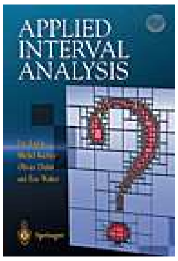
Nonlinear control of robots using interval constraints propagation

Département SI de Polytech' Nice-Sophia,Salle 310
le 9/11/05 à14h30



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ENSIETA, 2 rue François Verny, 29806 Brest Cédex 09

Objective of our team: Promote interval methods and constraint propagation within the robotics community (build solvers, solve applications, ...)

1 What is control theory ?

Many systems can be represented a state space equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

where $\mathbf{x}$ is the state vector and $\mathbf{u}$ is the control vector.

Control problem: Find a controller

$$\mathbf{u} = \mathbf{r}(\mathbf{x}, \mathbf{w}),$$

where $\mathbf{w}$ is the new input vector, such that the closed loop system behaves as desired.

More can be found on the book

Jaulin L. (2005) « Représentation d'état pour la modélisation et la commande des systèmes » (Coll. Automatique de base), Hermes , 198p

2 Interval constraints propagation

2.1 Interval arithmetic

If $\diamond \in \{+, -, ., /, \text{max}, \text{min}\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \,.$$

For instance,

$$\begin{array}{rcl} [-1,3] + [2,5] & = & [?,?], \\ [-1,3].[2,5] & = & [?,?], \\ [-1,3]/[2,5] & = & [?,?], \\ [-1,3] \vee [2,5] & = & [?,?]. \end{array}$$

If $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \, .$$

For instance,

$$\begin{array}{rl} [-1,3]+[2,5] & = [1,8], \\ [-1,3].[2,5] & = [-5,15], \\ [-1,3]/[2,5] & = [-\frac{1}{2},\frac{3}{2}], \\ [-1,3]\vee[2,5] & = [2,5]. \end{array}$$

If $f \in \{\text{cos, sin,sqr, sqrt, log, exp, } \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] \,.$$

For instance,

$$\begin{aligned}
\text{sin}([0,\pi]) &= [?,?], \\
\text{sqr}([-1,3]) &= [?,?], \\
\text{abs}([-7,1]) &= [?,?], \\
\text{sqrt}([-10,4]) &= [?,?], \\
\text{log}([-2,-1]) &= [?,?].
\end{aligned}$$

If $f \in \{$cos, sin,sqr, sqrt, log, exp, $\dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

For instance,

$$\begin{aligned}
\text{sin}([0,\pi]) &= [0,1], \\
\text{sqr}([-1,3]) &= [-1,3]^2 = [0,9], \\
\text{abs}([-7,1]) &= [0,7], \\
\text{sqrt}([-10,4]) &= \sqrt{[-10,4]} = [0,2], \\
\text{log}([-2,-1]) &= \emptyset.
\end{aligned}$$

2.2 Constraint projection

Let x, y, z be 3 variables such that

$$\begin{aligned} x &\in [-\infty, 5], \\ y &\in [-\infty, 4], \\ z &\in [6, \infty], \\ z &= x + y. \end{aligned}$$

The values < 2 for x, < 1 for y and > 9 for z are inconsistent.

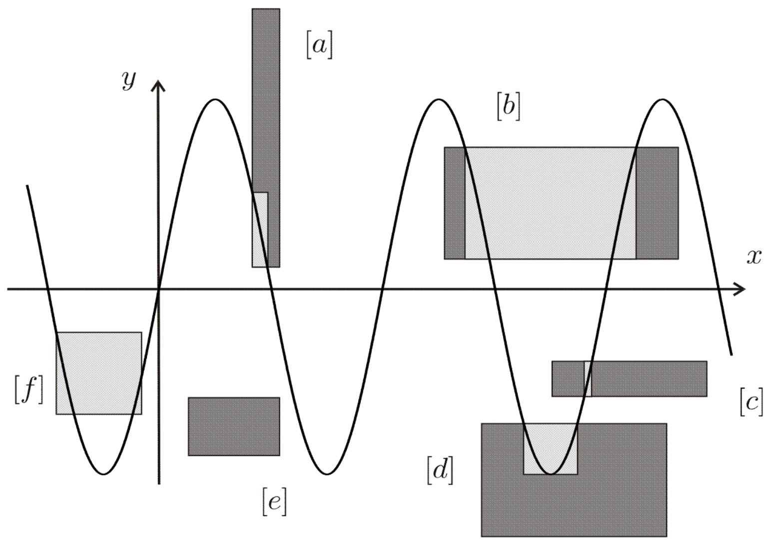
Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$
\begin{aligned}
z = x + y \Rightarrow \quad z \in \; & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \; & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \; & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}
$$

For the constraint

$$y = \sin x,\ \ x \in [x], y \in [y]$$

the problem is more difficult.



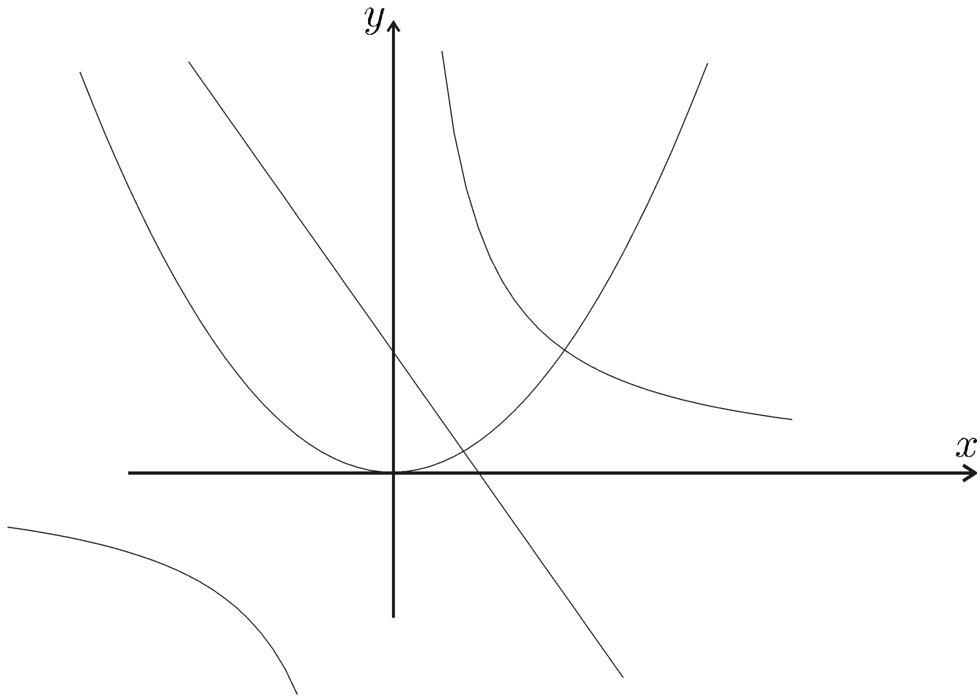
2.3 Constraint propagation

Consider the three constraints

$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$

To each variable we assign the domain $[-\infty, \infty]$.

Constraint propagation amounts to project all constraints until equilibrium.



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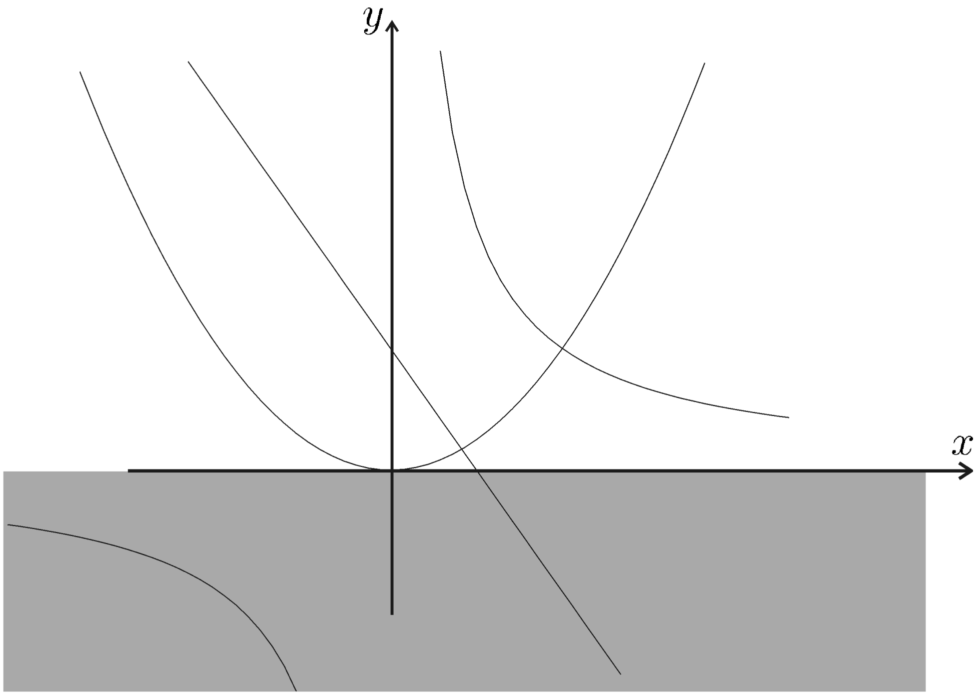

