# Localization and Imprecision Determination by Set Inversion

Arnaud Clerentin, Laurent Delahoche, Eric Brassart, Mélanie Delafosse

Université de Picardie Jules-Verne
IUT, département Informatique
Avenue des Facultés Le Bailly – 80025 AMIENS - FRANCE
{arnaud.clerentin,laurent.delahoche}@u-picardie.fr

**Abstract.** This paper describes the use of a set inversion algorithm to solve the localization problem. The method is based on the formalism of interval analysis. From a matching between the sensorial map and the theoretical map, the robot configuration is bracketed between two 3-D subpavings. The inner subpaving is supposed to contain the robot position in a guaranteed way. So the localization imprecision is naturally managed by this method.

## 1 Introduction

Localization is the process by which a mobile robot determines its own position and orientation with respect to a reference system of an environment. This is an essential capability for autonomous agents in several application domains. In order to act in a robust way and to increase the reliability in operation, the decisions should be made considering an uncertainty and an imprecision about the robot localization. Concerning the imprecision, it results from unavoidable imperfections of the sensors and of the environment map. The management of uncertainty and imprecision during the localization process is then a key element for the success of a mobile robotic mission.

Concerning imprecision, many localization methods use statistical state estimation techniques, for example the Extended Kalman Filter [4][5]. If we assume small variations and noise statistical modeling, this method is simple to use. But a major problem concerns the observation equation linearization made with the dead-reckoning prediction: the convergence of the E.K.F. estimation is assured only if the odometric error is not important.

An attractive alternative to these methods is set-membership estimation [6]. This formalism allows a natural representation of sensors imprecision by way of intervals.

This paper presents a localization method based on the interval analysis. So this method manages naturally imprecision.

This paper is organized as follows. In a first part, we will present our uncertainty management method based on the use of the Transferable Belief Model and its link with the localization imprecision. Then we will deal with our robot configuration determination method based on interval analysis and set inversion. The paper will end with the presentation of the experimental results.

## 2 Localization Uncertainty and Imprecision

### 2.1 Localization Uncertainty Estimation

The first part of our work has concerned the uncertainty management [9][10]. The originality of our study is its ability to propagate uncertainties from low level data in order to obtain a global uncertainty about the robot configuration. The key tool used is the Transferable Belief Model (TBM) of Smets [11], which is a variant of the belief functions theory [8]. This formalism enables to treat uncertainty easily since it permits to attribute mass not only on single hypothesis, but also on union of hypothesis. We can thus express ignorance.

This uncertainty propagation architecture is divided into four steps which are directly issued of the classical perception/navigation paradigm commonly used in mobile robotics. In the first step, we compute an uncertainty about the segments that compose the sensorial model. This sensorial model of the environment is built from a multi-sensor cooperation approach between an omnidirectional vision system and a panoramic range finder [9]. The segment uncertainty computation is done by considering a binary frame of discernment [9] and by taking into account several criteria.

The next step is to classify these segments in order to get high level primitives such as “corner”, “edge”, etc. The segments uncertainty is propagated to deduce the uncertainty of these primitives [10]. These significant landmarks are then used in our localization method based on multi-target tracking. This module uses the TBM in a framework called *extended open world* [7] and enables us to manage an uncertainty for each target.

The last step concerns the localization uncertainty computation. This uncertainty takes notably into account the targets uncertainties [10].

### 2.2 Study of the Correlation between the Uncertainty and the Imprecision

In order to try to establish a correlation between localization uncertainty and localization imprecision, we have first computed in a basic way the robot's configuration. This is done by considering the matchings we have performed in the multi-target tracking module between the sensorial primitives and the theoretical ones (the robot has in its possession a theoretical map of the environment). To this aim, we basically determine the translation and the rotation between the two maps. This enables us to get a configuration error between the "true" configuration and the computed configuration.

On 80 experimental results performed in an indoor environment, we have tried to determine if the error (i.e. the imprecision) is linked with the localization uncertainty computed in the previous paragraph. To this purpose, we have computed the correlation coefficient between the uncertainty and the localization error (Cartesian error, error in x, in y and in orientation). If the correlation coefficient is close to 1 or –1, this means that the two variables are correlated. If it is close to zero, the two variables are not correlated.

