Efficient method for guard set intersection in nonlinear hybrid reachability *

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∗. M. Maïga, N. Ramdani, and L. Travé-Massuyès. A fast method for solving guard set intersection in nonlinear hybrid reachability. Florence, Italy, 2013. In Proc. of 52nd IEEE Conference on Decision and Control, CDC 2013

Outline

ANR-Project : MAGIC-SPS

- Goal : To develop guaranteed methods and algorithms for integrity control and preventive monitoring of systems
- Different work package :
  - WP1 : Modelling and identification of systems with bounded uncertainties ;
  - WP2 : Identifiability and diagnosability of systems with bounded uncertainties ;
  - WP3 : Preventive monitoring of continuous systems with bounded uncertainties ;
  - WP4 : Preventive monitoring of hybrid systems with bounded uncertainties ;
  - WP5 : Dissemination
- Project duration = october 2012 to december 2014
- Partners

ANR-Project : MAGIC-SPS

Our work package : WP4

Given a Hybrid Dynamical System (HDS) on which we would like to do :

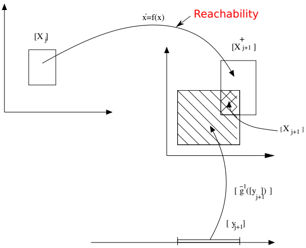
1. Nonlinear hybrid reachability ;
2. State estimation and diagnosis of HDS ;
3. Feasibility of fault prognosis for HDS

Outline

Why reachability ?

Continuous systems

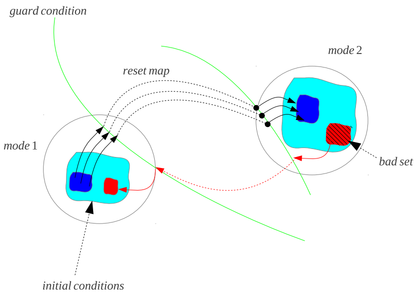
- State estimation ([Combastel 2003] ;[Raïssi, Ramdani 2004] ; [Alamo 2005, 2008] ; [Mojtaba,Vu Tuan 2013])
- Fault detection, diagnosis ([Ingimundarson 2008], [Sid-Ahmed 2013])
- Robust Control ([Kerrigan, 2004] ;[Bravo 2006] ; [Raimondo, Gonzalez, 2011] [Rakovi'c 2012])



Why reachability ?

Hybrid systems

- Verification
  (CheckMate [Chutinan 99] ; HyperTech [Henzinger 00] ;d/dt [Asarin, 2002] ; PHAVer, HyTech [Freshe 2005] ; SpaceEx [Freshe 2011])
- Synthesis
  Synthesis for timed automata [Asarin 98]
  Hamilton Jacobi Partial Diff. Eq.[Tomlin, 2003]



Hybrid system

Hybrid automaton (Alur, *et al.*, 95)

$$H = (\mathcal{Q}, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \mathrm{Inv}, \mathcal{F}),$$

Continuous transition

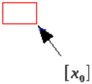
$$\begin{aligned} \mathrm{flow}(q):&\quad \dot{\mathbf{x}}(t) = f_q(\mathbf{x}, \mathbf{p}, t), \\ \mathrm{Inv}(q):&\quad \nu_q(\mathbf{x}(t), \mathbf{p}, t) < 0 \end{aligned}$$

Discrete transition

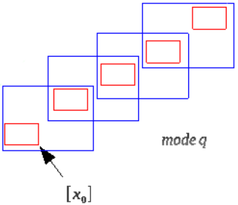
$$\begin{aligned} e:&\quad (q \rightarrow q') = (q, \mathrm{guard}, \sigma, \rho, q'), \\ \mathrm{guard}(e):&\quad \gamma_e(\mathbf{x}(t), \mathbf{p}, t) = 0 \end{aligned}$$

$$t_0 \leq t \leq t_N, \quad \mathbf{x}(t_0) \in \mathbb{X}_0 \subseteq \mathbb{R}^n, \quad \mathbf{p} \in \mathbb{P}$$