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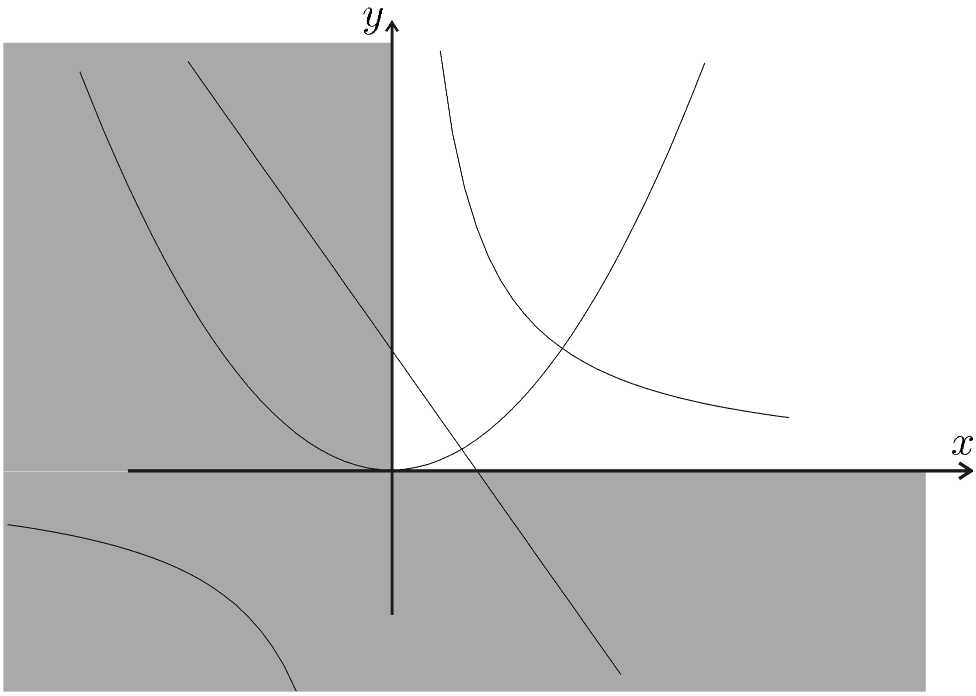
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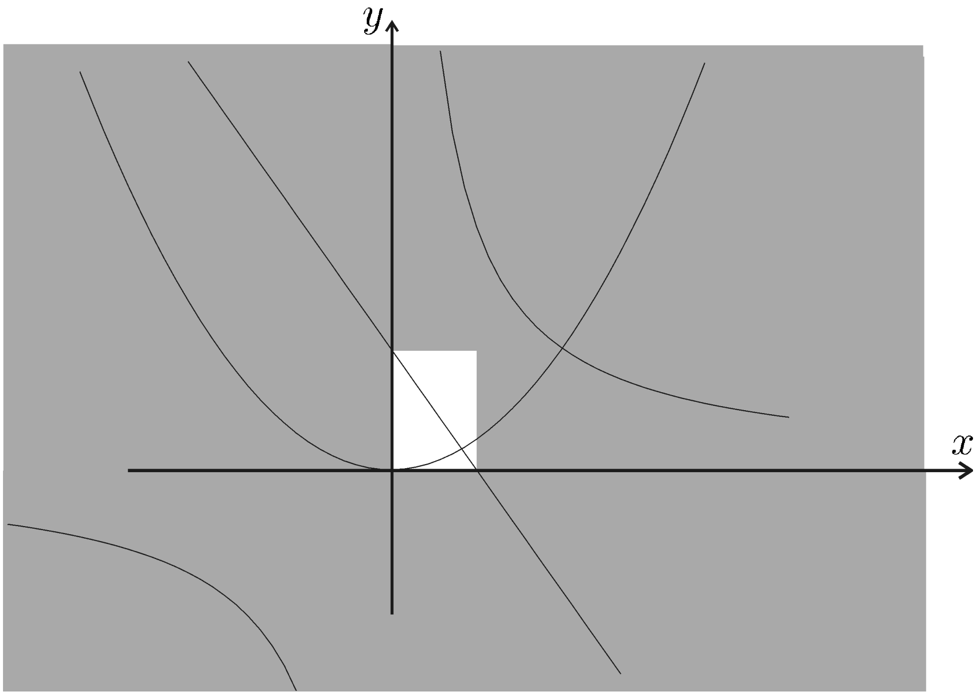

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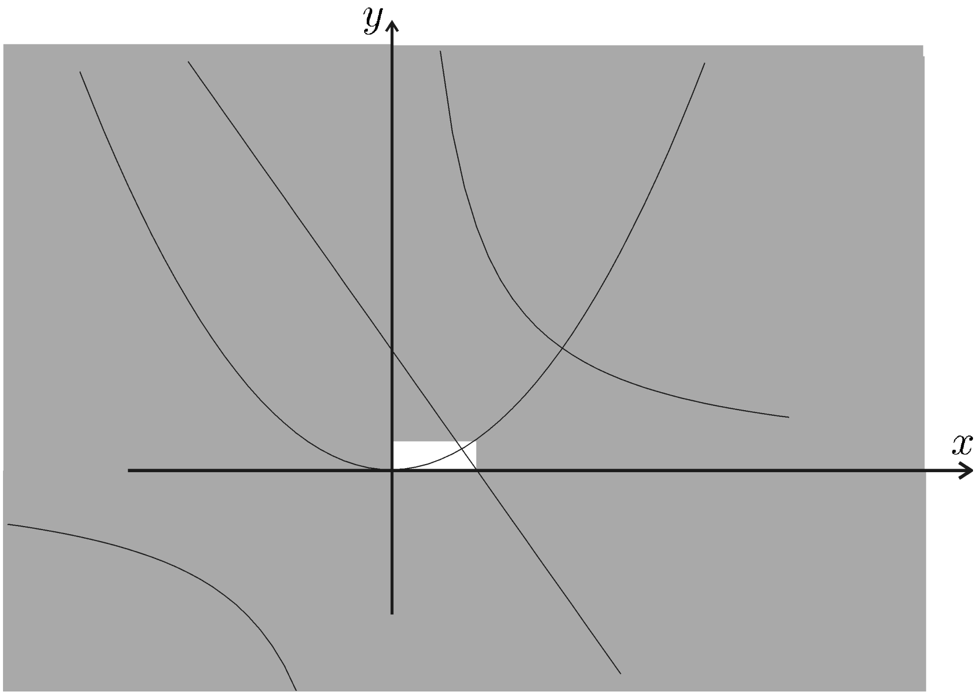
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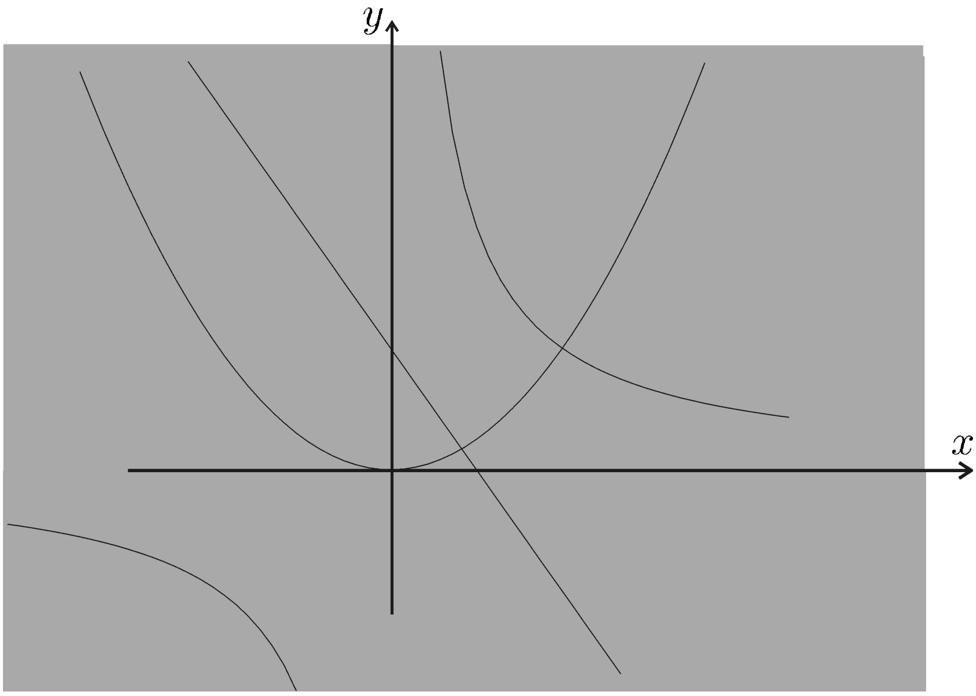
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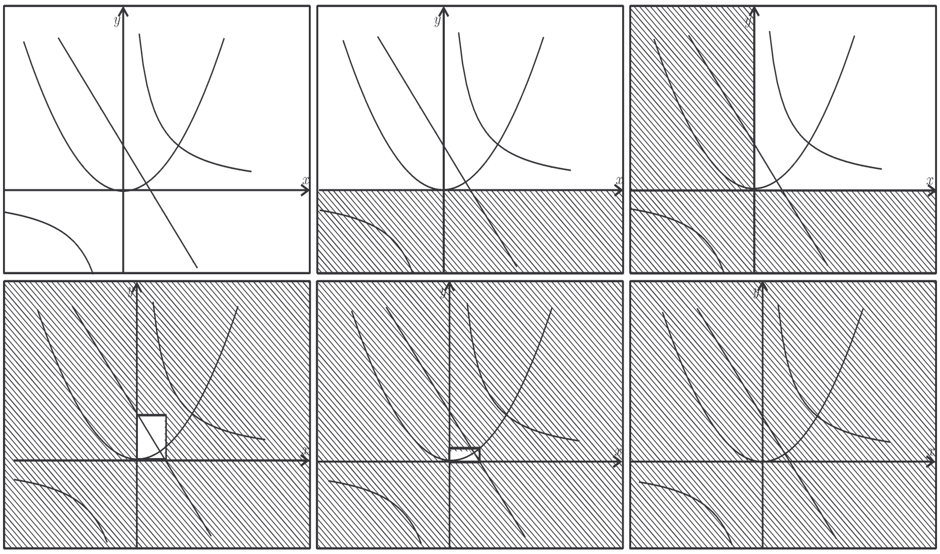

