Localization of a robot and guaranteed map building using interval analysis.

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Abstract. This paper deals with an original simultaneous localisation and map building paradigm (SLAM) based on the one hand on the use of an omnidirectional stereoscopic vision system and on the other hand on an interval analysis formalism for the state estimation. The first part of our study is linked to the problem of building the sensorial model. The second part is devoted to exploiting this sensorial model to localise the robot in the sense of interval analysis. The third part introduces the problem of map updating and deals with the matching problem of the stereo sensorial model with an environment map, (integrating all the previous primitive observations). The SLAM algorithm was tested on several large and structured environments and some experimental results will be presented.

1. Introduction

The stage of incremental construction of the robot's environmental map is preponderant for the increase of its autonomy (Guivant *et al.*, 2000). It consists in managing a coherent update of the cartographic primitives' state during the robots movement. This function is directly correlated to that of the localisation : the robustness of the cartographic primitives' state estimation is linked to that of the estimation of the robot's position. In this context it is necessary to take into account the interaction between both the localisation and the modelisation errors. The interval analysis formalism provides us with an answer to this problematic. Furthermore the soundness of the localisation's paradigm and the simultaneous modelisation are tightly linked to the quantity and quality of the sensorial data. The omnidirectional vision sensor's systems are, in this case, well adapted to this constraint, especially to a stereoscopic use.

In background literature, we can distinguish two main groups of methods used to build the evolution field of a robot: the "metric" methods and the "topologic" ones.

The first approach consists of managing the notion of distance and we can find principally two types of mapping paradigm in this context :

- The first ones consist in managing the notion of distance, where the Extended Kalman Filtering (EKF) is used to build a Cartesian representation of the environment (Crowley, 1989).

- The second where the occupational grid notion is used to provide a metric representation. These occupancy grids manage the "occupation", the "non-occupation" or the "potential occupation" of the group of cells representing the environment. (Elfes, 1987) (Boreinstein *et al*, 1991).

The second category of map representation is the topological one. This approach consists of determining and managing the location of significant places in the environment along with an order in which these places were visited by the robot. In the topological mapping step, the robot can generally observe whether or not it is at a significant place. The definition of significant places can be linked for example to the notion of "distinctive places" in the Spatial Semantic Hierarchy proposed in (Kuipers *et al*, 1991), and the notion of "meetpoints" in the use of Generalized Voronoi Graphs proposed in (Choset *et al*, 1995). This kind of method is interesting to use in complement with an occupancy grid, in order to take into account the semantic aspect.

In this paper we will present an alternative method to the two main ones mentioned above. Owing to the interval analysis formalism, the presented method guarantees the environment's representation. This way, the estimation of both the robot's state and the landmarks is characterised by subpaving.

2. Sensorial Model Building

We have developed a perception system called SYCLOP, which is similar to the COPIS system used by Yagi (Yagi *et al*, 1990). Our system is used to achieve both the localisation and the modelisation of the environment, based on the co-operation between two sensors. The SYCLOP prototype measures 60 cm in height and is composed of a conical mirror and a CCD camera. This vision system allows us to detect vertical parts in the environment with a 2D projection onto the camera's image plane (Delahoche *et al 1997*).

2.1. The omnidirectional and stereoscopic perception system

The idea behind this co-operation is that two image acquisitions are taken at two different positions separated by a known distance *d*. The translation between the two positions is achieved by two horizontal rails. These rails allow us to guarantee a known rigid in-line movement between these two previous positions (Figure 1).

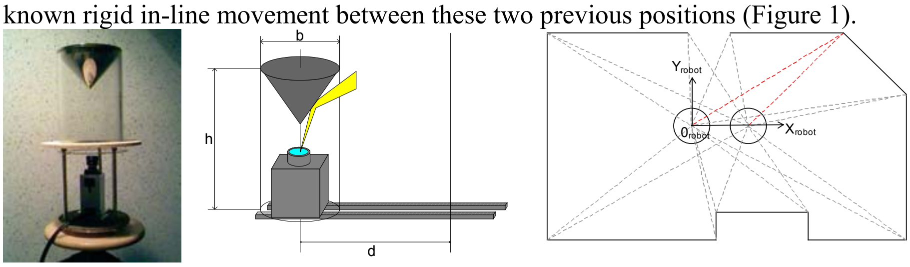

Fig. 1. Principal of the omnidirectional and stereoscopic sensor.

In each acquisition, a vertical landmark of the world (doors, corners, edges, …) is characterised onto the image plane by a strongly contrasting radial straight line.

