Guaranteed confidence region characterization for source localization using LSCR

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Source localization

- Using measurements of wave emitted by object
  - Time of arrival
  - Difference of time of arrivals
  - Received signal strength

System model

- n_a anchor nodes, with fixed and known locations θ_i, $i = 1, \ldots, n_a$,
- Agent with unknown location θ_0
  - periodically emits some electromagnetic or acoustic signal, received by the anchors.
  - $y(i,k)$: k-th RSS measurement by anchor node i
- Anchor nodes transmit RSS measurements to some central processing unit.
- Confidence region for estimator of θ_0 has to be derived from $y(i,k)$, $i = 1, \ldots, n_a$, $k = 1, \ldots, n$..

System model

k-th measurement obtained by anchor node i described by the Okumura-Hata model [?]

$$y(i,k) = P_0 - 10\gamma_\mathrm{P} \log_{10} \frac{\|\theta_0 - \theta_i\|}{d_0} + \varepsilon(i,k),$$

where

- P_0 is the signal power measured at some known reference distance d_0,
- γ_P is the path-loss exponent,
- $\varepsilon(i,k)$ is the measurement noise.

One assumes that

- γ_P is the same for all anchors.
- $\varepsilon(i,k)$s realizations of independently, not necessarily identically distributed random variables with a distribution symmetric around zero.

Parameters to estimate

- Consider parameter vector $\mathbf{p} = \left[\theta_0^T, P_0, \gamma_\mathrm{P}\right]^T$ for which a confidence region has to be estimated and from which a confidence region for θ_0 may be deduced.
- *True* value $\mathbf{p}^* = \left[\theta_0^T, P_0, \gamma_\mathrm{P}\right]^T$ of parameter vector, then

$$y(i,k) = y^\mathrm{m}(i,\mathbf{p}^*) + \varepsilon(i,k) \tag{1}$$

with

$$y^\mathrm{m}(i,\mathbf{p}^*) = P_0 - 10\gamma_\mathrm{P}\log_{10}\frac{\|\theta_0 - \theta_i\|}{d_0}. \tag{2}$$

- Search for $\mathbf{p}^*$ in $\mathbb{P}_0$.

Simulation conditions

- Five anchor nodes ($n_a = 5$) are placed in the corners and in the center of a square of $20\,\text{m} \times 20\,\text{m}$.
- $N = 32$ agents are regularly placed in the square
- Each agent broadcasts $n = 10$ times a message containing its identifier.
- $P_0 = 30$ dBm at $d_0 = 1$ m is the same for all agents.
- $\gamma_{\text{P}} = 4$.

Simulation conditions

Data are corrupted by two types of noise samples

- iid zero-mean Gaussian noise with $\sigma_0 = 2$ dBm.
- iid Gaussian-Bernoulli-Gaussian variables
  - with a probability $p_0 = 0.9$, $\sigma_0 = 2$ dBm
  - with a probability $p_1 = 0.1$, $\sigma_1 = 5$ dBm.

Three estimation problems are considered:

1. Only the location $\theta_{0,i}$, $i = 1, \ldots, N$ of each agent has to be estimated, γ and P_0 are assumed to be known.
2. $\theta_{0,i}$ and $P_{0,i}$, $i = 1, \ldots, N$ have to be determined for each agent.
3. $\theta_{0,i}$ and γ, $i = 1, \ldots, N$ have to be determined for each agent.

Simulation results

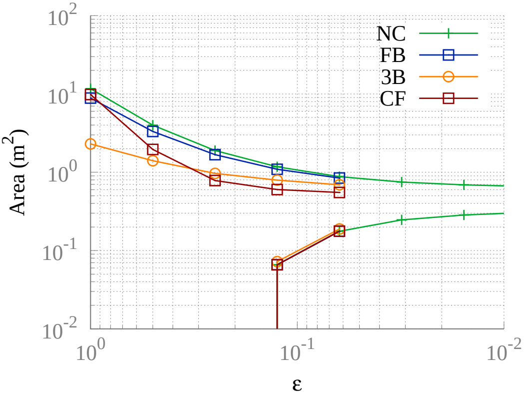


Figure: Size of the outer (top lines) and inner (bottom lines) approximation of the confidence region as defined by LSCR provided by different variants of SIVIA, without and with contractors for different values of the precision parameter ε

Simulation results

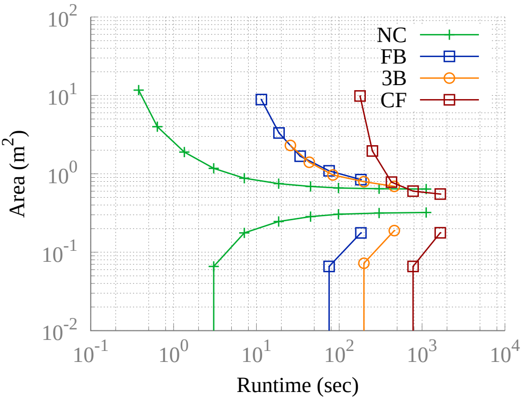


Figure: Size of the outer (top lines) and inner (bottom lines) approximation of the confidence region as defined by LSCR provided by different variants of SIVIA, without and with contractors as a function of the computing time (the curves are parametrized in ε)

