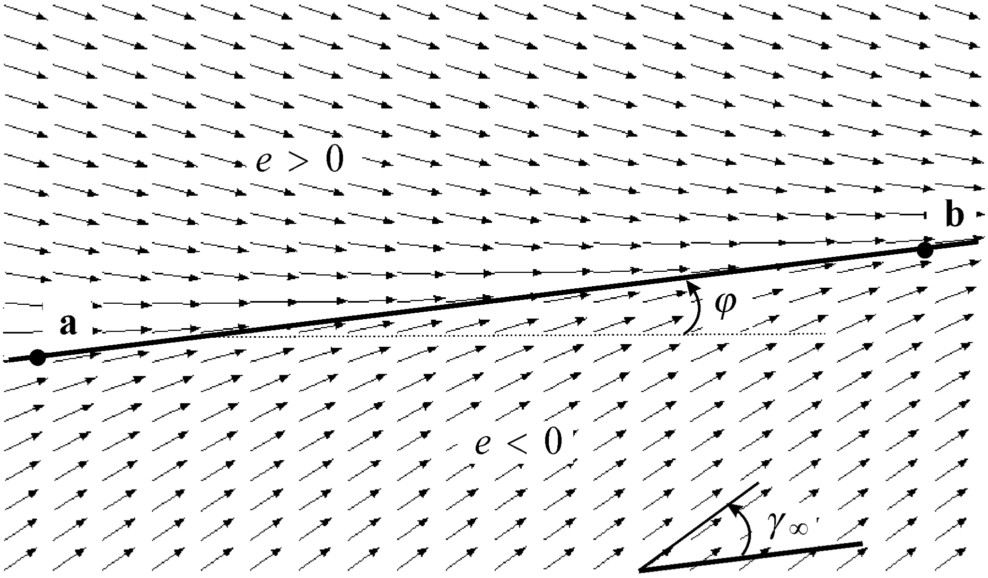
$$\begin{cases} \delta_g &= \begin{cases} \delta_g^{\mathsf{max}}.\,\mathsf{sin}\left(\theta - \bar{\theta}\right) & \text{if } \mathsf{cos}\left(\theta - \bar{\theta}\right) \geq 0 \\ \delta_g^{\mathsf{max}}.\mathsf{sign}\left(\mathsf{sin}\left(\theta - \bar{\theta}\right)\right) & \text{otherwize} \end{cases} \\ \delta_s^{\mathsf{max}} &= \frac{\pi}{2}.\left(\frac{\mathsf{cos}(\psi - \bar{\theta}) + 1}{2}\right) \end{cases}$$



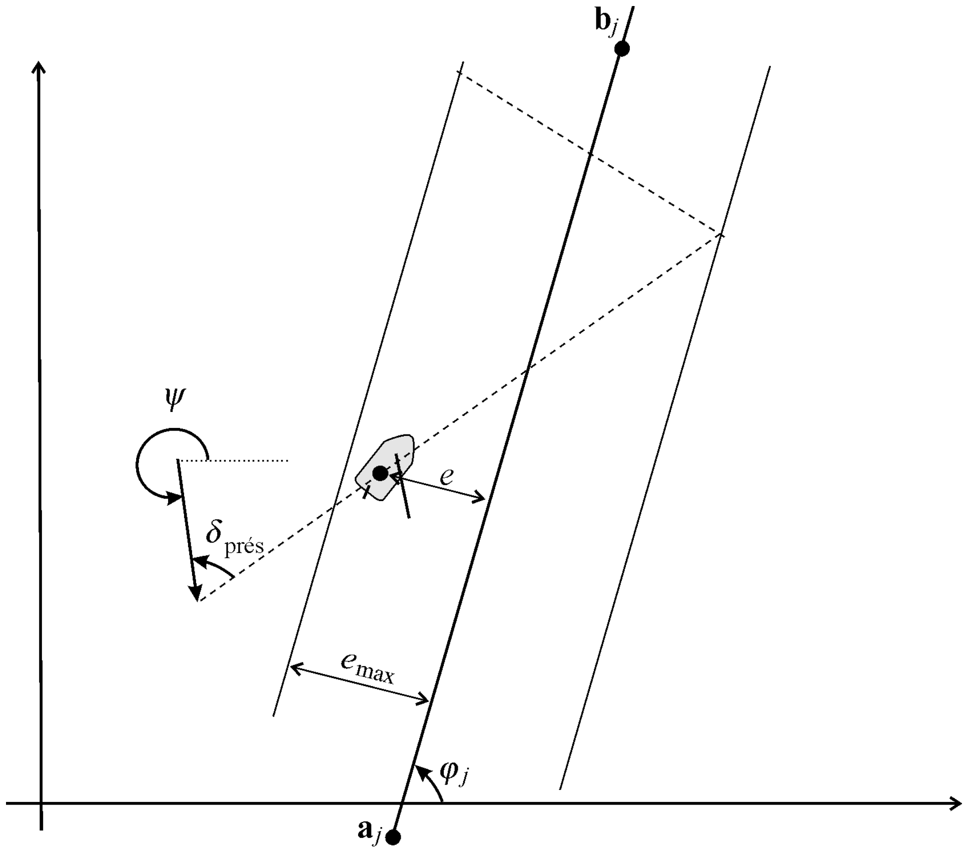
$\delta_{v\,\max}$
$\frac{\pi}{2}$
β
0
0
$\frac{\pi}{2}$
π
2π
$\psi - \theta$



