NEW SET-MEMBERSHIP TECHNIQUES FOR PARAMETER ESTIMATION IN PRESENCE OF MODEL UNCERTAINTY

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Abstract – This paper introduces new methods for estimating parameters and their uncertainty in the context of inverse problems. The new techniques are capable of dealing with both measurement and modelling errors but also with uncertainty in parameters of the model that are assumed known. All the uncertain quantities are taken as unknown but bounded. In such a bounded-error context, reliable set-membership techniques are used to characterize, in a guaranteed way, the set of the unknown physical parameters that are compatible with the collected experimental data, the model and the prior error bounds. This ensures that no solution is lost. The methodology described will be applied to the simultaneous identification of thermal conductivity and diffusivity of polymer materials by a periodic method from actual experimental data. The guaranteed approaches provide a natural description of the uncertainty associated with the identified parameters.

1 INTRODUCTION

This paper introduces new methods for estimating parameters and their uncertainty in the context of inverse problems. These methods are capable of dealing not only with both measurement and modelling errors but also with uncertainty in parameters of the model that are not to be estimated (nuisance parameters).

The parameter estimation problem is usually solved with the widespread least-square approach, which minimizes an eventually weighted quadratic norm of the difference between the vector of collected data and the model output. Because the models employed are often nonlinear with regard to the unknown parameters, this minimization is most often performed by local iterative search algorithms such as the Newton, Gauss-Newton, Levenberg-Marquardt, quasi-Newton or conjugate gradients technique [1], even though it is common knowledge that the resulting estimate is very sensitive to initialization. Indeed, the search method may get trapped near a local minimum or stop before reaching the actual global minimum. Alternative global optimization techniques based on random search may partly overcome this problem, but again the results are obtained with no guarantee.

Moreover, the measurement of physical parameters by identification should be regarded in a same way as any experimental measurement technique, which means that an uncertainty region for the estimated parameters should always be provided. The Cramèr-Rao bound, given by the inverse of the Fisher information matrix, is commonly used to quantify this uncertainty. It corresponds to the asymptotic variance of the maximum-likelihood estimate under the hypothesis that the data are corrupted by a noise with known probabilistic distribution and that the knowledge model is valid [1]. Unfortunately, the number of collected experimental data might be small, the measurement uncertainty may be partly due to some deterministic systematic errors, no credible probability distribution may be available for the noise, and the knowledge models are most often based on some important simplifying hypotheses (such that those about radiative or convective heat fluxes, for instance). It is therefore more natural to assume all the uncertain quantities as unknown but bounded. In such a bounded-error context, the solution is no longer a point but is the set of all acceptable values of the parameter vector, which render model output consistent with actual data and prior error bounds.

The first aim of this paper is to introduce this new approach, known as bounded-error estimation, or set membership estimation (see e.g. [2-6] and references therein) which characterizes the set of all acceptable values of the parameter vector. The size of this set quantifies naturally the uncertainty associated with the estimated parameters. Moreover, as all the acceptable values of the parameter vector are enclosed, this approach allows the prior identifiability study, i.e. the issue of identified parameters unicity, to be bypassed and potentially unidentifiable parameters to be estimated. This relatively new technique has been developed in the fields of control and signal processing and has been recently used in electrochemistry [7] and robot localization [8]. It has been investigated for the first time by the authors for a guaranteed characterization of a thermal experimental set-up [9-11].

The second aim of this paper is to deal with the fact that the knowledge-based model involves other uncertain parameters than those to be estimated. Usually, these parameters, to be called nuisance parameters from now on, are assumed to be known. In fact, they are subject to uncertainty corresponding to imprecise measurement, insufficient knowledge or strong modelling simplification. Obviously, this uncertainty can be expected to affect both the identified parameters and their uncertainty. In order to take into account this disturbance, Fadale [12] has proposed an extended maximum-likelihood estimator in which the above nuisance parameters are modelled as normal random variables with known variance. The uncertainty associated which the identified parameters is then derived from the asymptotic variance of the estimator. The use of the asymptotic variance with few experimental data has already been criticized above. In addition, the prior distribution of the nuisance parameters is not always known, except for the optimistic case where they are actually measured in repeated experiments. In the general case, only a range of values is available. Last, characterizing uncertainty by random variables may not be a valid approach as modelling error may induce systematic errors which cannot be taken into account by stochastic variables. Consequently, the uncertainty in the nuisance parameters will be also characterized by intervals in the following. We shall show that set-membership estimation can reliably account for this type of uncertainty.

