Verification of the convergence properties of non-holonomous robots using interval analysis and Lyapunov methods.

PLAN

- Problem Definition
- Tools used
- Problem approach
- Application with a non holonomous system
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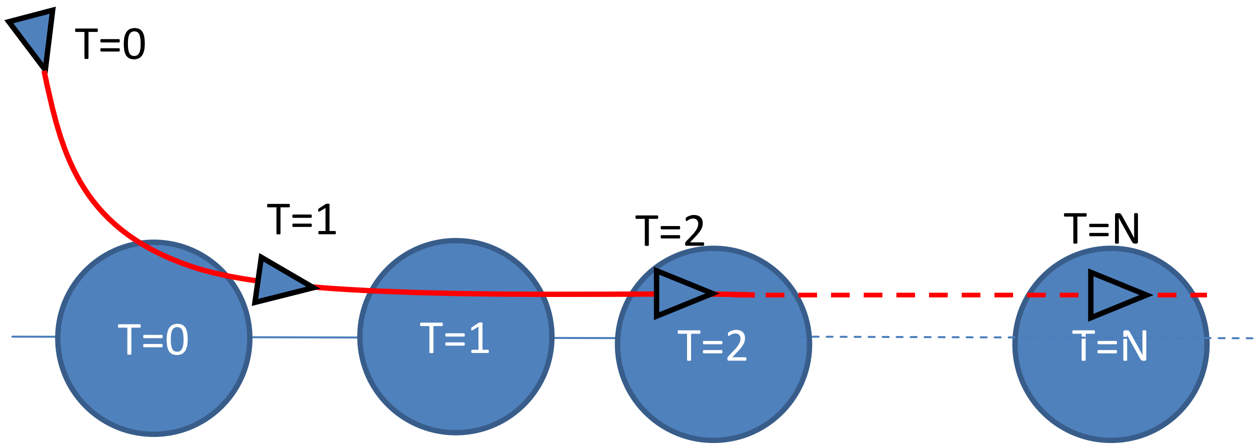
Problem Definition

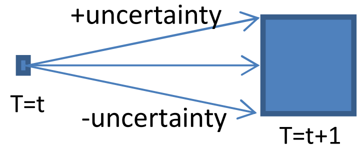
- Validate that the mobile will follow a time-dependent trajectory .
- Equivalent to say that the mobile will stay in a time-moving bubble.
- Validate the regulator and the trajectory.



Problem Definition

- Uncertainties occur in the system evolution → differential inclusions.

$$\dot{x} \in F(x)$$



- Allow to:

  - adjust parameters of the controller

  - specify the maximum uncertainties

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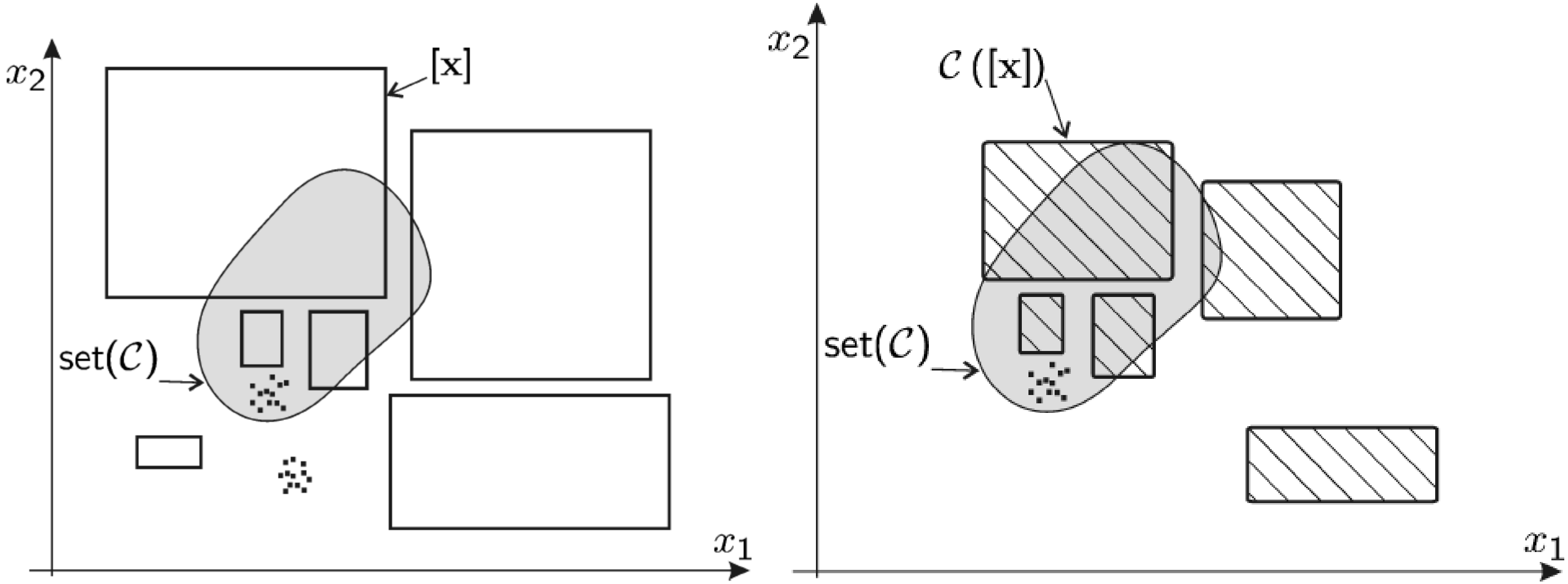
Contractors

The operator $\mathcal{C}_{\mathbb{X}} : \mathbb{IR}^n \longrightarrow \mathbb{IR}^n$ is a *contractor* for $\mathbb{X} \subset \mathbb{R}^n$ if

$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \begin{cases} \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)}, \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(completeness)}. \end{cases}$$

Contractors



SIVIA

- The algorithm used to solve the problem is a SIVIA (Set Inversion Via Interval Analysis) with contractors.
- Contractions and bissections

IBEX

- C++ librairy which allow easy contractors implementation.

http://www.emn.fr/z-info/ibex/

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V-Stability or Lyapunov method

$$S\text{: } \dot{x} = f(x)$$

Due to system evolution uncertainties , we get:

$$S\text{: } \dot{x} \in F(x)$$

which represent a differential inclusion

- Definition:
  - S is Lyapunov-stable if ∃ $V(x) > 0$ such that

$$\dot{V}(x) < 0 \; if \; x \neq 0,$$
$$V(x) = 0 \; iff \; x = 0$$

  - S is (V, v^+)−stable iff $V(x) \in [0, v^+] \rightarrow \; \dot{V}(x) < 0$

With V a Lyapunov function and $v^+ > 0$.

How to use V-Stability

- Define A= $\{x \in \mathbb{R}^n /\ V(x) < v^+\}$ and B=$\{x \in \mathbb{R}^n / V(x) < 0\}$.