Besides, we have analyzed several others criterion which can influence the imprecision. These criterion are :

- The number of primitives used in the localization process, i.e. the primitives which have been matched with a theoretical one in the multi-target tracking module [9].
- The number of high level primitives "corner' and "edge" used in the localization process [9].
- The angular repartition of the primitives used to localize the robot.
- The mean distance between the robot and the primitives. Indeed, our depth sensor becomes less accurate when the distance increases [9].

From 80 experimental acquisitions, we have obtained the correlation coefficients summarized in Table 1.

**Table 1.** Correlation coefficients between the imprecision and several criterion

| | Cartesian error | Error in X | Error in Y | Orientation error |
|---|---|---|---|---|
| Number of. primitives | -0.20 | -0.66 | 0.35 | 0.30 |
| Number of. primitives corner-edge | -0.21 | 0.09 | -0.30 | -0.06 |
| Angular repartition | -0.11 | -0.28 | 0.05 | 0.15 |
| Mean distance | -0.40 | 0.06 | -0.55 | 0.07 |
| Localization uncertainty | -0.15 | -0.55 | 0.30 | 0.11 |

So we can note that the uncertainty and the criterion are not strongly correlated to the error. So, we have decided to use an imprecision quantification formalism which is independent of the uncertainty. This formalism has to be able to determine a localization imprecision from the measurements imprecision. As we will see in the next paragraph, the formalism of interval analysis is adequate.

# 3 Localization by Set Inversion

## 3.1 Introduction

We consider here the localization problem of a mobile robot in a 2D-mapped environment.

The world map consists of four maps: a map of corners, of edges, of other primitives and a map of segments. These segments, which compose the high level primitives described before, are defined in the world reference frame by their endpoints.

The problem is to find the robot configuration $q$ considering the matching realized at the previous step (in the multi-target tracking module) between the sensorial primitives and the theoretical ones, and considering an imprecision on the sensors measurements.

We will firstly deal about the set inversion problem in the general case. Then, we will show that the localization problem is a set inversion problem.

## 3.2 Set Inversion and Interval Analysis

Consider a continuous computable function $f$ from $IR^n$ to $IR^p$. Consider $Y$ a set in the image space $IR^p$. The set inversion problem consists in determining the set $X$ in $IR^n$ so that $X$ is the reciprocal image of $Y$ by $f$ (Fig. 1). This set $X$ is defined by :

$$X = f^{-1}(Y) = \left\{ x \in IR^n \mid f(x) \in Y \right\}$$

The $f^{-1}$ function is the reciprocal image of the function $f$, $Y$ is the set to be inverted and $X$ is the solution set of the set inversion problem.

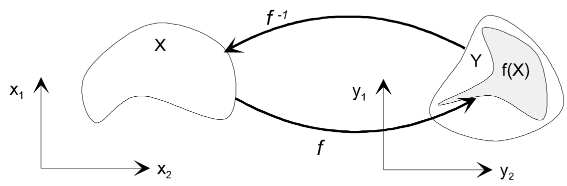


**Fig. 1.** The set inversion problem.

The interval analysis is a way to solve this problem. In this formalism, an imprecise number is represented by an interval which contains it in a guaranteed way. In particular, the SIVIA (Set Inversion Via Interval Analysis) algorithm developed by Jaulin and Walter [1] uses interval analysis to solve the set inversion problem and approximates the solution set by an union of boxes.

### 3.3 Localization is a Set Inversion Problem

**Problem statement**. The robot configuration estimation can be seen as a set inversion problem. Indeed, the localization problem from exteroceptive data is the inverse problem of the sensor simulation.