Section 2 details the framework of bounded-error set estimation. Section 3 introduces interval analysis and constraint propagation techniques, which allow the set estimation algorithms presented in section 4 to be reliable. The method is illustrated in section 5 with the simultaneous estimation of thermal conductivity and diffusivity of a polymeric sample material via a periodic method and actual experimental data.

2. SET-MEMBERSHIP ESTIMATION

In the sequel two types of parameters will be distinguished. The parameters of interest, i.e., those to be identified, are in the *parameter vector* $\mathbf{p}$. The other non-essential parameters are gathered in a vector $\mathbf{q}$ called the *nuisance parameter vector*. It is assumed that $\mathbf{p} \in \mathbb{P}$ and $\mathbf{q} \in \mathbb{Q}$, where $\mathbb{P}$ and $\mathbb{Q}$ are known prior domains.

Let $\mathbf{e}$ be the model output error $\mathbf{e} = \mathbf{y} - \mathbf{f}(\mathbf{p},\mathbf{q})$, where $\mathbf{y}$ is the vector of the collected data and $\mathbf{f}(.,.)$ the corresponding model output. In bounded-error estimation (or set-membership estimation), one looks for the set of all parameter vectors such that the error stays within some known feasible domain $\mathbb{E}$, *i.e.*, $\mathbf{e} \in \mathbb{E}$ (see [4-5] and the references therein). The set estimate then contains all values of the parameter vector that are acceptable, i.e., consistent with the model and the collected data $\mathbf{y}$, given what is deemed an acceptable error. The size of this set quantifies the uncertainty associated with the estimated parameters.

Assume first that the value $\mathbf{q}^*$ taken by the nuisance parameter vector $\mathbf{q}$ is known. The set $\mathbb{C}$ to be estimated is the set of all the acceptable parameter vectors $\mathbf{p}$

$$\mathbb{C} = \left\{ \mathbf{p} \in \mathbb{P}, \quad \mathbf{f}(\mathbf{p}, \mathbf{q}^*) \in \mathbb{Y} \right\} \tag{1}$$

where $\mathbb{Y} = \mathbf{y} + \mathbb{E}$. Characterizing $\mathbb{C}$ is a set-inversion problem, as (1) can be rewritten as

$$\mathbb{C} = \mathbf{g}^{-1}(\mathbb{Y}) \cap \mathbb{P} \tag{2}$$

where $\mathbf{g}(.) = \mathbf{f}(.,\mathbf{q}^*)$. It can be solved in a guaranteed way using the algorithm SIVIA [13-15], see Section 3.

Suppose now that $\mathbf{q}^*$ is unknown. One may of course choose to estimate the set

$$\mathbb{S} = \left\{ (\mathbf{p}, \mathbf{q}) \in \mathbb{P} \times \mathbb{Q} \mid \mathbf{f}(\mathbf{p}, \mathbf{q}) \in \mathbb{Y} \right\} \tag{3}$$

which can again be seen as a set-inversion problem. However, characterizing $\mathbb{S}$ will be much more difficult than estimating $\mathbb{C}$, since the dimension of $\mathbb{S}$ is larger than that of $\mathbb{C}$ and the volume of $\mathbb{S}$ may be very large, if the parameters in $(\mathbf{p}, \mathbf{q})$ are not identifiable.

Since the value of $\mathbf{q}$ is not considered essential, an alternative simpler approach is to characterize the set Π of all the acceptable parameter vectors $\mathbf{p}$ under the assumption that $\mathbf{q}$ belongs to its prior domain, *i.e.*,

$$\Pi = \left\{ \mathbf{p} \in \mathbb{P} \mid \exists \mathbf{q} \in \mathbb{Q}, \mathbf{f}(\mathbf{p}, \mathbf{q}) \in \mathbb{Y} \right\} \tag{4}$$

The estimation of the acceptable values of $\mathbf{q}$ is then given up to simplify computation. While $\mathbb{C}$ is a cut of $\mathbb{S}$, Π is the projection of $\mathbb{S}$ onto the $\mathbf{p}$-space (see Figure 1)

$$\Pi = proj_{\mathbb{P}} \mathbb{S} \tag{5}$$

Remark 1. The inclusion $\mathbb{C} \subset \Pi$, illustrates the fact that when $\mathbf{q}$ is uncertain, the uncertainty on $\mathbf{p}$ increases.