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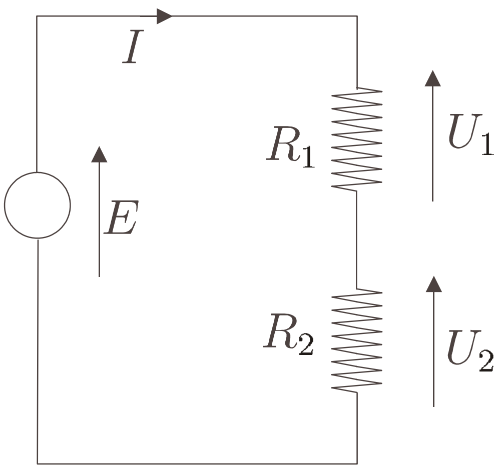
For more complex constraints, a decomposition is required. For instance,

$$\begin{array}{l} x + \mathsf{sin}(y) - xz \leq 0, \\ x \in [-1,1], y \in [-1,1], z \in [-1,1] \end{array}$$

can be decomposed into the following one.

$$\left\{\begin{array}{l} a = \mathsf{sin}(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array}\right. , \quad \begin{array}{ll} x \in [-1,1] & a \in]-\infty,\infty[\\ y \in [-1,1] & b \in]-\infty,\infty[\\ z \in [-1,1] & c \in]-\infty,\infty[\\ & d \in]-\infty, 0] \end{array}$$

2.4 Constraint propagation for estimation



Assume that

$$
\begin{aligned}
E &\in [23V, 26V],\ I \in [4A, 8A],\ U_1 \in [10V, 11V],\\
U_2 &\in [14V, 17V],\ P \in [124W, 130W],
\end{aligned}
$$

The constraints are

$$
\begin{aligned}
P &= EI;\ E = (R_1 + R_2)\, I;\\
U_1 &= R_1 I;\ U_2 = R_2 I;\ E = U_1 + U_2.
\end{aligned}
$$

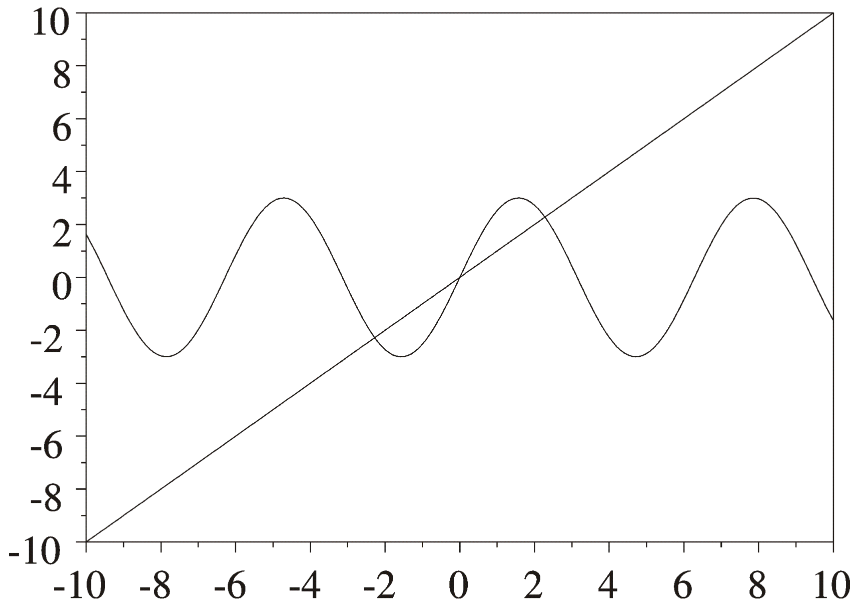
IntervalPeeler gets

$$
\begin{aligned}
R_1 &\in [1.84\Omega, 2.31\Omega]\,,\; R_2 \in [2.58\Omega, 3.35\Omega],\\
I &\in [4.769A, 5.417A]\,,\; U_1 \in [10V; 11V]\,,\\
U_2 &\in [14V; 16V]\,,\; E \in [24V; 26V]\,,\\
P &\in [124W, 130W]\,.
\end{aligned}
$$

2.5 Solving nonlinear equations

Consider the system

$$\begin{aligned} y &= 3\sin(x) \\ y &= x \end{aligned}$$



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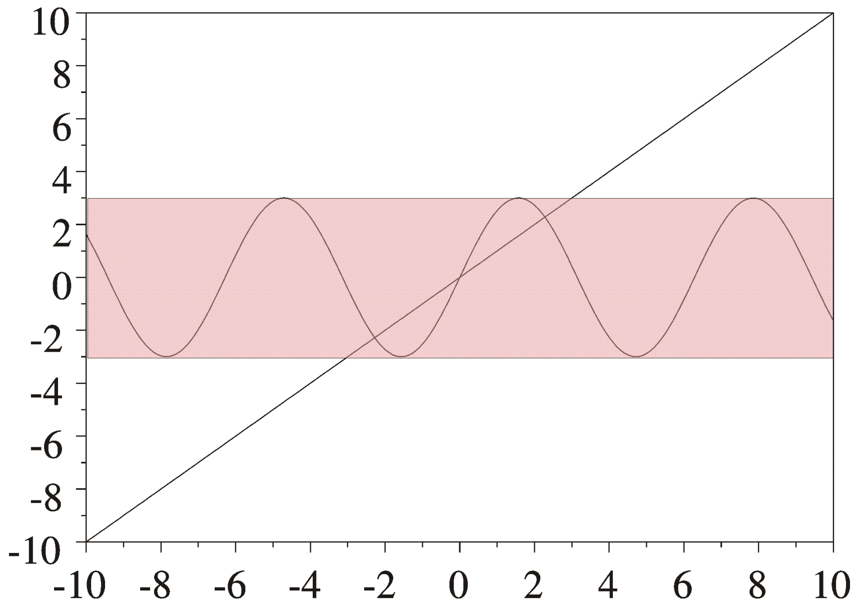

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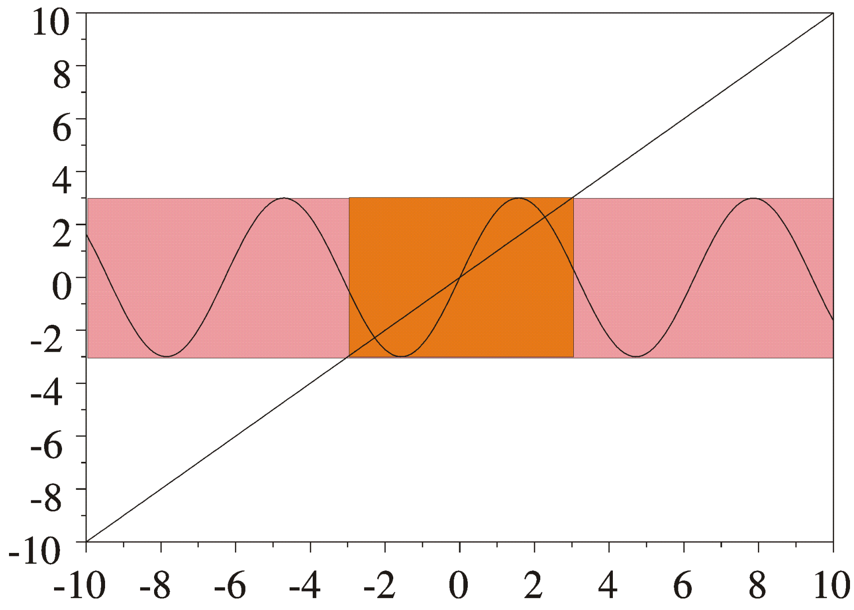