The sensor simulation problem is the following :

> Knowing the evolution world of the robot, its configuration $q$=($x_r$, $y_r$, $\theta_r$) and a modeling function $f$ of the sensor, compute the set $M$ of the sensor measurements $m_i$, image of $q$ by the function $f$

$$\begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix} \xrightarrow{f} \begin{pmatrix} m_1 \\ m_2 \\ \dots \\ m_n \end{pmatrix}$$

From this statement, the localization problem can be seen as follow:

> Knowing a set $M$ of sensors measurement which are matched with their corresponding primitives of the theoretical map, compute the set $Q$ of the configurations $q$ whose image by the function $f$ belongs to $M$

$$Q = \{ q \mid f(q) \in M \} = f^{-1}(M)$$

This is a set inversion problem:

- The set to inverse is $M$
- The function is $f$
- The solution set is $Q$

**Problem resolution**. Consider the robot configuration q=($x_r$, $y_r$, $\theta_r$) defined by the coordinates of the robot together with its orientation in a world reference frame (*Xe*, *Ye*). The robot sensor detects and matches *n* landmarks *Bi* (*i*=1..*n*) in the robot reference frame ($X_R$,$Y_R$). In order to take into account the sensor inaccuracy, the polar coordinates of each landmark are expressed as intervals:

- [$d_i$] for the distance from the sensor to the landmark.
- [$\phi_i$] for the azimuth angle of the landmark.

In the multi-target tracking module, the detected landmarks have been matched with their corresponding primitives *Bc* of the theoretical map whose coordinates in the world reference frame are ($x_c$, $y_c$) (see Fig. 2 for the example of one landmark).

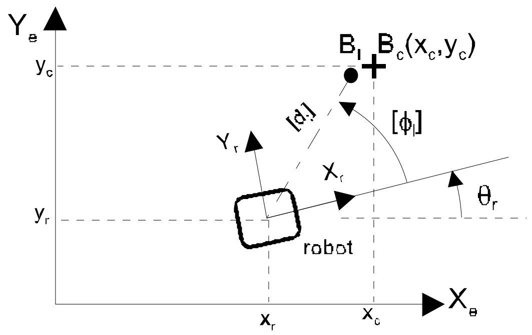


**Fig. 2.** The data of the problem.

The goal is to compute a subpaving which contains the robot configuration. This configuration is represented by a 3D box ([$x_r$], [$y_r$], [$\theta_r$]).

To solve this problem, we will first argue in the ideal case (i.e. perfect sensor) for one landmark. Then we will add the interval formalism, always for one landmark. Finally, we will consider all the landmarks.

*Ideal case, one landmark.* In the world reference frame (*Xe*, *Ye*), the distance $d_c$ between the robot and the landmark *Bc* is:

$$d_c = \sqrt{(x_r - x_c)^2 + (y_r - y_c)^2}$$

Always in the world reference frame, the angle $\phi_c$ between the robot and the landmark *Bc* in the robot reference frame is (Fig. 3):

$$\phi_c = \arctan\left(\frac{y_c - y_r}{x_c - x_r}\right) - \theta_r$$

Since ($x_r$, $y_r$, $\theta_r$) is the robot configuration, we have $d_c$=$d_i$ and $\phi_c$=$\phi_i$. This observation will be the test used by SIVIA to determine if the boxes are feasible or not.

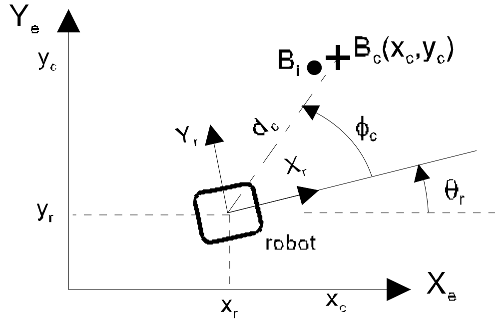


**Fig. 3.** The localization problem in the perfect case.

*Imprecise case, one landmark.* The robot configuration is now represented by a 3D box ($[x_r]$, $[y_r]$, $[\theta_r]$). The distance $d_i$ between the robot and the landmark and its azimuth angle $\phi_i$ are not known with precision. They are expressed in an interval way [$d_i$] and [$\phi_i$].

In the world reference frame (*Xe*, *Ye*), the distance [$d_c$] between the robot and the landmark *Bc* is now an interval:

$$[d_c] = \sqrt{([x_r] - x_c)^2 + ([y_r] - y_c)^2}$$

Always in the world reference frame, the angle [$\phi_c$] between the robot and the landmark *Bc* in the robot reference frame is also an interval :

$$[\phi_c] = \arctan\left(\frac{y_c - [y_r]}{x_c - [x_r]}\right) - [\theta_r]$$

Since the box ([$x_r$], [$y_r$], [$\theta_r$]) contains the robot position, the interval [$d_c$] is included in the interval [$d_i$] and the interval [$\phi_c$] is included in [$\phi_i$]. In other words, the box [$d_c$] [$\phi_c$] is included in the box [$d_i$] [$\phi_i$].

