

# Set membership approach for underwater exploration with swarms

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## Cible Océanique en Meute Tactique

Organismes financeurs



Partenaires



# Outline

- 1 Underwater exploration with swarms
- 2 Localization and explored area characterization
  - Localization
  - Explored area
  - Set-membership approach
- 3 Experimental results
  - AUV swarm simulation
  - Guaranteed explored area computation

# Underwater exploration with swarms.

It is sometimes necessary to explore a given zone, and to ensure that it has been entirely covered (mapping, mine hunting, search, ...)

AUV swarms:

- robustness through redundancy and diversity
- scalability
- cooperation

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## In this talk

- Localize the swarm of robots
- Characterize the explored area w.r.t localization uncertainty

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# Localization.

- Surface : GPS
- Underwater :  
No installed infrastructure for localization.
  - Dead reckoning
  - Acoustical localization system

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t)) \end{cases}$$

- dead reckoning
- GPS, acoustic ranges

Use boxes (interval vectors)  $[\mathbf{x}]$  and  $[\mathbf{y}]$  to represent positions and measurements, and their uncertainties.

# Inter-AUV communication for localization

- Time of arrival measurements  $\rightarrow$  ranges
- Emitter's position box is transmitted

Each communication is a constraint between two AUV positions.

- Communication enables contraction of position boxes
- Repeated contractions enhance swarm localization



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# What area has been explored by the swarm?

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) \\ \mathbb{V}(t) &= \{\mathbf{z} \in \mathbb{R}^2 : v(\mathbf{z}, \mathbf{x}(t)) \leq 0\} \\ \mathbb{M} &= \bigcup_{t \in [t]} \mathbb{V}(t) \end{cases}$$

- evolution
- observation
- visibility
- explored map

This formalism can represent the single AUV case as well as the full swarm:

- $\mathbf{x}$  is the state of the swam
- observations are GPS positions and inter-AUV range measurements.

# Visible area and explored area

The visible area at time  $t$  is represented by the set-valued function  $\mathbb{V}(t)$ :

$$\mathbb{V}(t) = \{z \in \mathbb{R}^2 : v(z, x(t)) \leq 0\}$$

where  $v(z, x(t))$  is the visibility function.

Let  $\mathbb{M}$  be the map of the explored area.  $\mathbb{M}$  is the union of the visible areas over the whole trajectory:

$$\mathbb{M} = \bigcup_{t \in [t]} \mathbb{V}(t)$$

# Bracketing of the visible area: guaranteed and possible

Guaranteed visible area  $\mathbb{V}^{\forall}$ : set of points that have necessarily been observed, regardless of the state uncertainty

$$\mathbb{V}^{\forall}(t) = \{ \mathbf{z} \in \mathbb{R}^2 : \forall \mathbf{x}(t) \in [\mathbf{x}](t), v(\mathbf{z}, \mathbf{x}(t)) \leq 0 \} \quad (1)$$

Possible visible area  $\mathbb{V}^{\exists}$ : set of points that may have been in the robot's field of view:

$$\mathbb{V}^{\exists}(t) = \{ \mathbf{z} \in \mathbb{R}^2 : \exists \mathbf{x}(t) \in [\mathbf{x}](t), v(\mathbf{z}, \mathbf{x}(t)) \leq 0 \} \quad (2)$$

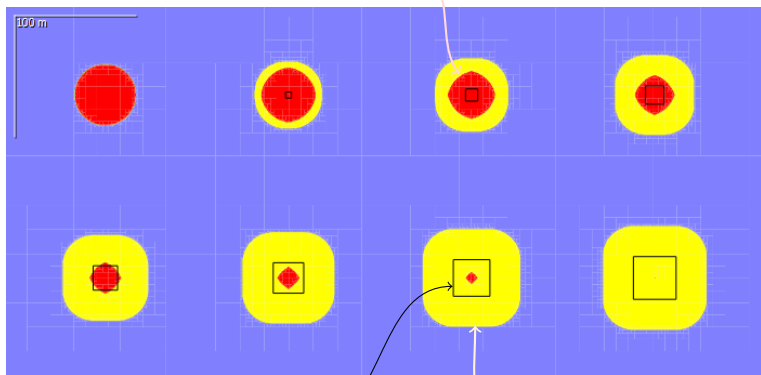
$\mathbb{V}^{\forall}(t)$  and  $\mathbb{V}^{\exists}(t)$  form a bracketing of the actual visible area  $\mathbb{V}(t)$ :

$$\forall t \in [t], \mathbb{V}^{\forall}(t) \subset \mathbb{V}(t) \subset \mathbb{V}^{\exists}(t)$$