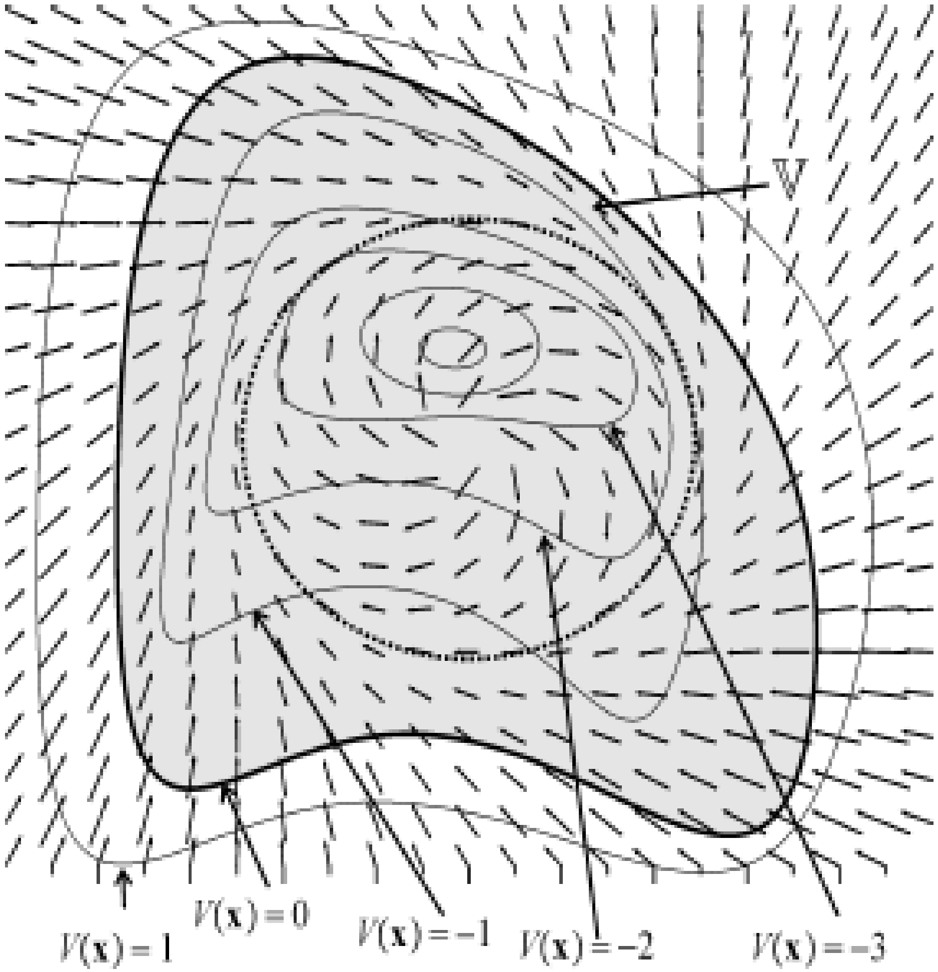
- If S is (V, v^+)−stable then

$$\begin{cases} \forall x(0) \in A, \exists t > 0\ tq\ x(t) \in B \\ \forall x(t) \in B, \forall \tau > 0, x(t+\tau) \in B \end{cases}$$

- Theorem :

$$\begin{cases} \dfrac{dV}{dx}.a \geq 0 \\ a \in F(x) \\ V(x) \in [0, v^+] = A \setminus B \end{cases} \quad inconsistent \Leftrightarrow \dot{x} \in F(x)\ est\ V\ stable$$