If the same radial straight line is matched in both conical images, it is quite simple to compute the location of the intersection point in the robot's reference frame. This point corresponds to a vertical landmark. This can be extended to all pairs of matched radial straight lines (Figure 1).

It is necessary to specify that the calibration of the vision system has been done before applying any sort of image processing.

The reader can find further information about the complete calibration of the SYCLOP sensor in (Cauchois *et al*, 1999).

2.2. The sensorial primitives calculation

Our goal is to match the angular sectors of homogenous grey levels in the two images. These sectors are delimited by the radial straight lines mentioned above.

All the radial straight lines in a conical image converge to a single point called $\boldsymbol{O}$ (the projection of the revolution axis of the cone onto the image plane). This means that only the angular reference determines a radial line in the image. Thus a 2D image processing can easily be reduced to a 1D computation.

We therefore consider a concentric circle of a grey level on the image, centred on the previous point $\boldsymbol{O}$. In order to obtain a maximal density of 1D signal information, this circle is designed on the periphery of the conical image. A 1D grey level signal is computed to characterise each image.

We have applied a segmentation algorithm based on a derivative filtering of the 1D grey level signal in order to proceed to the matching step. The reader can find more details on this method in (Drocourt *et al*, 1999). In our case, the matching phase consists in matching two by two all the detected grey level sectors of the two stereoscopic images. As the robustness of the matching is primordial, we will use several different complementary criteria. The criteria will be merged according to the Dempster-Shafer combination rules.

As the viewpoint is different for the two images (shifted by the distance $\boldsymbol{d}$), the landmarks in both images cannot be observed in the same way. We have retained four significant and robust criteria :

- The inclination of the approximate lines corresponding to the set of sector grey level,
- The average of the grey level of the sector
- The standard deviation of the grey level of the sector.
- The geometric constraints of the sector imposed by the view point ; which can be categorised as a "simplified epipolar geometry"

We use the Dempster Shafer theory to perform the fusion (Dempster, 1967) (Shafer, 1976). The Dempster-Shafer method also enables us to function with partial knowledge. A final example of matching is given in Figure 2, where we can see that a large number of sectors are correctly matched (Drocourt *et al*, 1999).

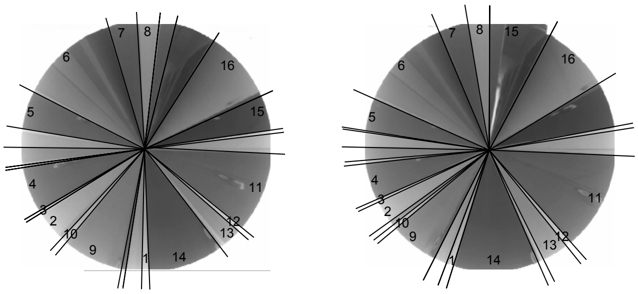

Fig. 2. Segmentation and final matching of sectors for an acquisition

Once the mutual matching of sectors has been achieved, all that we need to do is to calculate the co-ordinates of the segment points that they represent. We know the orientation angle of the two straight lines that border the sides of the angular sectors and the distance d that separates the two cones (the two images). The co-ordinates of all the points in the sensor's reference frame (situated on the centre of cone ***O***) are calculated through triangulation using the following formulas :

$$x = \frac{d \times \tan(\beta)}{\tan(\beta) - \tan(\alpha)} \qquad y = \frac{d \times \tan(\beta) \times \tan(\alpha)}{\tan(\beta) - \tan(\alpha)} \tag{1}$$

3. Localisation of a mobile robot using interval analysis

When the imprecision is not taken into account, the localisation / modelisation process is rendered incomplete, and therefore the influence of the error of the robot's position estimation on the estimation of the vertical landmarks' parameters cannot be processed, whilst this is a main factor. There actually is an obvious interaction between the committed errors with regards to the robot's position and those introduced by the calculation of the position of the landmarks. It is this interaction that – in the process of incremental construction – is at the origin of the cumulative errors. This is the reason why we wish to present an alternative that allows to integrate the imprecision notion as of the stage of localisation and therefore, we decided to use interval analysis method.

3.1. Localisation of a mobile robot using SIVIA

The SIVIA (*Set Inversion Via Interval Analysis*) algorithm was developed by Luc Jaulin and Eric Walter (Jaulin *et al*, 1997). It enables us to determine the solution of the set inversion problem *via* subpaving (rectangular-sub-sets). The subpaving gives an approximate but guaranteed solution.

The algorithm consists in sub-dividing an initial box into two boxes. They are then both examined to determine if they are to be kept or disregarded. If a box is not valid, it is eliminated. If it is valid, it is re-divided into two and so on and so forth until the boxes are of the required precision.

