Range-only SLAM with indistinguishable landmarks

Paris, november 13, 2013.

L. Jaulin
ENSTA-Brest, IHSEV, OSM, LabSTICC.

http://www.ensta-bretagne.fr/jaulin/

1 SLAM problem

Robot: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}),\ \mathbf{x}(0) = 0$.

Marks $\mathcal{M} = \{\mathbf{m}(1), \mathbf{m}(2), \dots\} \subset \mathbb{R}^q$.

(i) the map is static,

(ii) indistinguishable point marks

(iii) the marks are partially observable

Our SLAM is a *chicken and egg* problem of cardinality three:
(i) if the map and the associations are known, we have localization problem,
(ii) if the trajectory and the associations are known, we have a mapping problem
(iii) if the trajectory and the map are known we have an association problem.

The unknown variables have an heterogenous nature:

(i) marks $\mathbf{m}(j) \in \mathbb{R}^q$

(ii) trajectory $\mathbf{x}(t) : \mathbb{R} \to \mathbb{R}^n$,

(iii) the free space $\mathbb{F} \in \mathcal{P}(\mathbb{R}^q)$

(iv) the data associations is a graph $\mathcal{G}$.

2 Formalization

A *sector* $\mathbb{H}$ is a subset of $\mathbb{R}^q$ which contains a single mark.

Our SLAM problem:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) & \text{(evolution equation)} \\ (t_i, \mathcal{H}_i(\mathbf{x})) & \text{(sector list)} \end{cases}$$

where $t \in [0, t_{\mathsf{max}}]$, $\mathbf{u}(t) \in [\mathbf{u}](t)$.

Each set $\mathcal{H}_i(\mathbf{x}(t_i)) \subset \mathbb{R}^q$ contains a unique mark.

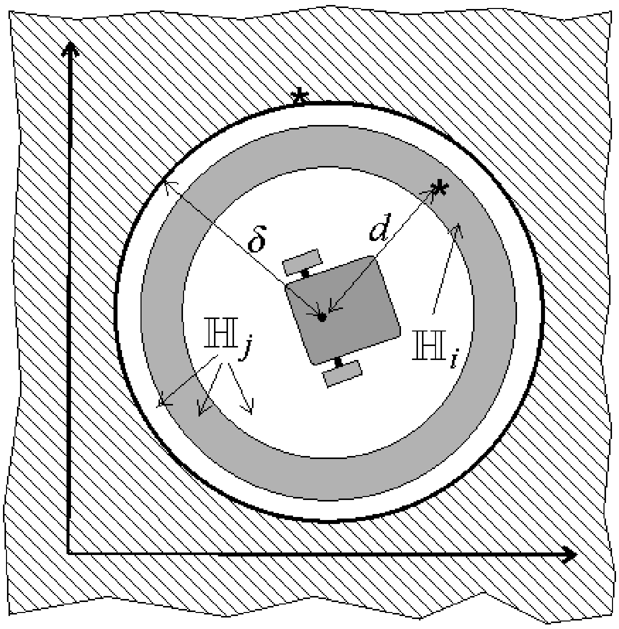
We have an egocentric representation.

We define $\mathbb{H}_i = \mathcal{H}_i(\mathbf{x}(t_i))$.

Example 1. A robot moving in a plane and located at (x_1, x_2). At t_3 the robot detects a unique mark at a distance $d \in [4, 5]$. We have

$$\mathcal{H}_3(\mathbf{x}) = \left\{ \mathbf{a} \in \mathbb{R}^2 | (x_1 - a_1)^2 + (x_2 - a_2)^2 \in [16, 25] \right\}.$$

