1 - Introduction
2 - Intervals
of fonctions
3 - Contractors
on tubes
4 - Simple example
5 - Spring-Mass example
6 - Conclusion
- Solve constraint satisfaction problem using tubes.
- Tubes are now implemented in IBEX (yesterday)
- Many applications involving constraints on trajectories.
- Trajectories of swarms of robot (Paladyn)
ENSTA
Bretagne
Solving non-linear constraint satisfaction
problem involving time-dependant function.
Aymeric BETHENCOURT
DGA

Prezi


ENSTA Bretagne
Solving non-linear constraint satisfaction problem involving time-dependant function.
Aymeric BETHENCOURT
DGA
4 - Simple example
5 - Spring-Mass example
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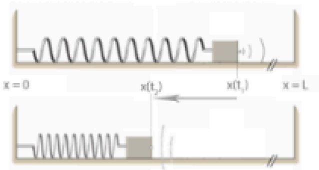
Prezi

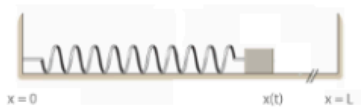
1 - Introduction

Mass-Spring-Sonar System



$$(L - x(t_1)) + (L - x(t_2)) = c.(t_2 - t_1)$$
$$\Leftrightarrow x(t_2) + x(t_1) = 2L - c.(t_2 - t_1)$$

Mass-Spring-Sonar System



$$m.\ddot{x} + \gamma.\dot{x} + \kappa.x - \beta x^3 = 0$$

Non-linear spring
No info on initial conditions

Problem

Non-linear constraint satisfaction problem involving time-dependant function.

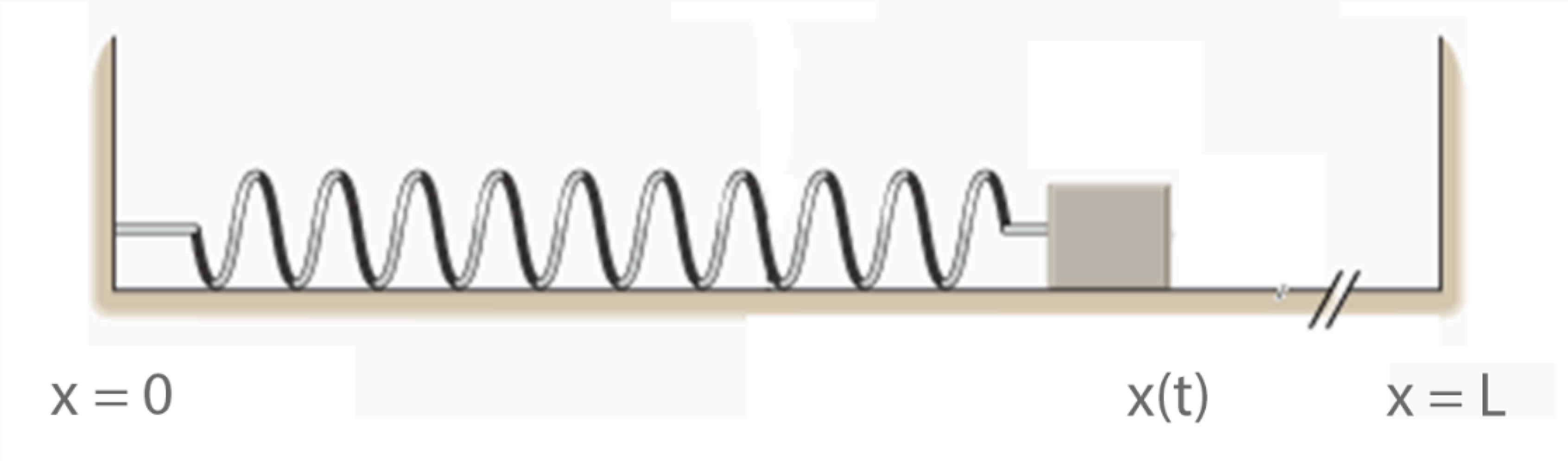
- Simple solving with EKF ?

- We are going to use interval of functions.

Plan

1. Introduction.
2. Interval of functions (Tubes).
3. Contractors on tubes.
4. Simple exemple.
5. Spring-Mass-Sonar example.
6. Conclusion

Mass-Spring-Sonar System



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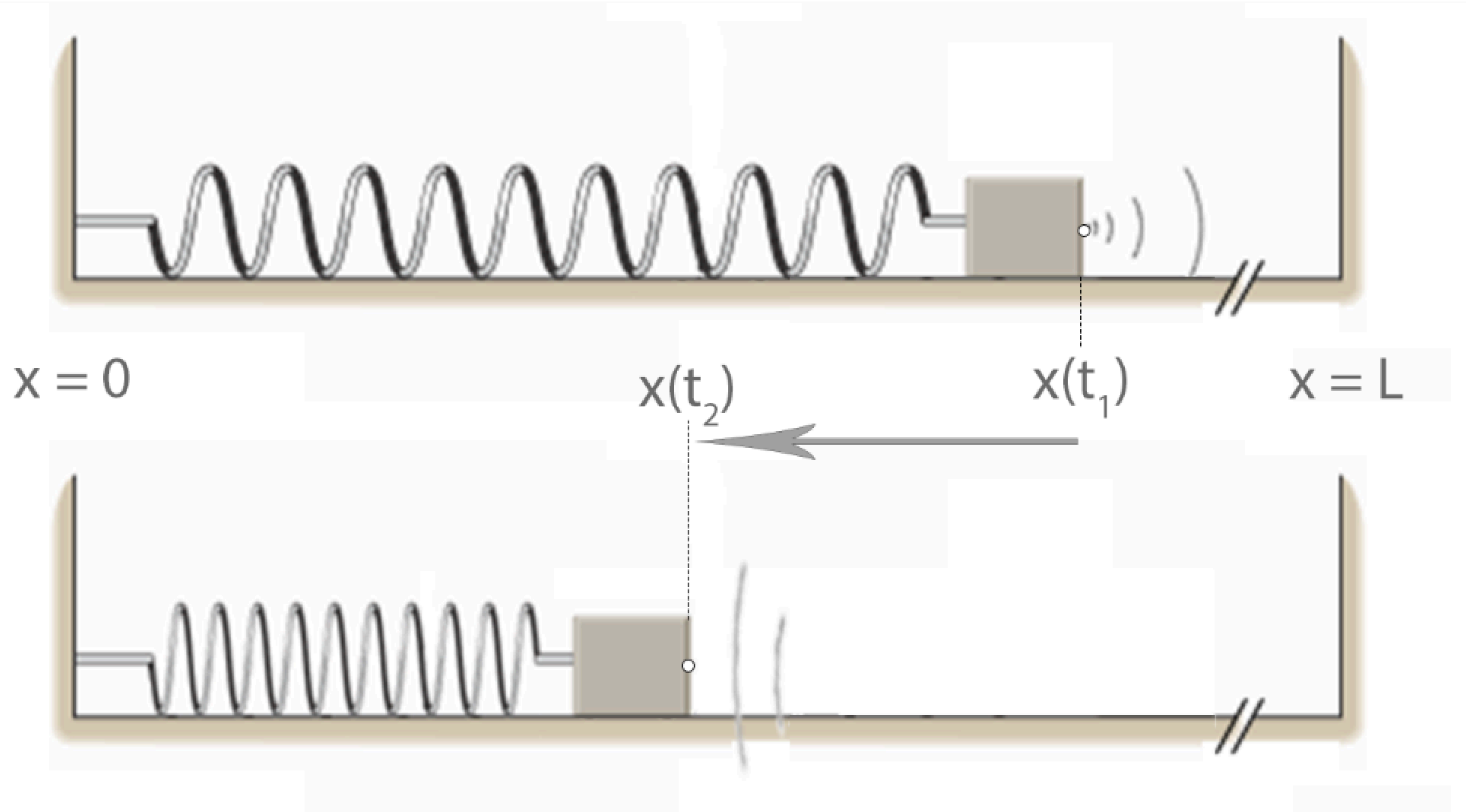
Mass-Spring-Sonar System



$$\begin{aligned}(L - x(t_1)) + (L - x(t_2)) &= c.(t_2 - t_1)\\ \Leftrightarrow x(t_2) + x(t_1) &= 2L - c.(t_2 - t_1)\end{aligned}$$

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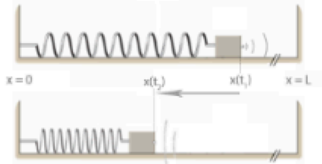
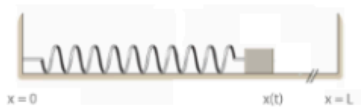
1 - Introduction

Mass-Spring-Sonar System



$$(L - x(t_1)) + (L - x(t_2)) = c.(t_2 - t_1)$$
$$\Leftrightarrow x(t_2) + x(t_1) = 2L - c.(t_2 - t_1)$$

Mass-Spring-Sonar System



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2 - Intervals of fonctions

Lattice

- We use classical interval analysis where the unknown variables are trajectories (function from ℝ to ℝ^n).
- Trajectories belong to a lattice.

Lattice

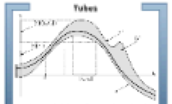
A lattice is a partially ordered set, closed under least upper and greatest lower bounds.

Infimum

Supremum

Tubes

Tubes



Interval video of a random signal

Operations

Extend operators on lattices to intervals of this lattice.

The extension is the smallest interval enclosing all possible values.

Operations

Theorem

The integral (monotonic operator) verify

Operators

Extensions of binary operators in the Minkowski sense.

Lattice

- We use classical interval analysis where the unknown variables are trajectories (function from $\mathbb{R}$ to $\mathbb{R}^n$).

- Trajectories belong to a lattice.

Lattice

A lattice $(\mathcal{E}, \leq)$ is a partially ordered set, closed under least upper and greatest lower bounds.

Infimum $x \vee y$

(The least upper bound)

Supremum $x \wedge y$

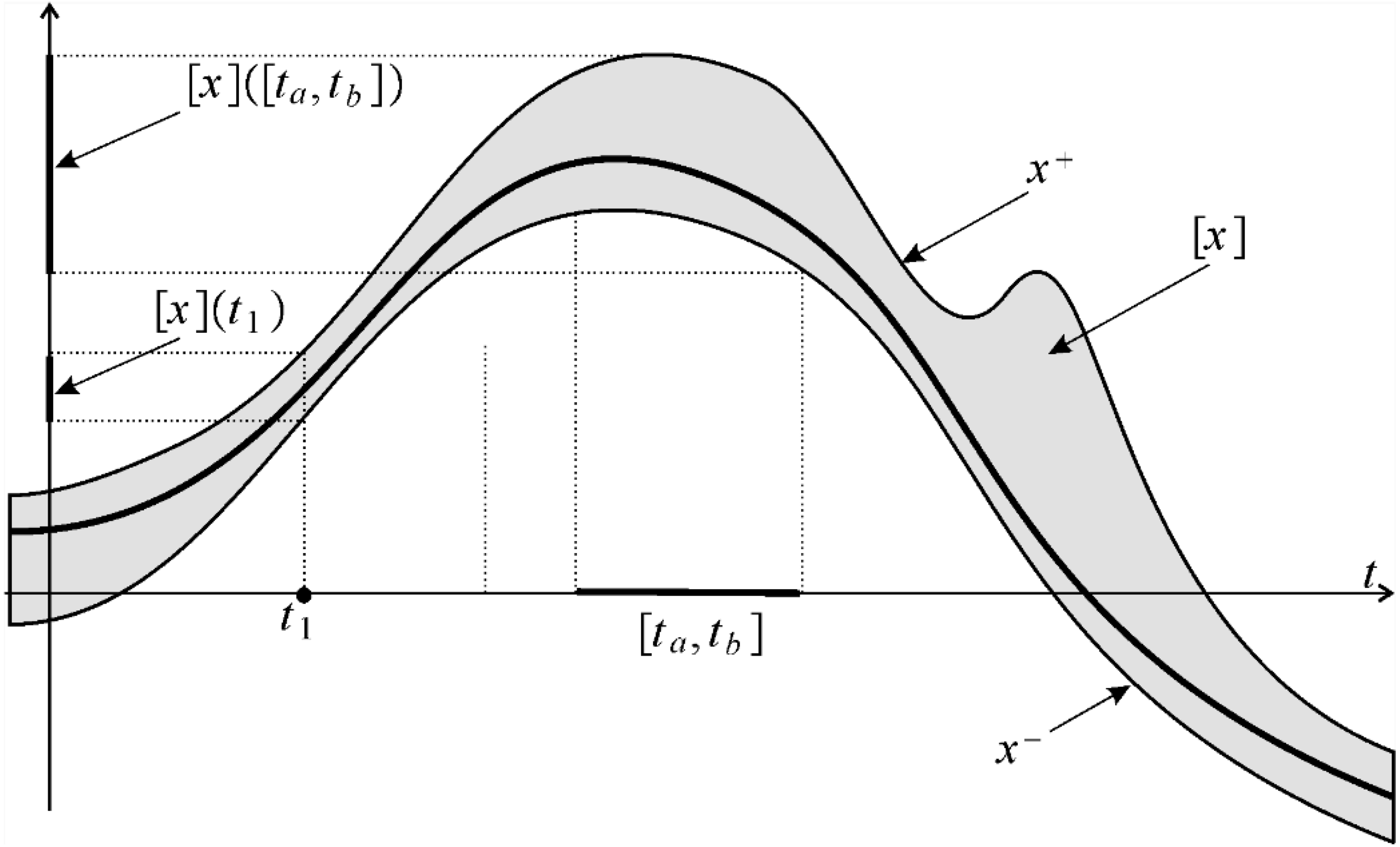
(The greatest lower bound)

Tubes

The tube $[\mathbf{x}]$ is an interval $[\mathbf{x}^-, \mathbf{x}^+]$ of $\mathcal{F}^n$

$$\mathbf{x}^-(t) \leq \mathbf{x}(t) \leq \mathbf{x}^+(t)$$

Tubes



Interval vision of a random signal

Operations

Extend operators on lattices to intervals of this lattice.

The extension is the smallest interval enclosing all possible values.

Operations

Theorem

The integral (monotonic operator) verify

$$\int_{t_1}^{t_2} [\mathbf{x}](\tau)\, d\tau = \left[\int_{t_1}^{t_2} \mathbf{x}^{-}(\tau)\, d\tau, \int_{t_1}^{t_2} \mathbf{x}^{+}(\tau)\, d\tau\right]$$

Operators

Extensions of binary operators in the Minkowski sense.

$$[x] + [y] = [x^- + y^-, x^+ + y^+]$$

2 - Intervals of fonctions

Lattice

- We use classical interval analysis where the unknown variables are trajectories (function from ℝ to ℝ^n).

- Trajectories belong to a lattice.

Lattice

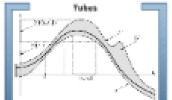
A lattice is a partially ordered set, closed under least upper and greatest lower bounds.

Infimum

Supremum

Tubes

Tubes



Interval video of a random signal

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Operations

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3 - Contractors on tubes

Definitions

Constraint

Associated contractor

Contracting Completeness

Proposition 1

The minimal contractor associated with

Proposition 2

The minimal contractor associated with

Proposition 5

The minimal contractor associated with a delay

Non-synchronized constraint

Proposition 4

The minimal contractor associated with a central symmetry at

Definitions

Constraint $\mathcal{L}(x).$

Associated contractor $[x] \xrightarrow{\mathcal{C}_{\mathcal{L}}} [y]$

Contracting Completness

$$\forall t, [x](t) \subset [y](t)$$

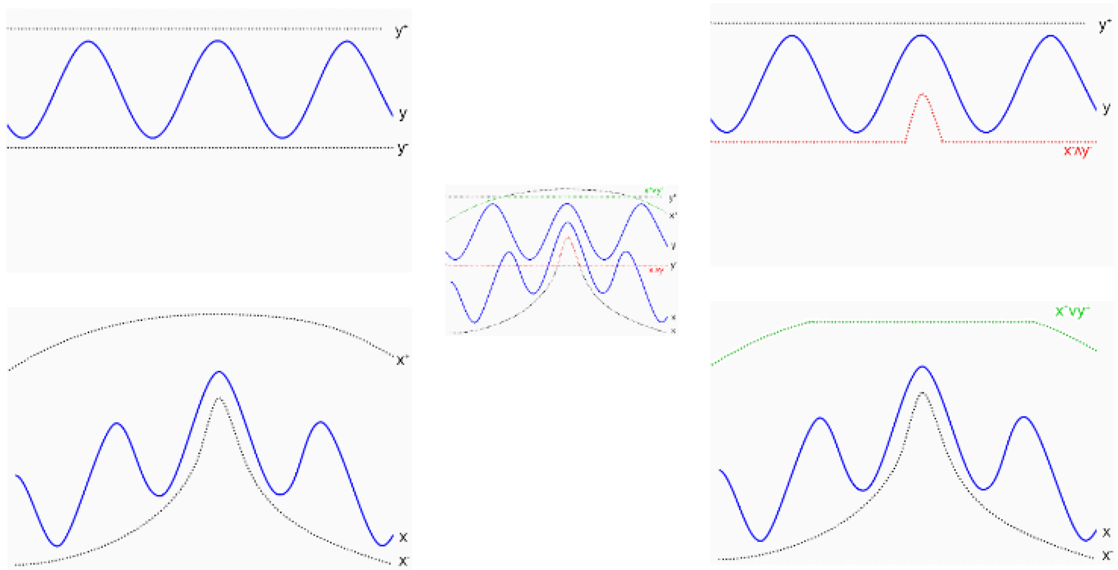
$$\begin{pmatrix} \mathcal{L}(x) \\ x \in [x] \end{pmatrix} \Longrightarrow x \in [y]$$