The basic tools for the characterization of Π will now be presented.
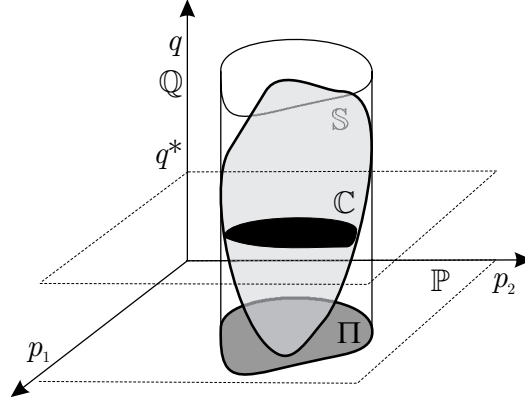
Figure 1 : The sets to be characterized

3. INTERVAL ANALYSIS

Interval analysis was initially developed to account for the quantification errors introduced by the rational representation of real numbers with computers and was extended to validated numerics [15-16-17]. A real interval $[a] = [\underline{a}, \overline{a}]$ is a connected and closed subset of $\mathbb{R}$. The set of all real intervals of $\mathbb{R}$ is denoted by $\mathbb{IR}$.

Real arithmetic operations are extended to intervals [16]. Consider an operator $\bullet \in \{+, -, *, /\}$ and $[a]$ and $[a]$ two intervals, then

$$[a] \circ [b] = \{x \circ y \mid x \in [a], y \in [b]\} \tag{6}$$

Let $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$, the range of the function $\mathbf{f}$ over an interval vector $[\mathbf{a}]$ is given by:

$$\mathbf{f}([\mathbf{a}]) = \{\mathbf{f}(\mathbf{x}) \mid \mathbf{x} \in [\mathbf{a}]\} \tag{7}$$

The interval function $[\mathbf{f}]$ from $\mathbb{IR}^n$ to $\mathbb{IR}^m$ is an inclusion function for $\mathbf{f}$ if

$$\forall [\mathbf{a}] \in \mathbb{IR}^n, \ \mathbf{f}([\mathbf{a}]) \subseteq [\mathbf{f}]([\mathbf{a}]) \tag{8}$$

An inclusion function of $\mathbf{f}$ can be obtained by replacing each occurrence of a real variable by its corresponding interval and by replacing each standard function by its interval evaluation. Such a function is called the natural inclusion function. In practice the inclusion function is not unique, it depends on how $\mathbf{f}$ is written.

For the sake of brevity, the same notation will be used for the ranges of the used functions and their inclusion functions.

Constraints satisfaction problem, contractors

Consider n variables $x_i \in \mathbb{R}$, $i \in \{1,2,\ldots,n\}$, linked by n_f relations of the form

$$f_j(x_1, x_2, \ldots, x_n) = 0 \tag{9}$$

and where each variable x_i is known to belong to a prior interval domain $[x_i]_0$; define the vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ and the function $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots f_{n_f}(\mathbf{x}))^T$, then equation (9) can be written as a *constraint satisfaction problem CSP* [15] :

$$H : \mathbf{f}(\mathbf{x}) = \mathbf{0}, \quad \mathbf{x} \in [\mathbf{x}]_0 \tag{10}$$

Interval CSPs can be solved with *contractors*. An operator C_H is a contractor for the CSP H defined by (10) if, for any box $[\mathbf{x}]$ in $[\mathbf{x}]_0$, it satisfies

   a) the *contractance* property : $C_H([\mathbf{x}]) \subset [\mathbf{x}]$

   b) and the *correctness* property : $C_H([\mathbf{x}]) \cap \mathbb{S} = [\mathbf{x}] \cap \mathbb{S}$

where $\cap$ is the intersection of two boxes and $\mathbb{S}$ a solution set for H. A *solver* for the CSP defined by (10) is an algorithm Ψ such that:

$$\mathbf{f}(\mathbf{x}) = \mathbf{0} \Leftrightarrow \mathbf{x} = \Psi(\mathbf{x}) \tag{11}$$

According to the fixed-point theorem and using (11), if the series $\mathbf{x}_{k+1} = \Psi(\mathbf{x}_k)$ converges towards $\mathbf{x}_\infty$, then $\mathbf{x}_\infty$ shall contain the solution of H. Several punctual solvers such as the Newton method, the Gauss-Siedel or the Krawczyk operators have been extended to intervals, and are used to solve efficiently even non-linear CSPs [15-17]. However, they remain limited to problems where the number of constraints is equal to the number of variables.