Running hybrid system



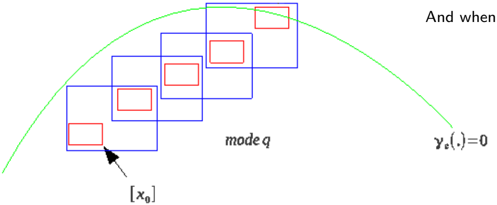
Running hybrid system



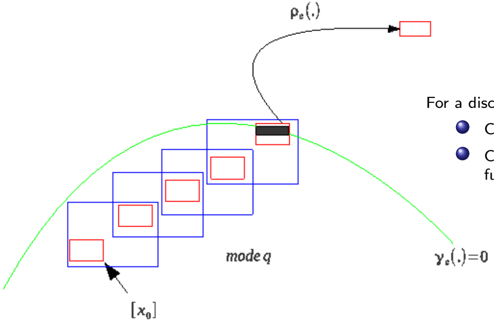
In a mode

- Continuous transition by using Interval Taylor Methods ;
- Compute flowpipe/invariant intersections.

Running hybrid system



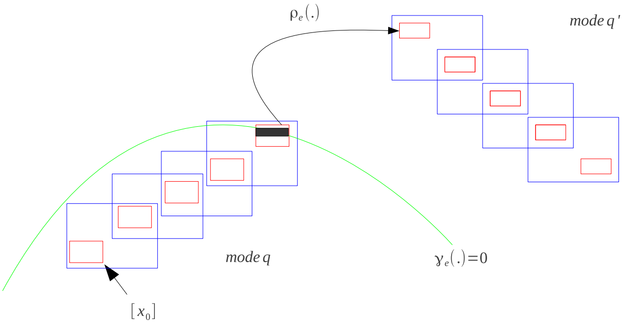
And when the flow reached guard condition.



For a discrete transition

- Compute flowpipe/guard intersections ;
- Compute the image of the sub-flowpipe by reset function.

Running hybrid system



After Jump, continuous expansion in new mode q' and so on...

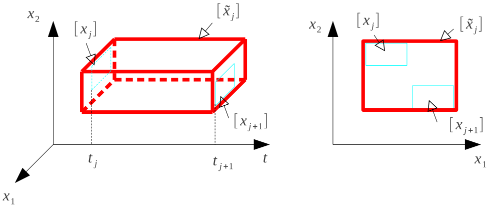
Outline

Guaranteed set integration with intervals Taylor methods

(Moore,66) (Eijgenraam,81) (Lohner,88) (Rihm,94) (Berz,98) (Nedialkov,99)

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, \mathbf{p}, t), \quad t_0 \le t \le t_N,\ \mathbf{x}(t_0) \in [\mathbf{x}_0],\ \mathbf{p} \in [\mathbf{p}]$$

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$



- Analytical solution for $[\mathbf{x}](t),\ t \in [t_j, t_{j+1}]$

$$[\mathbf{x}](t) = [\mathbf{x}_j] + \sum_{i=1}^{k-1} (t - t_j)^i \mathbf{f}^{[i]}([\mathbf{x}_j], [\mathbf{p}]) + (t - t_j)^k \mathbf{f}^{[k]}([\tilde{\mathbf{x}}_j], [\mathbf{p}])$$

Guaranteed set integration with Taylor methods

(Moore,66) (Eijgenraam,81) (Lohner,88) (Rihm,94) (Berz,98) (Nedialkov,99)

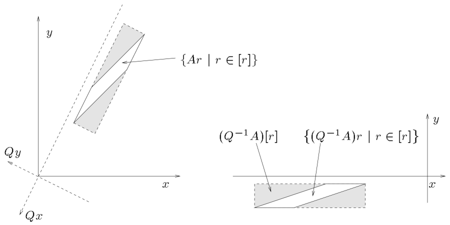
$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, \mathbf{p}, t), \quad t_0 \leq t \leq t_N,\ \mathbf{x}(t_0) \in [\mathbf{x}_0],\ \mathbf{p} \in [\mathbf{p}]$$

Mean-value approach

mean value forms + matrice preconditioning+ linear transforms

$$[\mathbf{x}](t) \in \left\{ \mathbf{A}(t)\mathbf{r}(t) + \mathbf{v}(t) \mid \quad \mathbf{r}(t) \in [\mathbf{r}](t),\ \mathbf{v}(t) \in [\mathbf{v}](t) \right\}.$$