90
1.1
120
60
0.9
0.7
150
30
0.4
0.2
180
0
210
330
240
300
270



$$\bar{\theta} = \varphi - \frac{2.\gamma_\infty}{\pi}.\mathsf{atan}\left(\frac{e}{r}\right)$$



$\mathbf{b}_j$
ψ
e
$\delta_{\text{prés}}$
$e_{\max}$
φ_j
$\mathbf{a}_j$

A course $\bar{\theta}$ is feasible if

$$\cos\left(\psi - \bar{\theta}\right) + \cos\zeta \geq 0$$

Function $\bar{\theta}\left(\mathbf{x}, \mathbf{a}, \mathbf{b}, \psi, \gamma_{\infty}, r, \zeta\right)$

$e = \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{x}-\mathbf{a}\right)$

$\varphi = \text{atan2}(\mathbf{b}-\mathbf{a})$

$\theta^* = \varphi - \frac{2.\gamma_{\infty}}{\pi}.\text{atan}\left(\frac{e}{r}\right)$

if $\cos\left(\psi - \theta^*\right) + \cos\zeta < 0$

  or $(|e| < r$ and $(\cos(\psi - \varphi) + \cos(\zeta) < 0))$

    then $\bar{\theta} = \pi + \psi - \zeta.\text{sign}(e)$;

    else $\bar{\theta} = \theta^*$;

end

Choose a frame $\mathcal{R}(\mathbf{a}, \mathbf{i}, \mathbf{j})$ based on the line.

Function $\bar{\theta}(\mathbf{x}, \psi, \gamma_\infty, r, \zeta)$

$\theta^* = -\frac{2.\gamma_\infty}{\pi}.\text{atan}\left(\frac{x_2}{r}\right)$

if $\cos(\psi - \theta^*) + \cos\zeta < 0$

    or $(|e| < r$ and $(\cos(\psi - \varphi) + \cos(\zeta) < 0))$

        then $\bar{\theta} = \pi + \psi - \zeta.\text{sign}(x_2)$;

        else $\bar{\theta} = \theta^*$;

end

$$\dot{\mathbf{x}} = \begin{pmatrix} \cos\bar{\theta}\,(\mathbf{x},\psi) \\ \sin\bar{\theta}\,(\mathbf{x},\psi) \end{pmatrix}$$

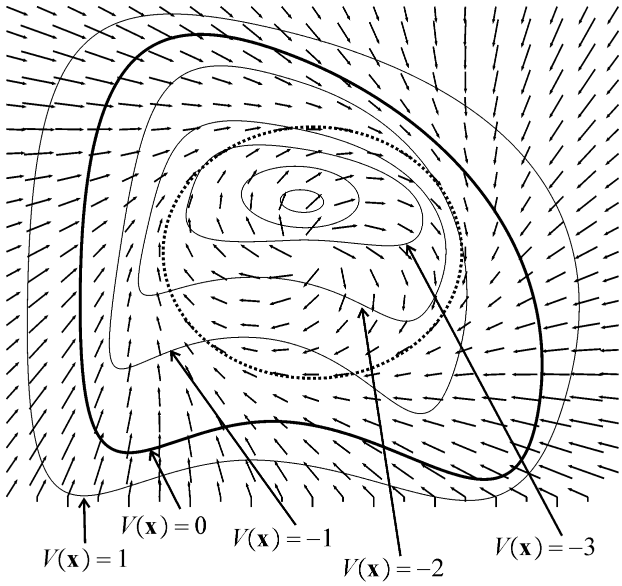
4 V-stability

The controlled robot:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

Definition. Consider a differentiable function $V(\mathbf{x})$. The system is V-stable if $\exists \varepsilon > 0$ such that