When the number of constraints and the number of variables are different, one can use an alternate contractor relying on interval propagation techniques. These techniques combine the constraint propagation techniques classically used in the domain of artificial intelligence [18] and interval analysis. They have been brought to automatic control in [19], for solving set inversion problems in a bounded-error context. The algorithm used for constraint propagation is based on the interval extension of the local Waltz filtering [18-19]. In fact, the relationships (9) between the variables can be viewed as a network where the nodes are connected with the constraints. In order to spread the consequences of each node throughout the network, the main idea is to deal with a local group of constraints and nodes and then record the changes in the network. Further deductions will make use of these changes to make further changes. The inconsistent values for the state vector are thus removed. If the network exhibits no cycles, then optimal filtering can be achieved by performing only one forward and one backward propagation: this is known as the *forward-backward contractor* [15-19]. Figure 2 illustrates several types of contractors.
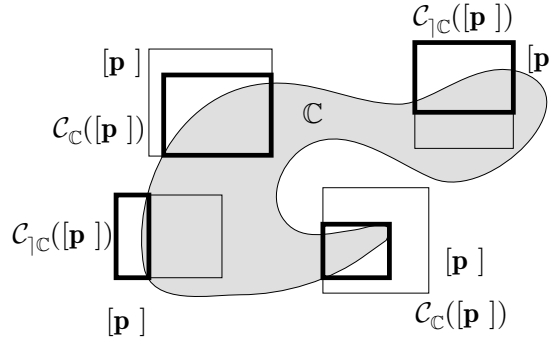

Figure 2 : Contractors.

Set inversion via interval analysis

Consider the problem of determining a solution set for the unknown quantities **u** defined by:

$$\mathbb{S} = \left\{ \mathbf{u} \in \mathbb{U} \mid \Psi(\mathbf{u}) \in [\mathbf{y}] \right\} = \Psi^{-1}\left([\mathbf{y}]\right) \cap \mathbb{U} \tag{12}$$

where $[\mathbf{y}]$ is known *a priori*, $\mathbb{U}$ is an *a priori* search set for **u** and ψ a nonlinear function not necessarily invertible in the classical sense. Equation (12) involves computing the reciprocal image of Ψ, it is a set inversion problem which can be solved using SIVIA (Set Inversion Via Interval Analysis). SIVIA [13-14] is a recursive algorithm which explores all the search space without losing any solution. This algorithm makes it possible to derive a guaranteed enclosure of the solution set $\mathbb{S}$ as follows

$$\underline{\mathbb{S}} \subseteq \mathbb{S} \subseteq \overline{\mathbb{S}} \tag{13}$$

The inner enclosure $\underline{\mathbb{S}}$ is composed of the boxes that have been proved feasible. To prove that a box $[\mathbf{u}]$ is feasible it is sufficient to prove that $\Psi([\mathbf{u}]) \subseteq [\mathbf{y}]$. Contrariwise, if it can be proved that $\Psi([\mathbf{u}]) \cap [\mathbf{y}] = \varnothing$, then the box $[\mathbf{u}]$ is unfeasible. Otherwise, no conclusion can be reached and the box $[\mathbf{u}]$ is said undetermined. The latter is then bisected and tested again until its size reaches a threshold $\varepsilon > 0$ to be tuned by the user. Such a termination criterion ensures that SIVIA terminates after a finite number of iterations. The outer enclosure $\overline{\mathbb{S}}$ is defined by

$$\overline{\mathbb{S}} = \underline{\mathbb{S}} \cup \Delta\mathbb{S} \tag{14}$$

where $\Delta\mathbb{S}$ is an *uncertainty* layer given by the union of all the undetermined boxes (with their widths not larger than ε).

Set projection via interval analysis

When only Π is to be characterized, one can use another algorithm called PROJECT [9,10,11,19]. This algorithm computes inner and outer approximations $\underline{\Pi}$ and $\overline{\Pi}$ of the set Π defined by (4). As only the **p**-space is partitioned, the memory and computational time required are much smaller than for a full characterization of $\mathbb{S}$. Obviously, the main difference between PROJECT and SIVIA lies in the tests to be implemented. In SIVIA, the outer approximation $[g]([\mathbf{p}])$ is directly used to test the acceptability of all elements of $[\mathbf{p}]$. Here, to characterize Π, $[\mathbf{p}]$ will be said acceptable if there exists $\mathbf{q} \in \mathbb{Q}$ such that $f([\mathbf{p}], \mathbf{q}) \subset \mathbb{Y}$. Feasible point finders then require specific approaches. In order to allow consideration of higher dimensions, the procedure implemented in PROJECT uses contractors (see [19] for details). As only the **p**-space is partitioned, the memory and computational time required are much smaller than for a full characterization of $\mathbb{S}$.