Our sensor works in the same way as a goniometre. In other words, sensorial data represents the observation angles of the environment's vertical landmarks. This means that they can not be linked to other elements on the map (such as horizontal landmarks). This is an advantage as it necessarily decreases the amount of matching combinations.

The localisation of a mobile robot using the theory of interval analysis has, of course, already been achieved, *e.g.* with telemetric sensors (Leveque, 1998) (Kieffer *et al*, 2000). In a parametric sense, it is easy to see that our sensorial data are of the same nature as telemetric data used by M. Kieffer. Thus we have extrapolated the error model of Kieffer to our problem. This error model is characterised by both a distance and angular error.

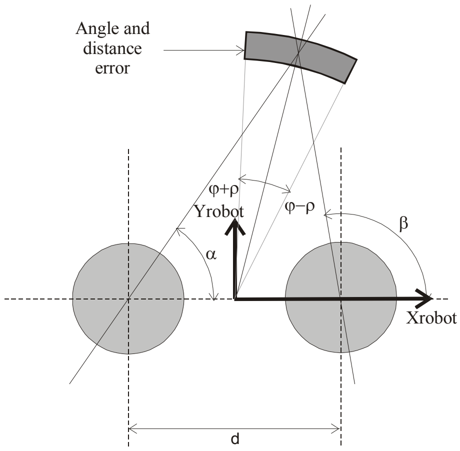

Fig. 3. Error modelisation Approach.

At this level, we assume that our sensor provides the positions of the environment's vertical landmarks contaminated by an angular and a distance error. This forms an emission cone that resembles the one obtained by a telemetric sensor. The apex of this cone lies in the middle of the two images and on the axis that runs through their centre. As we know the angles α and β, we have the co-ordinates of the landmark, which enables us to calculate φ, the landmark's observation angle, and $\boldsymbol{l}$, the distance measured (Figure 3).

If (x_r, y_r, θ_r) represents the robot's position, l the distance measured and φ the measured angle, then the computation of the co-ordinates of a point i on the map is calculated with the following formulas :

$$\begin{cases} x''_{si} = l_i \times \cos(\varphi_i) \\ y''_{si} = l_i \times \sin(\varphi_i) \end{cases} \tag{2}$$

We then apply a rotation in the robot's reference frame that is equal to the orientation θ_r of the robot, followed by a switch from the robot's reference frame to the world's reference frame. Stating $[l_i]=[l_i - \varepsilon, l_i + \varepsilon]$ and $[\varphi_i]=[\varphi_i - \rho, \varphi_i + \rho]$ and using the inclusion functions +, –, ×, ÷, *cos()* and s*in()* relative to the interval analysis, we obtain the following inclusion function:

$$
\begin{aligned}
[S_i] &= \begin{pmatrix} [x_{si}] \\ [y_{si}] \end{pmatrix} \\
&= \begin{pmatrix} \cos([\theta_r]) & -\sin([\theta_r]) \\ \sin([\theta_r]) & \cos([\theta_r]) \end{pmatrix} \times \begin{pmatrix} [l_i]\times\cos([\varphi_i]) \\ [l_i]\times\sin([\varphi_i]) \end{pmatrix} + \begin{pmatrix} [x_r] \\ [y_r] \end{pmatrix} \\
&= f^{1}([l_i],[\varphi_i],[x_r],[y_r],[\theta_r])
\end{aligned} \tag{3}
$$

It is this inclusion function that will be used with the SIVIA algorithm.

First of all, once this box $[S_i]$ that corresponds to a sensorial data is found, we need to test if one of the map's elements is actually in this box. Given the fact that we are trying to estimate the position of the environment's vertical landmarks using a subpaving in order to obtain the imprecision, a landmark j of the environment is not represented by a point P_j, but by a subpaving made out of n boxes, that we note down as $[[P]]_j = \{[T]_r \,/\, 1 \leq r \leq n\}$.

In order to obtain the Boolean inclusion function which will allow us to possess a global validity test, we apply this algorithm to the total of the sensorial data. The inclusion function used by SIVIA during this stage can be explained in the following way: For each localisation's box, we calculate if there is an intersection between the considered observation and the rectangular-set to be tested. As soon as the intersection is non-void, the function returns the undetermined value. During the initialisation of the map, the algorithm is limited, because there is no box representing the robot's localisation. Thus, we immediately have the subpaving that corresponds to the observation.