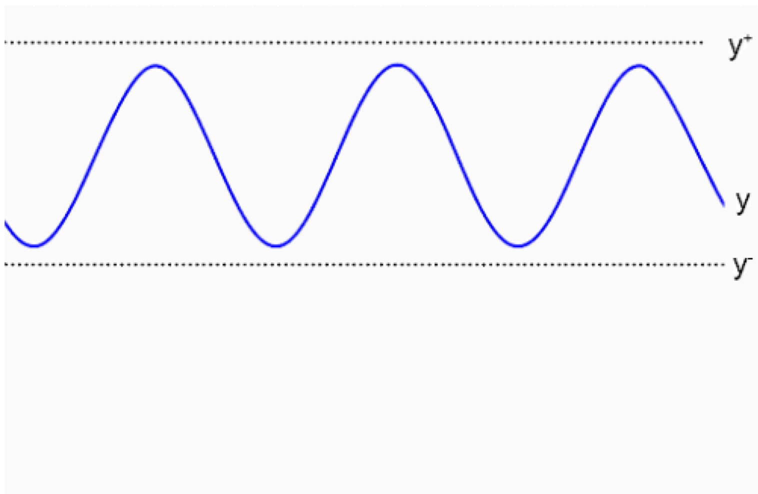
Proposition 1

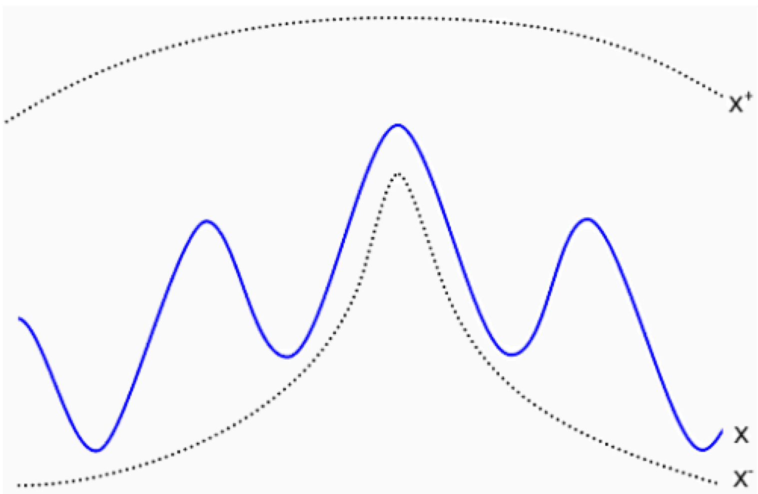
The minimal contractor associated with

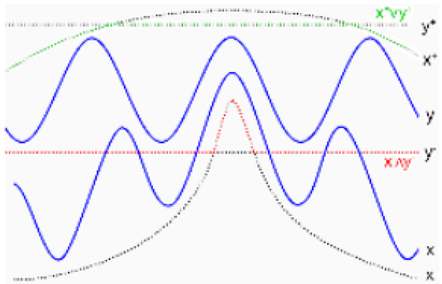
$$x \leqslant y$$

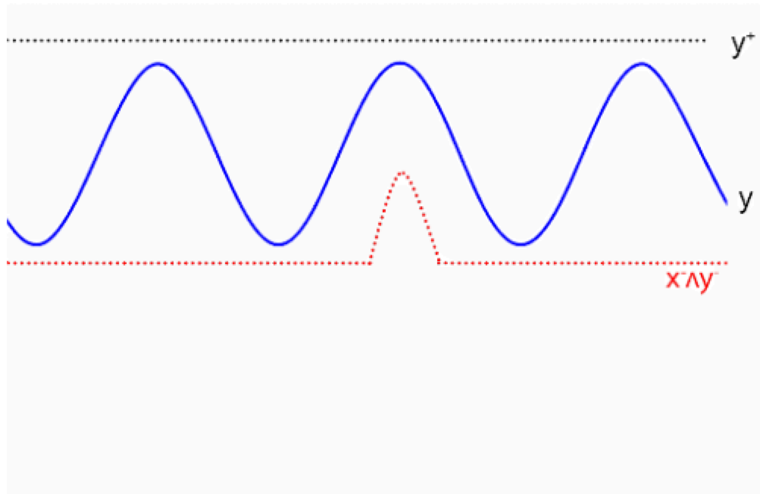
$$C_{\leqslant}\left(\begin{array}{c}[x]\\ [y]\end{array}\right)=\left(\begin{array}{c}[x^-, x^+ \vee y^+]\\ [x^- \wedge y^-, y^+]\end{array}\right)$$

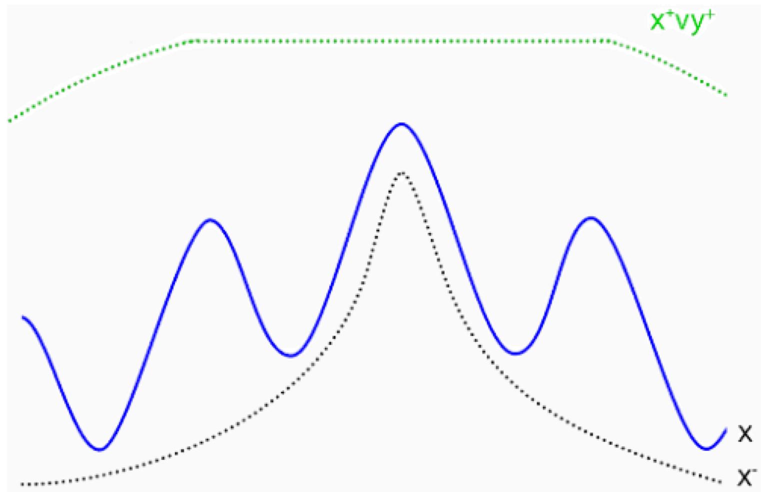



y⁺
y
y⁻


x⁺
x
x⁻


x⁺vy⁺
y⁺
x⁺
y
y⁻
x⁻∧y⁻
x
x⁻


y⁺
y
x⁻∧y⁻

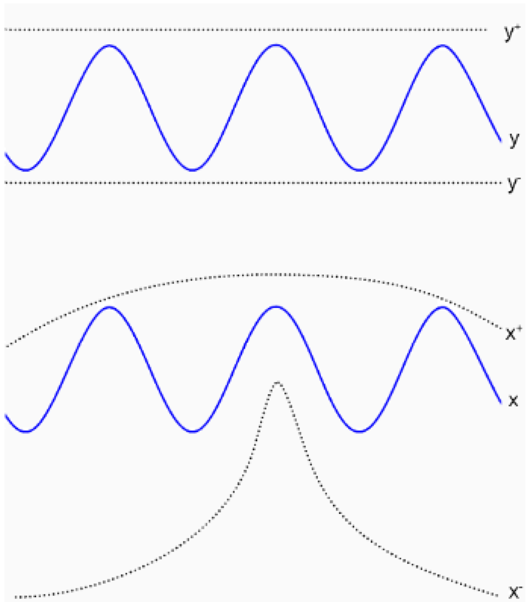

x⁺vy⁺
x
x⁻

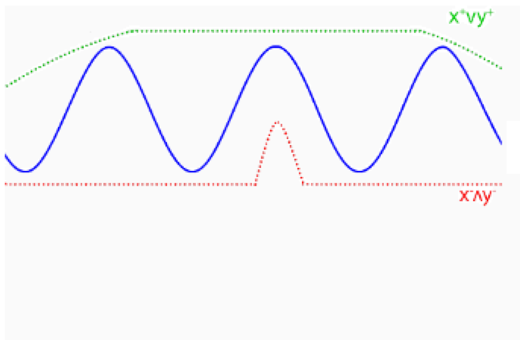
Proposition 2

The minimal contractor associated with

$$x = y$$

$$C_{=}\begin{pmatrix} [x] \\ [y] \end{pmatrix} = \begin{pmatrix} [x^- \wedge y^-, x^+ \vee y^+] \\ [x^- \wedge y^-, x^+ \vee y^+] \end{pmatrix}$$



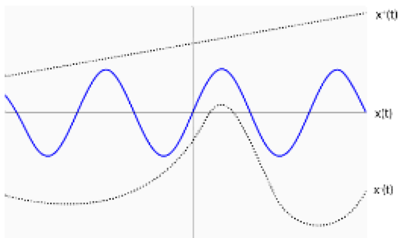


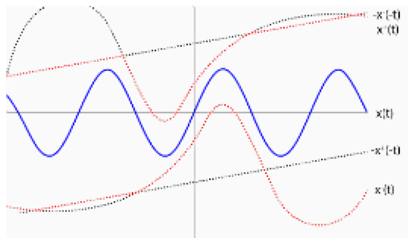
Proposition 4

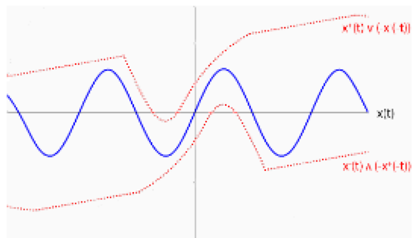
The minimal contractor associated with a central symetry at t=T

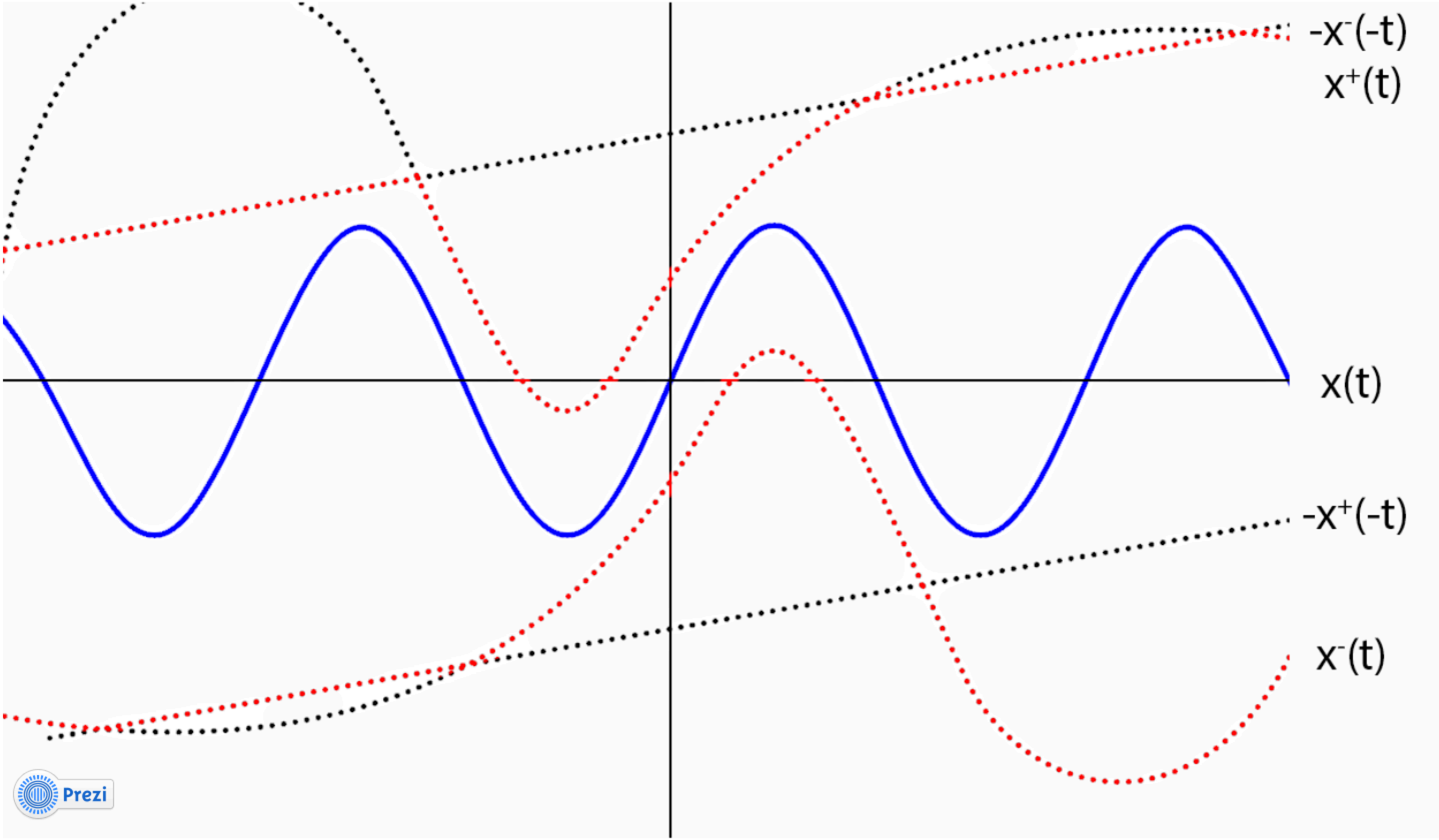
$$C_{CSym}([x(T+t)]) = \\ [x^-(T+t) \vee (-x^+(T-t)); x^+(T+t) \wedge (-x^-(T-t))]$$

$$C_{odd}([x(t)]) = [x^-(t) \vee (-x^+(-t)); x^+(t) \wedge (-x^-(-t))]$$







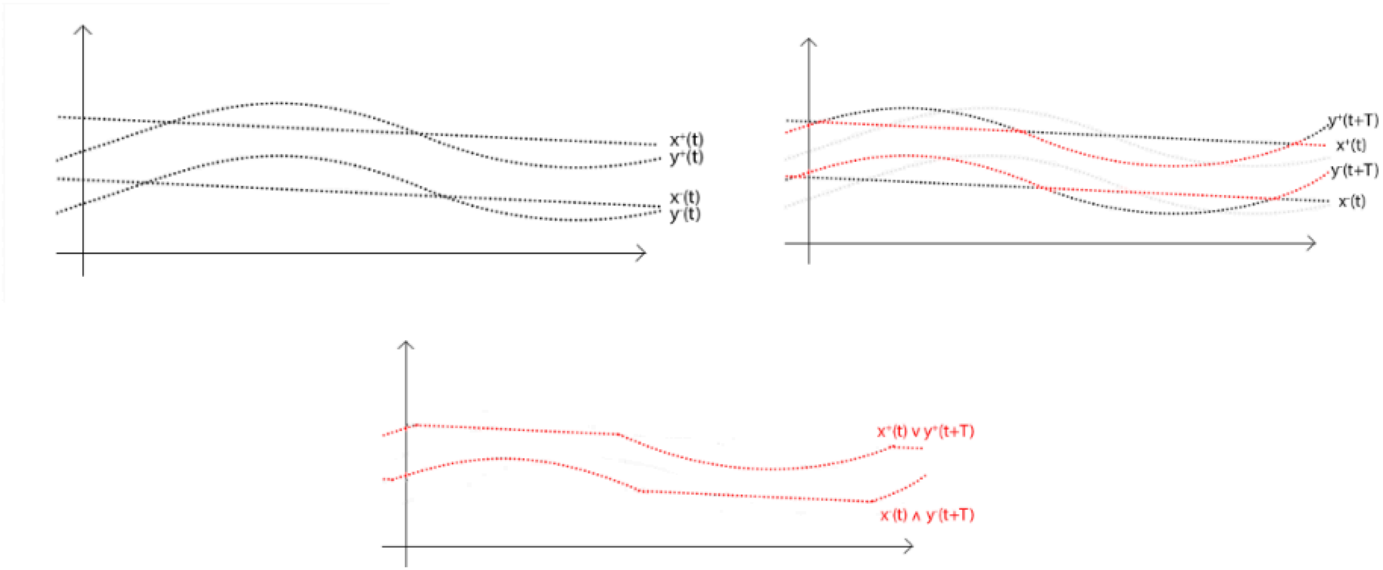


-x⁻(-t)
x⁺(t)
x(t)
-x⁺(-t)
x⁻(t)

Proposition 5

The minimal contractor associated with a delay x(t)=y(T+t)

$$C_{delay}\left(\begin{array}{c}[x]\\ [y]\end{array}\right)=\left(\begin{array}{c}[x^{-}\vee y^{-}(T+t),x^{+}\wedge y^{+}(T+t)]\\ [x^{-}\vee y^{-}(T+t),x^{+}\wedge y^{+}(T+t)]\end{array}\right)$$

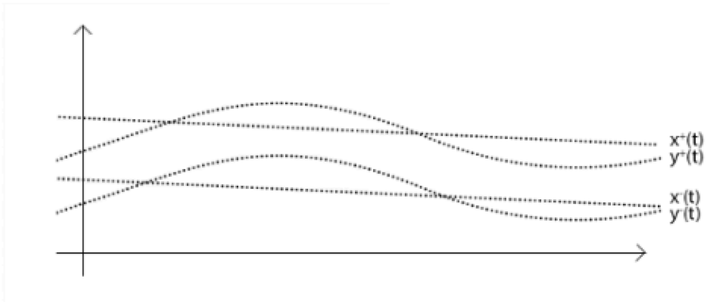


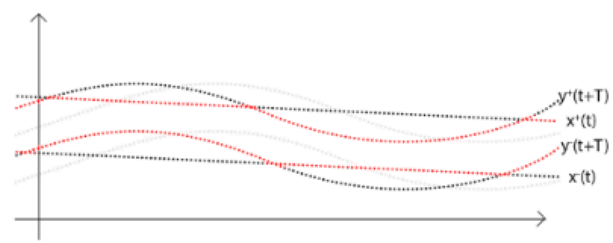
Non-synchronised constraint

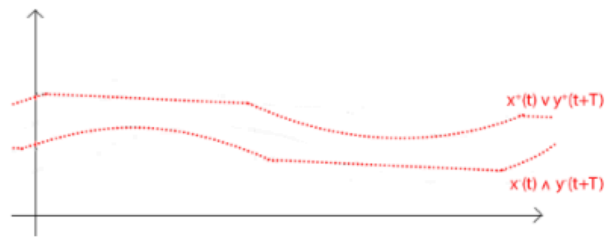
Proposition 5

The minimal contractor associated with a delay x(t)=y(T+t)

$$C_{delay}\left(\begin{array}{c}[x]\\ [y]\end{array}\right)=\left(\begin{array}{c}[x^-\vee y^-(T+t),x^+\wedge y^+(T+t)]\\ [x^-\vee y^-(T+t),x^+\wedge y^+(T+t)]\end{array}\right)$$