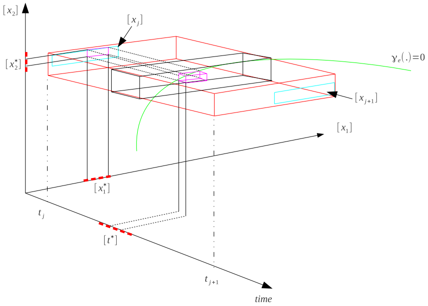
A=Q obtain via Lohner's QR-factorization (Lohner, 87)



Outline

Computing flow/guards intersection (Ramdani & Nedialkov, 2011)

Time grid $\rightarrow$ $t_0 < t_1 < t_2 < \cdots < t_N$



- **Interval Taylor methods**
  $\Rightarrow$ Analytical expressions for the boundaries of the continuous flows,
  $\Rightarrow$ Controlling wrapping effect
- **Interval constraint propagation techniques**
  $\Rightarrow$ Solve event detection/localization problems as an CSP (Exponential complexity)
- Find all combinations $[\mathbf{x}_j^\star] \times [\underline{t}^\star, \overline{t}^\star]$

Computing flow/guards intersection (Ramdani & Nedialkov, 2011)

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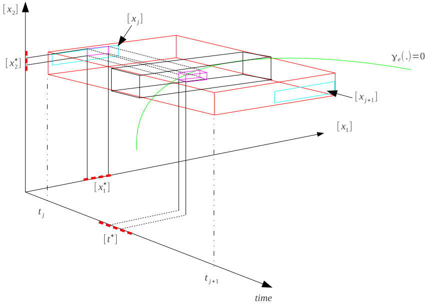

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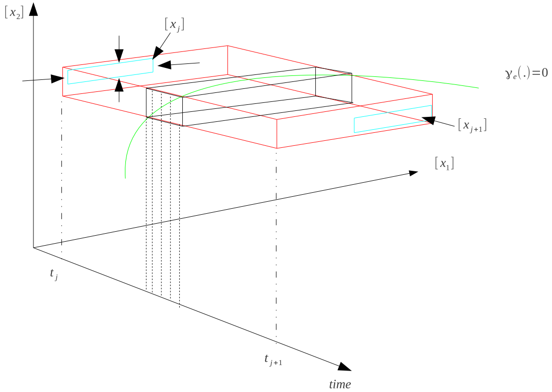
- Find all combinations $[\mathbf{x}_j^\star] \times [\underline{t}^\star, \overline{t}^\star] \rightarrow O(2^{n+1})$

In this talk

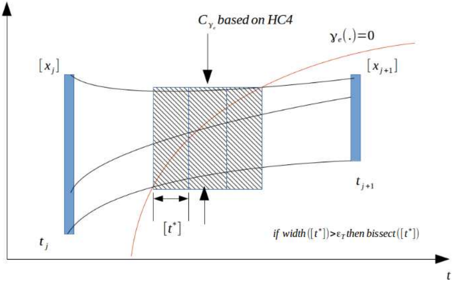
- Guaranteed relaxation of event detection/localization problems (scalable method) ;
- Test this new method on some benchmarks.

Our new method



- No need to find all the initial conditions ;

Our new method



- Main idea 1 :Bisection is performed only in the direction of the time variable ;
- Main idea 2 :The domain of the state variables are contracted upon each subinterval ;

Our new method

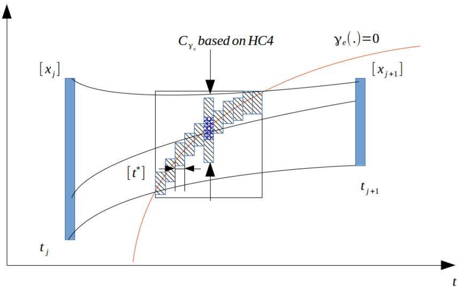


Figure: results after many contraction

Event $e = q \to q'$ occurs if :

- width($[t^*]$) $\leq \varepsilon_T$.