This situation only represents the case where there are no aberrant data. As a matter of fact, the algorithm successively tests all the boxes associated to the sensorial data and if only one is not valid, neither is the robot's position. Evidently, this situation presents several problems as it is quite common to have several aberrant data per acquisition.

Our solution to the problem is the same as the one adopted by M. Kieffer. It implements the algorithm whilst taking into account that there are no aberrant data. If no solution is found, the algorithm is repeated with one aberrant data, then two, etc.. This solution gives a result no matter the ratio of " aberrant data /valid data".

In this case, the boxes are always divided until the minimal size that is defined by the error has been attained. We solely have the exterior approximation of the robot's position. Nevertheless it is the unique information that we are interested in for ulterior processing-computations that we will implement.

When using SIVIA, the first step is the search for a solution from a box received as an argument that has to contain the real position of the robot. One solution is to initialise this root box using the dimensions of the environment. The problem we are faced with however, is that on the one hand the calculation time is higher and on the other hand, in relatively symmetrical environments, the solution can be multiple and may not even contain the real position of the robot.

Bearing these facts in mind, we decided to use dead-reckoning information to refine the search for solutions. One method to use this information is based on the same principle as Kalman filtering, *i.e.* using successive phases of prediction/correction. This method works but needs specific algorithms that use subpaving and binary trees to compute the predicted state. Furthermore, modified

SIVIA versions need to be used to take these particularities into account.

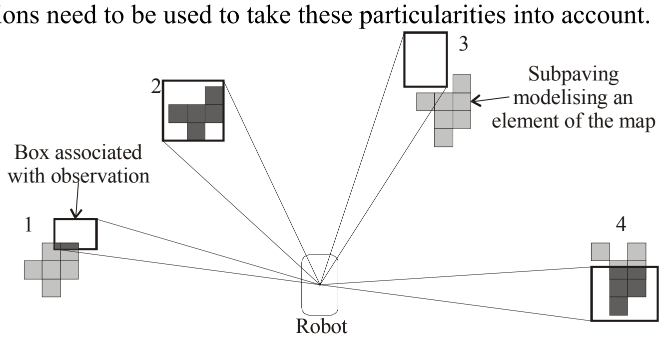

Fig. 4. Intersection test used in the localisation algorithm

Having the most precise prediction phase as possible is very useful when the number of boxes is relatively high, as in the case of use of telemetric sensors. In our case, sensorial data represent the vertical landmarks of the environment and therefore the imprecision will be smaller and the number of boxes will be relatively low (as can be seen from the experimental results).

This is why we decided to only use dead-reckoning in order to initialise the initial box P_0 that is used to start the search for the robot's actual positions. From a rectangular-subpaving that results from a localisation process, we compute the minimal box that draws round the subpaving. This box is then increased with the maximum dead-reckoning error, which is a function of the distance covered.

This method is purely an initialisation phase and as we raise the dead-reckoning error, this implies that the possible results, which are incompatible with the actual position of the robot do not need to be tested during the localisation process.

3.2. Modelisation of the environment

The representation of the data on the map is at the base of the SLAM paradigm. In our case, we need to focus on landmark's representation that is first of all compatible with the set interval analysis formalism and furthermore easy to use in an update phase. At this stage, the only solution that seems possible is a representation in subpaving.

The result of the localisation stage being a subpaving $[[L]]$, we can compute for each box $[L]_g$ (element of $[[L]]$) and for each sensorial data $[\varphi]_i$ and $[l]_i$, the box resulting in $f^{I}([L]_g, [\varphi]_i, [l]_i)$ thanks to the inclusion function that was already used in the localisation algorithm.

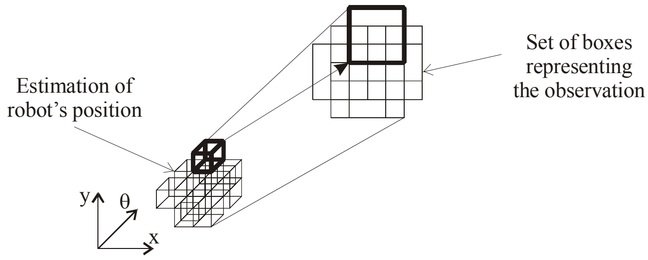

Fig. 5. Representation of all the rectangular-sets characterising a sensorial data

If we apply this inclusion function to the total of boxes rendered by the localisation stage, at the end of this process we obtain a set of boxes that correspond to each observation that can have a non-void mutual intersection and, therefore, do not constitute a subpaving (Figure 5).