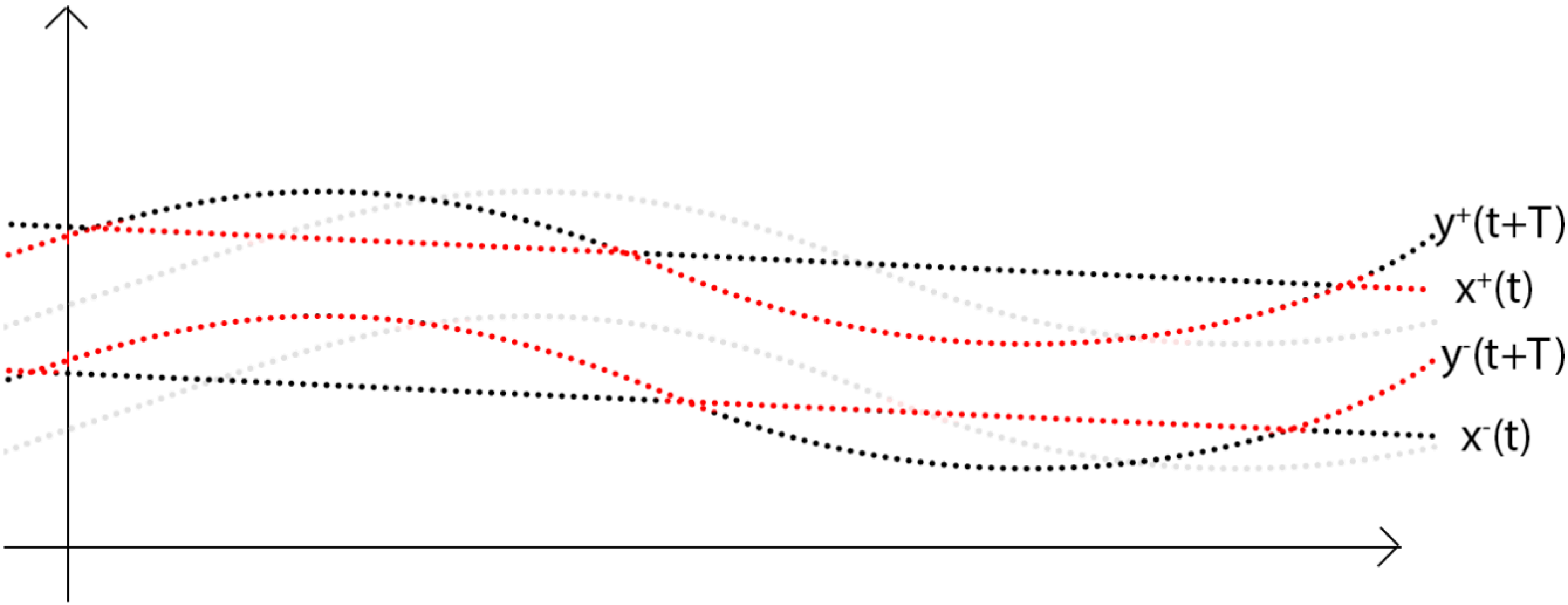






Non-synchronised constraint

Prezi



$y^+(t+T)$
$x^+(t)$
$y^-(t+T)$
$x^-(t)$

3 - Contractors on tubes

Definitions

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Proposition 2

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Proposition 5

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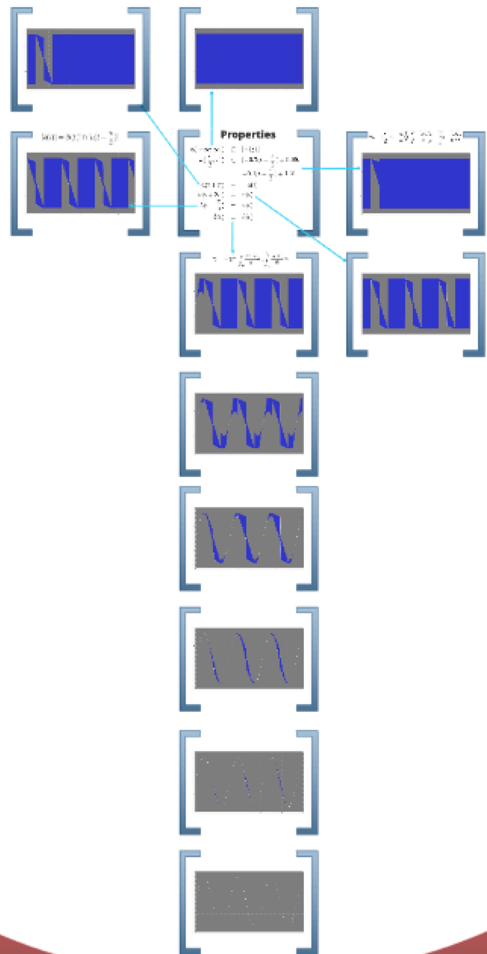
Non-synchronized constraint

Proposition 4

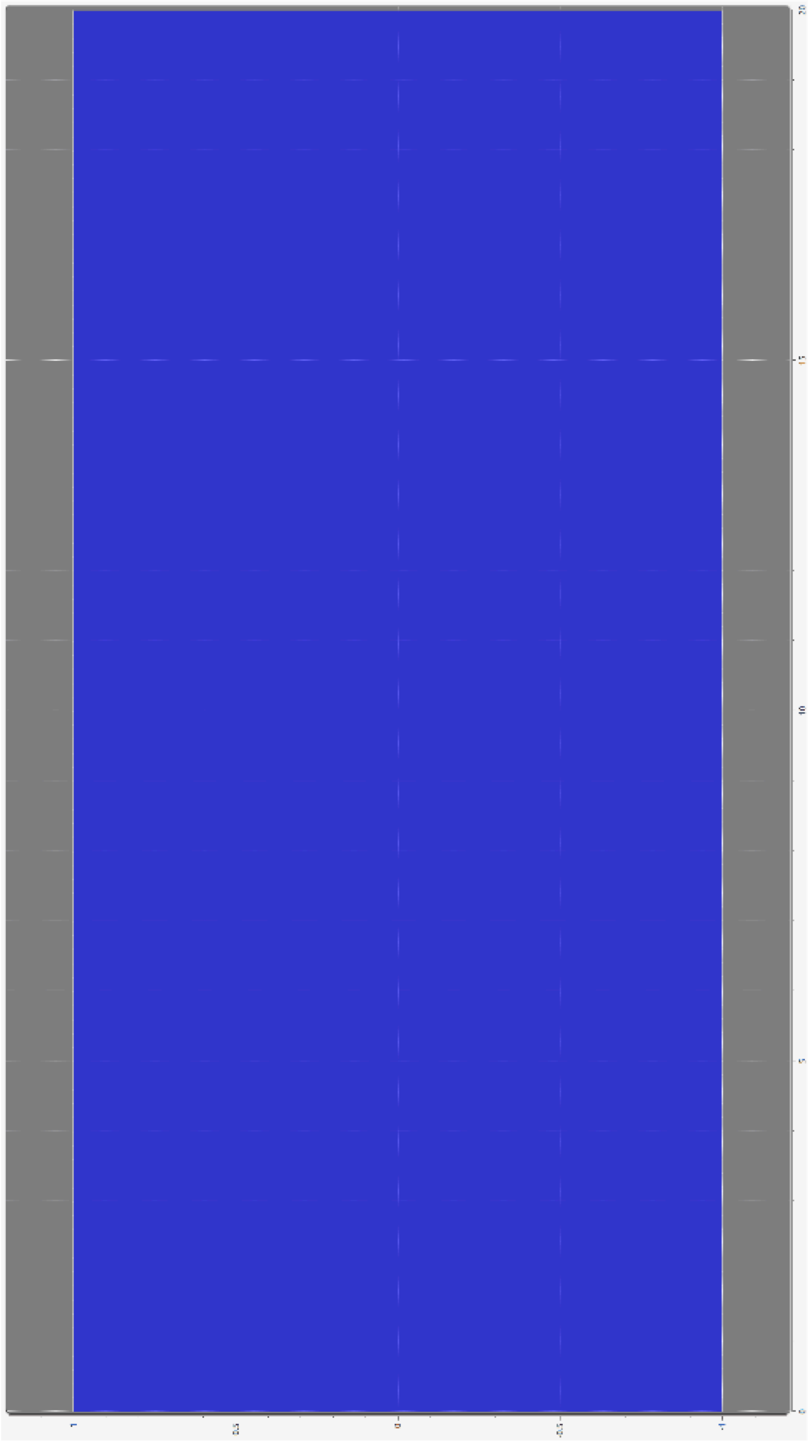
The minimal contractor associated with a control symetric at t=T

4 - Simple example

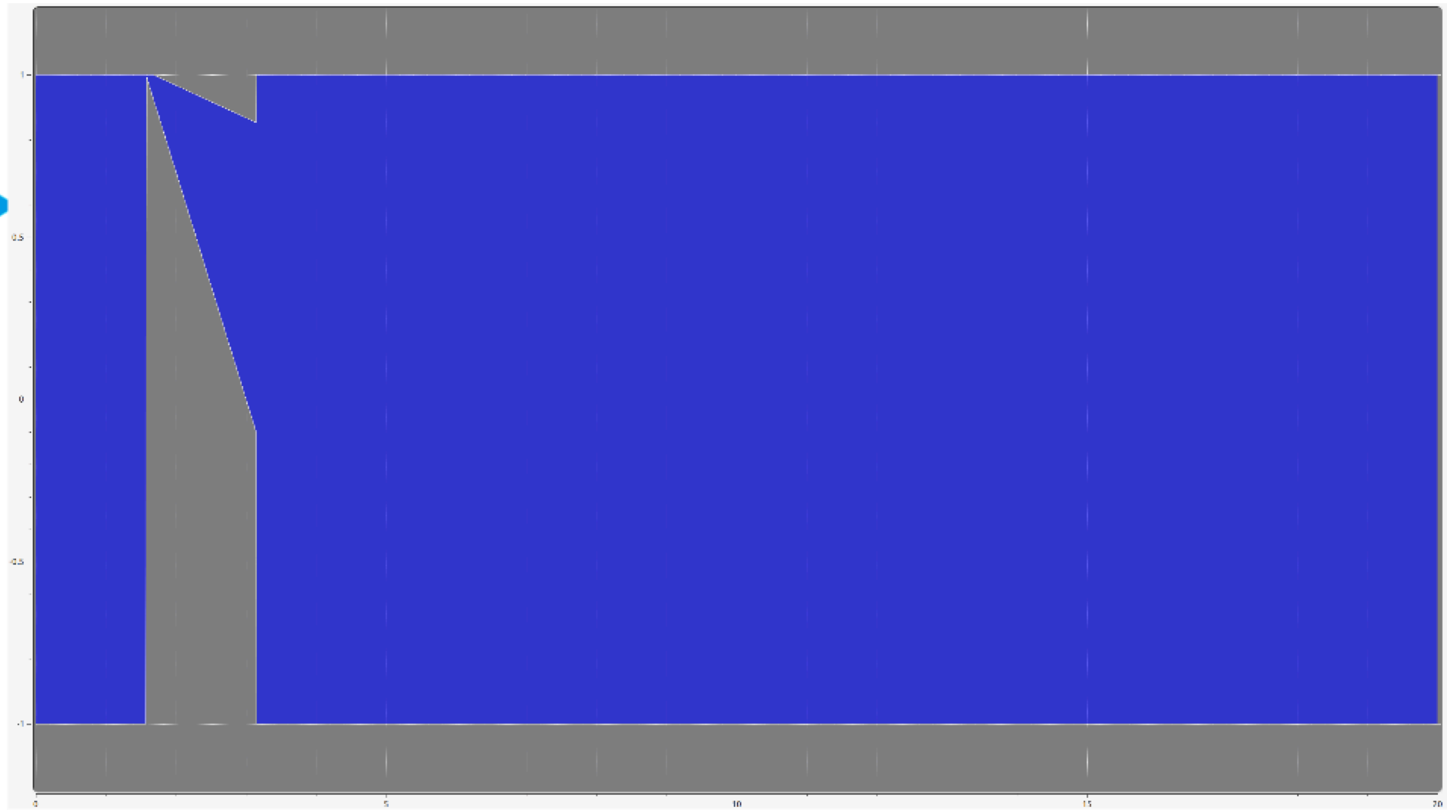


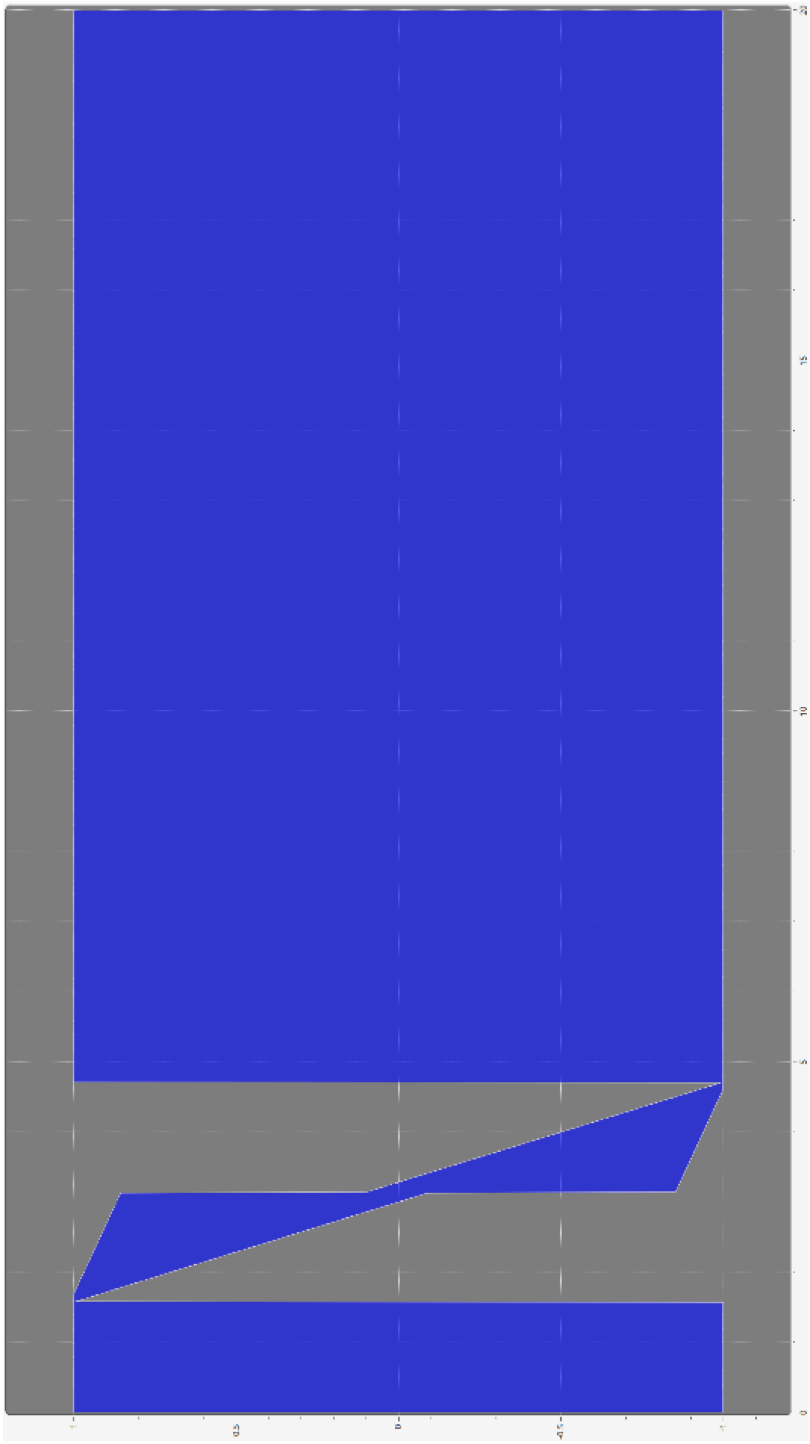
Properties

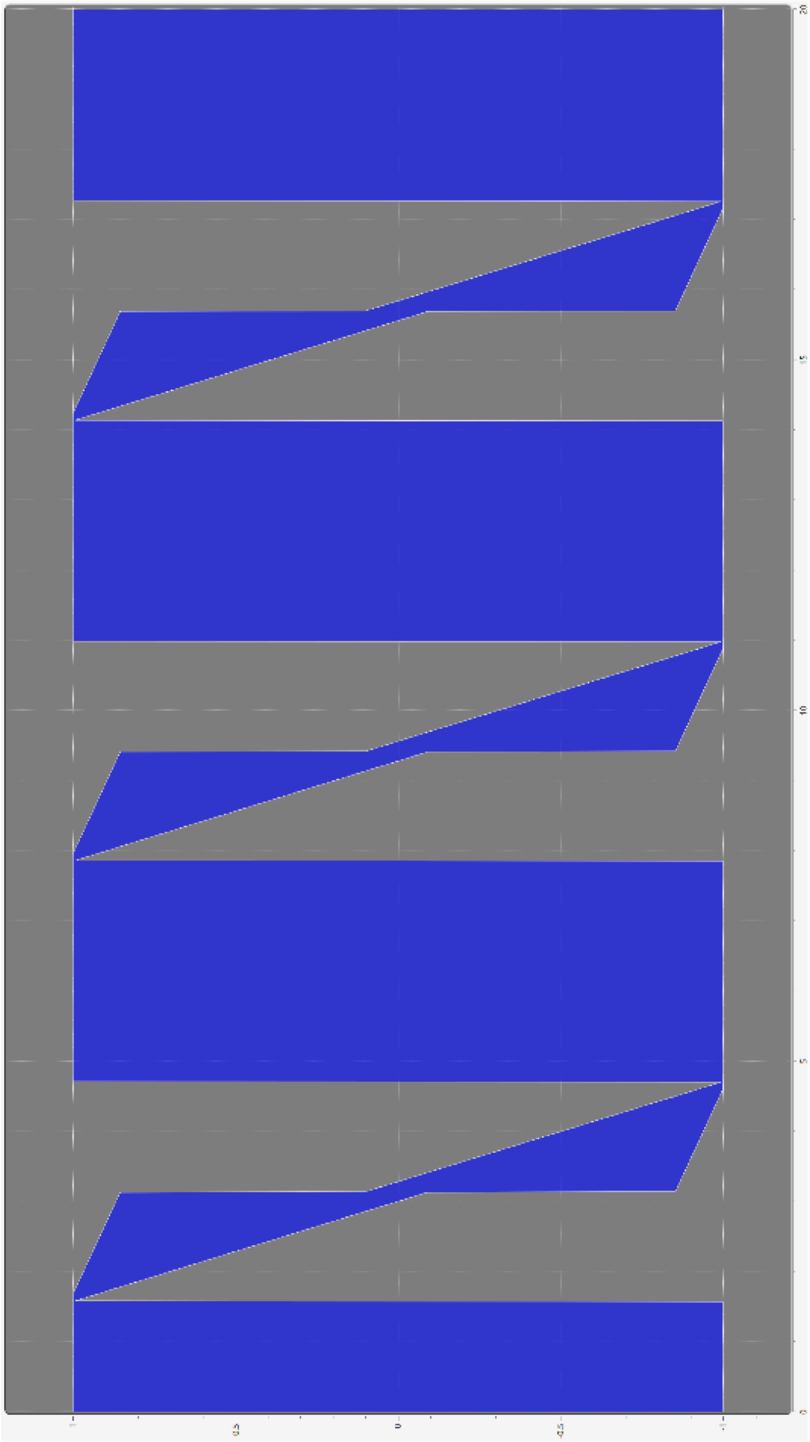
$$\begin{aligned}
a([-\infty;\infty]) &\subset [-1;1] \\
a([\frac{\pi}{2},\pi]) &\subset [-0.7(t-\frac{\pi}{2})+0.99, \\
&\quad -0.1(t-\frac{\pi}{2})+1.01] \\
a(t+\pi) &= -a(t) \\
a(t+2\pi) &= a(t) \\
b(t-\frac{\pi}{2}) &= a(t) \\
b(t) &= \dot{a}(t)
\end{aligned}$$