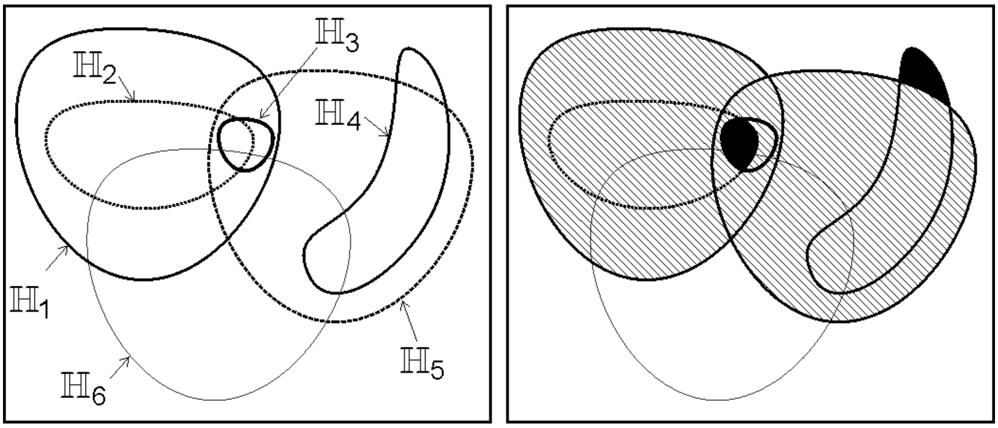
Example 2. We have two sectors $\mathbb{H}_i$ and $\mathbb{H}_j$.
Since $\mathbb{H}_i \subset \mathbb{H}_j$, $\mathbb{H}_j \backslash \mathbb{H}_i$ has no mark. Thus we can associate $\mathbb{H}_i$ with $\mathbb{H}_j$.



Theorem. Define the free space as $\mathbb{F} = \{\mathbf{p} \in \mathbb{R}^q \mid \mathbf{p} \notin \mathcal{M}\}$. Consider m sectors $\mathbb{H}_1, \ldots, \mathbb{H}_m$. Denote by $\mathbf{a}(i)$ the mark in $\mathbb{H}_i$. We have

$$\begin{array}{ll} \text{(i)} & \mathbb{H}_i \subset \mathbb{H}_j \Rightarrow \mathbf{a}(i) = \mathbf{a}(j) \\ \text{(ii)} & \mathbb{H}_i \cap \mathbb{H}_j = \emptyset \Rightarrow \mathbf{a}(i) \neq \mathbf{a}(j) \\ \text{(iii)} & \mathbb{H}_i \subset \mathbb{H}_j \Rightarrow \mathbb{H}_j \backslash \mathbb{H}_i \subset \mathbb{F}. \end{array}$$

Example.



The two black zones contain a single mark and no mark exists in the hatched area.

Association graph. Consider m detections $\mathbf{a}(1), \ldots, \mathbf{a}(m)$. The *association graph* is the graph with nodes $\mathbf{a}(i)$ such that $\mathbf{a}(i) \to \mathbf{a}(j)$ means that $\mathbf{a}(i) = \mathbf{a}(j)$.

3 Generalized contractors

3.1 Lattices

A *lattice* $(\mathcal{E}, \leq)$ is a partially ordered set, closed under least upper and greatest lower bounds.
The *join*: $x \vee y$.
The *meet*: $x \wedge y$.

Example 1 . The set $(\mathbb{R}^n, \leq)$ is a lattice.
We have $\mathbf{x} \wedge \mathbf{y} = (x_1 \wedge y_1, \ldots, x_n \wedge y_n)$ and $\mathbf{x} \vee \mathbf{y} = (x_1 \vee y_1, \ldots, x_n \vee y_n)$ where $x_i \wedge y_i = \min(x_i, y_i)$ and $x_i \vee y_i = \max(x_i, y_i)$.

Example 2. If $\mathbb{E}$ is any set, the powerset $\mathcal{P}(\mathbb{E})$ is a complete lattice with respect to the inclusion $\subset$. The meet corresponds to the intersection and the join to the union.

Intervals. An *interval* $[x]$ of a complete lattice $\mathcal{E}$ is a subset of $\mathcal{E}$ which satisfies

$$[x] = \{x \in \mathcal{E} \mid \wedge[x] \le x \le \vee[x]\}\,.$$

Both $\emptyset$ and $\mathcal{E}$ are intervals of $\mathcal{E}$.

Example 3. The set $\mathcal{F}$ of all functions from $\mathbb{R}$ to $\bar{\mathbb{R}}^n$ is a complete lattice with $\mathbf{f} \leq \mathbf{g} \Leftrightarrow \forall t \in \mathbb{R},\ \mathbf{f}(t) \leq \mathbf{g}(t)$.
An interval of $\mathcal{F}$ is called a *tube*.

3.2 Contractors

A CSP is composed of
variables $\{x_1, \ldots, x_n\}$,
constraints $\{c_1, \ldots, c_m\}$ and
domains $\{\mathbb{X}_1, \ldots, \mathbb{X}_n\}$.

The domains $\mathbb{X}_i$ should belong to a lattice $(\mathcal{L}_i, \subset)$.