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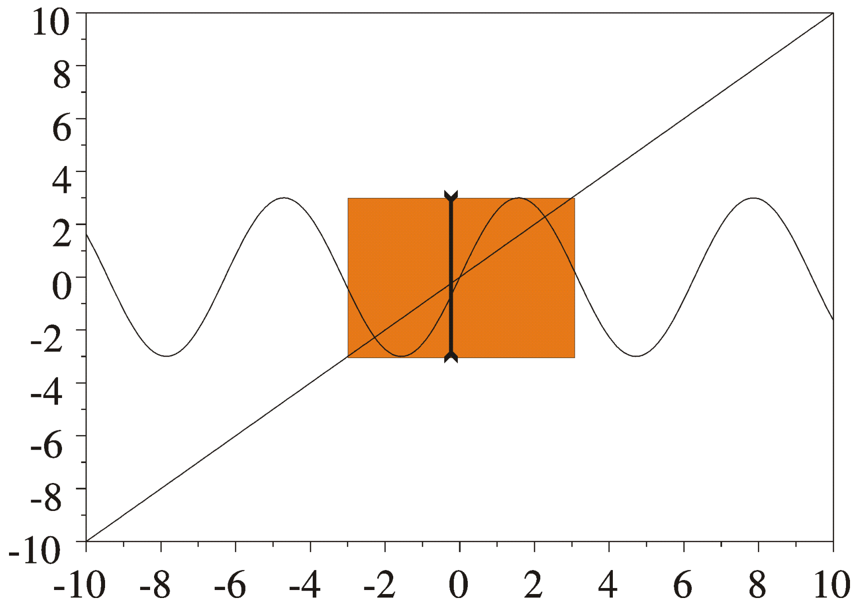
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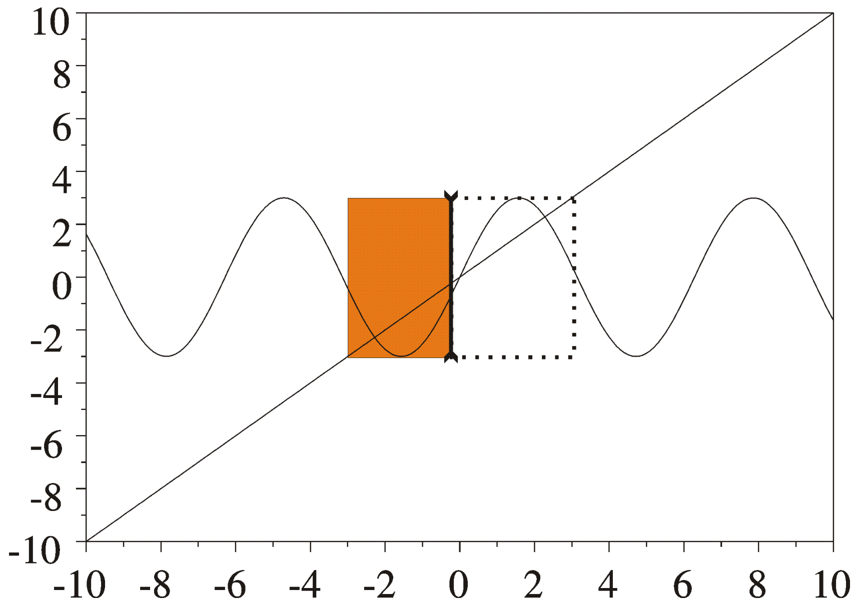

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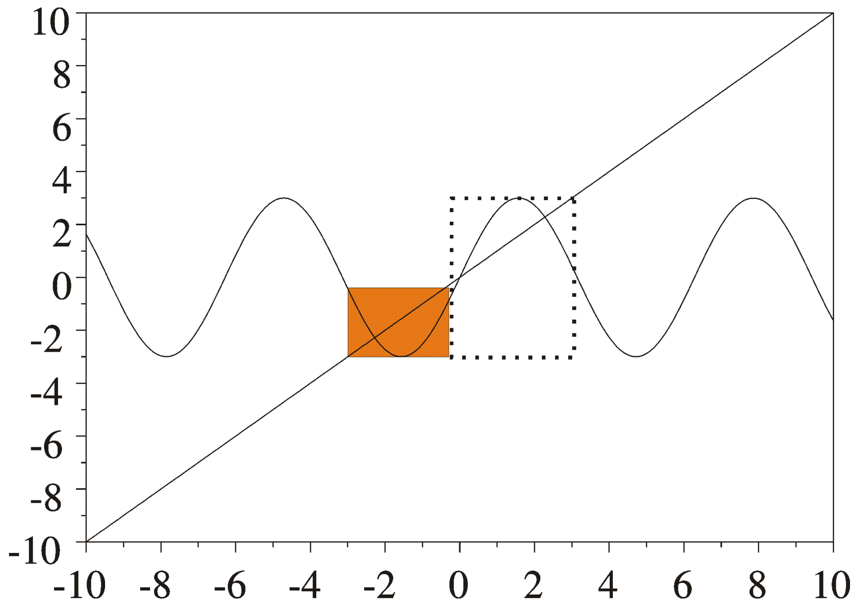
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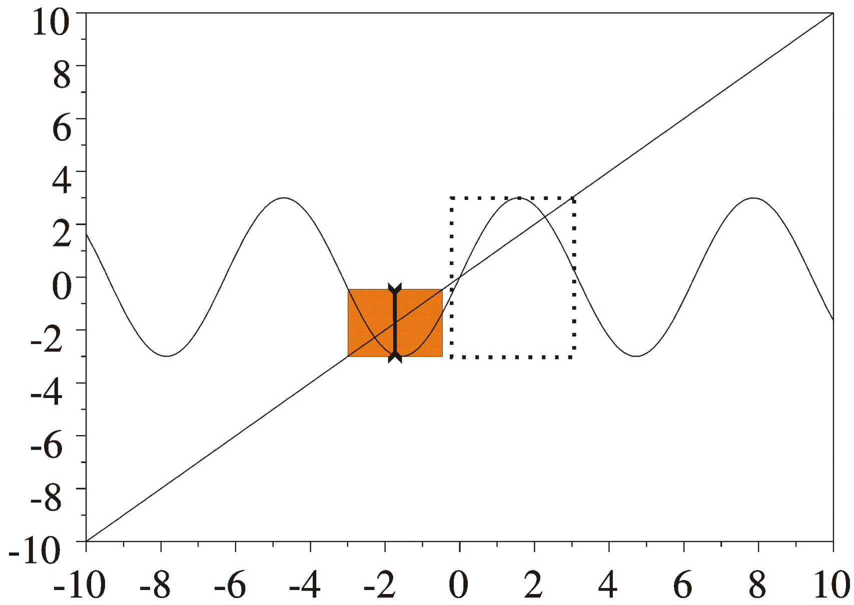

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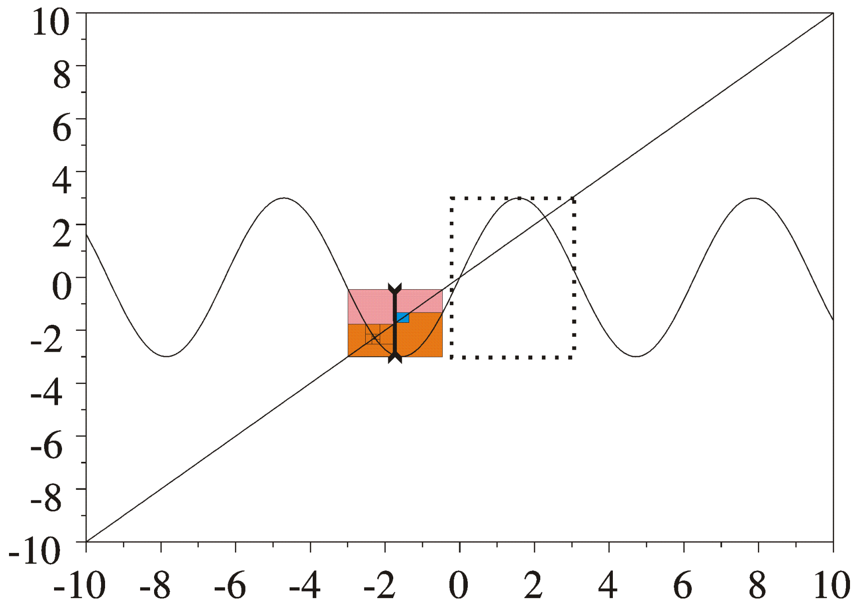