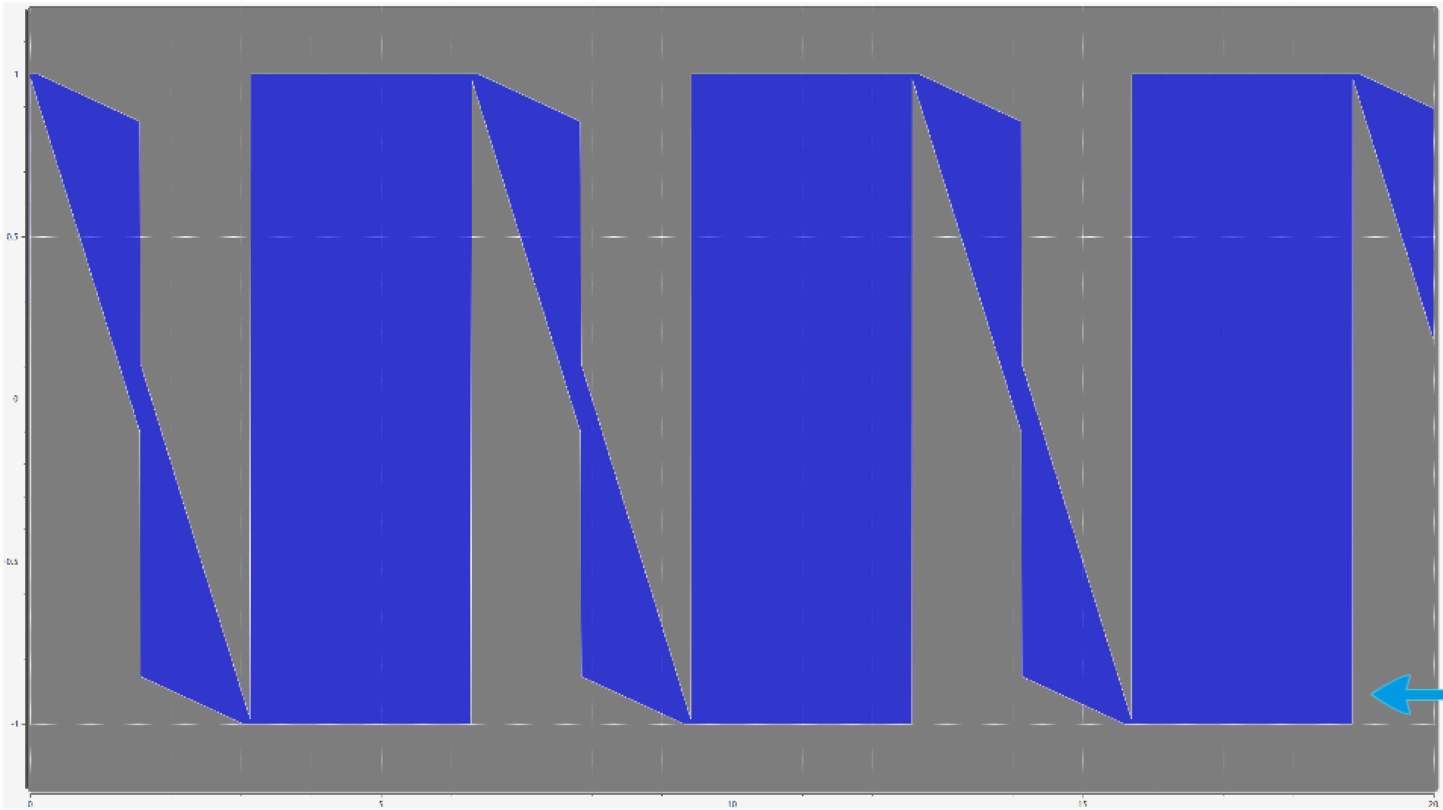
$$\forall t \in [\frac{\pi}{2}, \pi], \quad \begin{aligned} a(t) &\leqslant -0.1(t - \tfrac{\pi}{2}) + 1.01 \\ a(t) &\geqslant -0.7(t - \tfrac{\pi}{2}) + 0.99 \end{aligned}$$



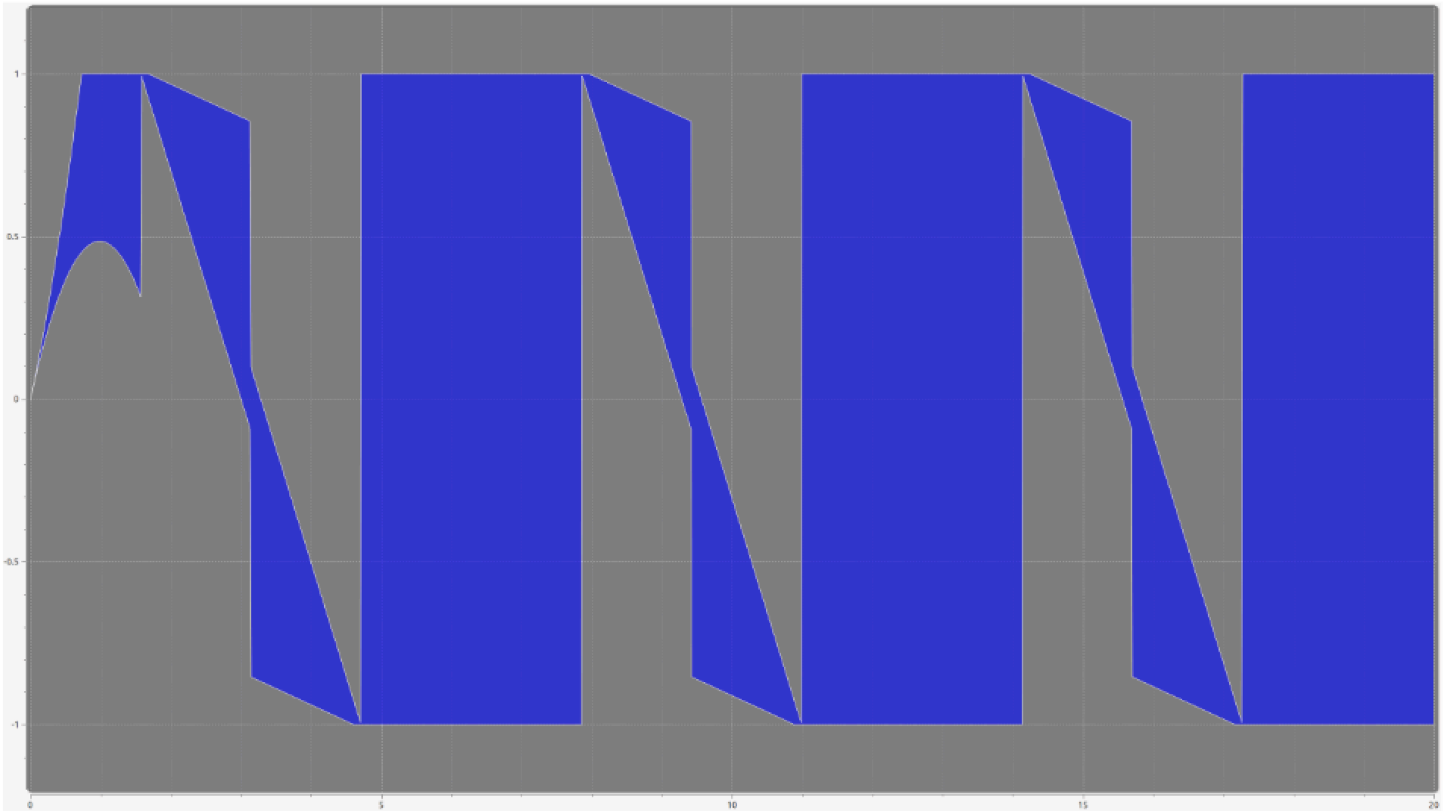


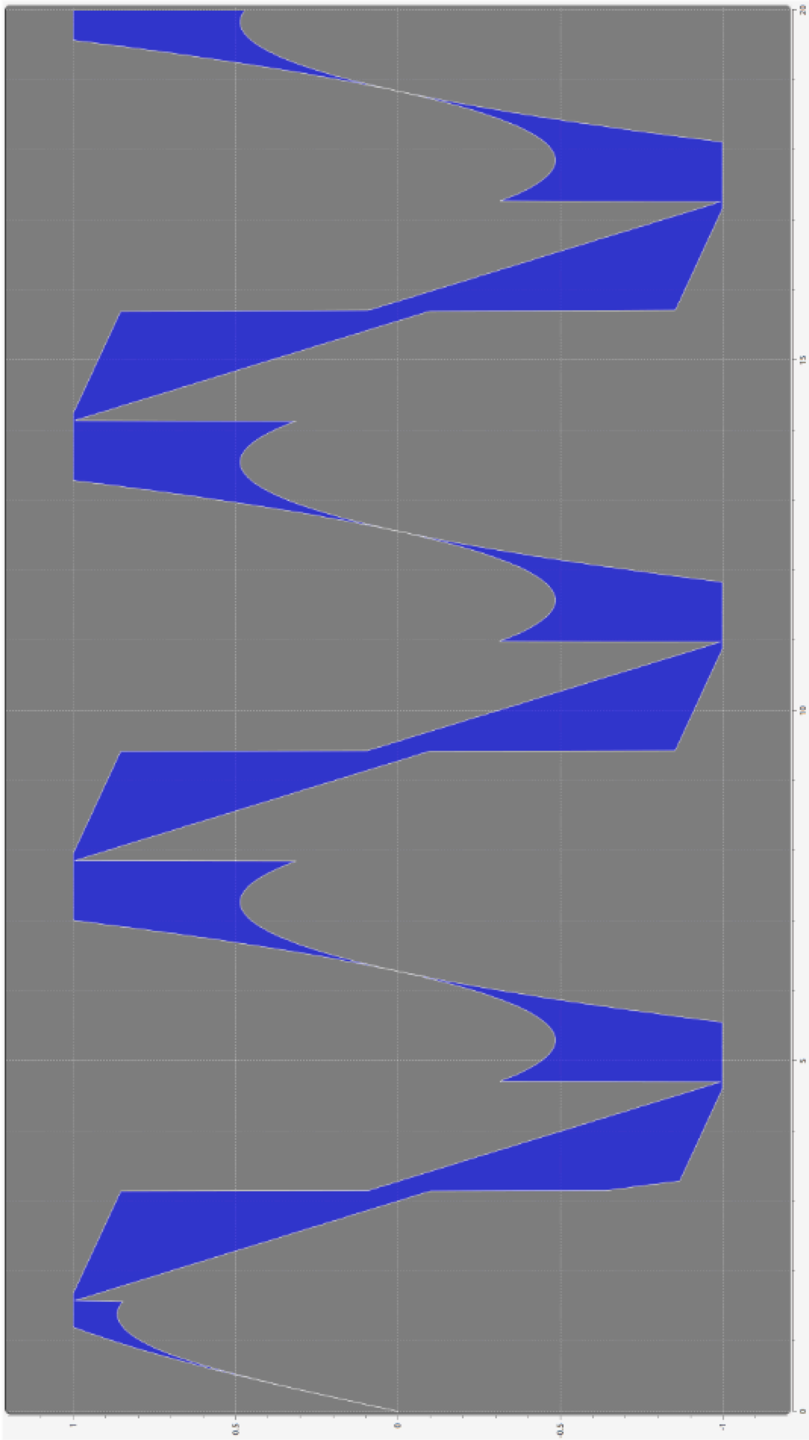


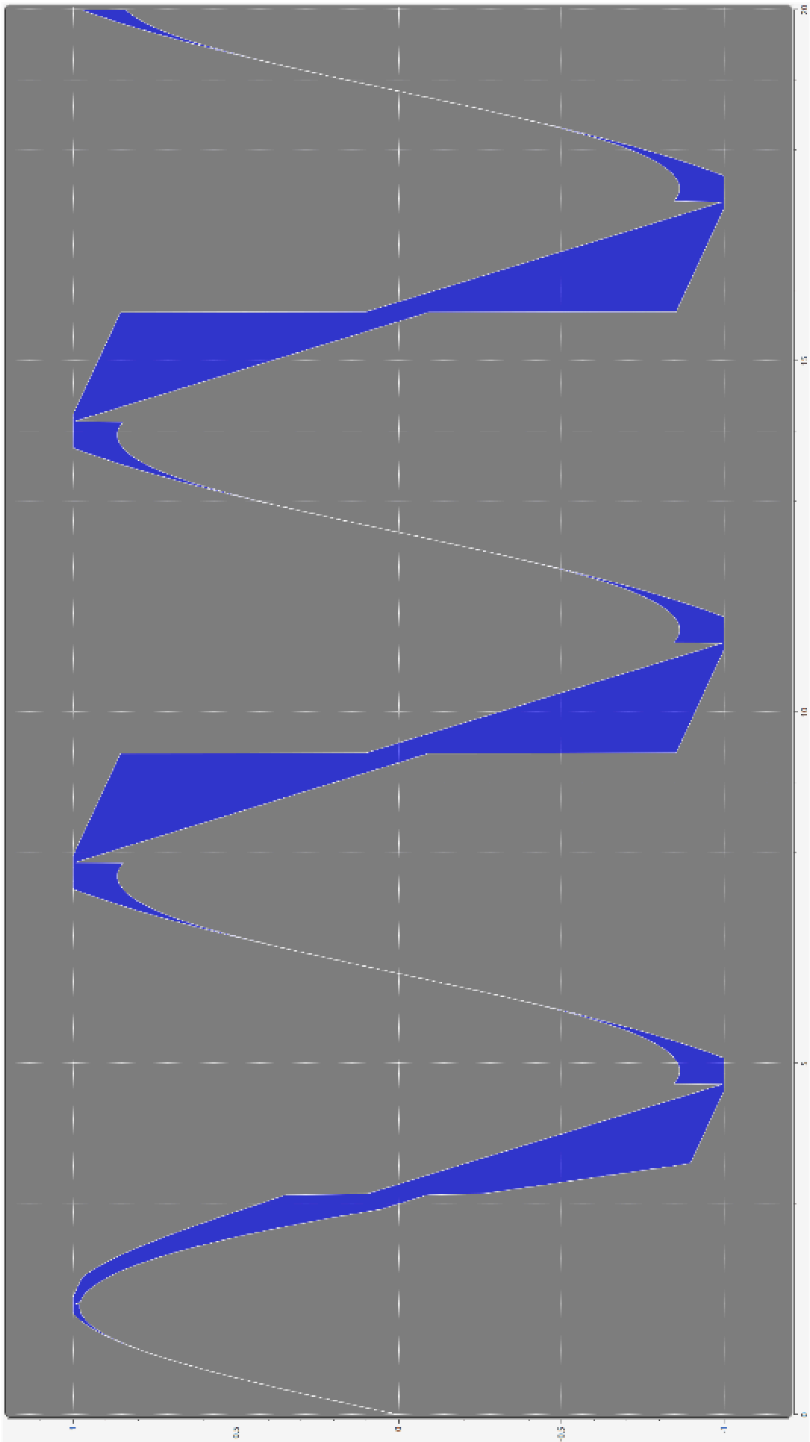
$$[b(t)] = [b(t)] \cap [a(t + \frac{\pi}{2})]$$

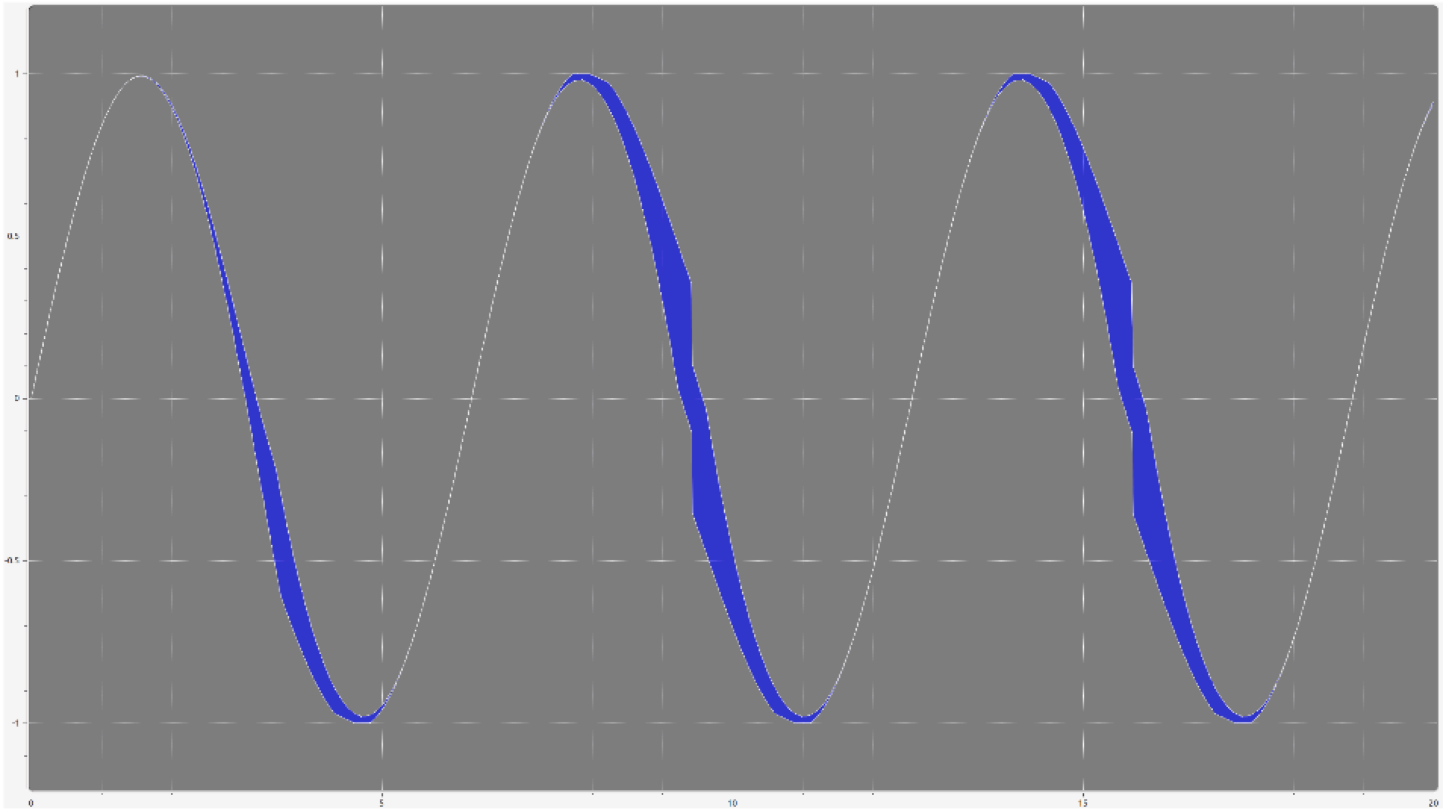


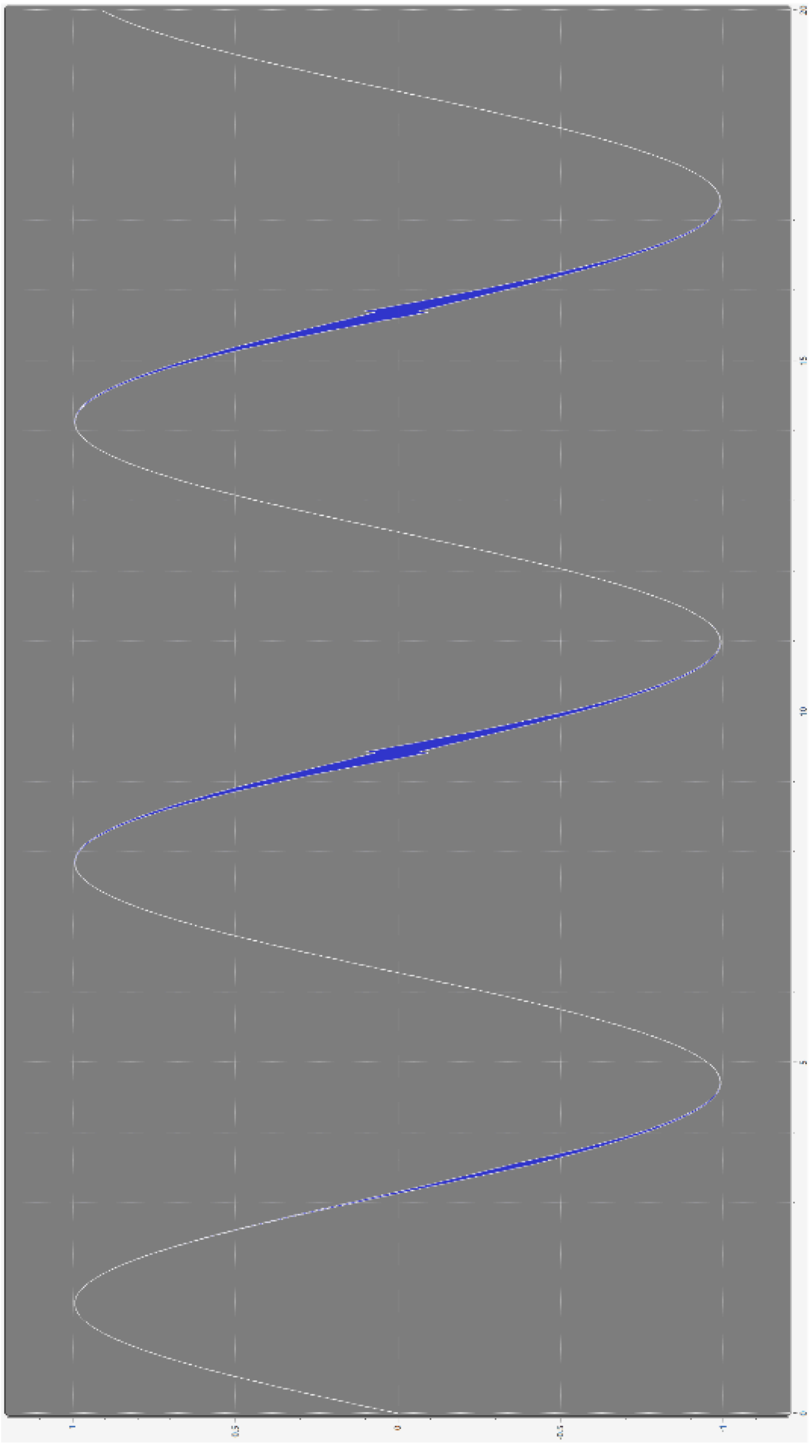
$$[a(t)] = [a(t)] \cap [\int_{\tau=0}^{t} \frac{db^{+}(\tau)}{dt} d\tau, \int_{\tau=0}^{t} \frac{db^{-}(\tau)}{dt} d\tau]$$