4. APPLICATION

Experimental procedure

The experimental procedure under analysis hereafter is devoted to the measurement of the thermal properties of materials: the thermal diffusivity, denoted a, and the thermal conductivity λ, of a sample are measured simultaneously by using a so-called *periodic* method, using multi-harmonic heating signals [20]. The experimental set-up is shown on figure 3. The sample under study is held in between metallic plates. A thermal grease layer ensures good thermal exchanges between the different elements. The front side of the first metallic plate, made of brass, is also fixed to a heating device. The rear side of the second metallic plate, made of copper, is in contact with air at ambient temperature and high vacuum.
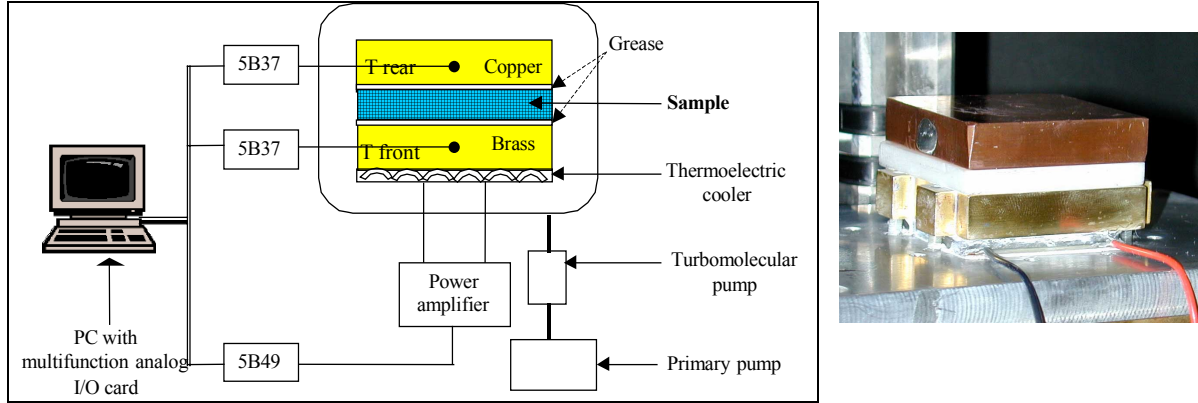

Figure 3: The experimental set-up.

The excitation voltage of the thermoelectric cooling device is a sum of five sinusoidal signals. The resulting signal is expressed as follows

$$V(t) = \sum_{n=1}^{5} V_n \cdot \sin\left(2\pi 2^{n-1} f_0 t\right) \tag{15}$$

where f_0 is the fundamental frequency, V_n is a partial voltage amplitude and t is the time.

An amplifier device feeds the thermoelectric cooler and is controlled by an analogical voltage provided by an I/O card via a signal conditioning block (Analog Devices, 5B49™). The signal provided by two thermocouples fixed on the front and rear plates is amplified and low-pass filtered with conditioning modules (Analog Devices, 5B37™). All conditioning modules are connected to a multifunction acquisition card (NI-6035E™) controlled by a Labview™ application.

In our case, the use of a sum of five sinusoidal signals allows us to obtain five times more information from one experience and reduces the measurement time. To obtain the thermal conductivity and diffusivity values, it is necessary to explore a large frequency range. In this case, several measurements are realised for different fundamental frequencies in order to study a large frequency range. Moreover, it is required to have large signal amplitudes for a better signal/noise ratio. However, the increase in amplitude is limited by the power of the generator and by the operating temperature of the thermoelectric cooling system. Generally, the temperature data obtained show a drift of the mean temperature of both front and rear plates (typically a few degrees). The signal mean value is subtracted from experimental data values to get the temperature variations only. After this rough signal correction, experimental data is given as the following frequency response

$$H_S(j\omega) = \frac{T_{rear}(j\omega)}{T_{front}(j\omega)} \tag{16}$$

where the temperature spectra are given by the Fourier transform of the time-history signals. Error bounds on the experimental transfer function are calculated prior to the identification, from measurements repeated 20 times. The experimental data are given in figure 4.