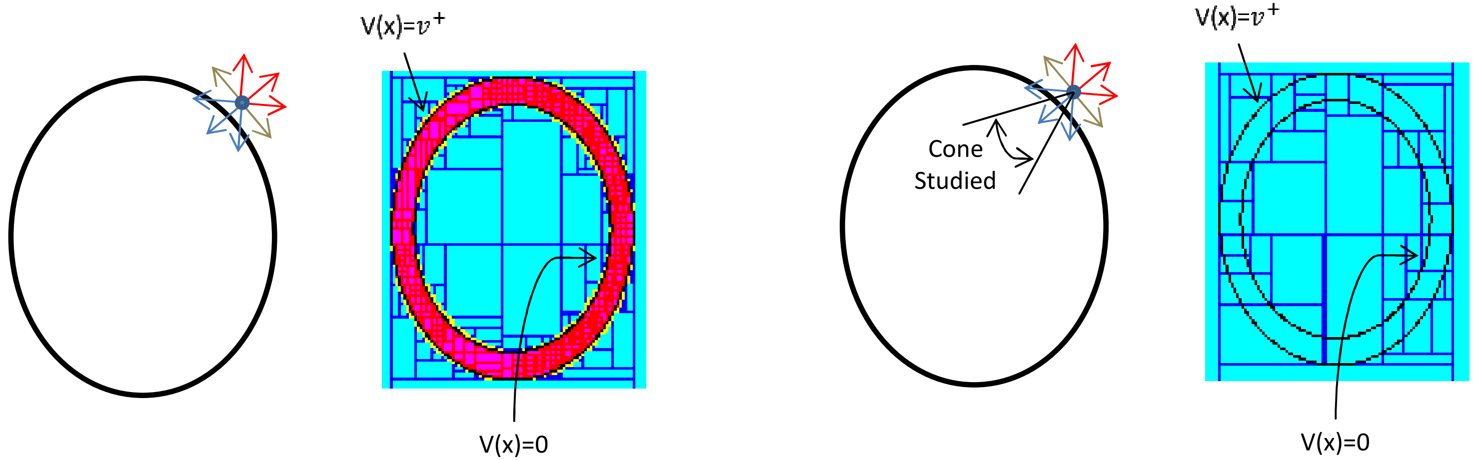
V-Stability or Lyapunov method



From V-stability to « A to B moving »

- Non-holonomous mobile and 2D-projection (x,y) of the results

→ Need to restrain the initial set A.



A to B moving

- Set A no more define with function V but with inequalities :

$$\mathbb{A} = \{x \mid \forall i \in \{1, \dots, \dim(B)\}, a_i(x) \leq 0\}$$
$$V(\mathbb{A}) \in \mathbb{R}^+$$
$$\mathbb{B} = \{x \mid \forall i \in \{1, \dots, \dim(B)\}, b_i(x) \leq 0\}$$

- « A to B moving » if

$$\forall x(0) \in A, \exists t_1 > 0 \ tq \begin{cases} x([0, t_1]) \subset \mathbb{A} \\ x(t_1) \in \mathbb{B} \end{cases}$$

- Condition $\dot{V} < 0$ is not enough to garanty « A to B moving ».

A to B moving

- Theorem 1: Assume that

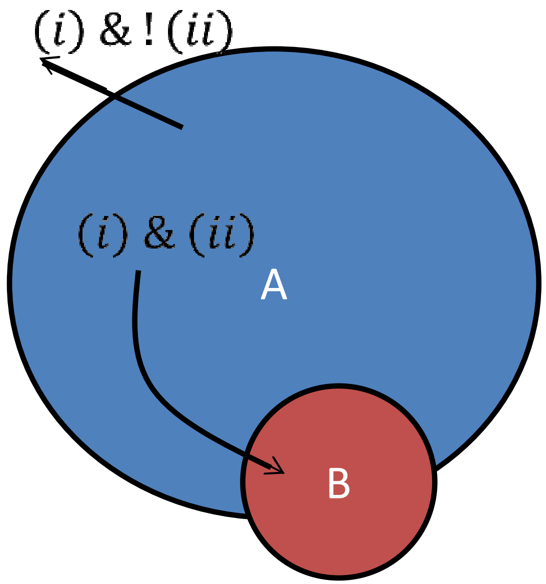
$$(i)\ x \in \mathbb{A} \setminus \mathbb{B} \Rightarrow \dot{V}(x) < 0$$

$$(ii)\ x \in \partial\mathbb{A} \setminus \mathbb{B}, a_i(x) = 0 \Rightarrow \langle f(x), \nabla a_i(x) \rangle < 0$$

Then the system is $\mathbb{A}\ to\ \mathbb{B}\ moving$

- $\begin{cases} (i) \Rightarrow x\ will\ leave\ \mathbb{A} \setminus \mathbb{B}\ at\ time\ t_1 \\ \qquad (ii) \Rightarrow x(t_1)\ will\ be\ in\ \mathbb{B} \end{cases}$