Here domains are
(i) subsets of $\mathbb{R}^n$ for the location of the marks,
(ii) tubes for the unknown trajectory and
(iii) intervals of subsets of $\mathbb{R}^n$ for the sectors and the free space.

Define $\mathcal{L} = \mathcal{L}_1 \times \cdots \times \mathcal{L}_n$.
An element $\mathbb{X}$ of $\mathcal{L}$ is the Cartesian product of n elements of $\mathcal{L}_i$: $\mathbb{X} = \mathbb{X}_1 \times \cdots \times \mathbb{X}_n$.
The set $\mathbb{X}$ will be called *hyperdomain*.

A *contractor* is an operator

$$\mathcal{C}: \begin{array}{ccc} \mathcal{L} & \rightarrow & \mathcal{L} \\ \mathbb{X} & \rightarrow & \mathcal{C}(\mathbb{X}) \end{array}$$

which satisfies

$$\begin{array}{ll} \mathbb{X} \subset \mathbb{Y} \Rightarrow \mathcal{C}(\mathbb{X}) \subset \mathcal{C}(\mathbb{Y}) & \text{(monotonicity)} \\ \mathcal{C}(\mathbb{X}) \subset \mathbb{X} & \text{(contractance)} \end{array}$$

3.3 Graph intervals

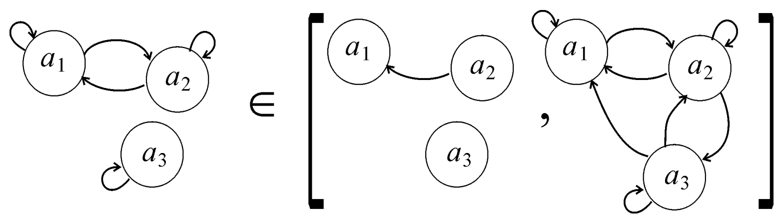
The set of graphs of $\mathcal{A}$ with the relation

$$\mathcal{G} \leq \mathcal{H} \Leftrightarrow \forall i, j \in \{1, \ldots, m\}\,,\ g_{ij} \leq h_{ij},$$

corresponds to a complete lattice. Intervals of graphs of $\mathcal{A}$ can thus be defined.

Example

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in \begin{pmatrix} [0,1] & [0,1] & 0 \\ 1 & [0,1] & [0,1] \\ [0,1] & [0,1] & [0,1] \end{pmatrix}$$



4 SLAM as a CSP

Variables

(i) the trajectory of the robot $\mathbf{x}$.

(ii) the sectors $\mathbb{H}_i$

(iii) the location of the mark $\mathbf{a}(i)$ detected at time t_i

(iv) the association graph $\mathcal{G}$

(v) the free space $\mathbb{F}$.

Domains

$$\mathbf{x} \in [\mathbf{x}] = \left[\mathbf{x}^{-}, \mathbf{x}^{+}\right]$$
$$\mathbf{a}\left(i\right) \in \mathbb{A}\left(i\right)$$
$$\mathbb{H}_i \in [\mathbb{H}_i] = \left[\mathbb{H}_i^{-}, \mathbb{H}_i^{+}\right]$$
$$\mathbb{F} \in [\mathbb{F}] = \left[\mathbb{F}^{-}, \mathbb{F}^{+}\right]$$
$$\mathcal{G} \in [\mathcal{G}] = \left[\mathcal{G}^{-}, \mathcal{G}^{+}\right].$$

Initialization

$[\mathbf{x}](t) = [-\infty, \infty]$ if $t > 0$ and $[\mathbf{x}](0) = \mathbf{0}$.

$\mathbb{A}(i) = \mathbb{R}^q$.

$\mathbb{H}_i \in [\emptyset, \mathbb{R}^q]$.