Physical model

The system under study is modelled with one-dimensional quadrupoles (two-port transfer functions). The quadrupole method is well known and extensively used in thermal science [21]. A "quadrupole" $\mathbf{Z}(s)$ is defined by

$$\mathbf{Z}(s) = \begin{bmatrix} \cosh\left(\sqrt{\tau s}\right) & \dfrac{R}{\sqrt{\tau s}}\sinh\left(\sqrt{\tau s}\right) \\ \dfrac{\sqrt{\tau s}}{R}\sinh\left(\sqrt{\tau s}\right) & \cosh\left(\sqrt{\tau s}\right) \end{bmatrix} \qquad (17)$$

where $\tau = \delta^2/a$, $R = \delta/\lambda$ and δ is material thickness. For the particular case of the grease layer, which is assumed with no inertia, the relationship uses the resistance only and becomes
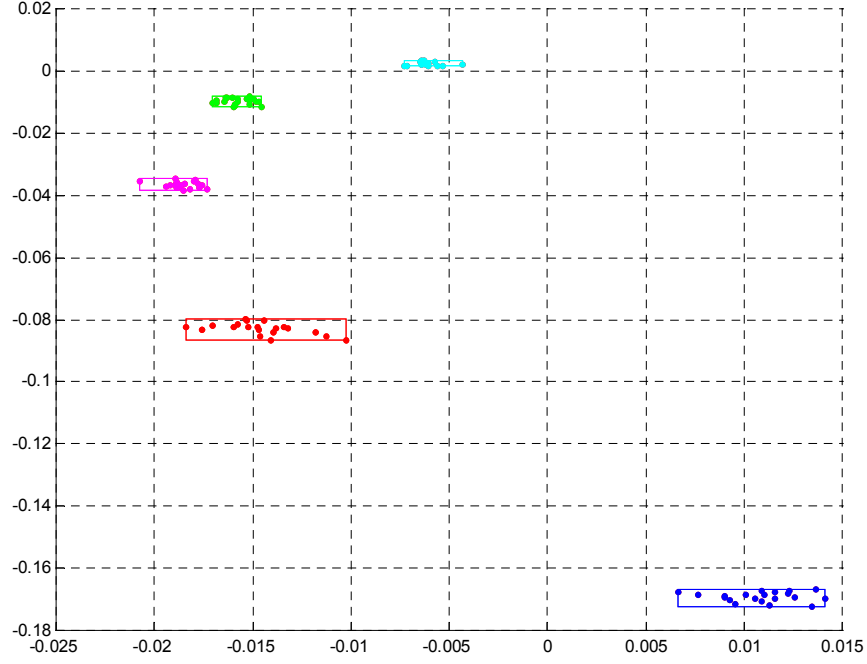

Figure 4: The experimental data. Nyquist plot of transfer function.

$$\mathbf{Z}(s) = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \qquad (18)$$

The model transfer function is then given by

$$H(s,\mathbf{p}) = \frac{T_{rear}(s)}{T_{front}(s)} \qquad (19)$$

where the front temperature is given by (the s symbol being removed, for convenience)

$$\begin{bmatrix} T_{front} \\ \varphi_{front} \end{bmatrix} = \mathbf{Z}_{Brass}\mathbf{Z}_{Gre}\mathbf{Z}_{Sample}\mathbf{Z}_{Gre}\mathbf{Z}_{Copp} \begin{bmatrix} T_0 \\ hT_0 \end{bmatrix} \qquad (20)$$

and the rear temperature is given by

$$\begin{bmatrix} T_{rear} \\ \varphi_{rear} \end{bmatrix} = \mathbf{Z}_{Copp\_half} \begin{bmatrix} T_0 \\ hT_0 \end{bmatrix} \qquad (21)$$

where h is a constant coefficient modelling surface heat exchanges with the fluid, and where T_0 is the temperature of the rear face of the copper plate. The nuisance parameters and their uncertainties are given in Table 1. The sample thickness is given by the interval $[1.95\,,\,2.05]\,10^{-3}m$

Material	Parameters	Scale	Nominal value	Uncertainty interval
Brass plate Front side	Diffusivity, a	10^{-6} m^2.s^{-1}	34.0	[33 , 35]
	Conductivity, λ	W.m^{-1}.K^{-1}	112.5	[100 , 125]
	Brass thermocouple – sample interface distance	10^{-3} m	5.0	[4.225 , 5.775]
Copper plate Rear side	Diffusivity, a	10^{-6} m^2.s^{-1}	115.5	[114 , 117]
	Conductivity, λ	W.m^{-1}.K^{-1}	395.5	[389 , 402]
	Copper thermocouple – sample interface distance	10^{-3} m	4.5	[3.725 , 5.275]
	Thickness	10^{-3} m	9.0	[8.89 , 9.01]
Grease	Thermal resistance	10^{-6} K.m^2.W^{-1}	115.0	[80 , 150]
Fluid	Surface heat exchange coefficient	W.m^{-2}.K^{-1}	5	[5 , 10]

Table 1: Nuisance model parameters.