A to B moving



A to B moving

- Theorem 2 :

$$\begin{cases} x \in \mathbb{A} \setminus \mathbb{B} \; and \; \dot{V}(x) \geq 0 \\ or \; \exists i, (\langle f(x), \nabla a_i(x) \rangle \geq 0 \; and \; x \in \partial\mathbb{A} \setminus \mathbb{B} \; and \; a_i(x) = 0 \end{cases} is \; inconsistent$$

$$\Rightarrow \; the \; system \; is \; \mathbb{A} \; to \; \mathbb{B} \; moving$$

- We use contractors to implement this system.

$$C = \left(C^{\mathbb{A}} \cap C^{\overline{\mathbb{B}}} \cap C^{\dot{V} \geq 0}\right) \cup \left(C^{\partial\mathbb{A}} \cap C^{\overline{\mathbb{B}}} \cap \bigcup_i \left(C^{\left\langle f(x), \frac{\partial}{\partial x} a_i(x) \geq 0 \right\rangle} \cap C^{a_i(x)=0}\right)\right)$$

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Application with a non holonomous system

- State equation : $\begin{cases} \dot{x} = v\cos\theta + \varepsilon_x \\ \dot{y} = v\sin\theta + \varepsilon_y \\ \dot{\theta} = u \end{cases}$
- Lyapunov function:

$$V(x, y, \theta, t) = \frac{1}{2}\left((x-t)^2 + y^2\right) - 4$$

Application with a non holonomous system

- Initial Set :

$$\mathbb{A}: \begin{cases} (i) \cos(\theta^* + \pi - \theta) < -\cos(Cone_{theta}) \\ (ii)\, V(x) \leq V^+ \end{cases}$$

- Target Set $\mathbb{B}$: $\{(x, y, \theta, t) \mid V(x) \leq 0\}$

Is it « A to B moving » ?

1. $\begin{cases} (x, y, \theta) \in \mathbb{A} \backslash \mathbb{B} \\ \dot{V}(x) > 0 \end{cases}$ is inconsistent

2. $\begin{cases} (x, y, \theta) \in \mathbb{A} \backslash \mathbb{B} \\ a(x) = \cos(\theta^{*} + \pi - \theta) = -\cos(Cone_{theta} \\ \langle \nabla a(x), f(x) \rangle > 0 \end{cases}$

is inconsistent

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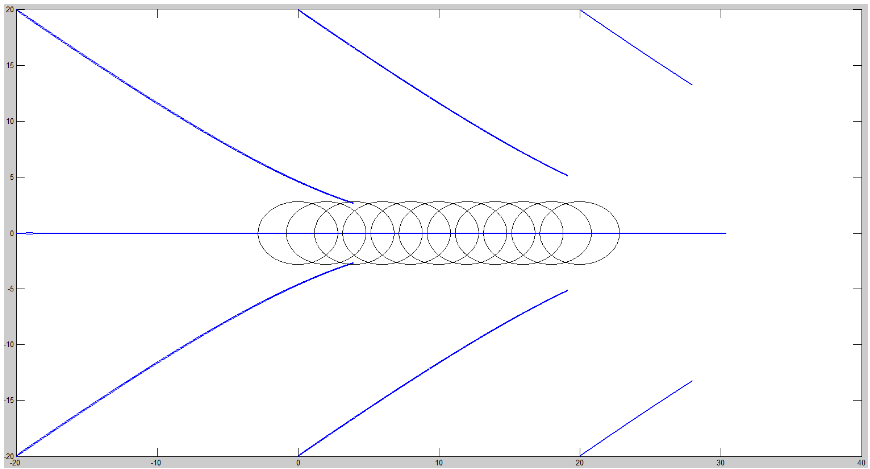
Outcomes