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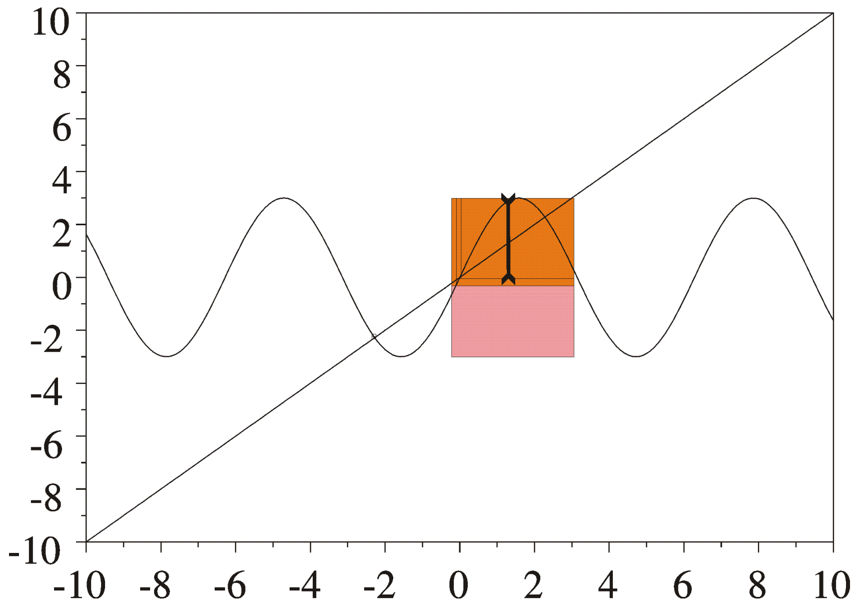
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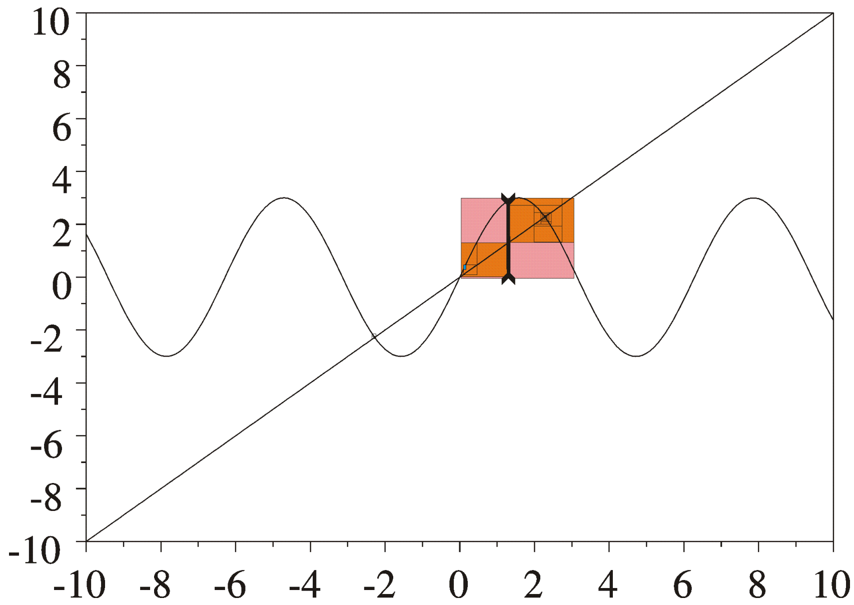
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2.6 Proving that a set of constraint is always satisfied

We use the negation of the constraints.

For instance, showing that

$$\forall x \in [x], \forall y \in [y],\ f(x,y) \leq 0 \text{ and } g(x,y) \leq 0,$$

amounts to proving that

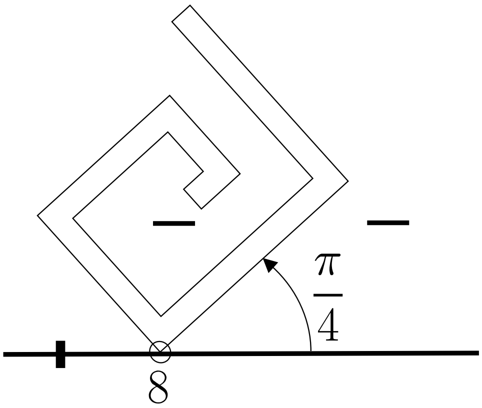
$$\{(x,y) \in [x] \times [y] \mid f(x,y) > 0 \text{ or } g(x,y) > 0\} = \emptyset$$

i.e.,

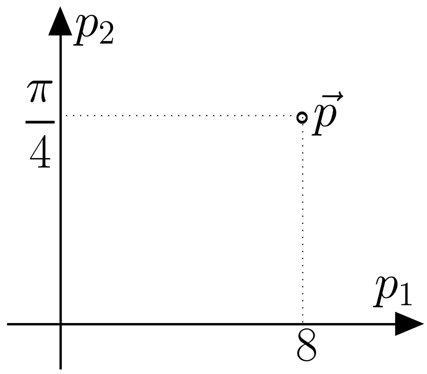
$$\{(x,y) \in [x] \times [y] \mid \max\left(f(x,y), g(x,y)\right) > 0\} = \emptyset.$$