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5. RESULTS

In this section, two cases are studied. First, the parameters are estimated while assuming the nuisance parameters perfectly known, then the latter are assumed uncertain. In both cases, the prior search space for the parameters is taken as:

$$\sqrt{\tau_p} \in [1, 30] \ \text{s}^{1/2} \text{ and } R_p \in \left[10^{-4}, 5\right] \text{m}^2.\text{K}.\text{W}^{-1} \tag{22}$$

Bounded-error identification with set inversion, nuisance parameters assumed known
SIVIA with a contractor, $\varepsilon = 0.001$ derives in 2 s the inner and outer approximations plotted in figure 5. The projection of the outer approximation $\overline{\mathbb{C}}$ onto the parameter axes provides an outer approximation of the uncertainty associated with each of the identified parameters

$$a = 8.167 \cdot 10^{-8} \pm 8.3\% \ \text{m}^2.\text{s}^{-1} \tag{23}$$

and
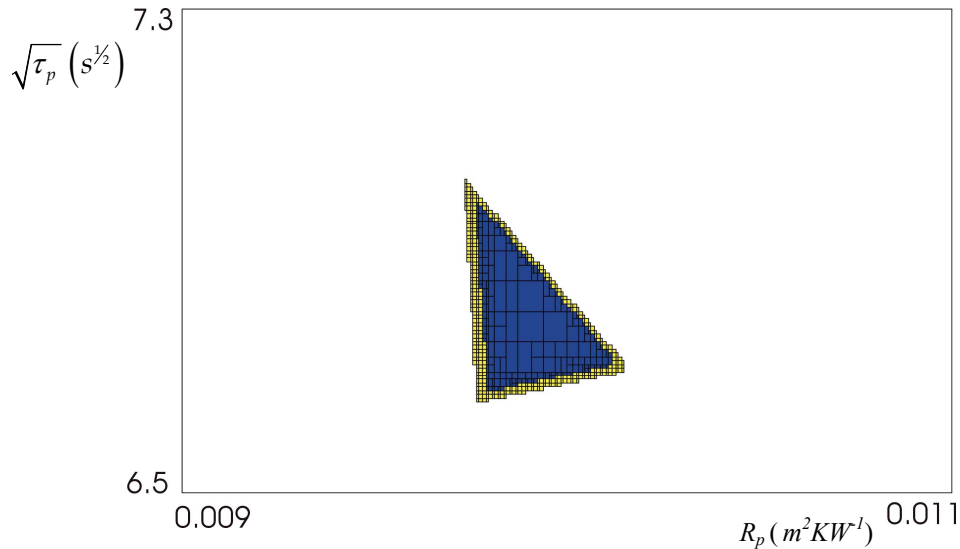$$\lambda = 0.201 \pm 4.6\% \ \text{W.m}^{-1}.\text{K}^{-1} \tag{24}$$

Figure 5: Inner (*dark boxes*) and outer approximation of posterior feasible set $\mathbb{C}$.
(*Light grey boxes are the uncertainty layer* $\Delta\mathbb{C}$)

Compare (23)-(24) with the estimated parameters given by least square estimation, *i.e.*

$$a = 8.767 \cdot 10^{-8} \pm 1.9\% \ \text{m}^2.\text{s}^{-1} \tag{25}$$

and
$$\lambda = 0.179 \pm 0.4\% \ \text{W.m}^{-1}.\text{K}^{-1} \tag{26}$$

The reader can see that the estimated uncertainty in the identified parameters is much smaller for least squares results. Still, the value identified for the diffusivity parameter a with set inversion is consistent with the one given by least squares.

However, the values identified for the conductivity parameter λ with both methods are significantly different. In order to explain this result, we have checked both identified models outputs: we found that the output of the model identified with least squares is not included in the prior bounds for experimental data at the lowest frequency, which is possible because this was not the purpose of least squares estimation. At the contrary, the output of the model given by set inversion is indeed included in the prior bounds for experimental data as this was the purpose of the bounded error estimation technique.

Bounded-error identification with set projection, nuisance parameters assumed uncertain

Now assume the nuisance parameters are uncertain, PROJECT derives in $300s$ the inner and outer approximations plotted in figure 6. The large thickness of the uncertainty layer is due to the pessimism of the contractors and inclusion functions employed, which limits the quality of the results.