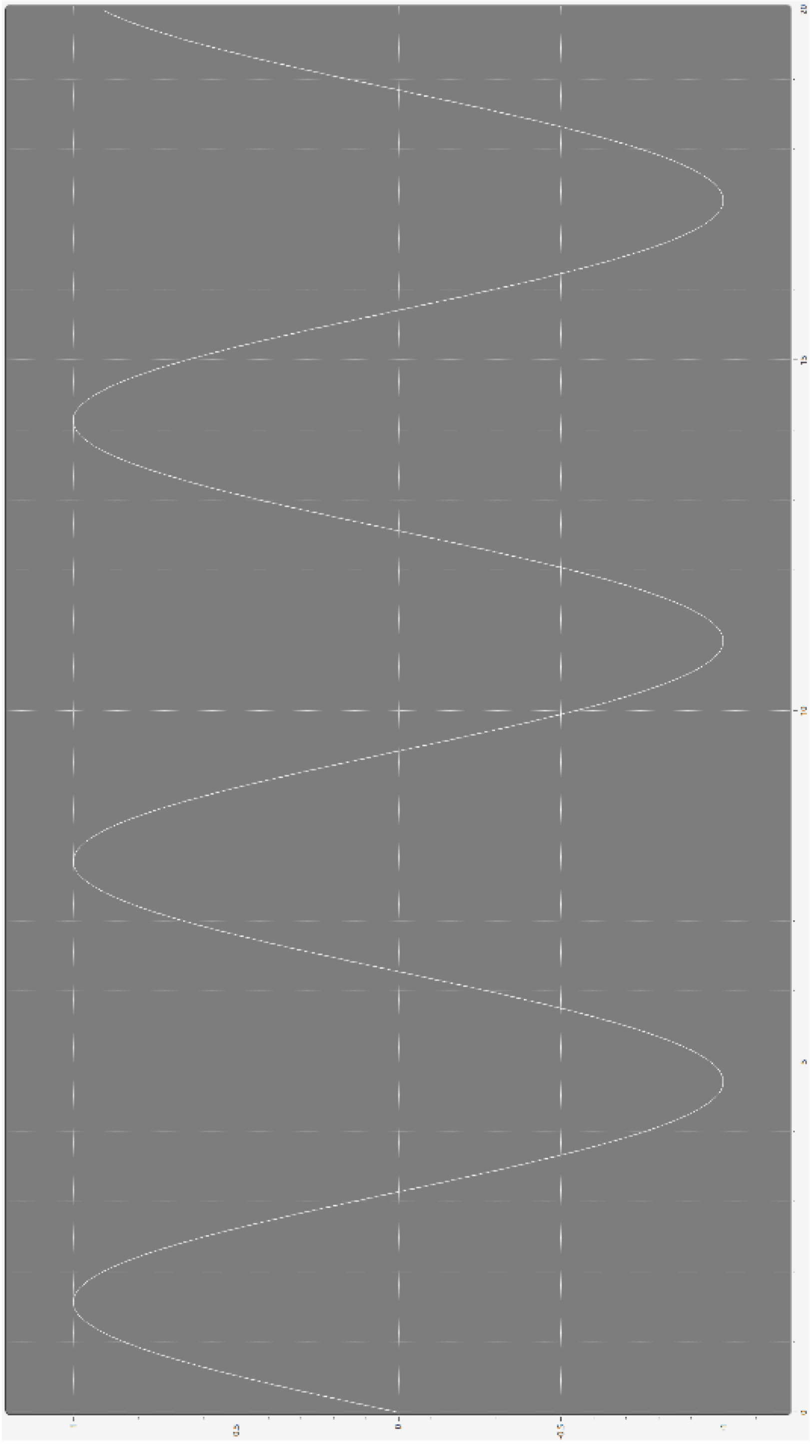



1
0.5
0
-0.5
-1
0
5
10
15
20

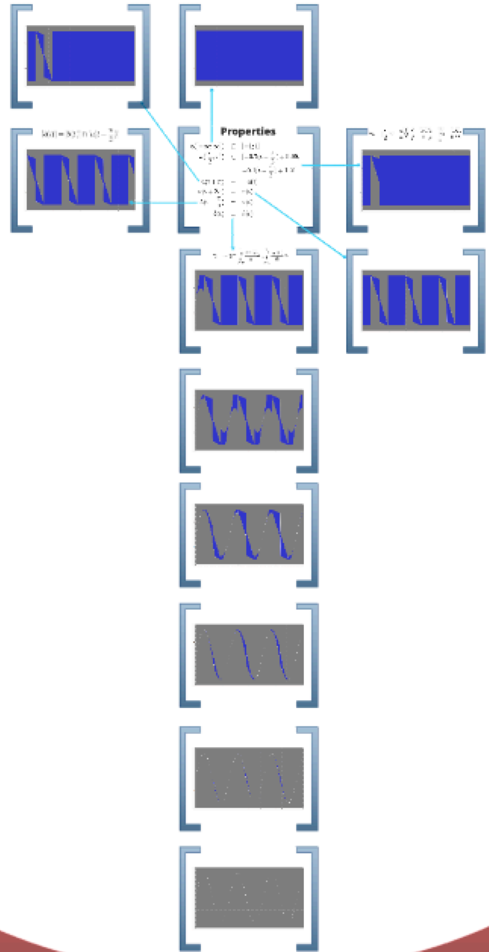



1
0.5
0
-0.5
-1
0
5
10
15
20

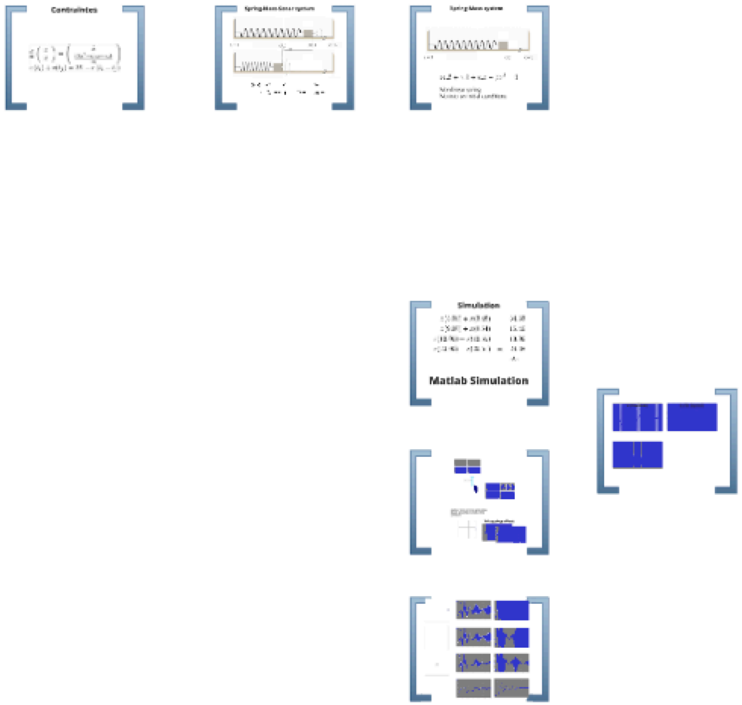




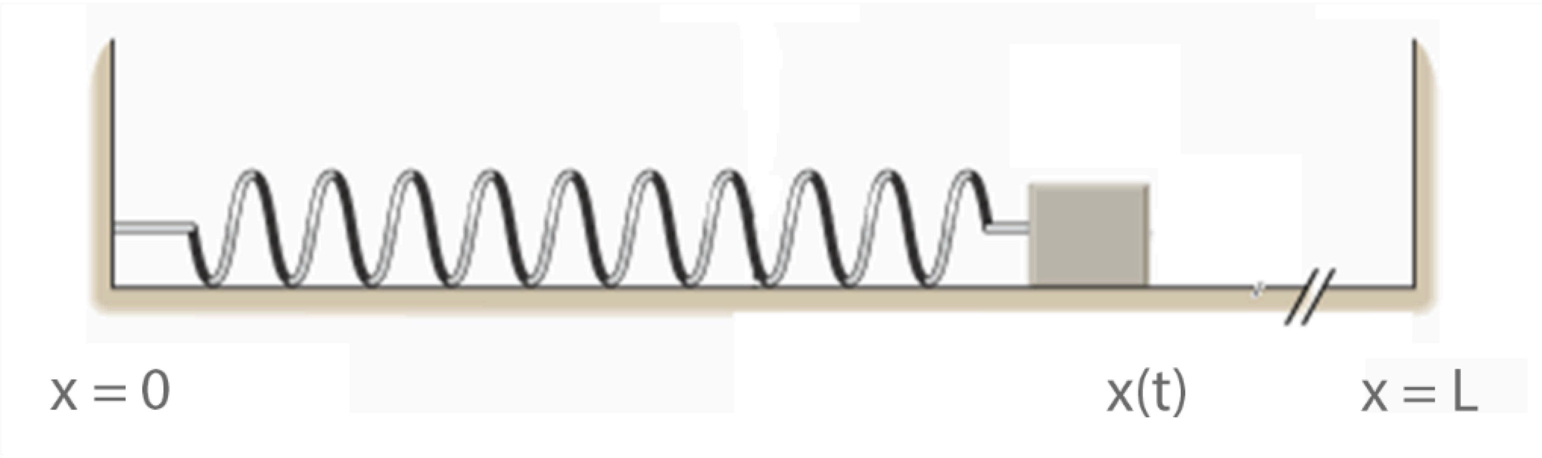
4 - Simple example



5 - Spring-Mass example



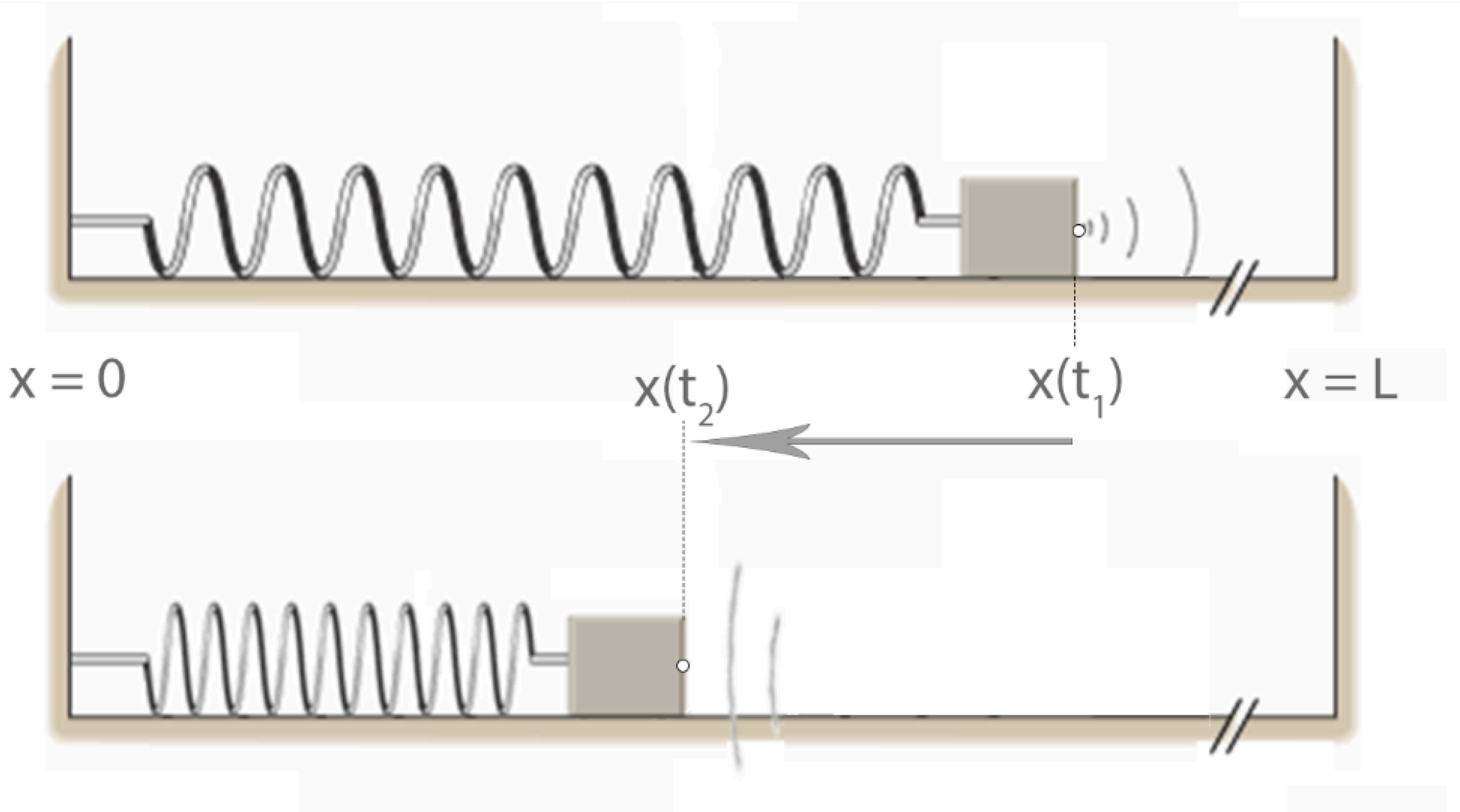
Spring-Mass system



$$m.\ddot{x} + \gamma.\dot{x} + \kappa.x - \beta x^3 = 0$$

Non-linear spring
No info on initial conditions

Spring-Mass-Sonar system



$$
\begin{aligned}
(L - x(t_1)) + (L - x(t_2)) &= c.(t_2 - t_1) \\
\Leftrightarrow x(t_2) + x(t_1) &= 2L - c.(t_2 - t_1)
\end{aligned}
$$

Contraintes

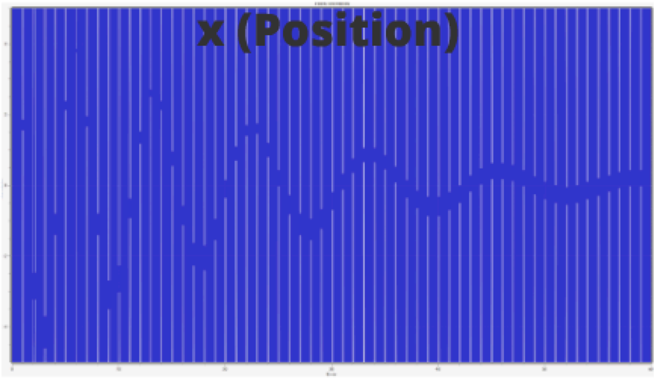
$$\frac{d}{dt}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \frac{(\beta x^2 - \kappa).x - \gamma.\dot{x}}{m} \end{pmatrix}$$

$$x(t_i) + x(t_j) = 2L - c.(t_i - t_j)$$

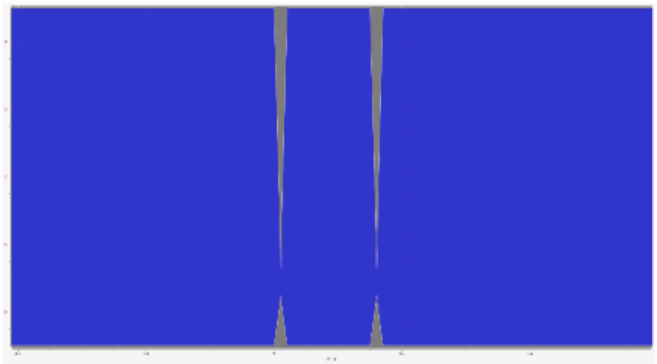
Simulation

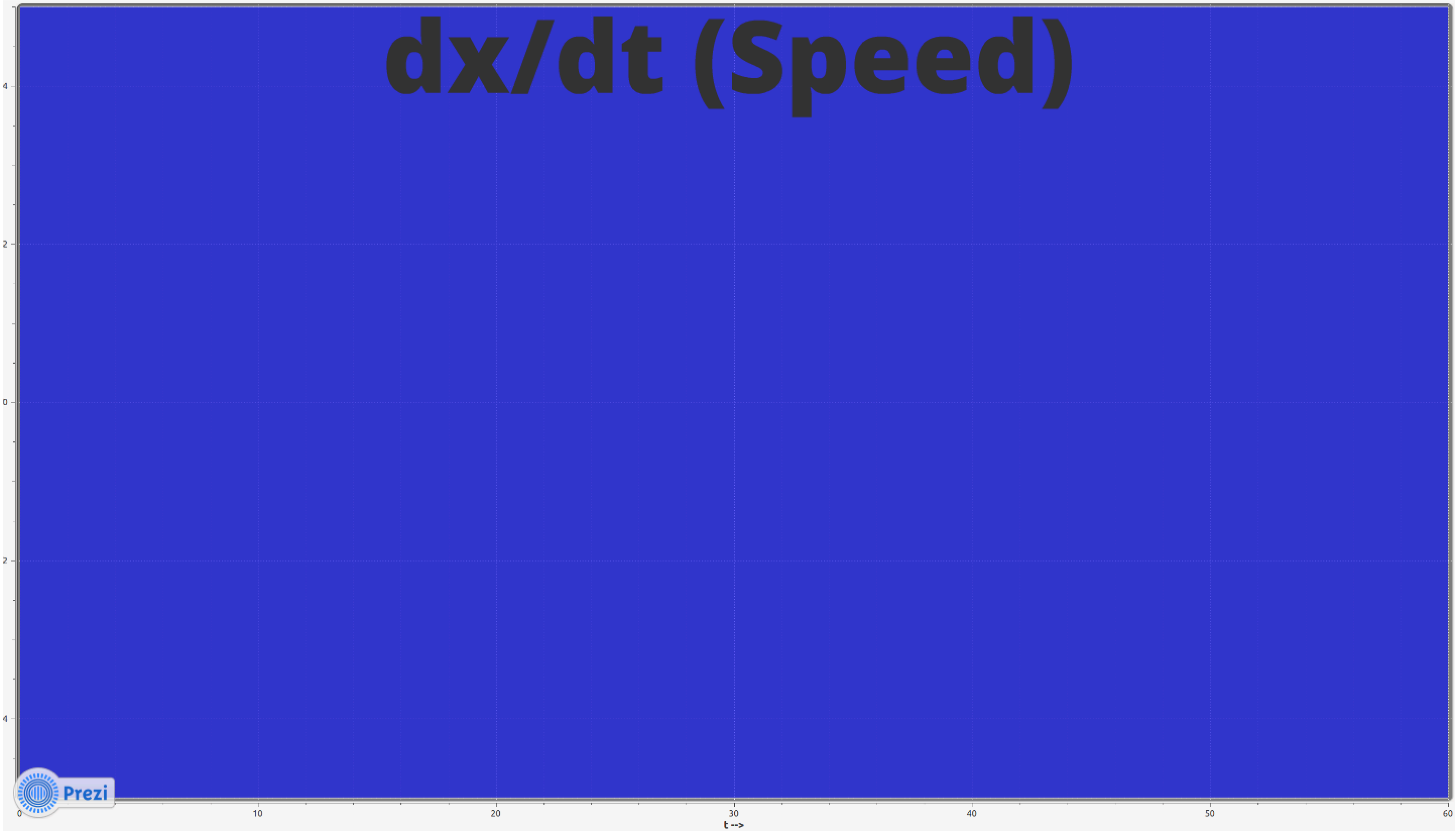
$$\begin{aligned} x(3.00) + x(3.48) &= 34.33 \\ x(9.00) + x(9.54) &= 15.46 \\ x(12.00) + x(12.55) &= 13.06 \\ x(20.00) + x(20.51) &= 24.38 \\ & \quad etc... \end{aligned}$$