This means that, if any box [$S$]=([$x$], [$y$], [$\theta$] ) contains the robot localization, we must have :

$$[d_c]\,[\phi_c] \subset [d_i]\,[\phi_i]$$

with: $[d_c] = \sqrt{([x] - x_c)^2 + ([y] - y_c)^2}$ (1)

$$[\phi_c] = \arctan\left(\frac{y_c - [y]}{x_c - [x]}\right) - [\theta] \quad (2)$$

The inclusion functions of our problem are the equations (1) and (2).

The algorithm starts with an initial box $[S_0]$ equal to the theoretical map. This enables us to be sure that the solution set is in this initial box. Then this initial box $[S_0]$ and the following boxes $[S]$ are split up according to the equations (1) and (2) by SIVIA. To this aim, SIVIA uses the two tests detailed in paragraph 3.3:

- If $f^{I}([S]) \subset Y$, i.e. if $[d_c]\,[\phi_c] \subset [d_i]\,[\phi_i]$, then $[S] \subset X$: the box $[S]$ is feasible.
- If $f^{I}([S]) \cap Y = \varnothing$, i.e. if $[d_c]\,[\phi_c] \cap [d_i]\,[\phi_i] = \varnothing$, then $[S] \cap X = \varnothing$ : the box $[S]$ is unfeasible.

In the other cases, the box $[S]$ is ambiguous.

*Imprecise case, several landmarks*. If $n$ landmarks have been detected and matched, the tests given by the equation (1) and (2) are performed for each landmark, i.e. $n$ times. The box [S] under analysis is then feasible if all the $n$ tests conclude that it is feasible.

The drawback of this strategy is that it doesn't allow any outliers. Our method to manage outliers is the following: if, after the algorithm, no feasible box is found, we restart it with one outlier allowed. This means that, if the box under analysis is unfeasible for one landmark, this box is not declared as unfeasible. But if a second landmark gives a conclusion "unfeasible", the landmark is now declared unfeasible. If no feasible box is found, the method is restarted with two outliers allowed, etc… If no feasible box is found considering $n/2$ outliers, we consider that no configuration can be found.

## 4 Experimental Results

We have tested our algorithm on several acquisitions made in an indoor environment (two trajectories in the end of a corridor).

The sensor imprecision on orientation is fixed at one degree. The imprecision in distance is proportional to the landmark distance. Indeed, our depth sensor is less precise when the distance increases [9].

The initial box $[S_0]$ is fixed to the size of the theoretical map, i.e. [-500 cm, 800 cm][-100 cm, 800 cm][0 degree, 360 degrees]. Fig. 4 shows several localization results. The gray boxes are the feasible ones, the yellow boxes are the ambiguous ones. The graduations on the $x$ axis and $y$ axis represent one meter.

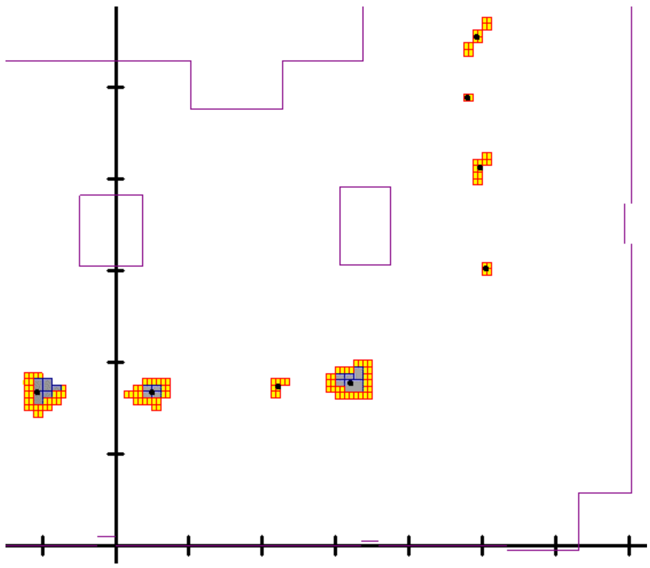

**Fig. 4.** Some experimental results**.**

On the major part of the cases, the subpavings are found considering no outliers. Only few cases admit one or two outliers. In all the cases, a subpaving is found (however, in certain cases, we have only an outer subpaving). The subpavings are coherent with reality. Finally, the error (distance between the true position and the center of gravity of the subapving) is acceptable: 15 cm and 6.2 degrees in orientation.