This problematic has already been broached by M.Kieffer. As a matter of fact, he developed the *ImageSP* algorithm, which auto-decomposes into three phases, just to be able to calculate the image of a subpaving :

- **Hashing** : Calculates a regular subpaving $[[A]]$ of which all the boxes have a size that is inferior to ε,
- **Evaluation** : Calculates the image of each of these boxes using the considered inclusion function f^I,
- **Regularisation** : Approximation of the union of these boxes $f^I([[A]])$ using a new subpaving $[[B]]$.

The first phase (*Hashing*) is unnecessary, given the fact that the subpaving $[[L]]$ obtained during the localisation process is already made up solely of boxes that are smaller than the expected precision.

Thanks to the former inclusion function, we can directly compute the resulting box for each of these boxes and for all the sensorial data in the *evaluation* phase..

Finally, *the Regularisation* consist in using the new algorithm SIVIA to obtain the desired subpaving. The representation that we chose to use is a set of boxes of identical size, equal to the fixed minimal precision that characterises the two preceding sets. The advantage of this representation is that no bisection will be necessary when we need to process such a set. The boxes will be either accepted or rejected, as they are all of an inferior size to the expected precision. Using this method simplifies the representation of data in the map, but also the calculations that will be applied in the following phases. Another method would be to use the exterior and interior approximation (Figure 6).

We now need to determine the inclusion function that will be used by the SIVIA algorithm during the addition of a new landmark in the map. As we want to obtain the set of boxes of an inferior size to the expected precision, this function should never render anything but two values: "true" or "undetermined". As a matter of fact, a "true" value rendered by this inclusion test would immediately stop the pending bisection of the box.

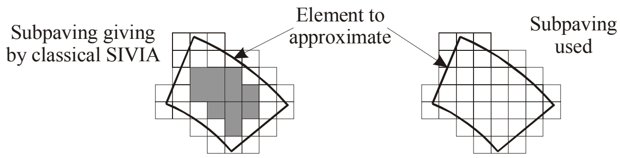

Fig. 6. Approximation of a set using the two former methods

This inclusion function plays a double role because it will be used to initialise the environmental map using the data issued from the first acquisition but also each time a new landmark is added to the map.

These two possibilities force us to differentiate between the two applications of this inclusion function. As a matter of fact, the robot's position is not a subpaving but a position during the initialisation phase, as it represents the origin of the map. However, when a new landmark is added the robot's position is defined by a set of boxes issued by the localisation phase. We will explain in detail our inclusion function in this second, more complicated case.

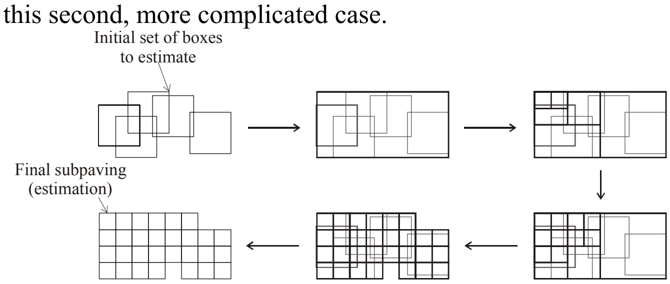

Fig. 7. Approximation of a set of rectangular-sets using SIVIA

At this stage, we have to remind the reader that, of course, the direct observation image from a subpaving issued from the localisation phase provides a set of boxes, but not necessarily disjointed. This means that we need to estimate it, using a more practical and representative subpaving. Its only intersections' zones are the boxes' borders. In order to compute this set, we will again use the SIVIA algorithm: starting with an initial box, this will provide us the required subpaving. This will allow us to estimate each new landmark to be inserted in the map. Therefore, before running SIVIA, we need to compute an initial box. This can easily be done when calculating the minima and maxima from an observation for each box (Figure 7).

3.3. Decision method for matching

3.3.1 Determination of the belief put in each association

We now need to determine which information will have to be merged and which will have to be added to the map as new primitives. The decision method used here consists in determining a belief for each association, using the Dempster-Shafer theory (Dempster, 1967) (Shafer, 1976). This part of the process is crucial and decisive in the localisation paradigm and the simultaneous modelisation. In fact, it is this stage that will condition the maintenance of the environmental map's coherence. A wrong choice between a new insertion or fusion will generally be at the root of an excess of primitives in the map, which will lead to cumulative errors and, hence, a divergence in the algorithm.

At the start of this phase, we have three imprecision's data at our disposal that will be uses:

- An environmental map made up of subpaving each representing the imprecision associated to the modelled landmark.
- A set of sensorial data characterised by information of the distance/angle type in the form of intervals, providing the imprecision in the measure,
- A subpaving resulting from the localisation stage, representing the imprecision associated to the robot's position.