Show SetDemo

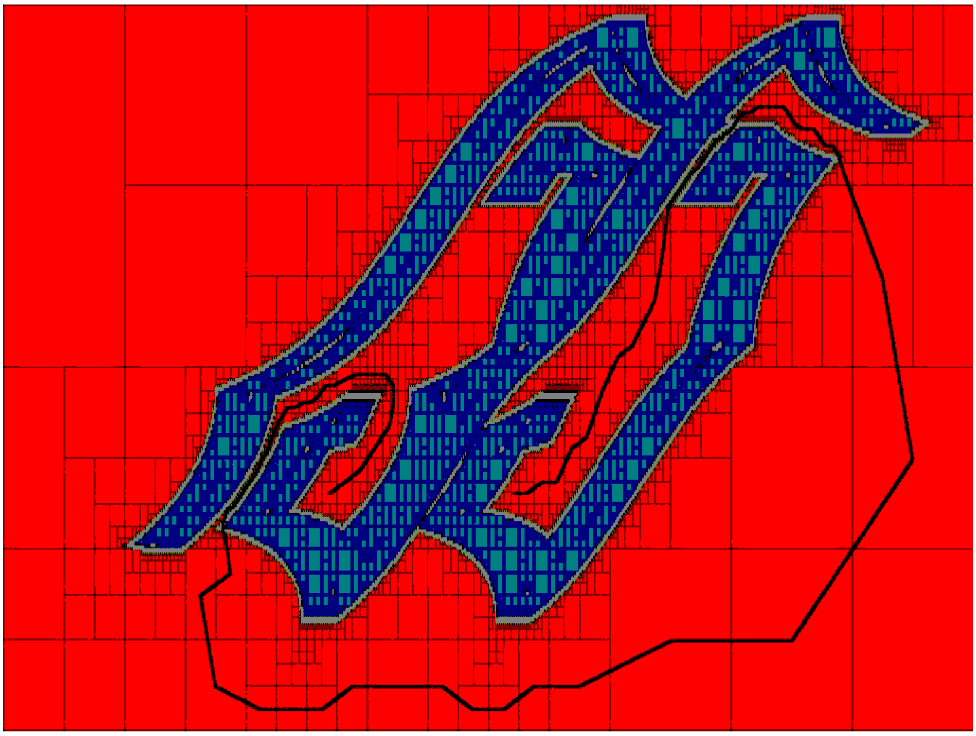
3 Application to path planning

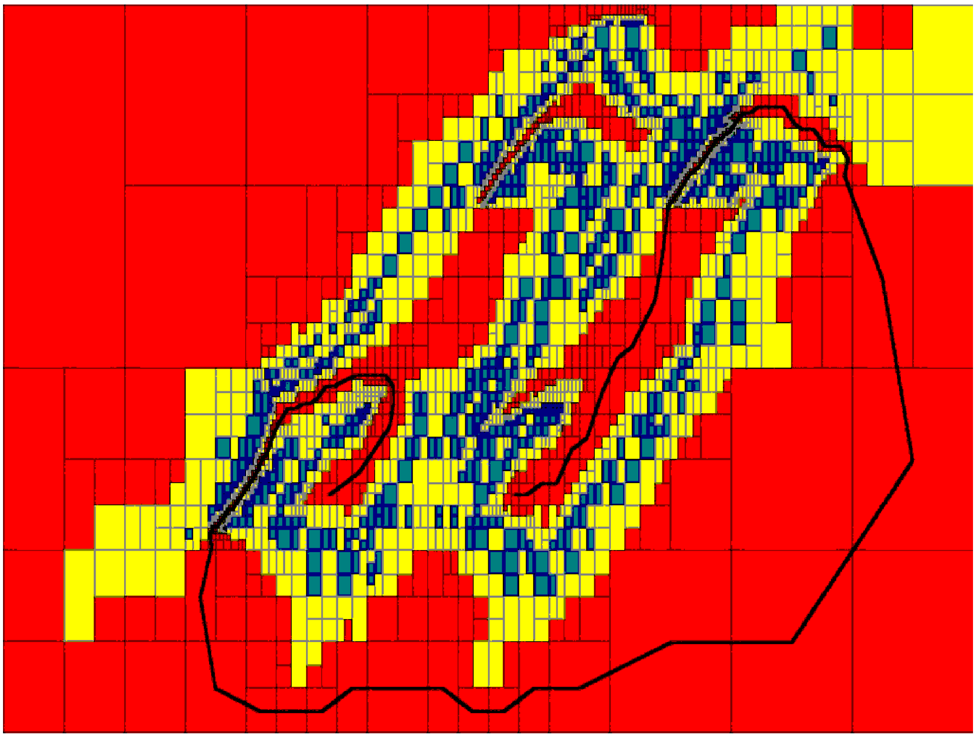


Room

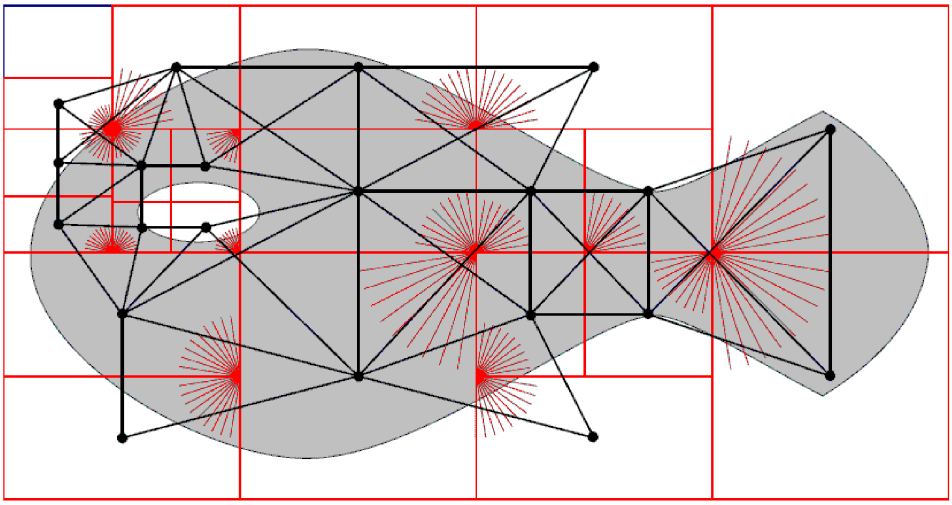


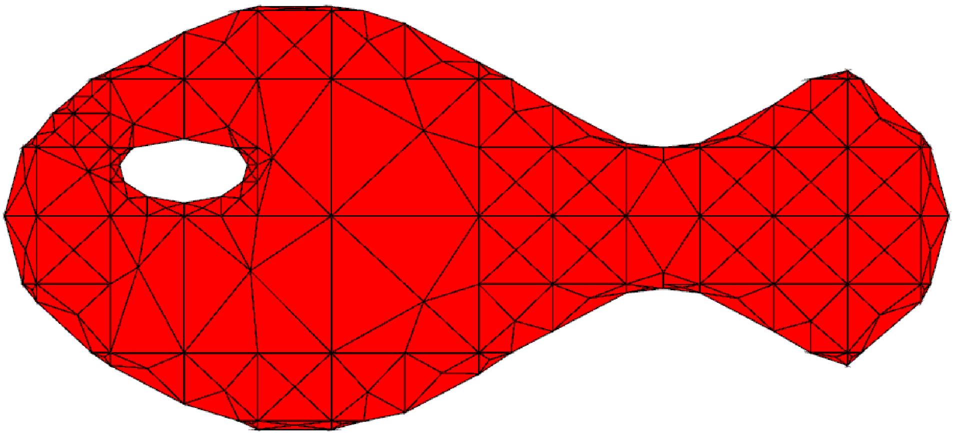
Configuration space





$$\mathbb{S} \stackrel{\text{def}}{=} \left\{ (x,y) \in \mathbb{R}^2 \mid \begin{pmatrix} x^2 + 4y^2 - 16 \\ 2\sin x - \cos y + y^2 - \frac{3}{2} \\ -(x+\frac{5}{2})^2 - 4(y-\frac{2}{5})^2 + \frac{3}{10} \end{pmatrix} \leq 0 \right\},$$





4 Control of a sailboat

(Collaboration with M. Dao, M. Lhommeau, P. Herrero,
J. Vehi and M. Sainz).

Projection of an equality.

Consider the set

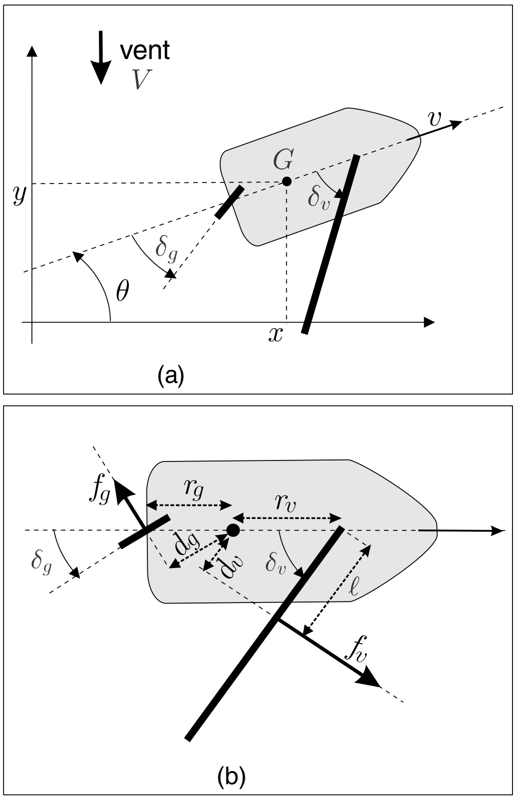
$$\mathbb{S} \overset{\text{def}}{=} \{\mathbf{p} \in \mathbf{P} \mid \exists \mathbf{q} \in \mathbf{Q}, f(\mathbf{p}, \mathbf{q}) = 0\}.$$

where $\mathbf{P}$ and $\mathbf{Q}$ are boxes and f is continuous.

Since $\mathbf{Q}$ is a connected set and f is continuous, we have

$$\mathbb{S} = \left\{\mathbf{p} \in \mathbf{P} \mid \exists (\mathbf{q}_1, \mathbf{q}_2) \in \mathbf{Q}^2, f(\mathbf{p}, \mathbf{q}_1) \leq 0,\ f(\mathbf{p}, \mathbf{q}_2) \geq 0\right\}.$$

Sailboat :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$$
\begin{cases}
\dot{x} &= v \cos\theta, \\
\dot{y} &= v \sin\theta - \beta V, \\
\dot{\theta} &= \omega, \\
\dot{\delta}_s &= u_1, \\
\dot{\delta}_r &= u_2, \\
\dot{v} &= \frac{f_s \sin\delta_s - f_r \sin\delta_r - \alpha_f v}{m}, \\
\dot{\omega} &= \frac{(\ell - r_s \cos\delta_s) f_s - r_r \cos\delta_r f_r - \alpha_\theta \omega}{J}, \\
f_s &= \alpha_s \left(V \cos(\theta + \delta_s) - v \sin\delta_s \right), \\
f_r &= \alpha_r v \sin\delta_r.
\end{cases}
$$

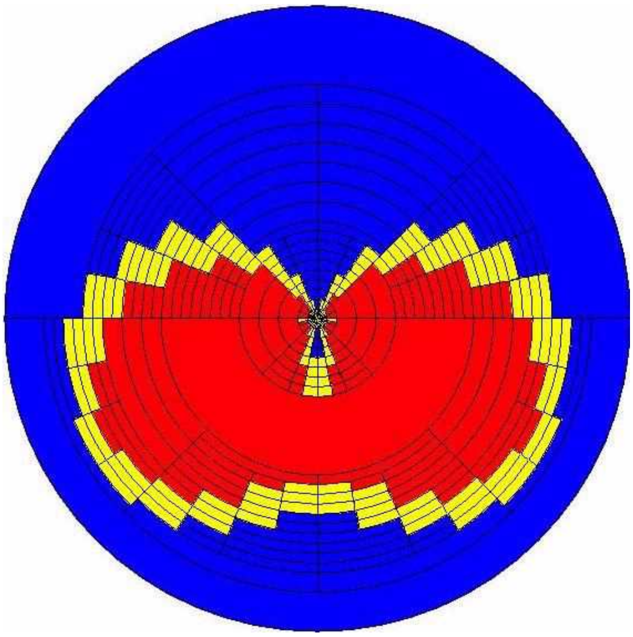
Polar speed diagram of a sailboat.