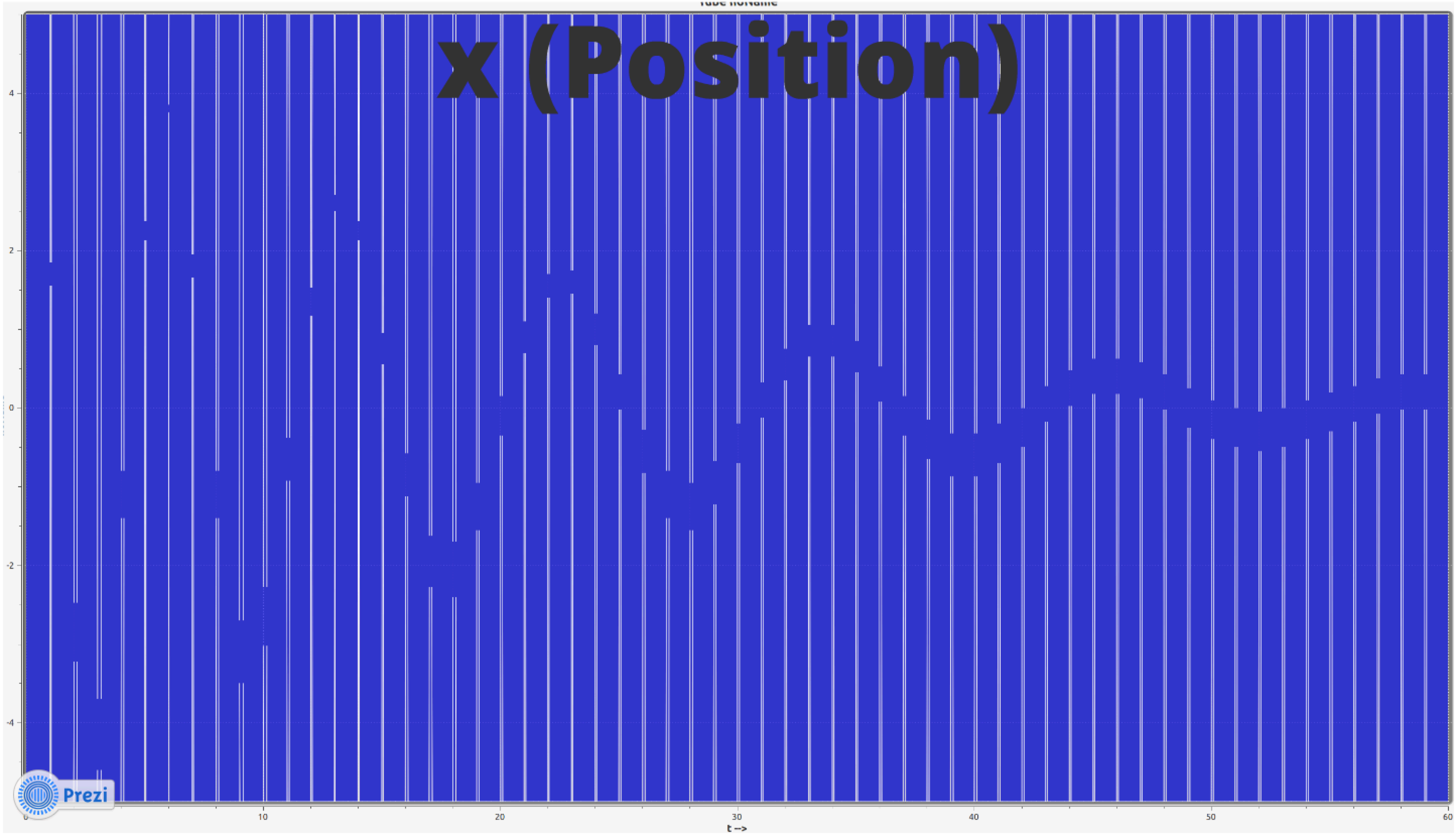
Matlab Simulation


x (Position)

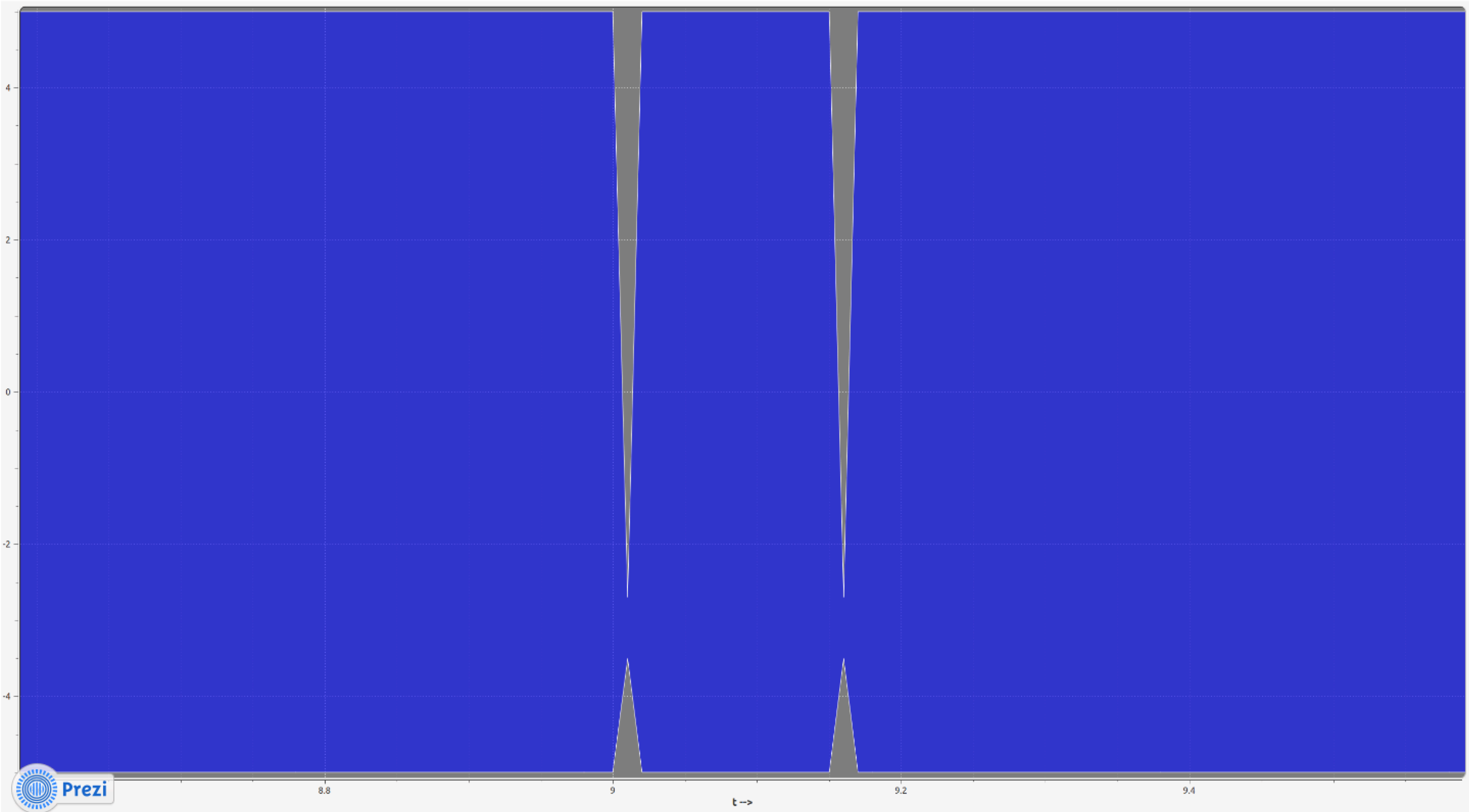

dx/dt (Speed)



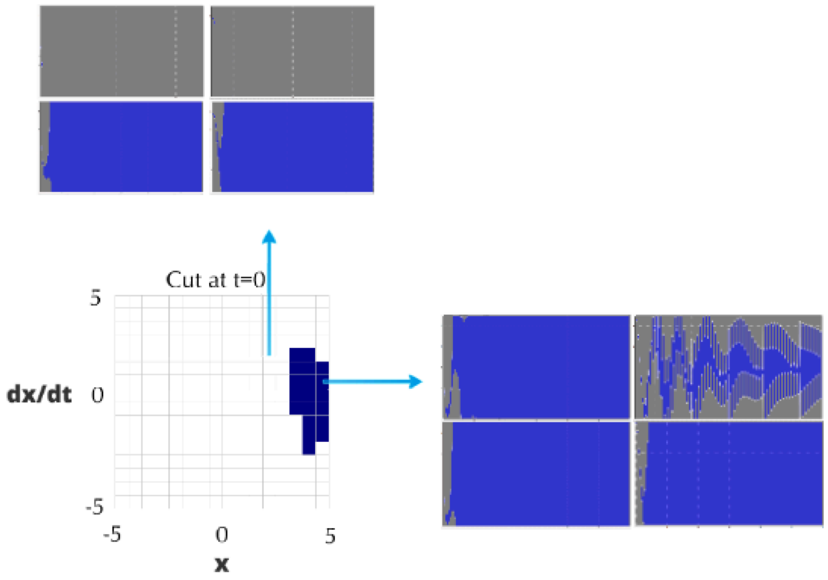

dx/dt (Speed)
4
2
0
2
4
0
10
20
30
40
50
60
t -->



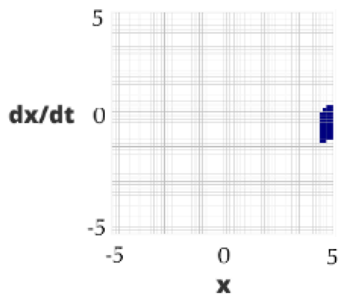
x (Position)
4
2
0
-2
-4
0
10
20
30
40
50
60
t -->



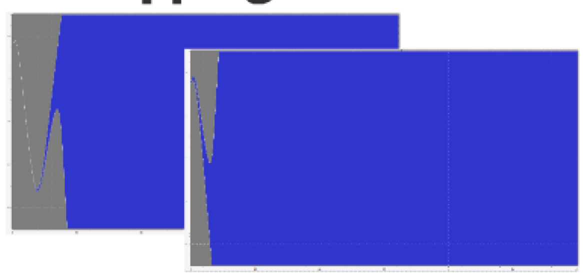
4
2
0
-2
-4
8.8
9
9.2
9.4
t -->

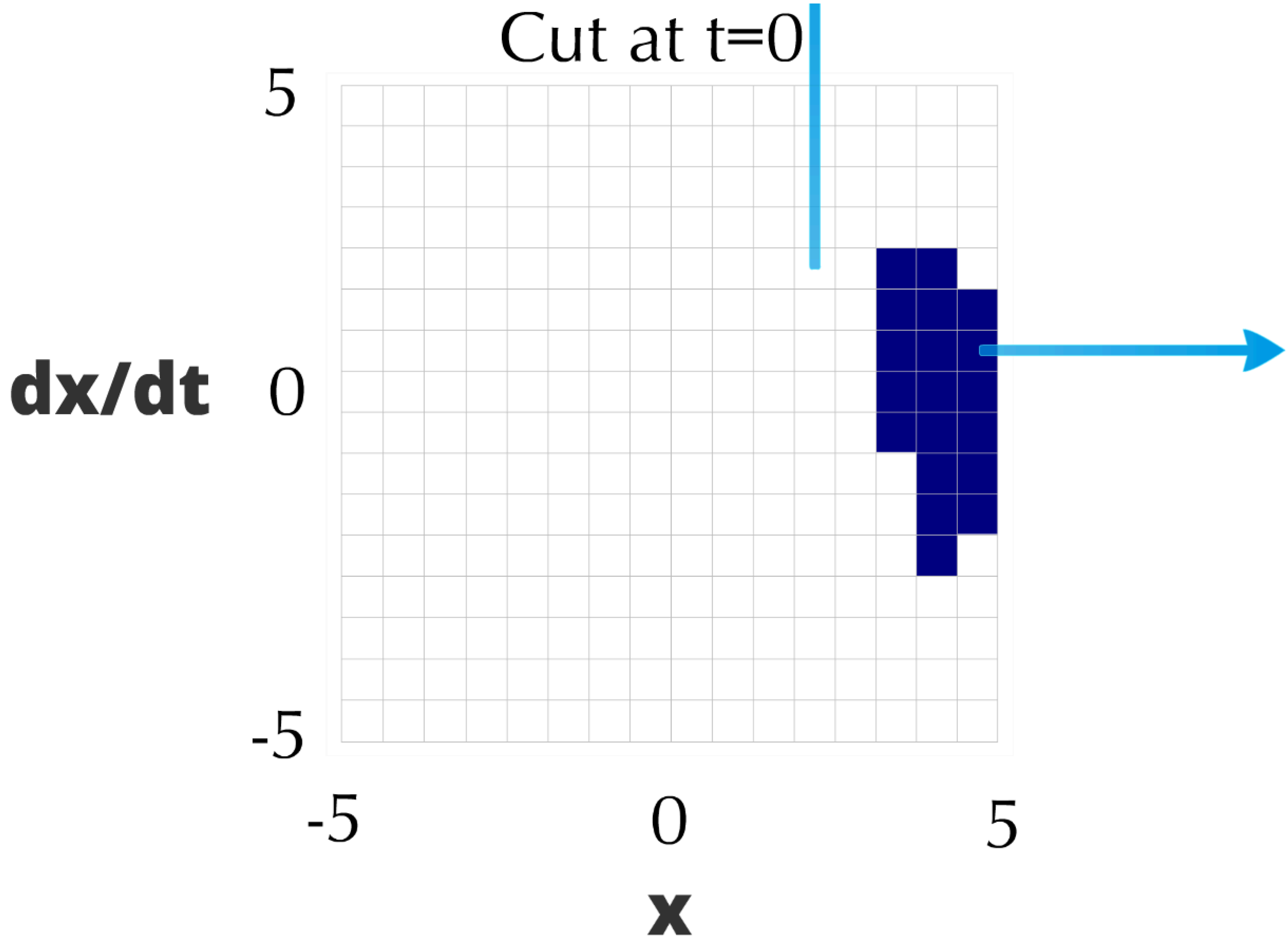


Solution : Union of all non-empty tubes
We get an interval around the initial conditions