Outline

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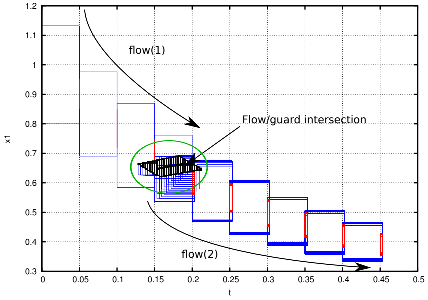
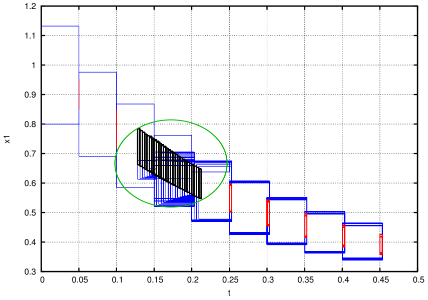
Consider a hybrid dynamical system (Brusselator), $q = 1, 2$ and one jump $e = 1 \to 2$ given by :

$$\begin{cases} \text{flow}(1) : f_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 - (b_1 + 1)x_1 + a_1 x_1^2 x_2 \\ b_1 x_1 - a_1 x_1^2 x_2 \end{pmatrix} \\ \text{inv}(1) : \nu_1(x_1, x_2) = -4x_1 + x_2 + 2 \\ \text{flow}(2) : f_2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 - (b_2 + 1)x_1 + a_2 x_1^2 x_2 \\ b_2 x_1 - a_2 x_1^2 x_2 \end{pmatrix} \\ \text{inv}(2) : \nu_2(x_1, x_2) = -\nu_1(x_1, x_2) \\ \text{guard}(1) : \gamma_1(x_1, x_2) = \nu_1(x_1, x_2) \\ \text{reset}(1) : \rho_1(x_1, x_2) = (\alpha_1 x_1, \alpha_2 x_2) \end{cases} \tag{1}$$

with $\alpha_1 = \alpha_2 = (1; 1)$, $a_1 = 1.5$, $a_2 = 3.5$, $b_1 = 1$, $b_2 = 3.5$ and $x_0 \in [1, 1.1] \times [0.1, 0.3]$. We took for this simulation a constant integration time step $h = 0.05$,$\varepsilon_Z = 1$ and $\varepsilon_T = 0.005$.



(a) x_1 flow with HC4 contractor

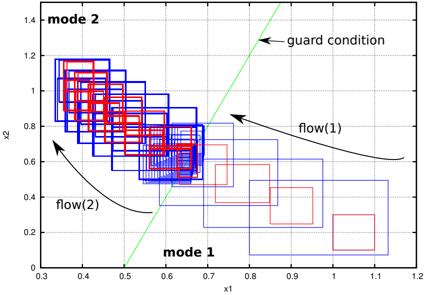


(b) x_1 flow without HC4 contractor

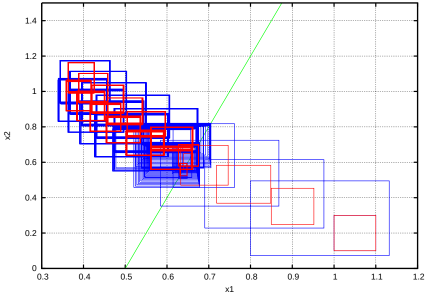
TCG † with contractor=0.148s

TCG without contractor=0.192s

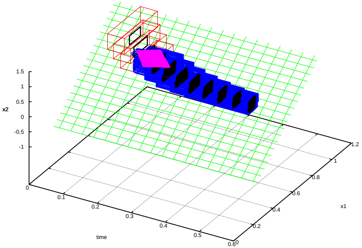
†. TOTAL TIME CROSSING GUARD CONDITION



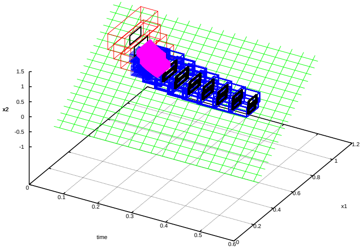
(c) phase plan $x_2 \times x_1$ with HC4 contractor