The projection of the outer approximation $\overline{\Pi}$ onto the parameter axes provides an outer approximation of the uncertainty associated with each of the identified parameters

$$a = 8.13 \cdot 10^{-8} \pm 22\% \ \text{m}^2.\text{s}^{-1} \tag{27}$$

and

$$\lambda = 0.205 \ \pm \ 15\% \ \text{W.m}^{-1}.\text{K}^{-1} \tag{28}$$

As expected, the uncertainty in the nuisance parameters leads to much larger uncertainties in the identified parameters.
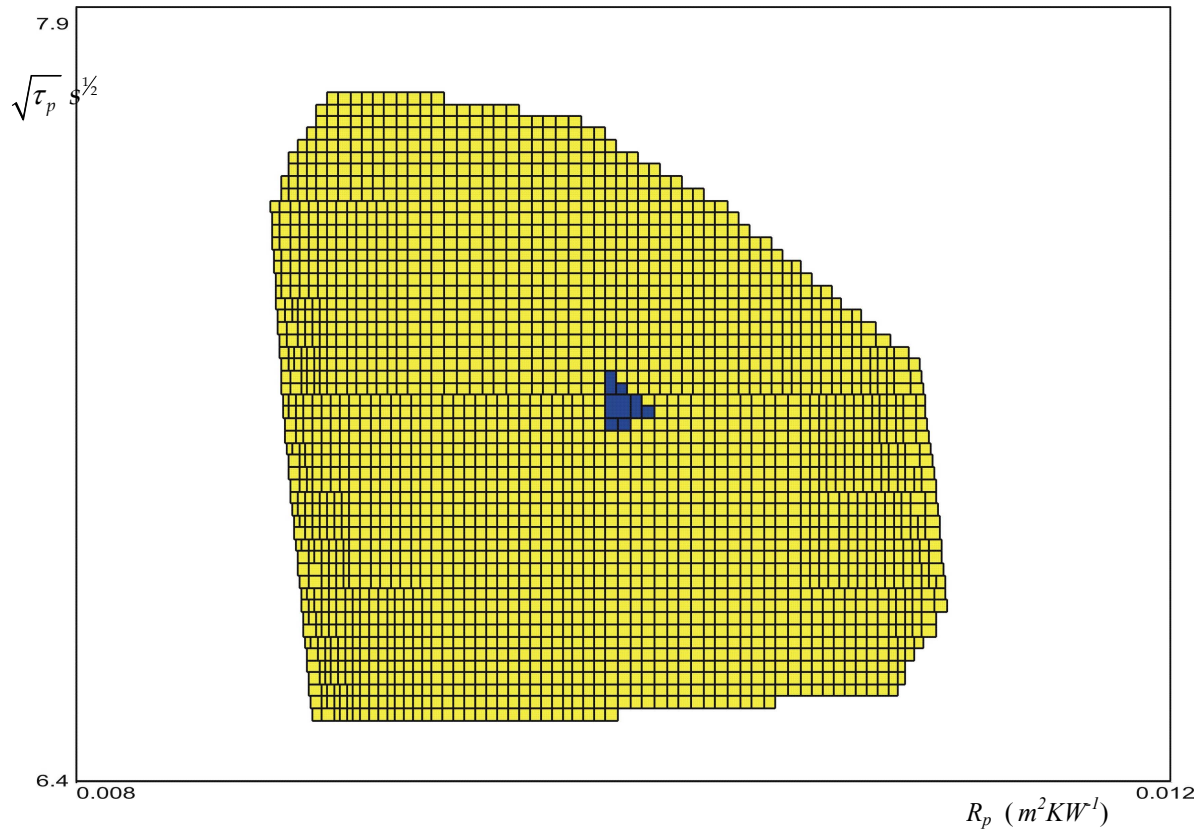

Figure 6: Inner (*dark boxes*) and outer approximation of posterior feasible set Π.
(*Light grey boxes are the uncertainty layer* $\Delta\Pi$).

CONCLUSION

In this paper we have addressed the problem of reliable parameter estimation in presence of model uncertainty resulting from nuisance parameters.

In the first part of this paper, we assumed that the nuisance parameter vector was accurately known. Assuming that the errors on model output were bounded but otherwise unknown, we have showed that estimating the parameters in such a framework is a set inversion problem. The algorithm SIVIA has been used with data taken from an actual experimental thermal set-up. The new method generates inner and outer approximations for the solution set which provides a simple evaluation of the estimation uncertainty.

In the second part of this paper, we have addressed the problem of estimating the feasible parameter set when the nuisance parameter are uncertain. We have shown that estimating the parameters in this framework amounts

to projecting the solution set onto the space of the parameters of interest. We characterized this projection with the algorithm PROJECT. The volume of this projected set is larger than the one derived when the nuisance parameters are assumed accurately known: taking into account uncertainty increases the uncertainty associated with the estimation. One is now capable of accounting for bounded uncertainty in nuisance parameters in a reliable and guaranteed way.

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