The set of feasible chosen input vectors is

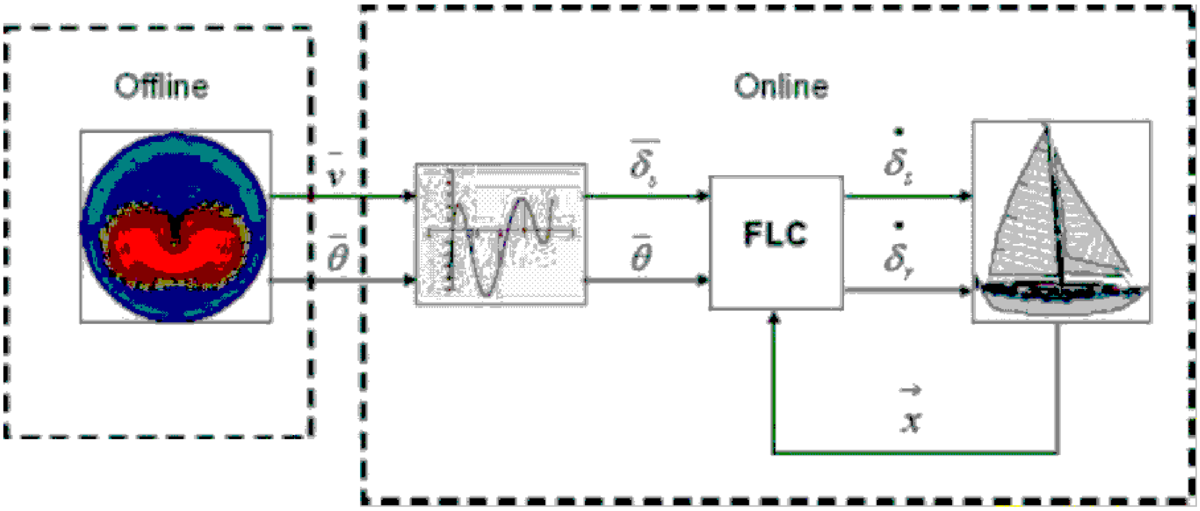
$$\mathbb{W} = \left\{ \quad (\theta, v) \mid \begin{array}{l} \exists (f_s, f_r, \delta_r, \delta_s) \\ 0 = \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ 0 = \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r}{J} \\ f_s = \alpha_s (V \cos(\theta + \delta_s) - v \sin \delta_s) \\ f_r = \alpha_r v \sin \delta_r \end{array} \right\}.$$

An elimination of f_s, f_r and δ_r yields

$$\mathbb{W} = \{ \quad (\theta, v) \mid \\ \exists \delta_s \in [-\frac{\pi}{2}, \frac{\pi}{2}], \\ \left(\frac{(\alpha_r + 2\alpha_f)v + 2\alpha_s v \sin^2\delta_s}{V} - 2\alpha_s \cos(\theta + \delta_s) \sin\delta_s\right)^2 \\ + \left(\frac{2\alpha_s}{r_r}(\ell - r_s \cos\delta_s)\left(\cos(\theta + \delta_s) - \frac{v}{V}\sin\delta_s\right)\right)^2 \\ -\alpha_r^2 \frac{v^2}{V^2} = 0 \ \}$$



This picture has been obtained using modal interval techniques.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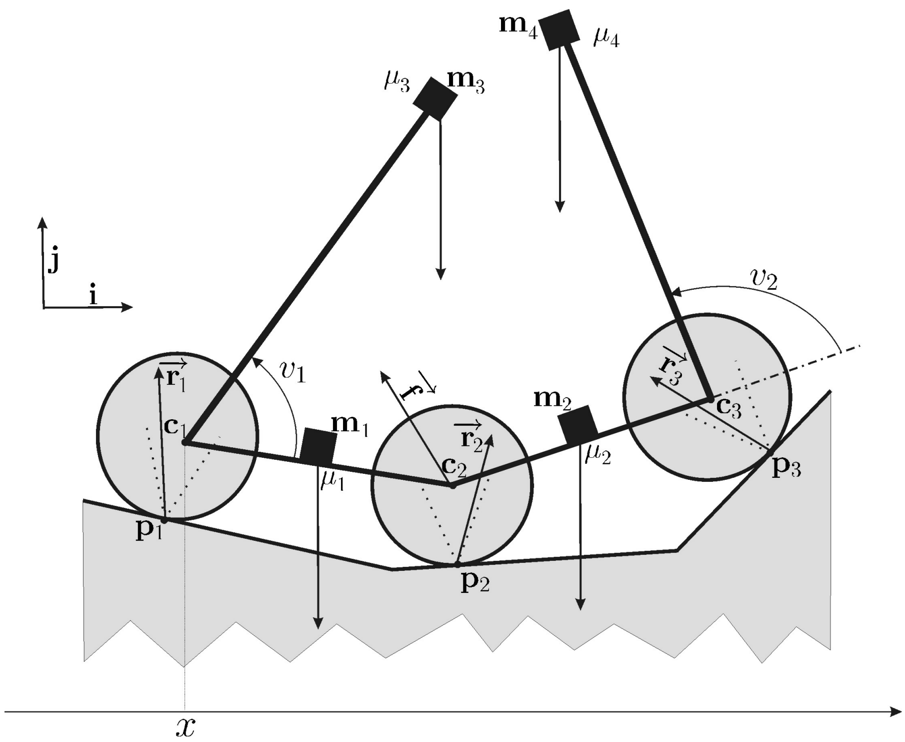
5 Control of a wheeled stair-climbing robot

(Collaboration with students and colleagues from ENSIETA)

Consider the class of constrained dynamic systems:

$$\begin{array}{ll} \text{(i)} & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \text{(ii)} & (\mathbf{x}(t), \mathbf{v}(t)) \in \mathbb{V}, \end{array}$$

where $\mathbf{v}(t) \in \mathbb{R}^{n_v}$ is the *viable input vector* and $\mathbb{V}$ is the *viable set*.



Assume that the robot has a quasi-static motion.

1) When the robot does not move, we have

$$\left\{\begin{array}{lcl} -\overrightarrow{\mathbf{p_1 m_1}} \wedge \mu_1 \mathbf{j} + \overrightarrow{\mathbf{p_1 c_2}} \wedge \overrightarrow{\mathbf{f}} - \overrightarrow{\mathbf{p_1 m_3}} \wedge \mu_3 \mathbf{j} & = & 0 \\ -\overrightarrow{\mathbf{p_2 m_2}} \wedge \mu_2 \mathbf{j} - \overrightarrow{\mathbf{p_2 c_2}} \wedge \overrightarrow{\mathbf{f}} + \overrightarrow{\mathbf{p_2 p_3}} \wedge \overrightarrow{\mathbf{r}}_3 & & \\ -\overrightarrow{\mathbf{p_2 m_4}} \wedge \mu_4 \mathbf{j} & = & 0 \\ \overrightarrow{\mathbf{r}}_1 - (\mu_1 + \mu_3)\mathbf{j} + \overrightarrow{\mathbf{f}} & = & 0 \\ \overrightarrow{\mathbf{r}}_2 - \overrightarrow{\mathbf{f}} - (\mu_2 + \mu_4)\mathbf{j} + \overrightarrow{\mathbf{r}}_3 & = & 0, \end{array}\right.$$

This system can be written into a matrix form as

$$\mathbf{A}_1(x).\mathbf{y} = \mathbf{b}_1(x),$$

where

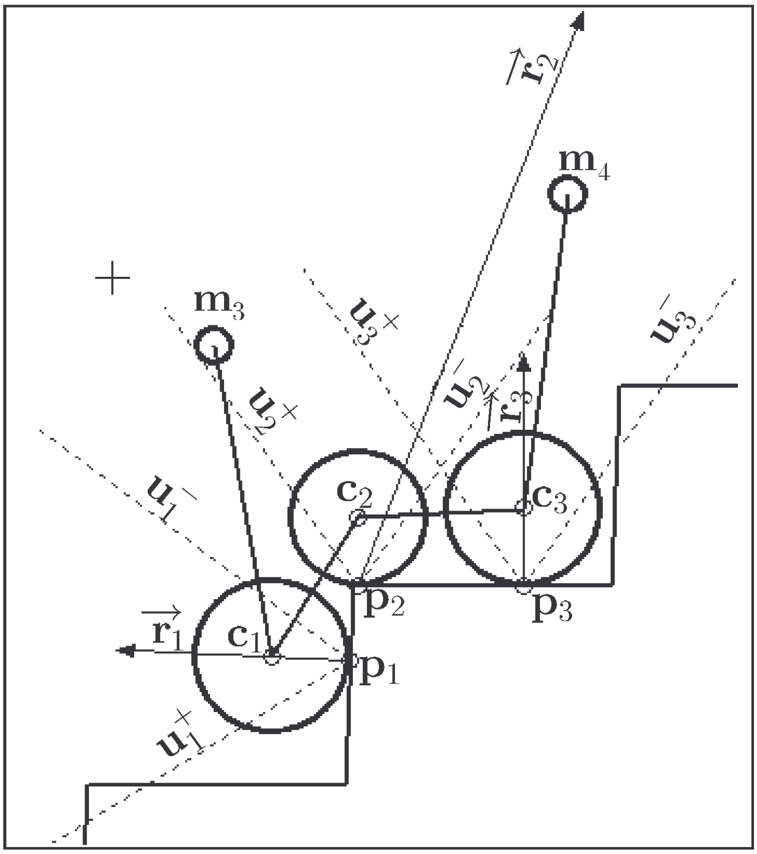
$$\mathbf{y} = \left(r_{1x}, r_{1y}, r_{2x}, r_{2y}, r_{3x}, r_{3y}, f_x, f_y, m_{3x}, m_{4x}\right)^{\mathsf{T}}.$$