Explored area

# Guaranteed visible area depends on position accuracy

Guaranteed visible area  $\mathbb{V}^v$



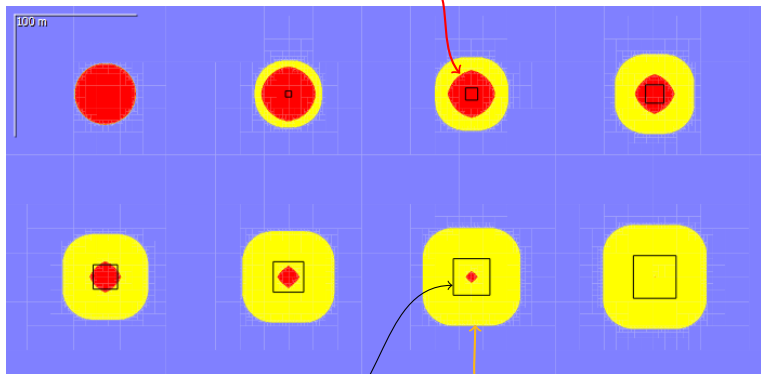
Position uncertainty box  $[x]$

Possible visible area  $\mathbb{V}^v$

Explored area

# Guaranteed visible area depends on position accuracy

Guaranteed visible area  $\mathbb{V}^v$



Position uncertainty box  $[x]$

Possible visible area  $\mathbb{V}^3$

# Guaranteed and possible explored area

Guaranteed explored area  $M^{\forall}$ : union of all the guaranteed visible areas during the mission

$$M^{\forall} = \bigcup_{t \in [t]} V^{\forall}(t), \quad (3)$$

Possible explored area  $M^{\exists}$ : union of all the possible visible areas over time

$$M^{\exists} = \bigcup_{t \in [t]} V^{\exists}(t). \quad (4)$$

A bracketing of the actual explored area  $M$  is given by

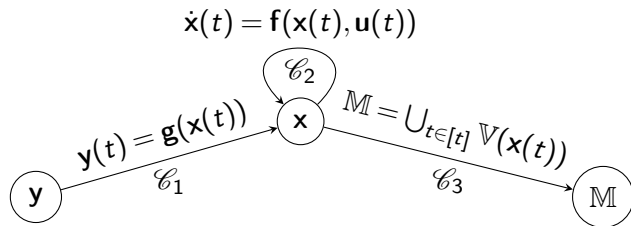
$$M^{\forall} \subset M \subset M^{\exists}.$$

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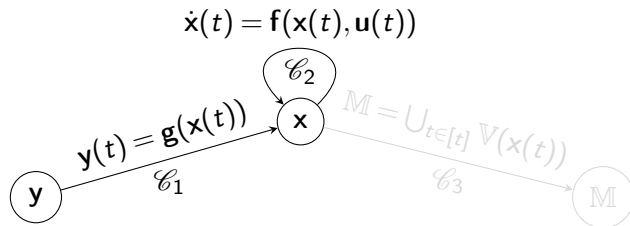
## Constraint satisfaction problem



Variables:  $\mathbf{x}$ ,  $\mathbf{u}$ ,  $\mathbf{y}$ ,  $M$ .

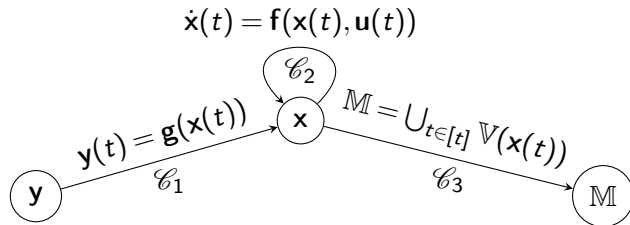
Domains: Tubes (function intervals)  $[\mathbf{x}]$ ,  $[\mathbf{y}]$  and  $[\mathbf{u}]$ . Set interval  $[M]$ .

## Contraction process



- 1 Contract the trajectory tube until a fixed point
  - observations
  - differential equation
- 2 Compute a bracketing of the map (union of SIVIA)

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  - **AUV swarm simulation**
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# AUV swarm simulation.



Play

Simulate a swarm of AUVs equipped with

- GPSs
- Speed and depth sensors
- INS
- Acoustical communication and ranging

Mission: exploration and sonar mapping of an 1.5 km x 2 km area

# AUV swarm: exploration of an area.

Play

# AUV localization uncertainty.



Play

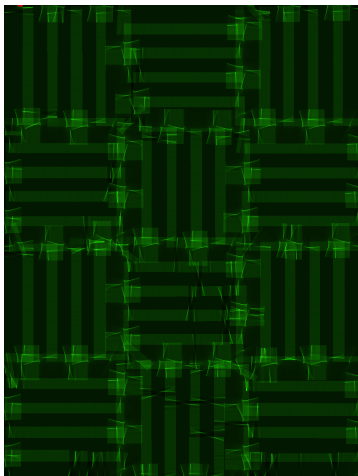
- Boxes represent position uncertainty
- Control done wrt box centers
- Range measurements contract boxes

AUV swarm simulation

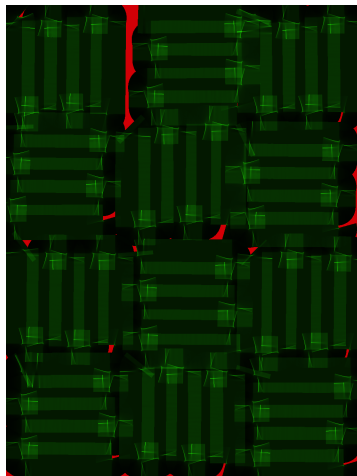
# Communication improves accuracy and coverage.