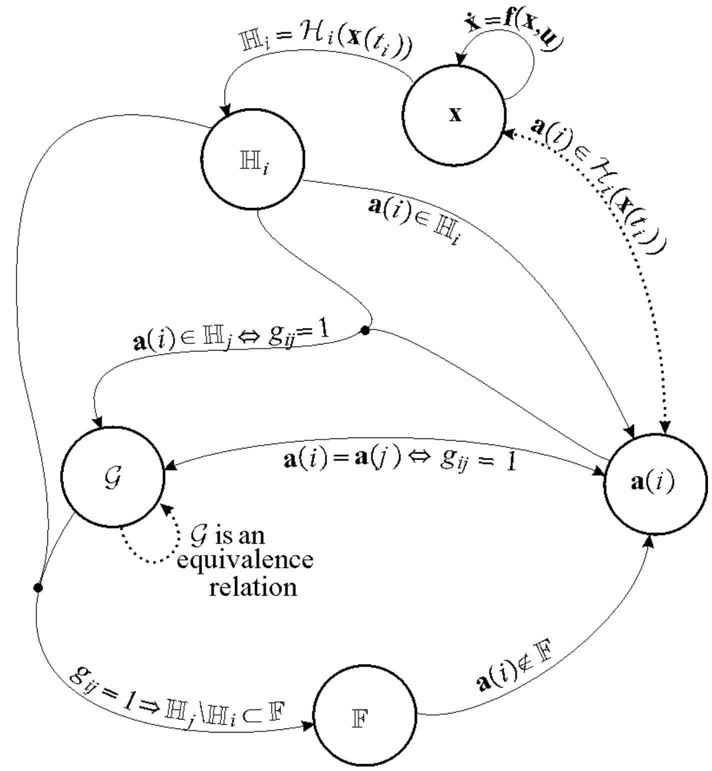
$\mathbb{F} \in [\emptyset, \mathbb{R}^q]$.

$\mathcal{G} \in [\emptyset, \top]$

Constraints

(i)	$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$
(ii)	$\mathbb{H}_i = \mathcal{H}_i(\mathbf{x}(t_i))$
(iii)	$\mathbf{a}(i) \in \mathbb{H}_i$
(iv)	$\mathbf{a}(i) = \mathbf{a}(j) \Leftrightarrow g_{ij} = \mathbf{1}$
(v)	$\mathbf{a}(i) \in \mathbb{H}_j \Leftrightarrow g_{ij} = \mathbf{1}$
(vi)	$g_{ij} = \mathbf{1} \Rightarrow \mathbb{H}_j \backslash \mathbb{H}_i \subset \mathbb{F}$
(vii)	$\mathbf{a}(i) \notin \mathbb{F}$

Contractors graph



5 Test-case

Generation of the data.

A simulated robot follows a cycloid for 100sec.

10 landmarks inside $[-8,8] \times [-8,8]$.

A rangefinder collects the distance $\tilde{d}$ to the nearest mark.

Resolution. The robot is

$$\begin{cases} \dot{x}_1 & = & u_1 \cos u_2 \\ \dot{x}_2 & = & u_1 \sin u_2. \end{cases}$$

We define the set-valued sector functions

$$\begin{array}{lcl} \mathcal{H}_i\left(\mathbf{x}\left(t_i\right)\right) & = & \{\mathbf{a} \mid \; \|\mathbf{a}-\mathbf{x}\left(t_i\right)\| \; \in \; [d_i]\} \\ \mathcal{H}_{i+1}\left(\mathbf{x}\left(t_{i+1}\right)\right) & = & \{\mathbf{a} \mid \; \|\mathbf{a}-\mathbf{x}\left(t_{i+1}\right)\| \; < \; \delta_{i+1}\} \end{array}$$