2) None of the wheels will slide if all $\overrightarrow{\mathbf{r}}_i$ belong to their corresponding Coulomb cones:

$$\det(\overrightarrow{\mathbf{r}}_i, \mathbf{u}_i^-) \leq 0 \text{ and } \det(\mathbf{u}_i^+, \overrightarrow{\mathbf{r}}_i) \leq 0,$$

where $\mathbf{u}_i^-$ and $\mathbf{u}_i^+$ denote the two vectors supporting the ith Coulomb cone $\mathcal{C}_i$. These inequalities can be rewritten into

$$\mathbf{A}_2(x).\mathbf{y} \leq \mathbf{0}.$$



$\vec{r}_2$
m_4
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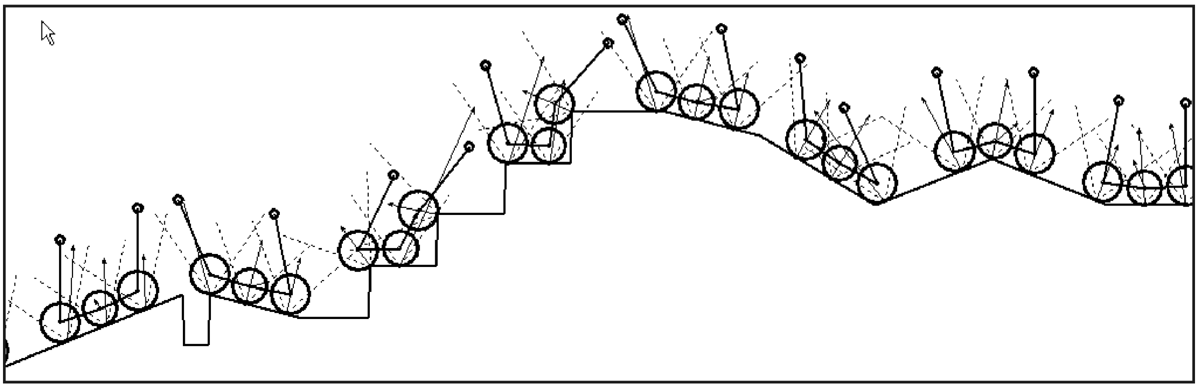
u_3^-
u_2^-
u_2^+
$\vec{r}_3$
u_1^-
c_2
c_3
p_2
p_3
$\vec{r}_1$
c_1
p_1
u_1^+

3) There is a relation between $\mathbf{y}$ and $\mathbf{v}$ of the form $\mathbf{v} = \mathbf{c}(\mathbf{y})$.

Finally,

$$\begin{aligned} \mathbf{A}_1(x).\mathbf{y} &= \mathbf{b}_1(x) \\ \mathbf{A}_2(x).\mathbf{y} &\leq \mathbf{0}. \\ \mathbf{v} &= \mathbf{c}(\mathbf{y}) \end{aligned}$$



The figure below represents the robot built by the robotics team of the ENSIETA engineering school that has won the 2005 robot cup ETAS. The robot can be seen as a three-dimensional version of the robot treated above. It has been proven to be very competitive on irregular grounds but failed to cross over some compulsory obstacles (such as stairs).

6 Localization of an AUV

Collaboration with the GESMA (Groupe d'Etude Sous-Marine de l'Atlantique). The sensors are composed by a DGPS when the AUV is at the surface of the ocean, an accelerometer and a camera oriented toward the bottom

The state space equations are given by

$$\begin{cases} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= u_1 \\ \dot{v} &= u_2 \end{cases}$$

An estimation of the state vector can be obtained by integration :

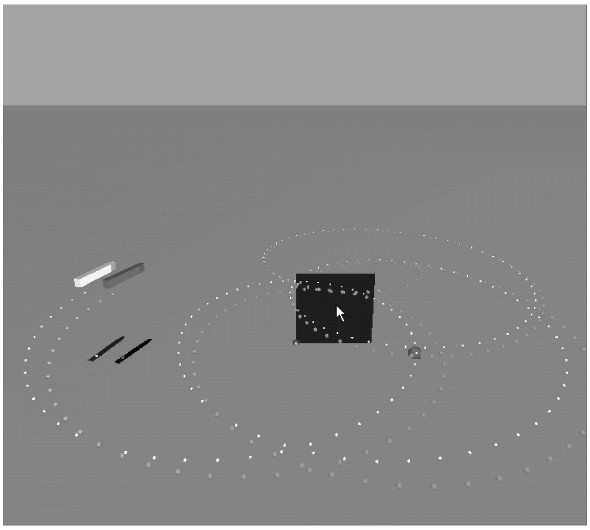
$$\frac{d}{dt}\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{\theta} \end{pmatrix} = \begin{pmatrix} v\cos\hat{\theta} \\ v\sin\hat{\theta} \\ u_1 \end{pmatrix}.$$

This state estimation is used by our controller.

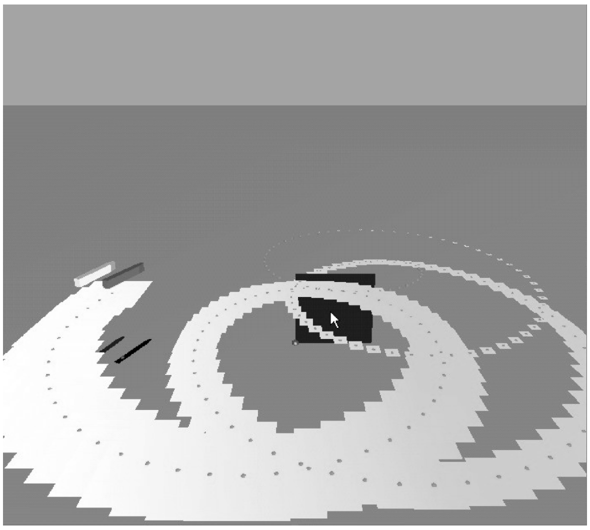
The controller to be used is given by

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\frac{\sin\hat{\theta}}{v} & \frac{\cos\hat{\theta}}{v} \\ \cos\hat{\theta} & \sin\hat{\theta} \end{pmatrix} \begin{pmatrix} \frac{x_d-\hat{x}}{4} + \dot{x}_d - v\cos\hat{\theta} + \ddot{x}_d \\ \frac{y_d-\hat{y}}{4} + \dot{y}_d - v\sin\hat{\theta} + \ddot{y}_d \end{pmatrix}.$$

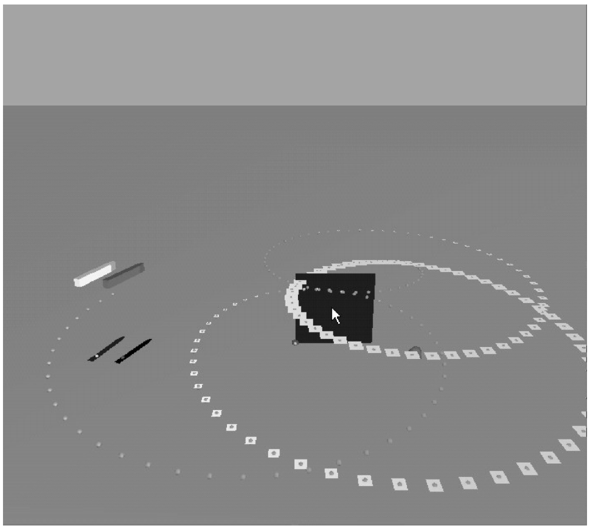
The figure below represents the actual trajectory in the situation where the desired trajectory is a cycloid. The white AUV represents the location where the AUV thinks it is located. The grey AUV represents the actual location.



An envelope containing the actual trajectory can be obtained using an interval simulation.



In the case where the output location is considered for the propagation, we get the following envelope.



If the camera is taken into account, we get :