We therefore have to resolve two principal problems:

- Define and use the set resulting from the association of the localisation and the measure imprecision;
- Find a comparison criteria that can be implemented to determine the belief attributed to the fusion of this set with a map's subpaving.

These two problems are tightly linked and in order to know if an observation can indeed be associated to a mapped primitive, we need to find a comparison criteria between the two: the intersection of the two subpavings. In fact, the more the set associated to an observation contains the subpaving that represents a point on the map, the more certain we are that it represents the same information, which implies that they have to be merged.

Given the fact that the set of boxes of an observation can overlap, several of them can have a non-void intersection with one of box representing a point on the map. This is why we cannot directly use the intersection notion between these different boxes to calculate the volume. In fact, if we were to consider the three sets A, B and C so that $A \cap B \cap C \neq \varnothing$, we would obtain the following inequation:

$$\mathrm{Volume}(A \cap C) + \mathrm{Volume}(B \cap C) > \mathrm{Volume}(A \cap B \cap C) \quad (4)$$

This signifies that the volume that corresponds with the intersection of the sets A and B with C is counted twice in the left part of the inequation. The chosen solution is then the same as when adding a new observation to the map. In other words, we calculate the image of the subpaving issued from a localisation and then SIVIA is applied to obtain a subpaving associated to the observation that we note as $[[S]]_i = \{ [K]_q \,/\, 1 \leq q \leq m \}$ with $1 \leq i \leq s$ and m representing the amount of boxes constituting the subpaving.

Our comparison criterion is therefore based on the value:

$$\tau = \left(Volume\left([[P]]_j\right) - Volume\left([[P]]_j \cap [[S]]_i\right)\right) \times 100 \,, \quad (5)$$

that represents the percentage of $[[P]]_j$ included in $[[S]]_i$.

The formalism used to determine the certainty associated to a fusion is based on the search of the maximum of belief compared to the application of the Dempster-Shafer rules. We therefore have to determine our discernment-frame constituted out of two elements: $\Theta = \{YES, NO\}$

- "YES" the observation *i* needs to be merged with the element j on the map
- "NO" the observation *i* should not be merged with the element j on the map

If the subpaving issued from an observation contains more than 50 % of the boxes that define a landmark, we consider that the belief must be the highest. Thus, we use the Basic Probability Assignment (B.P.A.) as represented in figure 8.

All we now need to do is to compute the intersection volume that exists between each subpaving $[[S]]_i$ issued from an observation and each subpaving $[[P]]_j$ that represents a landmark on the map.

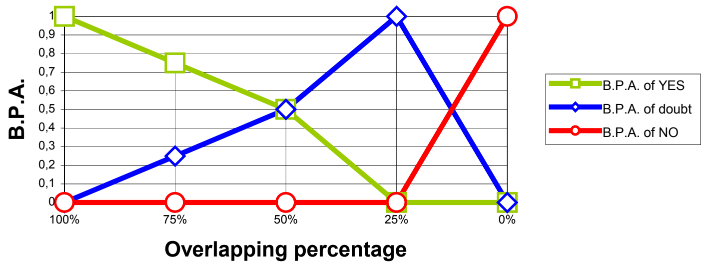

Fig. 8. Matching functions for the fusion stage

For an observation S_i, we now have p triplets:

$$\begin{array}{lll} m_{i,1}(P_1) & m_{i,1}(\overline{P_1}) & m_{i,1}(\Theta_1) \\ m_{i,2}(P_2) & m_{i,2}(\overline{P_2}) & m_{i,2}(\Theta_2) \\ \ldots & \ldots & \ldots \\ m_{i,p}(P_p) & m_{i,p}(\overline{P_p}) & m_{i,p}(\Theta_p) \end{array}$$

We can now compute these p triplets for the s observations, which will give us $s \times p$ triplets. The problematic introduced at this level resides in the fusion of all the information, in order to be able to choose. We resolved this problem by using the generalisation of the combination operator of Dempster-Shafer introduced by D. Gruyer and V. Cherfaoui (Gruyer *et al*, 1999).

3.3.2 Decisional Algorithm

The decisional algorithm that we use is based on the maximum of the probability obtained in the Dempster-Shafer sense. The precedent phase allowed us to calculate for each observation, p triplets that correspond to the match with each element on the map. We can now apply the generalised Dempster-Shafer operator in order to obtain a matrix of belief with the dimensions $s \times (p+2)$. The hypothesis "*" signifies that the observation S_i does not correspond with any element on the map. This means we work in a extended open world.