Red=unexplored. Green intensity=number of scans

- With communication



- Without communication





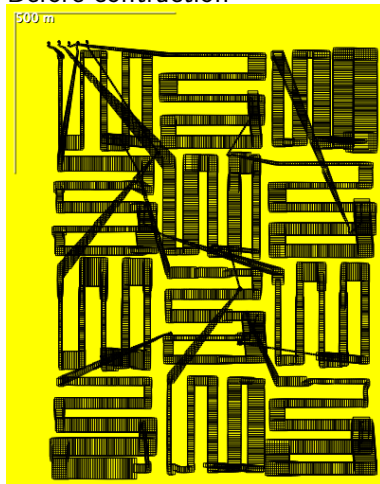
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Guaranteed explored area computation

# Position refining.

## Before contraction

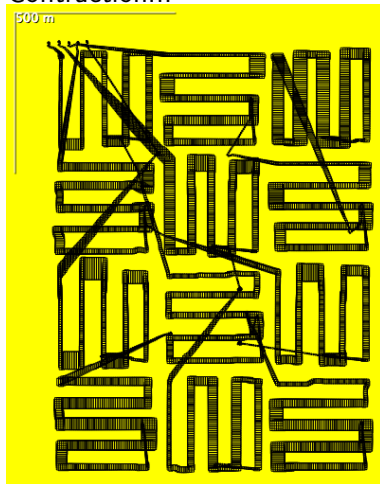


- Constraint propagation with distance measurements
- Forward-backward constraint propagation over trajectory with evolution equation

Guaranteed explored area computation

# Position refining.

## Contraction...

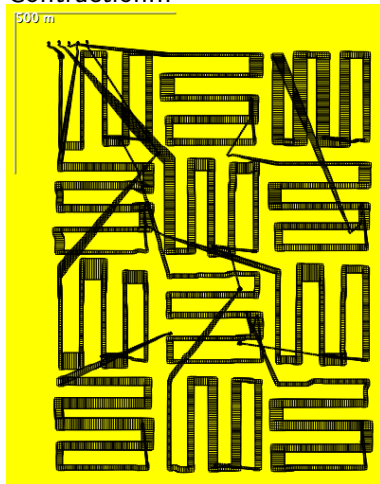


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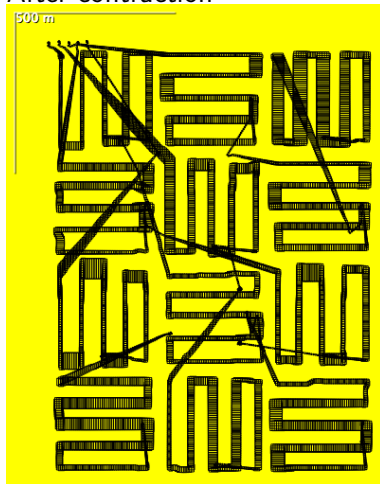


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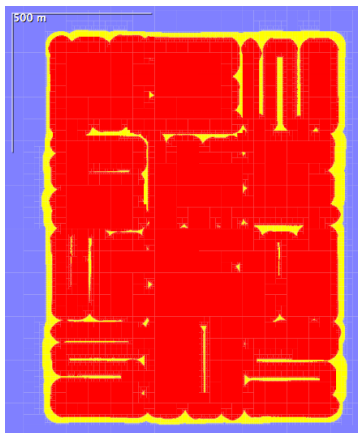
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Guaranteed explored area computation

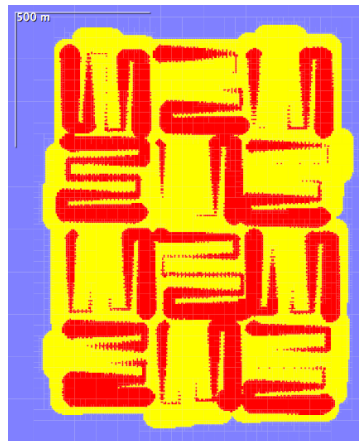
## Explored area computation.

Red=guaranteed ( $M^V$ ), Yellow=possible ( $M^E$ )

## ● With communication



## ● Without communication



# Summary

- Set membership approach for localization of AUVs in a swarm
- Guaranteed bracketing of the explored area
- Inter-AUV communication and ranging improves exploration coverage, and validation
  
- Outlook
  - Tighten the explored area set interval.
  - Deal with erroneous measurements and prevent rumour propagation in the swarm.
  - Swarm of different robots