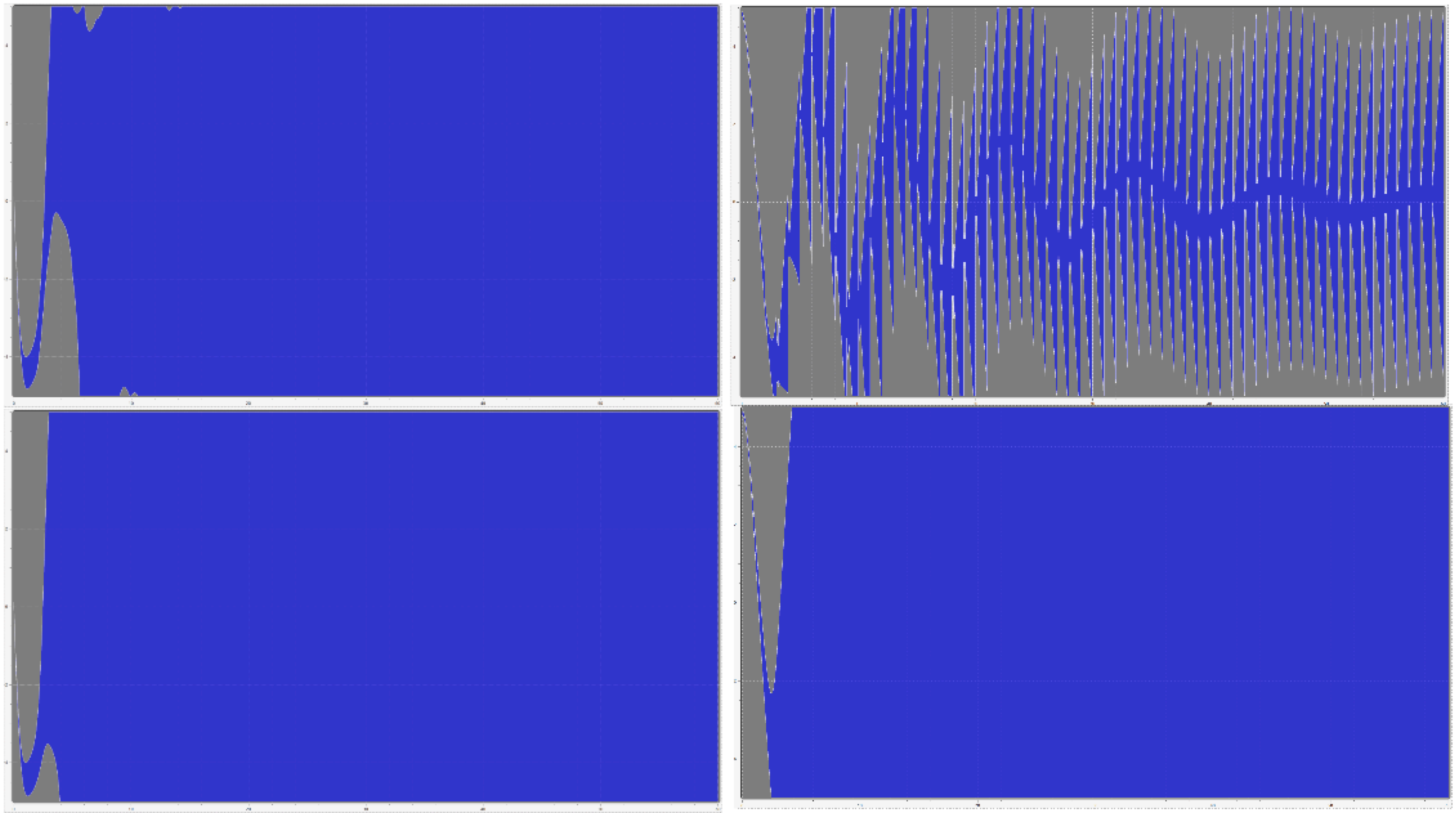


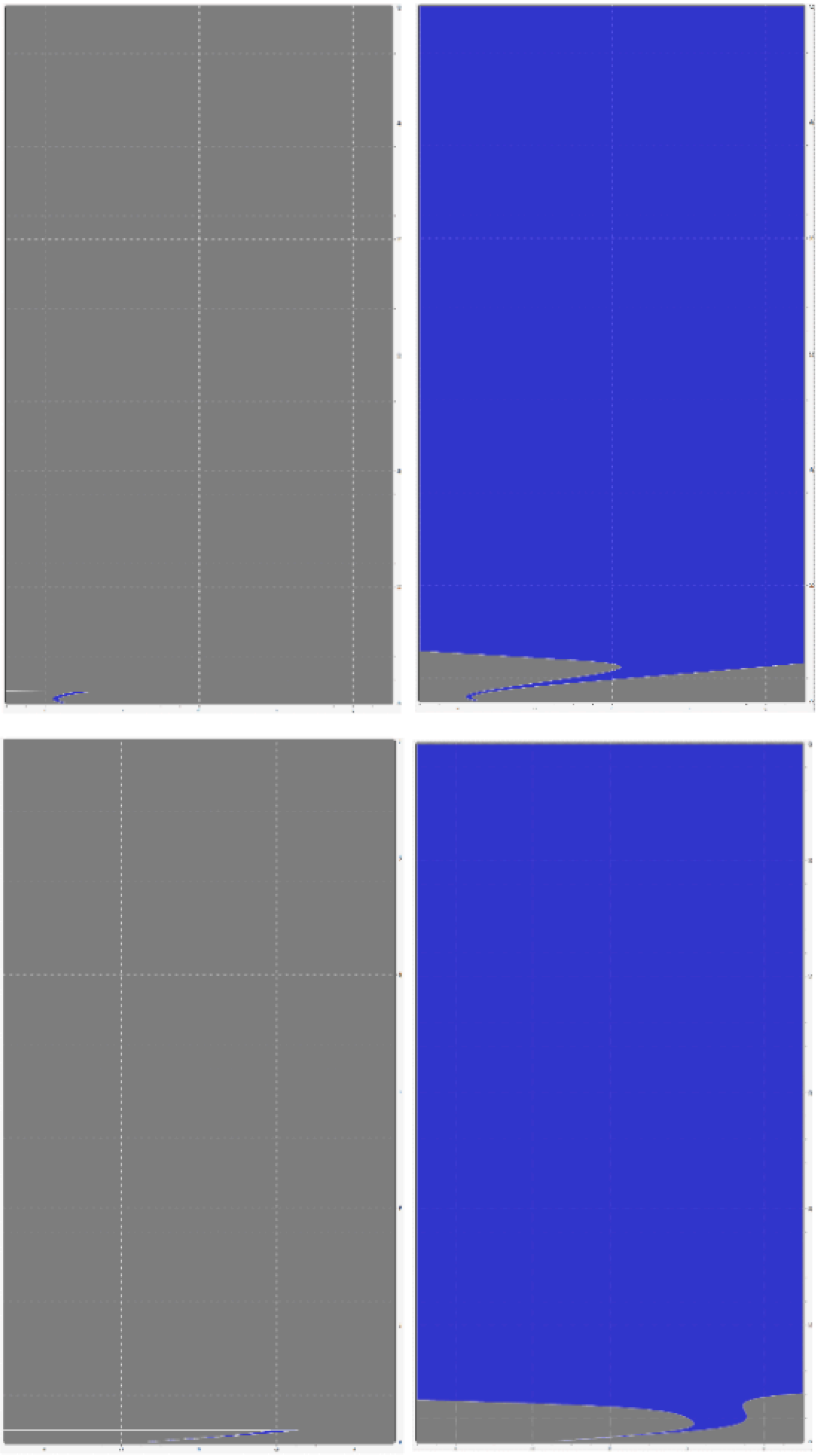
Wrapping effect

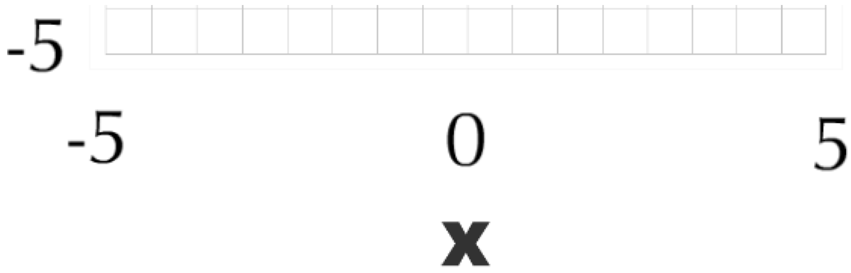




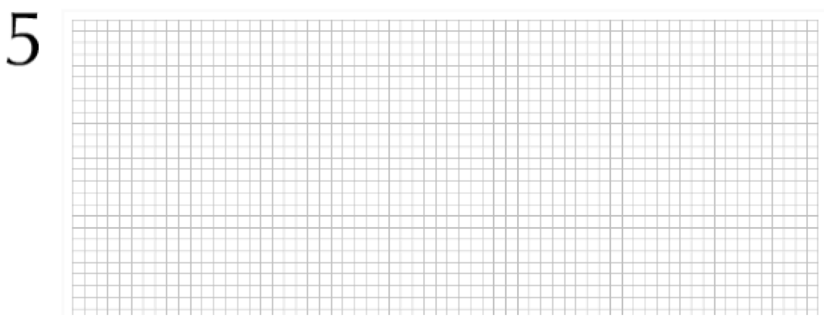
Cut at t=0
5
0
-5
dx/dt
-5
0
5
x



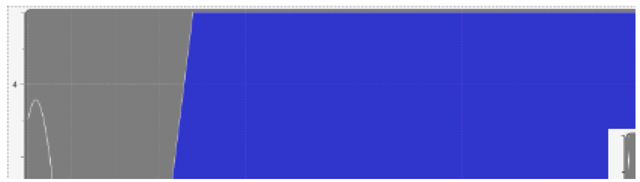


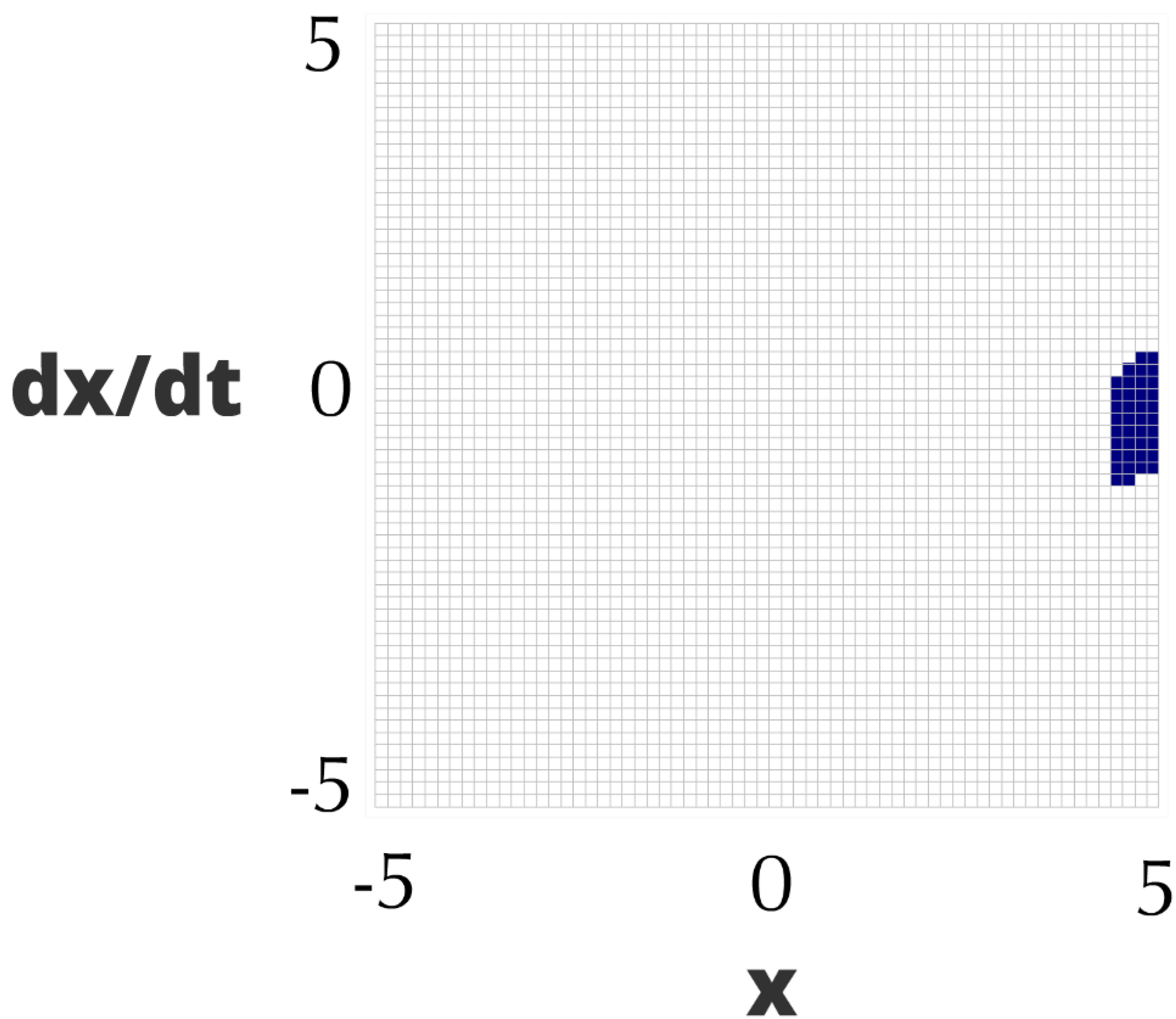


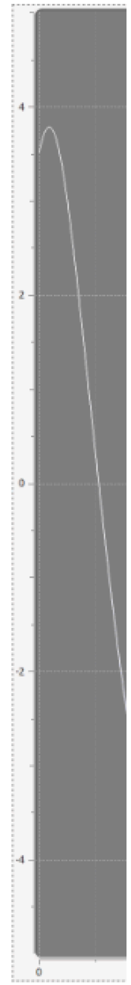
Solution : Union of all non-empty tubes
We get an interval around the initial conditions