$$\left(V(\mathbf{x}) \geq 0 \ \Rightarrow \ \dot{V}(\mathbf{x}) \leq -\varepsilon \right).$$



$V(\mathbf{x}) = 0$
$V(\mathbf{x}) = -1$
$V(\mathbf{x}) = -3$
$V(\mathbf{x}) = 1$
$V(\mathbf{x}) = -2$

Theorem. If the system is V-stable then

(i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
(ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0,\ V(\mathbf{x}(t+\tau)) < 0$.

Now,

$$
\begin{aligned}
& \left(V(\mathbf{x}) \geq 0 \;\Rightarrow\; \dot{V}(\mathbf{x}) < -\varepsilon\right) \\
\Leftrightarrow\;& \left(V(\mathbf{x}) \geq 0 \Rightarrow \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}).\mathbf{f}(\mathbf{x}) + \varepsilon < 0\right) \\
\Leftrightarrow\;& \forall \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}).\mathbf{f}(\mathbf{x}) + \varepsilon < 0 \text{ or } V(\mathbf{x}) < 0 \\
\Leftrightarrow\;& \forall \mathbf{x}, \min\left(\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}).\mathbf{f}(\mathbf{x}) + \varepsilon, V(\mathbf{x})\right) < 0 \\
\Leftrightarrow\;& \forall \mathbf{x}, g_\varepsilon(\mathbf{x}) < 0 \\
\Leftrightarrow\;& g_\varepsilon^{-1}([0,\infty[) = \emptyset.
\end{aligned}
$$

Proving the V-stability is thus a set-inversion problem.

5 Interval analysis

$$
\begin{aligned}
[-1,3]+[2,5] &= [1,8],\\
[-1,3].[2,5] &= [-5,15],\\
[-1,3]/[2,5] &= [-\tfrac{1}{2},\tfrac{3}{2}],
\end{aligned}
$$

If $\mathbf{f}$ is given

Algorithm $\mathbf{f}$(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: $\mathbf{y} = (y_1, y_2)$)

$z := x_1$;
for $k := 0$ to 100
    $z := x_2(z + kx_3)$;
next;
$y_1 := z$;
$y_2 := \sin(zx_1)$;

Its interval extension is

Algorithm $[\mathbf{f}]$(in: $[\mathbf{x}] = ([x_1], [x_2], [x_3])$, out: $[\mathbf{y}] = ([y_1], [y_2])$)

$[z] := [x_1]$;
for $k := 0$ to 100
    $[z] := [x_2] * ([z] + k * [x_3])$;
next;
$[y_1] := [z]$;
$[y_2] := \sin([z] * [x_1])$;

6 Parametric case

The system now depends on $\mathbf{p}$

$$\mathcal{S}(\mathbf{p}): \quad \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}).$$

Define the feasible set for $\mathbf{p}$ by

$$\mathbb{P} = \{\mathbf{p}, \mathcal{S}(\mathbf{p}) \text{ is } V\text{-stable}\}.$$

The system also depends on a perturbation $\mathbf{w} \in \mathbb{W}$

$$\mathcal{S}\left(\mathbf{p}\right): \quad \dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{p}, \mathbf{w}\right).$$

Define

$$\mathbb{P} = \left\{\mathbf{p}, \forall \mathbf{w} \in \mathbb{W}, \mathcal{S}\left(\mathbf{p}\right) \text{ is } V\text{-stable}\right\}.$$

7 Test-case

Assumption. The heading controller generates an actual heading of $\theta + w$ with $w \in \mathbb{W}$.