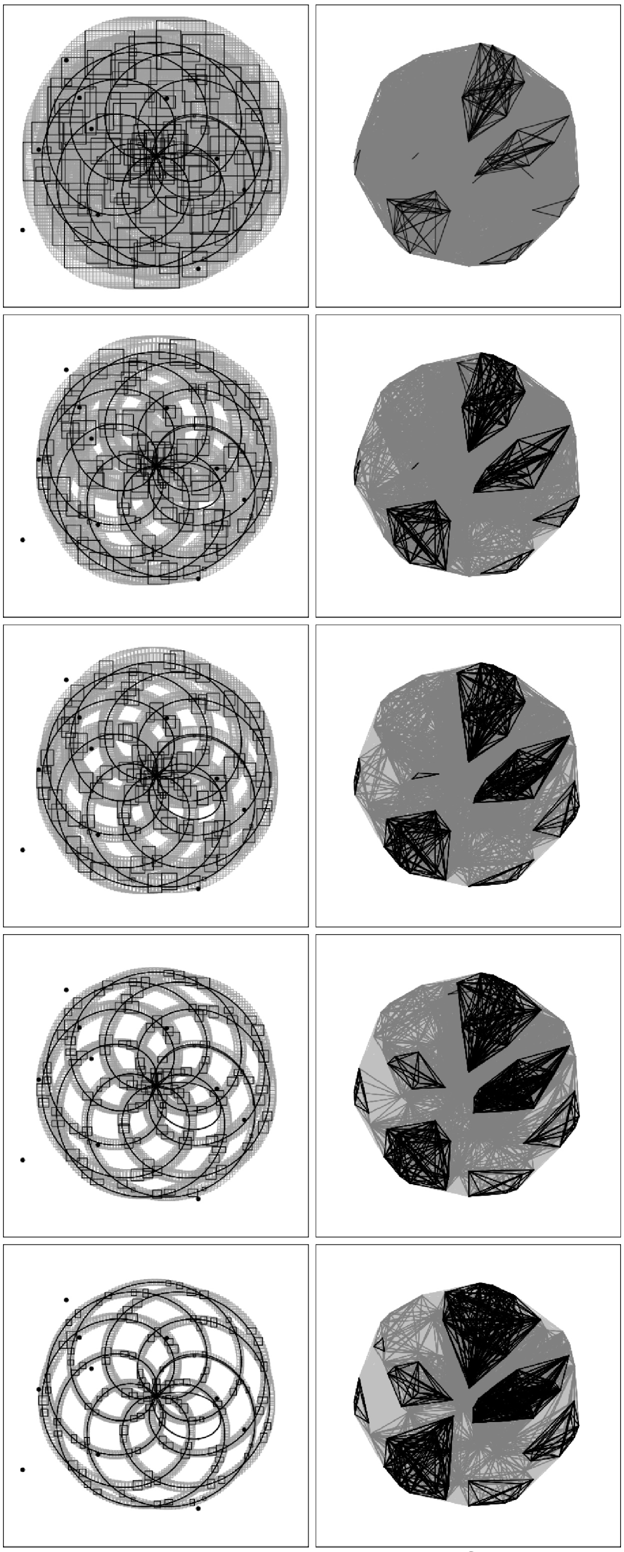
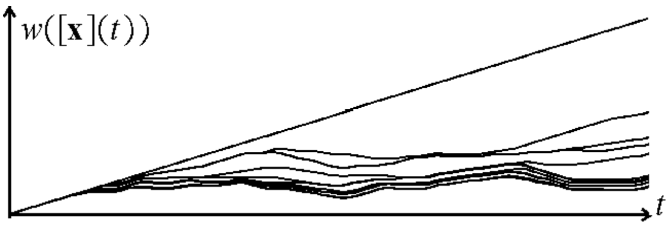
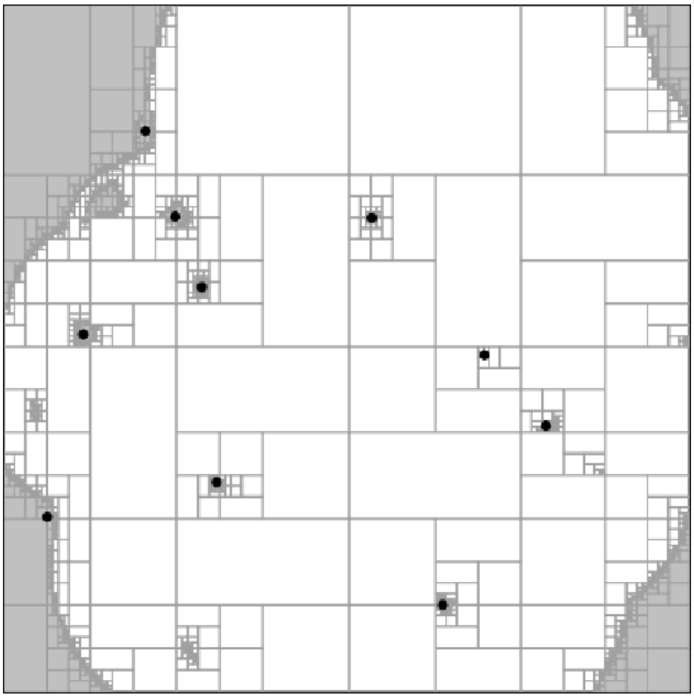


Illustration of the propagation. Left: the tube becomes more and more accurate. Right: The association graph has more and more arcs.



Superposition of the width of the tube $[\mathbf{x}](t)$

Associations. At the fixed point, 3888 associations have been found, 29128 pairs $(\mathbf{a}(i), \mathbf{a}(j))$ have been proven disjoint and 5400 pairs $(\mathbf{a}(i), \mathbf{a}(j))$ have not been classified.



Free space $\mathbb{F}$.