(d) phase plan $x_2 \times x_1$ without HC4 contractor



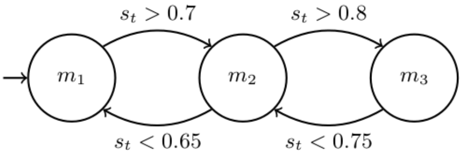
(e) 3D with HC4 contractor



(f) 3D without HC4 contractor

Outline

Evaluation on Benchmarks : Vehicle Model ‡



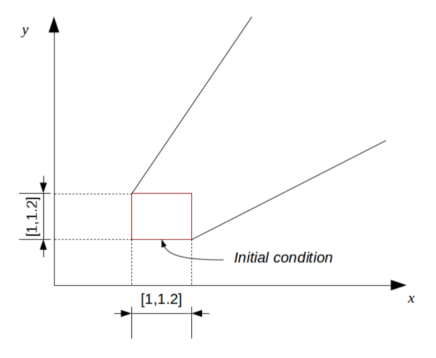
$$\frac{dx}{dt} = vc_t; \frac{dy}{dt} = vs_t; \frac{dv}{dt} = u_1$$
$$\frac{dc_t}{dt} = \sigma v^2 s_t; \frac{ds_t}{dt} = -\sigma v^2 c_t; \frac{d\sigma}{dt} = u_2$$

$$x \in [1, 1.2] \quad y \in [1, 1.2] \quad v \in [0.8, 0.81]$$
$$s_t \in [0.7, 0.71] \quad c_t \in [0.7, 0.71] \quad \sigma = [0, 0.05]$$

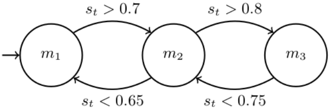
‡. Bench proposed by Sriram Sankaranarayanan and Xin Chen

Goal of this benchmark ?

Find all positions reached by vehicule over t$\in [0, 10]$

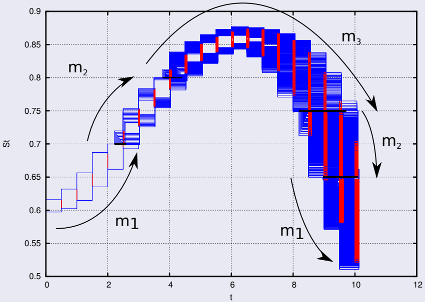


$$x \in [1, 1.2] \quad y \in [1, 1.2] \quad v \in [0.8, 0.81]$$
$$s_t \in [0.7, 0.71] \quad c_t \in [0.7, 0.71] \quad \sigma = [0, 0.05]$$

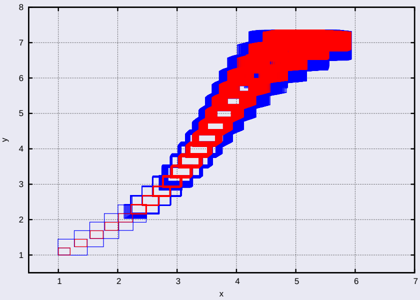


Results : Vehicle Model

$\sigma = [0, 0.01]$ and h=0.5

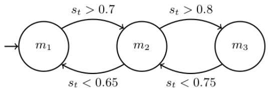


(g) St×t



(h) Y×X space

CPU times=87s with HC4 contractor
CPU times$>$ 1h without HC4 contractor



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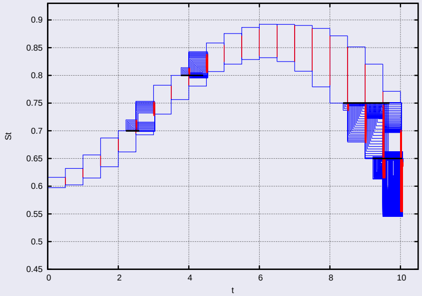
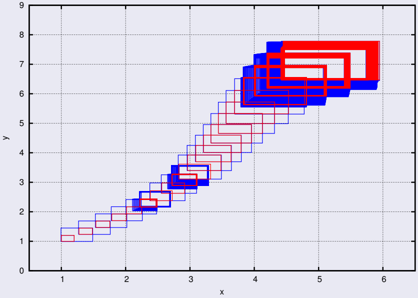


(i) St×t



(j) Y×X space

CPU times=87s with HC4 contractor
CPU times=7s with HC4 contractor and zonotope enclosure of boxes
(M. Maiga, C. Combastel, N. Ramdani, L. Travé-Massuyès, submitted to ECC14 Strasbourg)

Outline

Concluding remarks

Conclusion

- Tight overapproximation of flow/guard intersection ;
- Positive impact on computation times.

Current works : develop scalable methods

→ Merging boxes without over-approximation (submitted to ECC14 Strasbourg)

→ Building a scalable methods for nonlinear hybrid reachability analysis (submitted to HSCC 2014 Berlin) ;

→ All developed methods for reachability will be used for state estimation and diagnosis of SDH.

Thank You !

Questions ?