The result of our matrix of belief provides a belief onto the singletons hypothesis, *i.e.* a rule of decision based on the maximum pignistic probability will not add anything here because this last one use a group of elements. Furthermore, the values of this matrix are directly credibilist measures. This is why we have based our decisional criterion on the maximum credibility of this matrix.

The algorithm used is based on the search for the maximum value in the matrix previously built. The value that is found this way allows us to determine if the observed point is in relation with an existing point or if a new point has been created. In case of doubt (maximum credibility on "Θ"), we choose to create a new point defined by a subpaving.

Once this match has been carried out, all the elements of the line that contain the maximal value are put on 0, as well as those of the colon but only if this last one is different from "*"and from "Θ". In fact, the initialisation of all the elements of the line to 0 signifies that an observed element cannot be in relation to one single element on the map. The same applies for the colon that corresponds to the fact that several observations cannot be matched to the same point on the map. On the other hand, several observations can be new points ("*") just like the ignorance can be maximal in several ("Θ") observations. The algorithm is reiterated as long as there are positive values.

Finally, this algorithm gives us two sets. The first is made up of observations that need to be merged with an element on the map. The second is made up of new landmarks that need to be added. The processing and management of these two sets will be presented in the next part.

3.4. Incremental update of the environment's map

The former decisional integration/fusion stage, has provided us with two sets of points: the first contains those that need to be merged and the second those that need to be added to the map. The integration of a new element on the environment's map has already been given previously.

The last stage that needs to be processed is the fusion between an element from the map and an observation. Here, the data are defined by sets and as we find ourselves in a context of bounded error, the actual position of the landmark has to belong to the two sets. The result of the fusion of an observation with an element of a map is therefore the intersection of the two sets.

At this level we need to resolve a problem. In fact, each set is defined by several boxes. The one that represents the observation even contains boxes that can overlap. The calculation of the intersection is brought back to processing the problem of multiple intersections of disjointed boxes. It is far from a trivial problem.

In order to overcome this difficulty, we part from the following fact: as the solution belongs to both sets, one of the two can first be considered. Then, we can check if each box from the first set, is an element of the second set. If this is the case the box is kept, otherwise it is eliminated. The set of boxes most adapted to be the first set is then the one that represents the landmark on the map, as it is uniquely made of separate boxes, *i.e.* a subpaving (Figure 9).

We can observe at this point that the result of our fusion method can only contain a reduction of subpaving representing the imprecision of a landmark on the map. No matter the set associated with the observation, after fusion there can only be an addition of information in the sense that the subpaving of the landmark cannot increase.

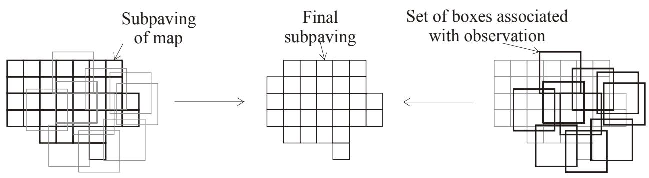

Fig. 9. Example of fusion between observation and element on the map

In order to validate our approach, we present the experimental results in the next part. These results were obtained in two distinctive environments.

3.5. Experimental results

We have tested our SLAM method in two types of structured environments.

The first series of 8 acquisitions has enabled us to validate our paradigm of localisation and simultaneous modelling in a small environment.

The second series of measures contains 45 acquisitions realised during a trajectory consisting of a return trip. The distance covered is approximately fourteen meters. Here, we use dead-reckoning to reduce the size of the box that looks for the possible positions of the robot. We remind the reader that the dead-reckoning error is maximised in order to serve uniquely in the initialisation of the SIVIA algorithm.

Certain elements of the map can be eliminated as we go along updating the localising. In fact, the filtering that we have developed and that we use here, allows us to keep the elements in the map that have already been observed several times.