Simulation results

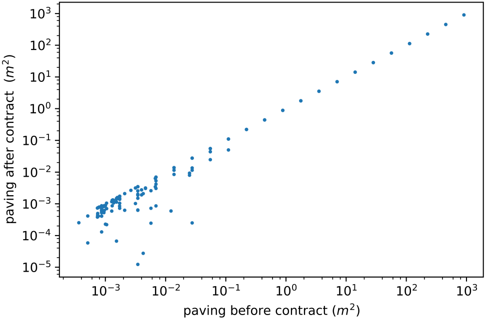
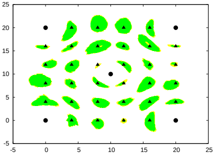
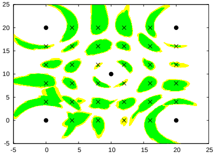
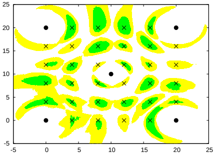
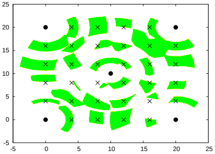
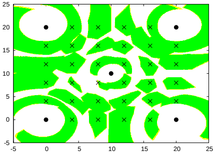
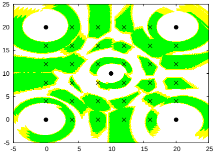
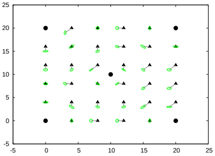
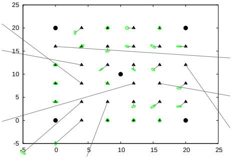
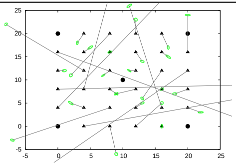
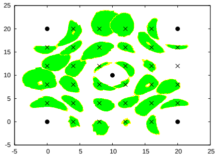
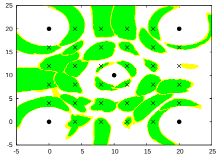
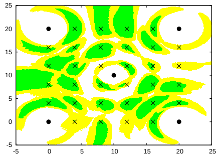
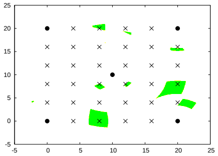
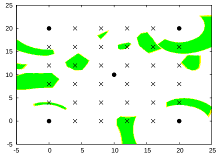
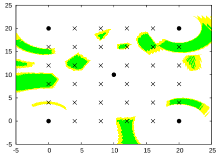
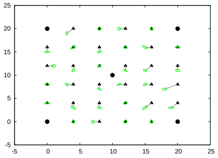
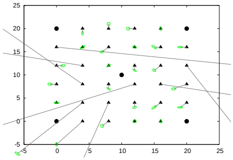
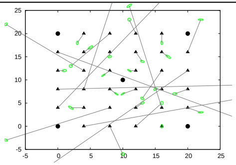


Figure: Evolution of the surface of boxes processed by SIVIA with the CF contractor

Comparison with alternative techniques

	θ_0	θ_0, P_0	θ_0, γ_P
LSCR			
BE			
MLE			

Comparison with alternative techniques

	θ_0	θ_0, P_0	θ_0, γ_P
LSCR			
BE			
MLE			

Comparison with alternative techniques

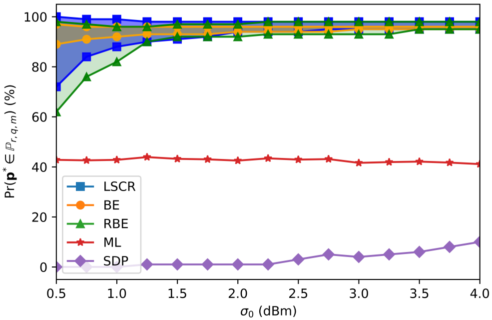


Figure: Proportions of agents for which the true value of the agent location is contained in the projection on the (θ_1, θ_2)-plane of the NACRs, the set estimates or the confidence region derived from the CRLB

Comparison with alternative techniques

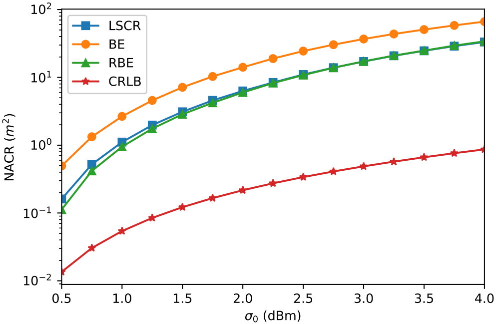


Figure: Evolution of the average surface of the projection on the (θ_1, θ_2)-plane of the NACRs, the set estimates or the confidence region derived from the CRLB

Comparison with alternative techniques

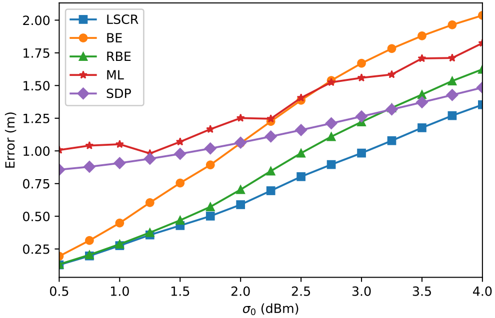


Figure: Location error