- Simulation regulator :

$$v = v_{min} + \frac{2\,\mathrm{atan}(t - x) + \pi}{2\pi}(v_{max} - v_{min})$$

$$u = K(\theta^* - \theta)$$

$$avec\ \theta^*(y) = -2 * \frac{\gamma_{inf}}{\pi} * \mathrm{atan}\left(\frac{y}{r}\right)$$



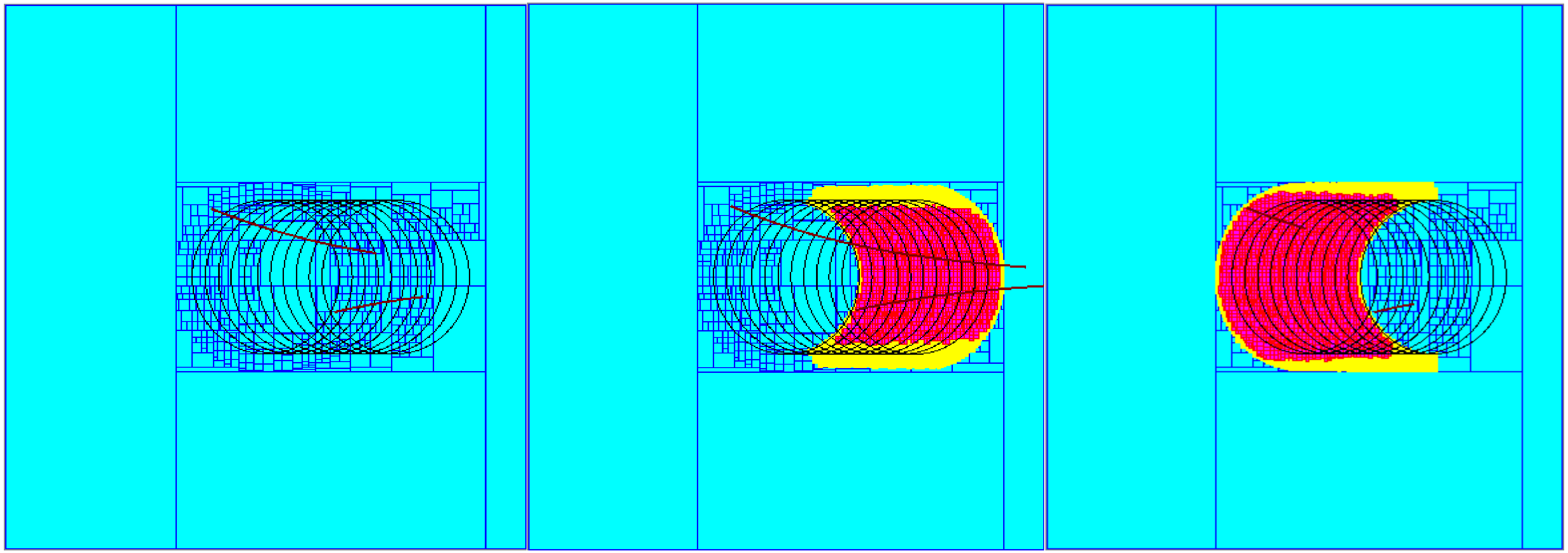
Outcomes

- Simulation values :

$$Cone_{theta} = \frac{\pi}{16}$$
$$\gamma_{inf} = \frac{\pi}{4}$$
$$K = 5$$
$$\varepsilon_x = 0.02$$
$$\varepsilon_y = 0.02$$
$$V^+ = 2$$
$$t = 5$$

- Simulation parameters : $\varepsilon_{boxes} = 0.2$

Outcomes



$v_{min} = 0.5$ $v_{\max} = 1.5$ $v_{min} = 2.0$ $v_{\max} = 3.0$ $v_{min} = 0.2$ $v_{\max} = 0.5$

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To go further

- 3D mobile evolution (x,y,z,t)
- Curve following and obstacle avoidance.
- Position dependant uncertainties.
- Multi-vehicule

Thank you for your attention