The system is

$$\dot{\mathbf{x}} = \begin{pmatrix} \cos\left(\bar{\theta} + w\right) \\ \sin\left(\bar{\theta} + w\right) \end{pmatrix}, \text{ with } \bar{\theta} = \bar{\theta}\left(\mathbf{x}, \psi, \gamma_{\infty}, r, \zeta\right).$$

Property 1. If $|e(\mathbf{x})| < r_{\max}$ then, it will be the case for ever.

Property 2. If $|e(\mathbf{x})| > r_{\max}$ then $|e(\mathbf{x})|$ will decrease until $|e(\mathbf{x})| < r_{\max}$.

Property 3. The course should be feasible, i.e., $\cos\left(\psi - \bar{\theta}\right) + \cos\zeta \geq 0$.

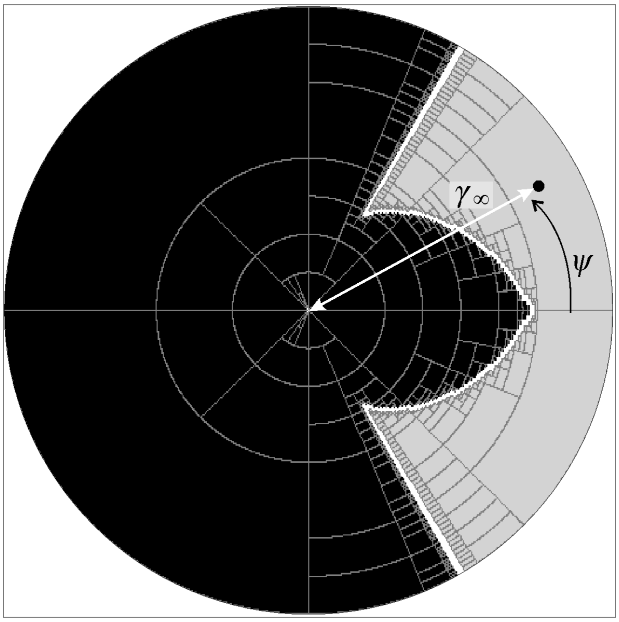
Property 4. The robot always moves toward the right direction, i.e. $\dot{x}_1 > 0$.

The parameter vector is $\mathbf{p} = (\gamma_\infty, \psi)$.

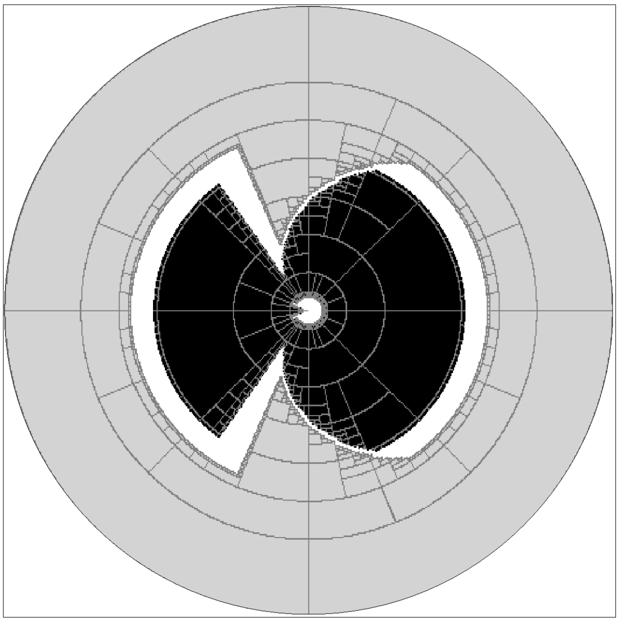
Case 1. To take into account Properties 1,2 and 3. Take $V(\mathbf{x}) = x_2^2 - r_{\text{max}}^2$. We have

$$\begin{aligned}\dot{V}(\mathbf{x}) &= \frac{\partial V}{\partial \mathbf{x}}.\dot{\mathbf{x}} = \begin{pmatrix} \frac{\partial V}{\partial x_1} & \frac{\partial V}{\partial x_2} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \\ &= 2x_2 \sin\left(\bar{\theta}(\mathbf{x}, \psi, \gamma_\infty, r, \zeta) + w\right).\end{aligned}$$

With $\zeta = \frac{\pi}{3}$, $\mathbb{W} = \{0\}$, we get.



Case 2. Assume that we want also that $\dot{x}_1 > 0$. Moreover, $\mathbb{W} = \pm 5^{\circ \circ} = \pm 0.085$rad.



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