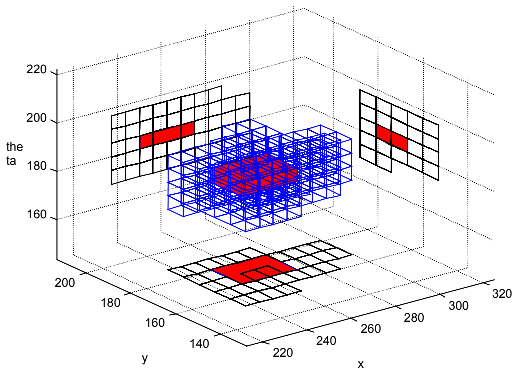


**Fig. 5**. An example of 3D subpavings

We show on Fig. 5 a 3D-view of one localization. The red boxes are the feasible ones and the white boxes are the ambiguous ones. The x axis and the y axis are graduated in cm and represent the position of the robot. The “theta” axis is graduated in degree and represents the robot orientation.

## 5 Conclusion

We have presented in this article a localization method based on interval analysis. This formalism is adequate to quantify in a natural way imprecision. Indeed, we have noted on experimental results that the uncertainty is not correlated to imprecision. That's why we have decided to treat the imprecision in an independent way. The landmarks coordinates are then represented as intervals. We have shown that the localization problem can be seen as a set inversion problem. So we have used the SIVIA algorithm which enables to solve the set inversion problem by the way of interval formalism. The result is a robot configuration bracketed by two 3-D subpavings.

On two paths made in an indoor environment, we have tested our algorithm and we have remarked that the experimental results are coherent. Besides, the localization error is weak. A consequent advantage of this method is to supply a guaranteed error domain of the robot's configuration.

### References


1. L. Jaulin and E. Walter "Set inversion via interval analysis for nonlinear bounded-error estimation", Automatica, vol 29(4), pp 1053-1064, 1993
2. R.E. Moore "Methods and applications of interval analysis", SIAM, Philadelphia, 1979
3. L. Jaulin, M. Kieffer, O. Didrit, E. Walter "Applied interval analysis", Springer-Verlag, 2001
4. J. Leonard and H. Durrant-Whyte, "Mobile robot localization by tracking geometric beacons" - IEEE Trans. on Robotics and Automation, Vol. 7, n°3, June 1991, pp. 89-97.
5. J.A. Castellanos, J. Monteil, J. Neira, "The smap: a probabilistic framework for simultaneous localization and map building", IEEE Trans. on Rob. & Aut., vol. 15, n. 5, pp. 948-952, 1999.
6. P. Bouron, D. Meizel, P. Bonnifait, "Set-membership non-linear observers with application to vehicle localisation.", ECC'01, European Control Conference, Porto, Portugal, Sept. 2001
7. C. Royère, D. Gruyer, V. Cherfaoui, "Data association with belief theory", 3rd int. conf. on information fusion FUSION 2000, Paris, France, 2000
8. G.A. Shafer, "A mathematical theory of evidence", Princeton : university press, 1976.
9. A. Clerentin, L. Delahoche, E. Brassart "Omnidirectional sensors cooperation for multi-target tracking", IEEE Int . Conf. on Multisensor Fusion and Integration for Intelligent Systems. (MFI 2001), Baden-Baden, August 2001
10 A. Clerentin, L. Delahoche, E. Brassart, C. Cauchois, "An Uncertainty Propagation Architecture for the Localization Problem", Workshop on Performance Metrics for Intelligent Systems PerMIS2002, NIST, Washington, USA, August 2002
11 Smets Ph., "The Combination of Evidence in the Transferable Belief Model", IEEE Trans. PAMI 12 (1990) 447-458