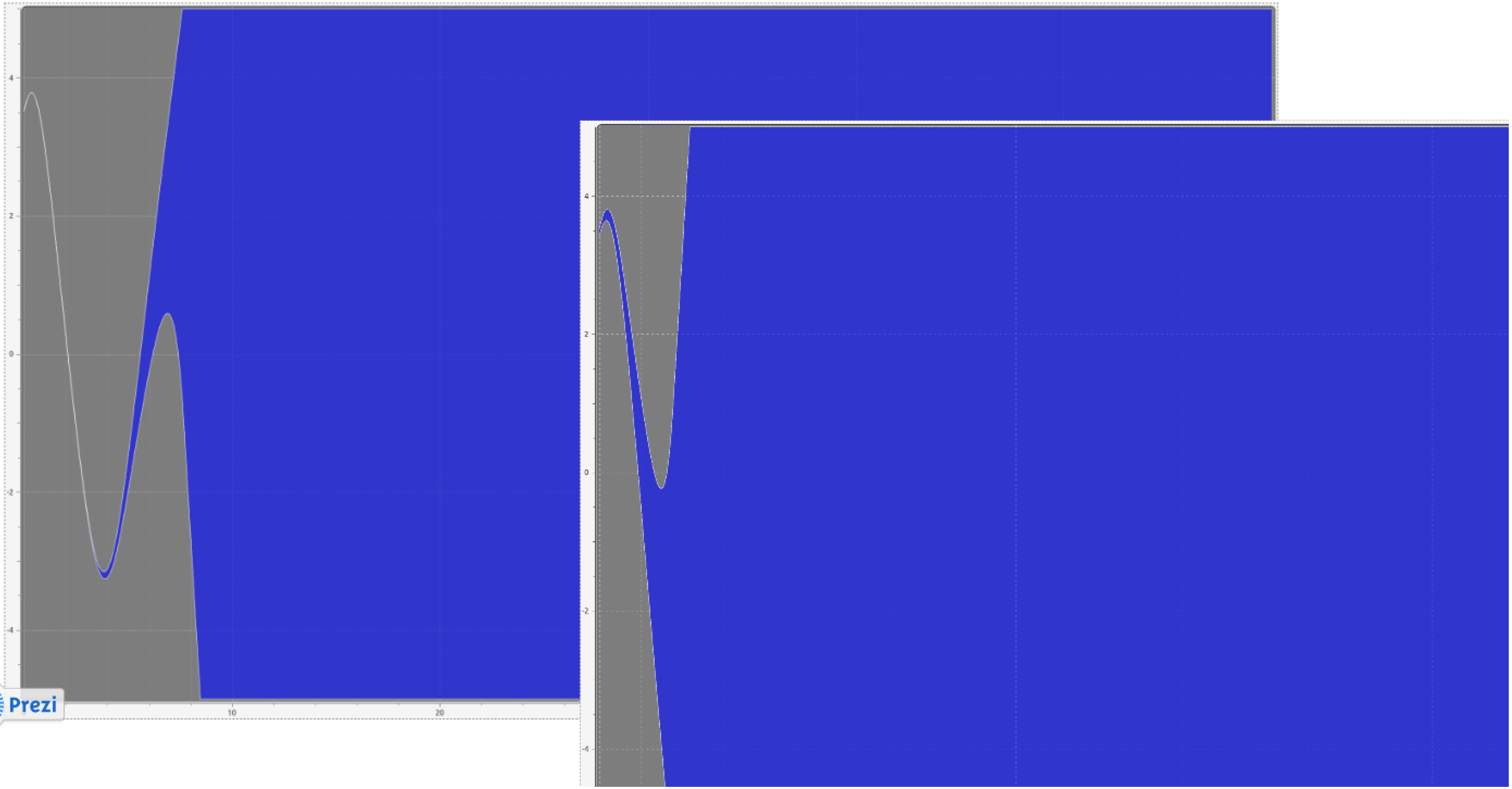
Wrapp

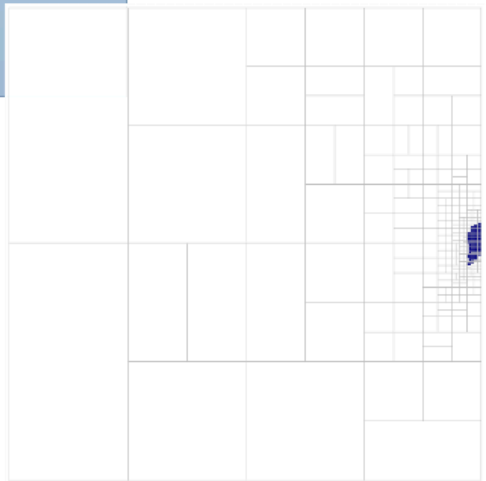


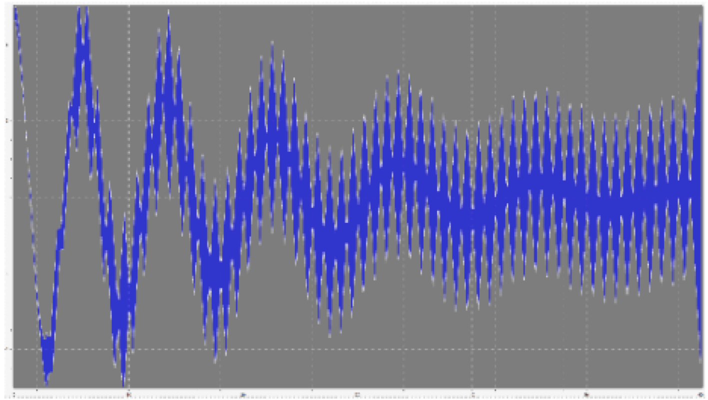

5
0
-5
dx/dt
-5
0
5
x

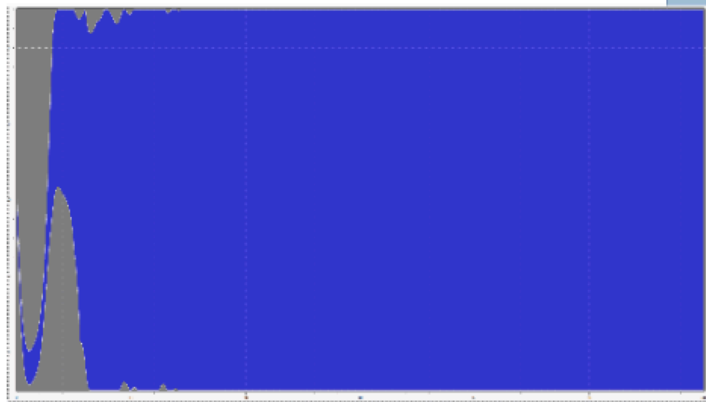


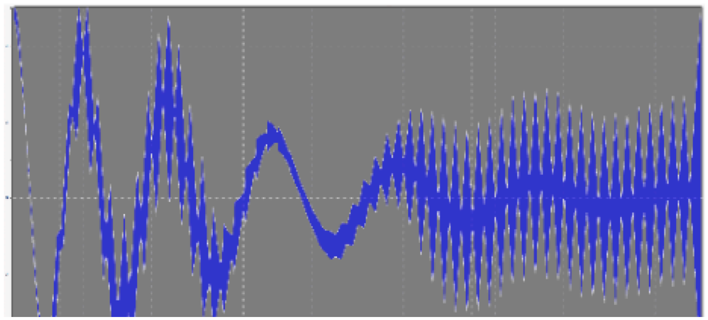
Wrapping effect

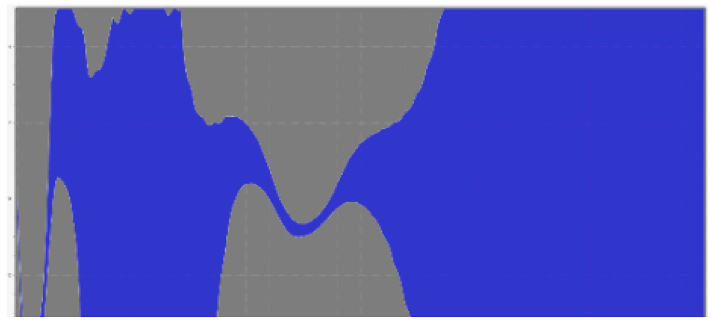


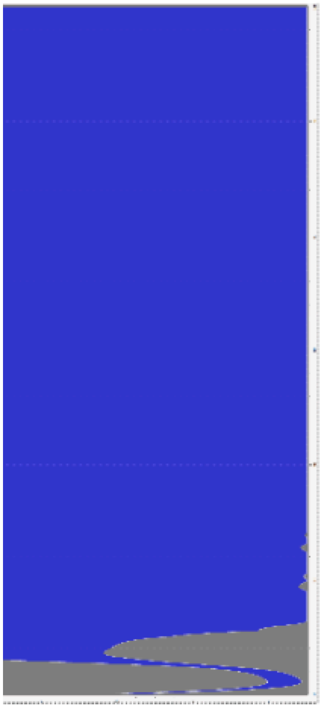
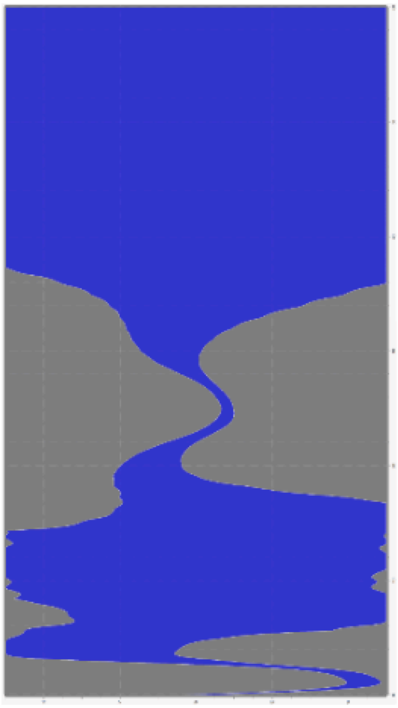
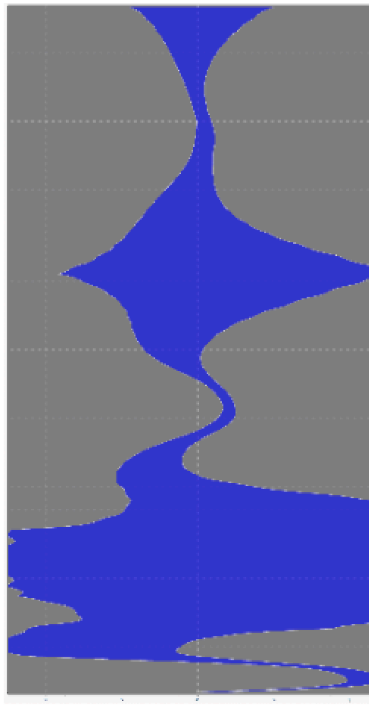
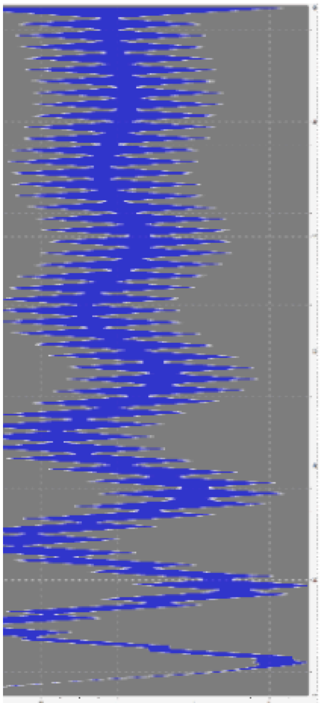
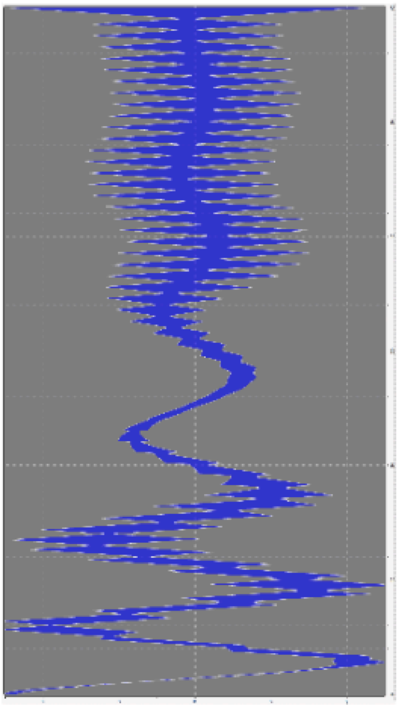
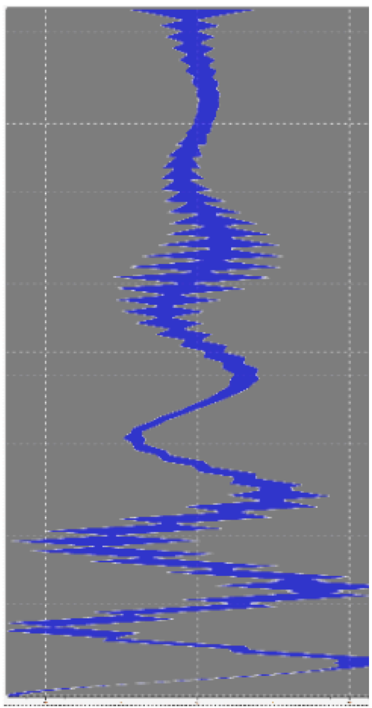
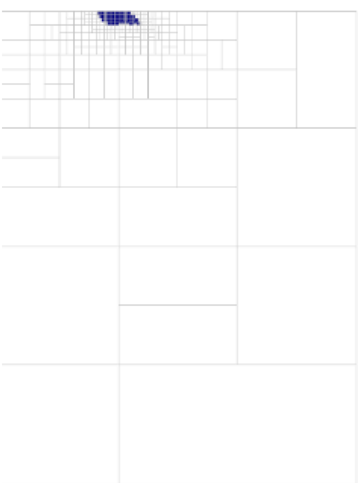
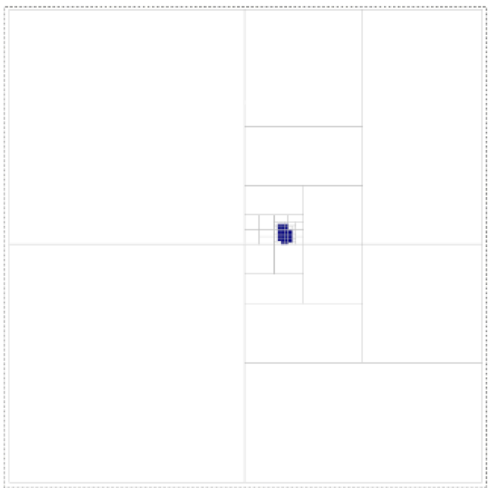
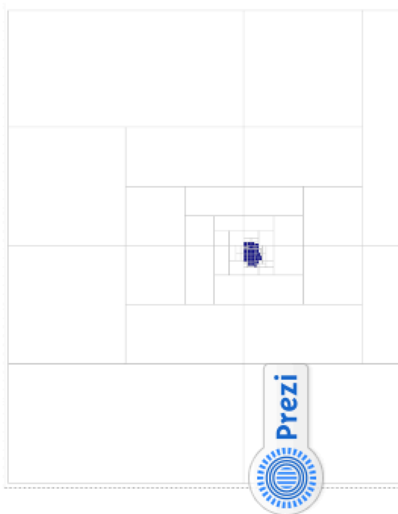


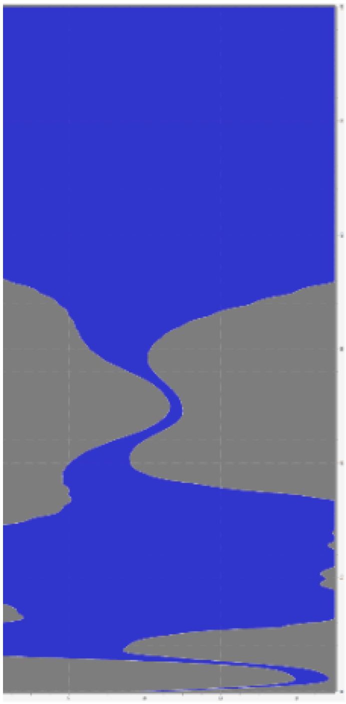
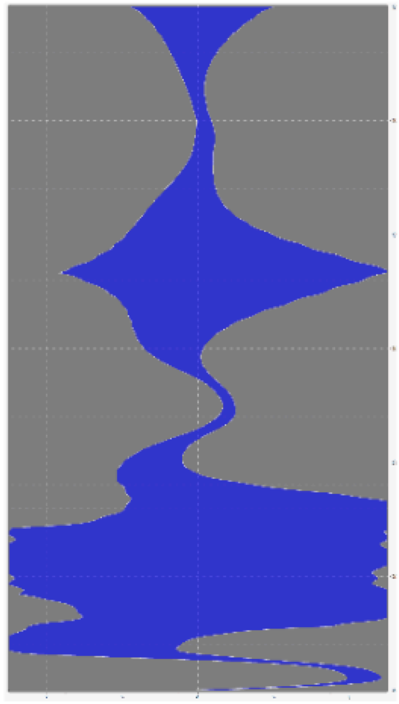
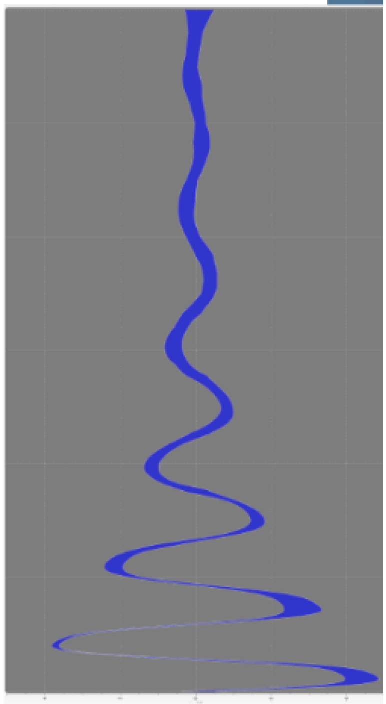
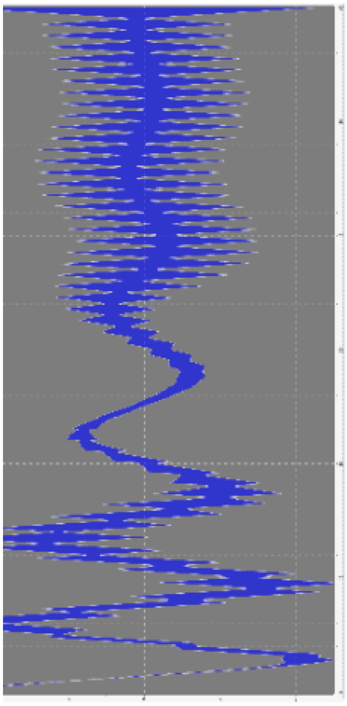
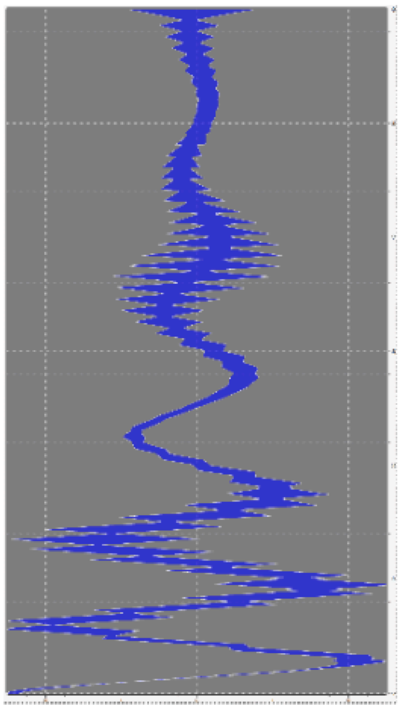
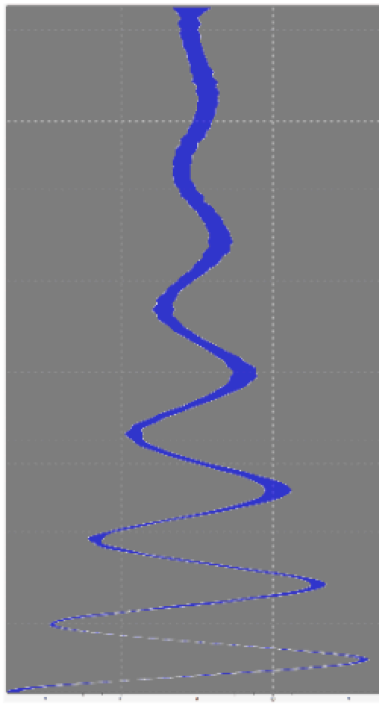
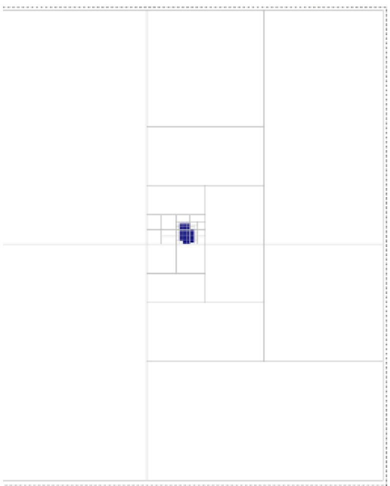
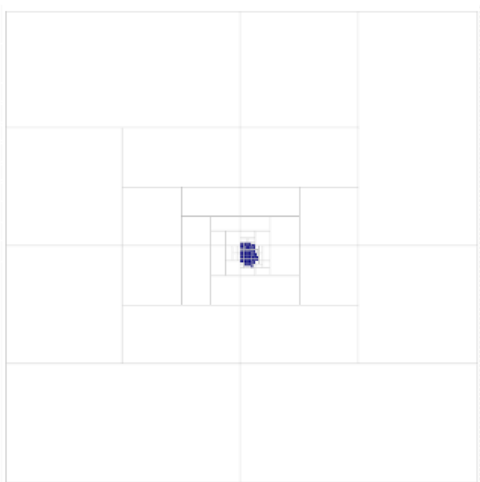


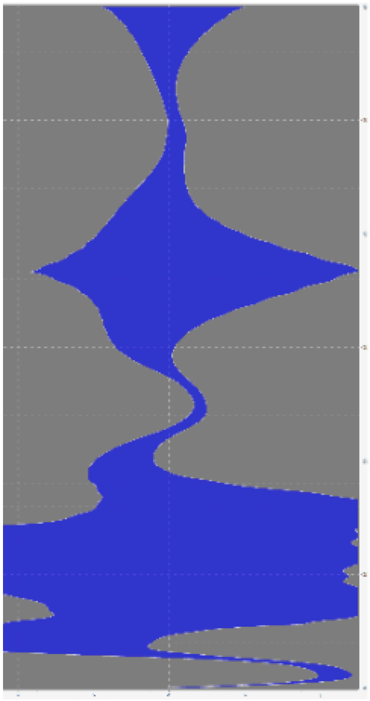


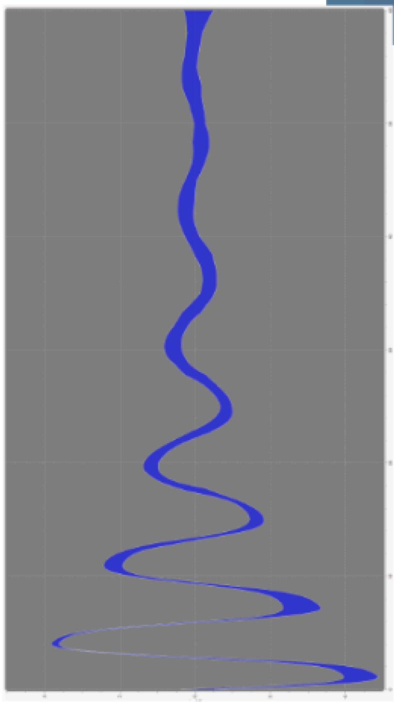


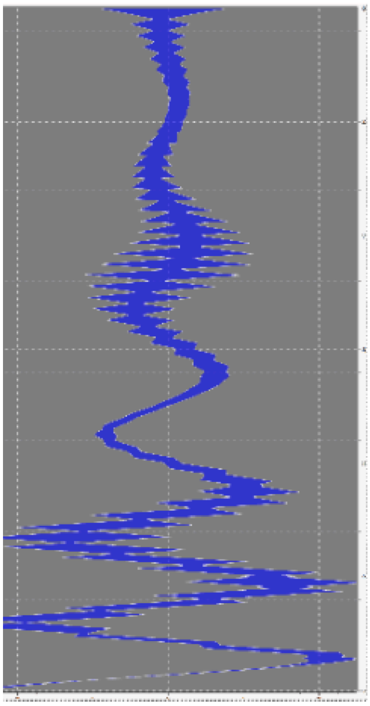


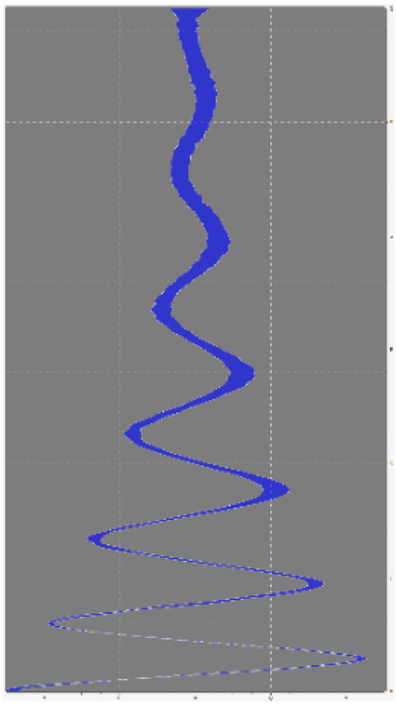










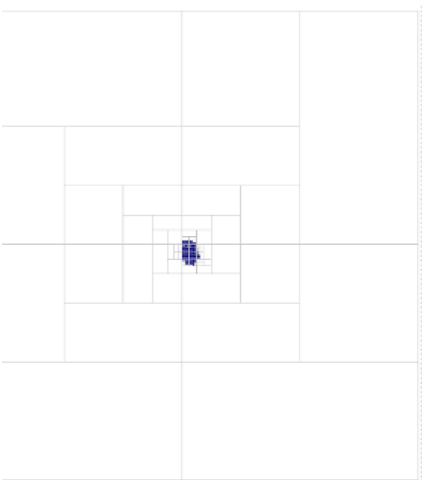



















5 - Spring-Mass example

6 - Conclusion

- Solve constraint safisfation problem using tubes.
- Tubes are now implementeted in IBEX (yesterday)
- Many applications involing constraints on trajectories.
- Trajectories of swarms of robot (Paladyn)