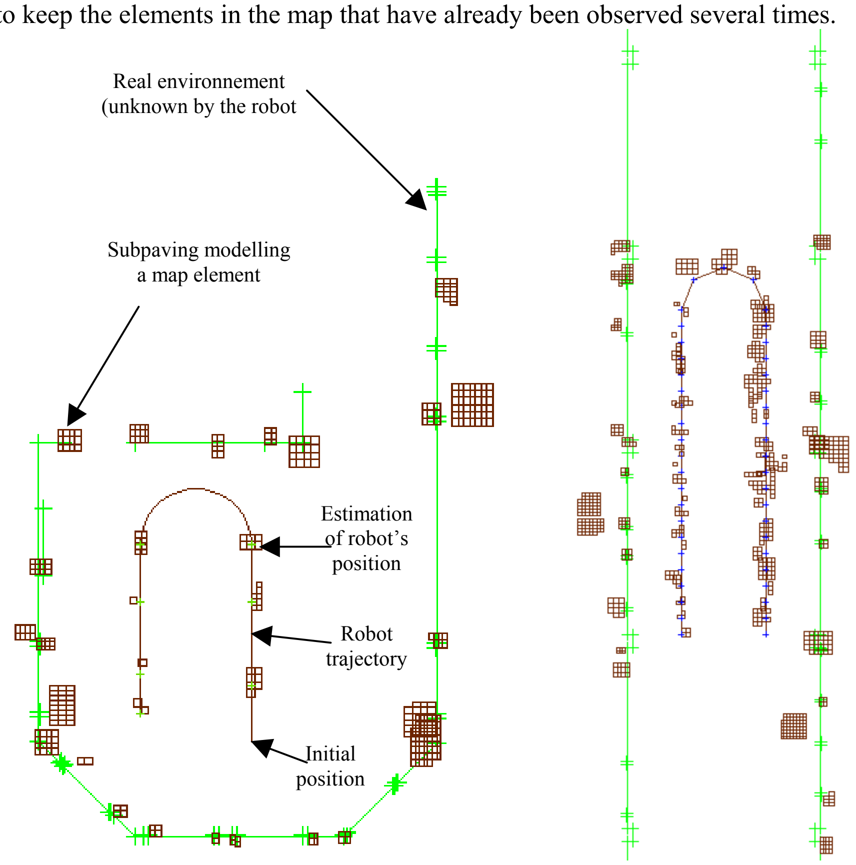

Fig. 10. Results of the environment's modelling

First, from a general viewpoint, the simultaneous process of localisation and modelling provides coherent results in terms of precision and in terms of robustness. Furthermore, we can see that the SLAM process does not diverge. In fact, the 45 acquisitions result in a coherent construction of the environmental map with no preliminary knowledge.

From a localisation viewpoint, we can affirm that the absence of the preliminary knowledge had not effected the estimation phase of the robot's configuration using interval analysis. The coherence of the localisation phase is also proved by the variation during the movement of the robot. We see that the subpaving decreases on the way back (in other words after the U-turn) than on the way there. From a modelling viewpoint, and still linked to the observations of a general order, we can affirm that the map generated is coherent in comparison with the actual terrain. The amount of cartographic integrated primitives is coherent, proving the validity of the fusion and integration process.

The evolution of the subpavings during the incremental modelling process is robust and coherent. Two points can justify this statement: First of all, the contribution of sensorial data is accompanied by a reduction of the size of the error domain and by a convergence of subpaving to the actual position of landmarks. Secondly, the interaction between the localisation error and the modelling error is taken into account because the higher the localization precision, the more the subpavings on the cartographic primitives are significantly reduced. This decisive factor allows the process of simultaneous localisation and modelling not to diverge after a certain amount of acquisitions. This test on large environments is important as it is put forth by several works, such as those of Dieter Fox (Fox *et al.*, 1999).

Rather than an alternative, the interval analysis approach is proposed as a solution that allows us to integrate intrinsically the imprecision notion. The fact that we can manage the imprecision implies the possibility to take the interactions into account, which is not possible with other formalisms. It is this rigorous management of these interactions that leads to a successful outcome of the map process generation on long distances.

4. Conclusion

In this work, we have developed a method of localisation and simultaneous modelling (SLAM) of the environment based on the use of the interval analysis. This method is different from classical algorithms found in literature and that are generally probabilistic. The novelty of the proposed formalism resides in the fact that the obtained imprecision domains linked to the state's estimation are equiprobable and guaranteed.

We have given preference to the use of the Dempster-Shafer rules, that allow us to manage a belief in different cases that can appear in a map-generation process from each observation (fusion, insertion or rejection).

The strategy to integrate primitives carried over is the reduction of the subpaving matched to the examined and mapped primitive. This technique processes rapidly but first needs all the elements to be inserted as subpavings reduced to the minimum. In other words, the size of each box has to be inferior to the expected precision.

We have seen that the method developed provides excellent results. First of all the paradigm, validated on a trajectory in a long corridor, gives a high precision on the localisation and the estimation of the landmarks' position. Secondly, no localisation drift has been observed.

Here we have a system that can simultaneously localise the robot from a non-reliable map and at the same time incrementally model the robot's evolution in the environment in a relatively precise way. These two stages being intimately linked, the quality of the one depends on the precision of the other. The use of the interval analysis has allowed us to propagate the imprecision introduced during each stage of our method on the next phases.

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